Small open economy models

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Small open economy models

- Preliminary model
- Adjustment costs to capital
- Closing the open economy
- Adding money

Preliminary model

- The Household
- Maximization problem

$$\max_{\{c_t,h_t,k_{t+1},b_t\}} E_t \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + Bh_t\right]$$

  where

$$B = \frac{A\ln(1-h_0)}{h_0}$$

- Budget constraint with b_t = holdings of risk free international bonds

$$b_t + k_{t+1} + c_t = w_t h_t + r_t k_t + (1-\delta)k_t + (1+r^f)b_{t-1}$$

- No "Ponzi" game condition

$$\lim_{t\to\infty} \frac{b_t}{(1+r^f)^t} = \lim_{t\to\infty} \beta^t b_t = 0$$

Preliminary model

- Assume $r^f = 1/\beta - 1$ (to have stationary state)

- Household first order conditions and budget constraints

$$\begin{aligned}
B &= -\frac{w_t}{c_t} \\
\frac{1}{c_t} &= E_t \frac{1}{c_{t+1}} \\
\frac{1}{\beta} &= E_t \frac{c_t}{c_{t+1}} \left[r_{t+1} + (1-\delta) \right] \\
b_t + k_{t+1} + c_t &= w_t h_t + r_t k_t + (1-\delta) k_t + (1+r^f) b_{t-1}
\end{aligned}$$

and

$$\lim_{t \to \infty} \frac{b_t}{(1+r^f)^t} = 0$$

Preliminary model

- The Firm
- The production function

$$f(\lambda_t, k_t, h_t) = \lambda_t k_t^{\theta} h_t^{1-\theta}$$

where

$$\lambda_{t+1} = \gamma \lambda_t + \varepsilon_{t+1}$$

with $0 < \gamma < 1$ and $E_t \varepsilon_{t+1} = 1 - \gamma$.

- Conditions for real wages and rentals (factor market equilibrium)

$$r_t = \theta \lambda_t k_t^{\theta-1} h_t^{1-\theta}$$

and

$$w_t = (1-\theta) \lambda_t k_t^{\theta} h_t^{-\theta}$$

Preliminary model

- Equilibrium conditions

$$\begin{aligned}
C_t &= c_t \\
K_t &= k_t \\
H_t &= h_t
\end{aligned}$$

and

$$B_t = b_t$$

Preliminary model

- Stationary state: the model

$$\begin{aligned} B &= -\frac{\overline{w}}{\overline{C}} \\ \frac{1}{\beta} &= \overline{r} + (1-\delta) \\ \overline{C} &= \overline{w}\overline{H} + (\overline{r}-\delta)\,\overline{K} + r^f\overline{B} \\ \overline{r} &= \theta\overline{K}^{\theta-1}\overline{H}^{1-\theta} \\ \overline{w} &= (1-\theta)\,\overline{K}^{\theta}\overline{H}^{-\theta} \end{aligned}$$

Preliminary model

- Stationary state: finding the stationary state

$$\overline{r} = \frac{1}{\beta} - (1-\delta)$$

$$\overline{w} = (1-\theta)\left(\frac{\theta}{\overline{r}}\right)^{\frac{\theta}{1-\theta}}$$

$$\overline{C} = -\frac{\overline{w}}{B}$$

$$\left[\frac{\overline{r}}{\theta} - \delta\right]\overline{K} = \overline{C} - r^f\overline{B}$$

$$\overline{H} = \frac{(1-\theta)}{\theta}\frac{\overline{r}}{\overline{w}}\overline{K}$$

Preliminary model

- Stationary state
- Given h_0, the hours worked for a family who works, write

$$\overline{C} = \overline{w}\left[1 + \frac{(\overline{r}-\delta)\,\theta}{(1-\theta)\,\overline{r}}\right]\overline{H} + r^f\overline{B}$$

- The maximum foreign debt possible (a negative number) in a stationary state is

$$\overline{B} = \frac{\overline{C} - \overline{w}\left[1 + \frac{(\overline{r}-\delta)\theta}{(1-\theta)\overline{r}}\right]h_0}{r^f}$$

- Each value of $\overline{B}$ above this (negative) amount generates a stationary state

