A Taylor rule

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.A Taylor rule

- J.B. Taylor proposed a rule for monetary policy of the form

$$r_t^f = a\left(Y_t - \overline{Y}\right) + b\left(\pi_t - \overline{\pi}\right) + \overline{r}^f$$

- The central bank sets an interest rate according to the difference of output and inflation from their stationary state values.
- In these models, a choice of a target inflation rate, $\overline{\pi}$, implies a target short term (lending) interest rate, $\overline{r}^f$
- The central bank achieves the target interest rate by adjusting its injections and withdrawals of money from the financial system in the equation

$$N_t + \left(g_t^M - 1\right) M_{t-1} = P_t w_t H_t$$

.Where does a money shock come from?

- Putting a Taylor rule in our model with Financial intermediaries implies there is no monetary shock from the central bank (it follows a rule)
- One could put noise in the variables that the central bank sees

$$Y_t^{CB} = Y_t + \varepsilon_t^Y$$

and

$$\pi_t^{CB} = \pi_t + \varepsilon_t^\pi$$

and the Taylor rule would be

$$r_t^f = a\left(Y_t^{CB} - \overline{Y}\right) + b\left(\pi_t^{CB} - \overline{\pi}\right) + \overline{r}^f$$

- ε_t^Y and ε_t^π are bounded and relatively small

.Where does a money shock come from?

- Another way would be to have a fiscal authority inject money via direct, lump sum transfers to the public
- This is just like in the Cooley-Hansen model
- The central bank then follows a monetary policy given by the Taylor rule
- Some comments:
- Timing is not innocent in these models
- It matters if the Taylor rule is

$$r_t^f = a\left(Y_t - \overline{Y}\right) + b\left(\pi_t - \overline{\pi}\right) + \overline{r}^f$$

or

$$r_t^f = a\left(Y_{t-1} - \overline{Y}\right) + b\left(\pi_{t-1} - \overline{\pi}\right) + \overline{r}^f$$

or

$$r_t^f = a\left(Y_t - \overline{Y}\right) + b\left(E_t\pi_{t+1} - \overline{\pi}\right) + \overline{r}^f$$

not the least for stability (see Carlstrom and Fuerst (2000))

.A model with a Taylor rule and money shocks

- Money shocks come from fiscal transfers to the households, so the cash-in-advance constraint for household j is

$$P_s c_s^j \leq m_{s-1}^j + \left(g_t^f - 1\right) M_{s-1} - N_s^j$$

where we assume that g_t^f follows

$$\ln g_t^f = \pi^g g_{t-1}^f + \varepsilon_t^f.$$

- Now our money shock is just like the one in the Cooley-Hansen model
- Everything else is just like in the Financial Intermediaries model except instead of a money growth rule, the central bank follows the rule

$$r_t^f = a\left(Y_t - \overline{Y}\right) + b\left(\pi_t - \overline{\pi}\right) + \overline{r}^f$$

- Note that in a stationary state, $\overline{g}^f = 1$, so the stationary states are the same as in the FI model
- Money growth is now

$$M_t = \left(g_t^f + g_t^M - 1\right) M_{t-1}$$

Log-linear version of the model

$$
\begin{aligned}
0 &= \widetilde{w}_t + \widetilde{P}_t - E_t\widetilde{P}_{t+1} - E_t\widetilde{C}_{t+1}, \\
0 &= \widetilde{w}_t - E_t\widetilde{w}_{t+1} + \beta\overline{r}E_t\widetilde{r}_{t+1}, \\
0 &= \widetilde{r}_t^n - \widetilde{w}_t + \widetilde{C}_t, \\
0 &= \overline{C}\widetilde{C}_t - \frac{\overline{M/P}}{\overline{g}^M}\widetilde{g}_t^f - \frac{\overline{M/P}}{\overline{g}^M}\widetilde{M}_{t-1} + \overline{N/P}\widetilde{N}_t + \overline{C}\widetilde{P}_t, \\
0 &= \overline{M/P}\widetilde{M}_t + \left[\overline{r}^n\overline{N/P} - \overline{M/P}\right]\widetilde{P}_t + \overline{K}\widetilde{K}_{t+1} - \overline{w}\overline{H}(\widetilde{w}_t + \widetilde{H}_t) \\
&\quad -\overline{r}\overline{K}\widetilde{r}_t - (\overline{r}+1-\delta)\overline{K}\widetilde{K}_t - \overline{r}^n\overline{N/P}\widetilde{N}_t - \overline{r}^n\overline{N/P}\widetilde{r}_t^n, \\
0 &= \widetilde{w}_t + \widetilde{r}_t^f - \widetilde{\lambda}_t - \theta\widetilde{K}_t + \theta\widetilde{H}_t,
\end{aligned}
$$

widthheighteqnarray* $0 = \widetilde{r}_t - \widetilde{\lambda}_t - (\theta-1)\,\widetilde{K}_t - (1-\theta)\,\widetilde{H}_t,$