Preliminary model: log-linear version

$$
\begin{aligned}
B &= -\frac{w_t}{C_t} \\
\frac{1}{C_t} &= E_t \frac{1}{C_{t+1}} \\
\frac{1}{\beta} &= E_t \frac{C_t}{C_{t+1}} \left[r_{t+1} + (1-\delta) \right] \\
B_t + K_{t+1} + C_t &= w_t H_t + r_t K_t + (1-\delta) K_t + (1+r^f) B_{t-1} \\
r_t &= \theta \lambda_t K_t^{\theta-1} H_t^{1-\theta} \\
w_t &= (1-\theta)\, \lambda_t K_t^{\theta} H_t^{-\theta}
\end{aligned}
$$

and

$$\lim_{t\to\infty} \frac{B_t}{\left(1+r^f\right)^t} = 0$$

Preliminary model: log-linear version

- One can reduce the model to the single equation

$$
\begin{aligned}
\overline{B}\widetilde{B}_t + \overline{K}\widetilde{K}_{t+1} &= \left[\left[\overline{r} + (1-\delta) \right] \overline{K} + \overline{w}\overline{H} \right] \widetilde{K}_t + (1+r^f) \overline{B}\widetilde{B}_{t-1} \\
&\quad - \left[\frac{\gamma \overline{C}}{(1-\theta)} - \frac{(1-\gamma)\left[\overline{w}\overline{H} + \overline{r}\overline{K} \right]}{\theta} \right] \widetilde{\lambda}_t
\end{aligned}
$$

- This is a sort of policy equation: given $\widetilde{K}_t$, $\widetilde{B}_{t-1}$, and $\widetilde{\lambda}_t$, one can determine the sum $\overline{B}\widetilde{B}_t + \overline{K}\widetilde{K}_{t+1}$
- Individual values for $\widetilde{B}_t$ and $\widetilde{K}_{t+1}$ cannot be determined
- Problem of indeterminacy

Model 2: adding adjustment costs (to solve indetermancy?)

- Adding capital adjustment costs to try and get $\widetilde{K}_{t+1}$ determined
- Adding capital adjustment costs:

$$\frac{\kappa}{2} \left(k_{t+1} - k_t \right)^2$$

- Household budget constraint becomes

$$
\begin{aligned}
& b_t + k_{t+1} + \frac{\kappa}{2} \left(k_{t+1} - k_t \right)^2 + c_t \\
= \quad & w_t h_t + r_t k_t + (1-\delta) k_t + (1+r^f) b_{t-1}
\end{aligned}
$$

Model 2: adding adjustment costs

- Household first order conditions and budget constraints are

$$\begin{aligned}
B &= -\frac{w_t}{c_t} \\
\frac{1}{c_t} &= E_t \frac{1}{c_{t+1}} \\
&\quad \frac{1}{\beta}\left(1+\kappa\left(k_{t+1}-k_t\right)\right) \\
&= E_t \frac{c_t}{c_{t+1}}\left[r_{t+1}+(1-\delta)+\kappa\left(k_{t+2}-k_{t+1}\right)\right] \\
&\quad b_t + k_{t+1} + \frac{\kappa}{2}\left(k_{t+1}-k_t\right)^2 + c_t \\
&= w_t h_t + r_t k_t + (1-\delta)k_t + (1+r^f)b_{t-1}
\end{aligned}$$

Model 2: adding adjustment costs

- Firm side of the problem is exactly the same as before

$$\begin{aligned}
w_t &= (1-\theta)\lambda_t K_t^{\theta} H_t^{-\theta} \\
r_t &= \theta \lambda_t K_t^{\theta-1} H_t^{1-\theta}
\end{aligned}$$

- Stationary state is the same as before because

$$\frac{\kappa}{2}\left(\overline{K}-\overline{K}\right)^2 = 0$$

Model 2: Log-linear version

- The full log linear model is

$$\begin{aligned}
0 &= \widetilde{C}_t - \widetilde{w}_t \\
0 &= \widetilde{C}_t - E_t \widetilde{C}_{t+1} \\
0 &= \widetilde{C}_t - E_t \widetilde{C}_{t+1} + \beta \overline{r} E_t \widetilde{r}_{t+1} - \beta \kappa \overline{K} E_t \widetilde{K}_{t+2} \\
&\quad - (1+\beta)\kappa \overline{K} \widetilde{K}_{t+1} + \kappa \overline{K} \widetilde{K}_t \\
0 &= \overline{B}\widetilde{B}_t + \overline{K}\widetilde{K}_{t+1} + \overline{C}\widetilde{C}_t - \overline{w}\overline{H}\widetilde{w}_t - \overline{w}\overline{H}\widetilde{H}_t - \overline{r}\overline{K}\widetilde{r}_t \\
&\quad - \left[\overline{r}+(1-\delta)\right]\overline{K}\widetilde{K}_t - (1+r^f)\overline{B}\widetilde{B}_{t-1} \\
0 &= \lambda_t + (\theta-1)K_t + (1-\theta)H_t - \widetilde{r}_t \\
0 &= \lambda_t + \theta K_t - \theta H_t - \widetilde{w}_t
\end{aligned}$$

Model 2: Log-linear version

- This system can be reduced to the two equation system

$$\begin{aligned}
0 &= \frac{\gamma(1-\gamma)}{(1-\theta)}\widetilde{\lambda}_t \\
&\quad + \xi\left[\beta E_t \widetilde{K}_{t+3} - (1+2\beta)E_t \widetilde{K}_{t+2} + (2+\beta)\widetilde{K}_{t+1} - \widetilde{K}_t\right]
\end{aligned}$$

and

$$
\begin{aligned}
0 \;=\;& \\
& \overline{B}\widetilde{B}_t - \left((1-\gamma)\left[\overline{w}\overline{H} + \overline{r}\overline{K}\right] - \frac{\gamma\overline{C}}{(1-\theta)}\right)\widetilde{\lambda}_t \\
& - \left(\left[\overline{C} + (1-\theta)\left[\overline{w}\overline{H} + \overline{r}\overline{K}\right]\right]\xi(1-\beta) - \overline{K}\right)\widetilde{K}_{t+1} \\
& + \left(\left[\overline{C} + (1-\theta)\left[\overline{w}\overline{H} + \overline{r}\overline{K}\right]\right]\xi(1-\beta)\right. \\
& \left. - \left[\overline{r} + (1-\delta)\right]\overline{K} - \overline{w}\overline{H}\right)\widetilde{K}_t \\
& -(1+r^f)\overline{B}\widetilde{B}_{t-1}
\end{aligned}
$$

- Note: this system is recursive. Find $\widetilde{K}_{t+1}$ then find $\widetilde{B}_t$.

Model 2: Log-linear version (solving)

- Writing the solution as

$$\widetilde{K}_{t+1} = P_{11}\widetilde{K}_t + Q_1\widetilde{\lambda}_t$$

- One can write

$$
\begin{aligned}
E_t\widetilde{K}_{t+2} &= P_{11}\widetilde{K}_{t+1} + Q_1 E_t\widetilde{\lambda}_{t+1} \\
&= P_{11}\left[P_{11}\widetilde{K}_t + Q_1\widetilde{\lambda}_t\right] + Q_1\gamma\widetilde{\lambda}_t \\
&= (P_{11})^2\,\widetilde{K}_t + Q_1(P_{11}+\gamma)\,\widetilde{\lambda}_t
\end{aligned}
$$

and

$$
\begin{aligned}
E_t\widetilde{K}_{t+3} &= P_{11}\widetilde{K}_{t+2} + Q_1 E_t\widetilde{\lambda}_{t+2} \\
&= P_{11}\left[P_{11}\widetilde{K}_{t+1} + Q_1\widetilde{\lambda}_{t+1}\right] + Q_1\gamma\widetilde{\lambda}_{t+1} \\
&= P_{11}\left[P_{11}\left[P_{11}\widetilde{K}_t + Q_1\widetilde{\lambda}_t\right] + Q_1\gamma\widetilde{\lambda}_t\right] + Q_1\gamma^2\widetilde{\lambda}_t \\
&= (P_{11})^3\,\widetilde{K}_t + Q_1\left((P_{11})^2 + \gamma P_{11} + \gamma^2\right)\widetilde{\lambda}_t
\end{aligned}
$$

Model 2: Log-linear version (solving)