$0 = \widetilde{Y}_t - \widetilde{\lambda}_t - \theta\widetilde{K}_t - (1-\theta)\,\widetilde{H}_t,$

$0 = \widetilde{r}_t^n + \widetilde{N}_t - \widetilde{P}_t - \widetilde{r}_t^f - \widetilde{w}_t - \widetilde{H}_t,$

$$
\begin{aligned}
0 &= \overline{N/P}\widetilde{N}_t + \overline{M/P}\left(1 - \frac{1}{\overline{g}}\right)\widetilde{M}_{t-1} \\
&\quad -\overline{w}\overline{H}\widetilde{P}_t + \overline{M/P}\widetilde{g}_t^M - \overline{w}\overline{H}\widetilde{w}_t - \overline{w}\overline{H}\widetilde{H}_t \\
0 &= \widetilde{M}_t - \frac{1}{\overline{g}^M}\widetilde{g}_t^f - \widetilde{g}_t^M - \widetilde{M}_{t-1} \\
0 &= a\overline{Y}\widetilde{Y}_t + b\overline{g}^M\widetilde{P}_t - b\overline{g}^M\widetilde{P}_{t-1} - \overline{r}^f\widetilde{r}_t^f
\end{aligned}
$$

The policy matrices are
[5cm]
6cm

$$
P = \begin{bmatrix} 0.9588 & -0.0576 & 0.0576 \\ 0.0560 & 0.7025 & 0.2975 \\ -0.2667 & 0.7219 & 0.2781 \end{bmatrix}
$$

$$
R = \begin{bmatrix} -0.8450 & -0.2949 & 0.2949 \\ 0.5194 & -0.0312 & 0.0312 \\ 0.1550 & -0.2949 & 0.2949 \\ 0.4317 & 0.3975 & -0.3975 \\ -0.3203 & -0.4608 & 0.4608 \\ -0.1995 & 0.8557 & 0.1443 \\ 0.0877 & -0.4287 & 0.4287 \\ -0.0441 & 0.1971 & -0.1971 \\ 0.0560 & -0.2975 & 0.2975 \end{bmatrix}
$$

4cm

$$
Q = \begin{bmatrix} 0.1367 & -0.0501 \\ -0.2800 & 0.8878 \\ -0.9570 & 1.4391 \end{bmatrix}
$$

$$S = \begin{bmatrix} 1.6184 & -0.6316 \\ 0.1480 & -0.0104 \\ 1.6184 & -0.6316 \\ 0.2913 & -0.1579 \\ 0.9663 & -0.9869 \\ 0.8047 & 0.6601 \\ -0.1433 & 0.1475 \\ 0.5041 & 0.3657 \\ -0.2800 & -0.0831 \end{bmatrix}$$

To what should we compare this economy?

- To standard FI economy?
  - Monetary shocks are very different
- To standard Cooley Hansen model
  - Does not have FI so the effects of a fiscal shock will be different
- Recommendation: to a FI model with constant money supply rule
  - Constant money supply rules are commonly recommended
  - It includes both kinds of shocks (money and technology)

Constant money supply growth rule

- Central bank policy
  $$g_t^M - \overline{g}^M = -\left(g_t^f - 1\right)$$
- Central bank counters the fiscal policy injections
- Around its stationary state money growth rate
- Log-linear version of rule
  $$0 = \overline{g}^M g_t^M + g_t^f$$
  replaces the Taylor rule
- Rest of the economy just the same
- Note: one might want to add a lag in the information that the central bank has, so that the central bank policy becomes
  $$g_t^M - \overline{g}^M = -\left(g_{t-1}^f - 1\right)$$
  In this case, the central bank corrects with a lag and the dynamics are more interesting