- Put these into the capital equation and get

$$
\begin{aligned}
0 &= \gamma(1-\gamma)\,\widetilde{\lambda}_t + \xi\left[\beta P_{11}^3 - (1+2\beta)\,P_{11}^2 + (2+\beta)\,P_{11} - 1\right]\widetilde{K}_t \\
&\quad + \xi Q_1\left[\beta\,(P_{11})^2 + \beta^2\gamma P_{11} + \beta^2\gamma^2 + P_{11} + \gamma - 2 - \beta\right]\widetilde{\lambda}_t
\end{aligned}
$$

- For this to hold for all $\widetilde{K}_t$ and $\widetilde{\lambda}_t$, need

$$\beta P_{11}^3 - (1+2\beta)\,P_{11}^2 + (2+\beta)\,P_{11} - 1 = 0$$

and

$$
\begin{aligned}
&\frac{\gamma(1-\gamma)}{\xi(\theta-1)} \\
= \quad & Q_1\left[\beta\left(P_{11}^2+\gamma P_{11}+\gamma^2\right)-(1+2\beta)(P_{11}+\gamma)+(2+\beta)\right]
\end{aligned}
$$

.Model 2: Log-linear version (solving)

- Solutions for P_{11} are
$$P_{11}=1 \text{ or } P_{11}=\frac{1}{\beta}$$
- and
$$Q_1=-\frac{\gamma}{\xi(1-\theta)(1-\beta\gamma)} \text{ or } Q_1=-\frac{\gamma}{\xi(1-\theta)\beta(1-\gamma)}$$
- Note
$$
\begin{aligned}
P_{21} &= \frac{[\overline{r}-\delta)]\overline{K}+\overline{w}\overline{H}}{\overline{B}} \\
&\text{and} \\
P_{22} &= 1+r^f
\end{aligned}
$$
- Problem: solution either random walk or explosive $\Longrightarrow$ SS not valid

.Closing the open economy

- Interest rates and country risk
- Assume: foreign interest rate a function of total international debt (or savings)
- Functional form: $r_t^f=r^*-aB_t$
- Household first order conditions

$$
\begin{aligned}
B &= -\frac{w_t}{C_t} \\
\frac{1}{C_t} &= \beta E_t\frac{1}{C_{t+1}}(1+r^*-aB_t) \\
&\frac{1}{\beta}(1+\kappa(K_{t+1}-K_t)) \\
&= E_t\frac{C_t}{C_{t+1}}\left[r_{t+1}+(1-\delta)+\kappa(K_{t+2}-K_{t+1})\right] \\
B_t+K_{t+1}+C_t &= w_tH_t+r_tK_t+(1-\delta)K_t \\
&-\frac{\kappa}{2}(K_{t+1}-K_t)^2+(1+r^*-aB_{t-1})B_{t-1}
\end{aligned}
$$

.Closing the open economy: Stationary states

- In a stationary state, $\overline{C} = C_t = C_{t+1}$ so the second equation is

$$\frac{1}{\overline{C}} = \beta \frac{1}{\overline{C}} (1 + r^* - a\overline{B}),$$

or

$$\overline{B} = \frac{r^* + 1 - \frac{1}{\beta}}{a}.$$

- In the stationary state, the holding of foreign bonds or debt is exactly that which makes the real interest rate that the country pays (or receives) for foreign borrowing (or lending) equal to $1/\beta$.

.Closing the open economy: Stationary states

- For an economy with the standard values for the other parameters and with $a = .01$ and $r^* = .03$, the stationary state is

$\overline{K}$	$\overline{B}$	$\overline{C}$	$\overline{r}$	$\overline{w}$	$\overline{H}$	$\overline{Y}$
12.3934	1.9899	.9187	.0351	2.3706	.3262	1.2084

- For the same economy but where $r^* = 0$, the stationary state is

$\overline{K}$	$\overline{B}$	$\overline{C}$	$\overline{r}$	$\overline{w}$	$\overline{H}$	$\overline{Y}$
12.8114	-1.0101	.9187	.0351	2.3706	.3372	1.2491

- Notice that $\overline{K}$ and $\overline{B}$ are determined

.Closing the open economy: Log-linear version

$$\begin{aligned}
0 &= \widetilde{C}_t - \widetilde{w}_t \\
0 &= \widetilde{C}_t - E_t\widetilde{C}_{t+1} - \beta a \overline{B}\widetilde{B}_t \\
0 &= \beta \overline{r} E_t \widetilde{r}_{t+1} + \widetilde{C}_t - E_t\widetilde{C}_{t+1} + \beta\kappa\overline{K}E_t\widetilde{K}_{t+2} \\
&\quad - (1+\beta)\,\kappa\overline{K}\widetilde{K}_{t+1} + \kappa\overline{K}\widetilde{K}_t \\
0 &= \overline{B}\widetilde{B}_t + \overline{K}\widetilde{K}_{t+1} + \overline{C}\widetilde{C}_t - \overline{w}\overline{H}\left(\widetilde{w}_t + \widetilde{H}_t\right) \\
&\quad - \overline{r}\overline{K}\widetilde{r}_t - \frac{\overline{K}}{\beta}\widetilde{K}_t - \left((1+r^*)\,\overline{B} - 2a\overline{B}^2\right)\widetilde{B}_{t-1} \\
0 &= \widetilde{\lambda}_t + (\theta - 1)\,\widetilde{K}_t + (1-\theta)\,\widetilde{H}_t - \widetilde{r}_t \\
0 &= \widetilde{\lambda}_t + \theta\widetilde{K}_t - \theta\widetilde{H}_t - \widetilde{w}_t
\end{aligned}$$

.Closing the open economy: Log-linear version

- Let $x_t = \left[\widetilde{K}_{t+1}, \widetilde{B}_t\right]'$ be the vector of state variables

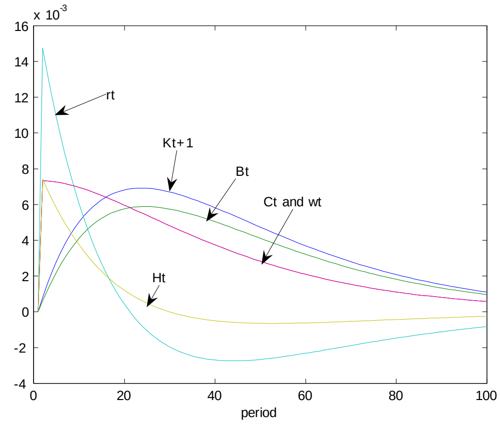

Figure 1: Impulse response functions to technology shock, $r^* = .03$

- $y_t = \left[\widetilde{C}_t, \widetilde{r}_t, \widetilde{w}_t, \widetilde{H}_t\right]'$ be the vector of jump variables
- $z_t = \left[\widetilde{\lambda}_t\right]'$ be the one stochastic variable
- We can write the system as

$$\begin{aligned} 0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t, \\ 0 &= E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right], \\ z_{t+1} &= Nz_t + \varepsilon_{t+1}. \end{aligned}$$

.Closing the open economy: Log-linear version

- Solution when $a = .01$ and $r^* = .03$ is

$$x_{t+1} = Px_t + Qz_t \quad \text{and} \quad y_t = Rx_t + Sz_t$$

where

$$P = \begin{bmatrix} 0.9572 & 0.0072 \\ 0.1419 & 0.8019 \end{bmatrix} \qquad Q = \begin{bmatrix} 0.0797 \\ 0.0606 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.3741 & 0.0933 \\ -0.6650 & -0.1658 \\ 0.3741 & 0.0933 \\ -0.0391 & -0.2591 \end{bmatrix} \qquad S = \begin{bmatrix} 0.7331 \\ 1.4745 \\ 0.7331 \\ 0.7414 \end{bmatrix}$$

.Closing the open economy: Impulse response function
.Closing the open economy: Log-linear version