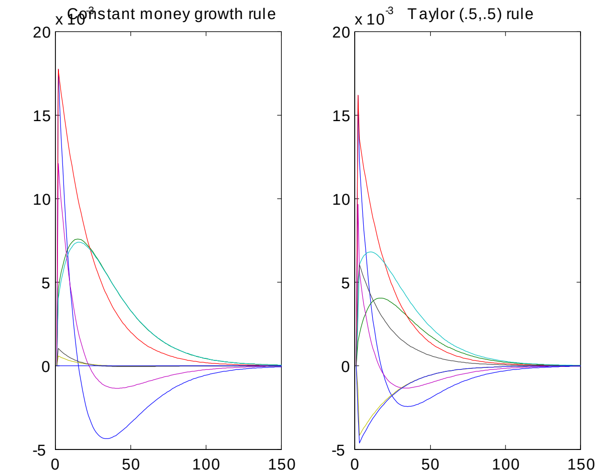

Figure 1: Responses of real variables

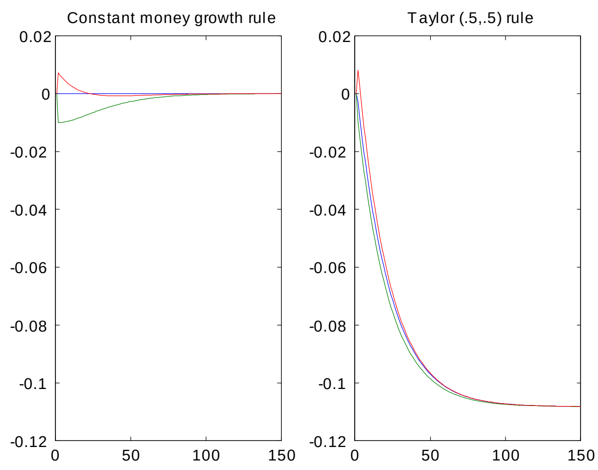

Figure 2: Response of nominal variables

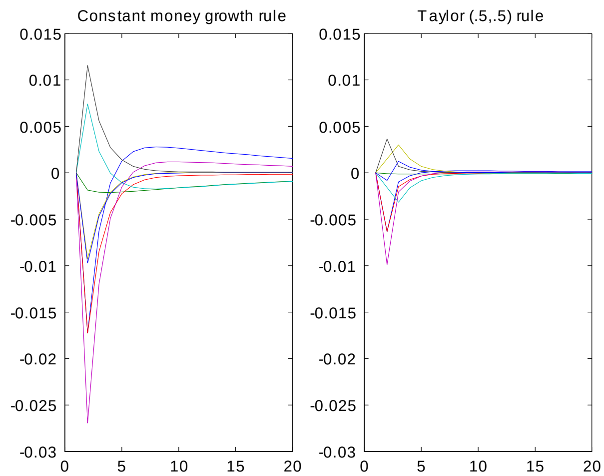

Figure 3: Responses of real variables

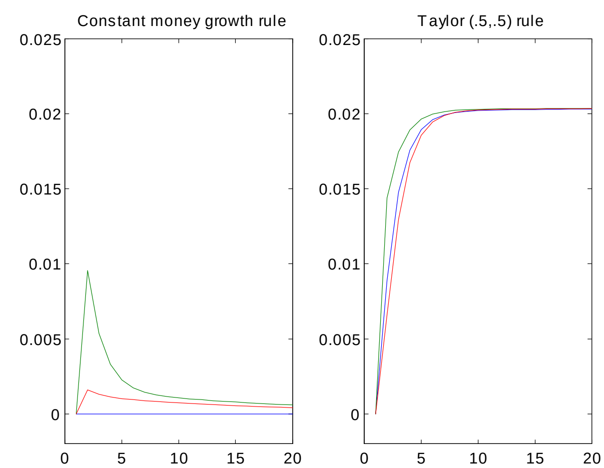

Figure 4: Responses of nominal variables

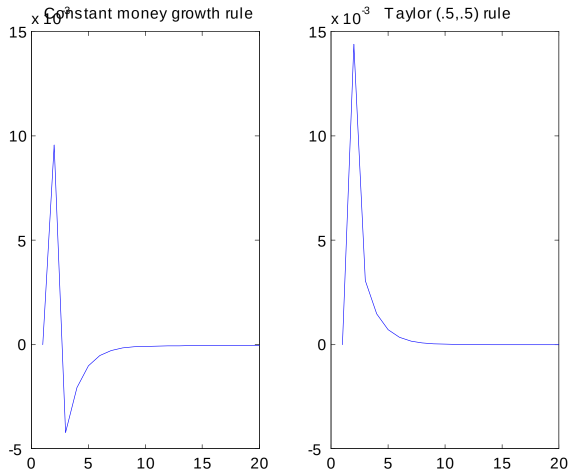

Figure 5: Responses of inflation

Results: Tech shock
Results: Tech shock
Results: fiscal money supply shock
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