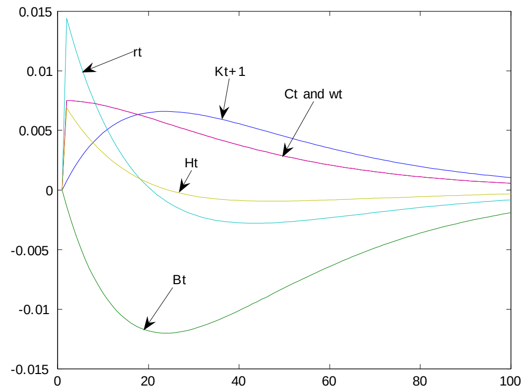

Figure 2: Impulse response functions to technology shock, $r^* = 0$

- Solution when $a = .01$ and $r^* = .00$ is

$$x_{t+1} = Px_t + Qz_t \quad \text{and} \quad y_t = Rx_t + Sz_t$$

where

$$P = \begin{bmatrix} 0.9567 & -0.0036 \\ -0.2718 & 0.8146 \end{bmatrix} \quad Q = \begin{bmatrix} 0.0759 \\ -0.1357 \end{bmatrix}$$

and

$$R = \begin{bmatrix} 0.3853 & -0.0514 \\ -0.6849 & 0.0914 \\ 0.3853 & -0.0514 \\ -0.0702 & 0.1429 \end{bmatrix} \quad S = \begin{bmatrix} 0.7518 \\ 1.4413 \\ 0.7518 \\ 0.6895 \end{bmatrix}$$

.Closing the open economy: Impulse response function
.Adding money to "Closed" economy

- Cash in advance money
- Foreign bonds are nominal in foreign currency
- Need to add exchange rate

.The open economy conditions

- Foreign market clearing condition

$$B_t - (1 + r^f_{t-1})B_{t-1} = P^*_t X_t$$

- Country risk rule

$$r_t^f = r^* - a\frac{B_t}{P_t^*}$$

- The foreign price level follows a stochastic process of

$$P_t^* = 1 - \gamma^* + \gamma^* P_{t-1}^* + \varepsilon_t^*$$

- We assume purchasing power parity so the exchange rate, e_t, is defined in terms of units of the local currency per unit of the foreign currency as

$$e_t = \frac{P_t}{P_t^*}$$

Households

- Households max

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\ln c_{t+j}^i + Bh_j^i\right]$$

- The cash-in-advance condition for domestic household i in period t is

$$P_t c_t^i = m_{t-1}^i + (g_t - 1) M_{t-1}$$

- The flow budget constraint for household i in period t is

$$\begin{aligned} \frac{m_t^i}{P_t} + \frac{e_t b_t^i}{P_t} + k_{t+1}^i + \frac{\kappa}{2}\left(k_{t+1}^i - k_t^i\right)^2 &= w_t h_t^i + r_t k_t^i \\ + (1-\delta) k_t^i & \quad + \frac{e_t(1+r_{t-1}^f) b_{t-1}^i}{P_t} \end{aligned}$$

Households contributiion to the model

- FOCs

$$\begin{aligned} 0 &= E_t \frac{e_t}{P_{t+1} c_{t+1}^i} - \beta E_t \frac{e_{t+1}(1+r_t^f)}{P_{t+2} c_{t+2}^i} \\ 0 &= E_t \frac{P_t}{P_{t+1} c_{t+1}^i}\left[1 + \kappa\left(k_{t+1}^i - k_t^i\right)\right] \\ & \quad - \beta E_t \frac{P_{t+1}}{P_{t+2} c_{t+2}^i}\left(r_{t+1} + (1-\delta) + \kappa\left(k_{t+2}^i - k_{t+1}^i\right)\right) \\ 0 &= \frac{B}{w_t} + \beta E_t \frac{P_t}{P_{t+1} c_{t+1}^i} \end{aligned}$$

- budget constraints

$$0 = P_t c_t^i - m_{t-1}^i - (g_t - 1)\, M_{t-1}$$

  and

$$\begin{aligned} 0 &= \frac{m_t^i}{P_t} + \frac{e_t b_t^i}{P_t} + k_{t+1}^i + \frac{\kappa}{2}\left(k_{t+1}^i - k_t^i\right)^2 \\ &\quad - w_t h_t^i - r_t k_t^i - (1-\delta)\, k_t^i - \frac{e_t (1 + r_{t-1}^f) b_{t-1}^i}{P_t} \end{aligned}$$

Firms

- production function,

$$Y_t = \lambda_t K_t^{\theta} H_t^{1-\theta}$$

- The equilibrium conditions for the domestic labor market

$$w_t = (1-\theta)\, \lambda_t K_t^{\theta} H_t^{-\theta}$$

- and for the domestic capital market is

$$r_t = \theta \lambda_t K_t^{\theta - 1} H_t^{1-\theta}$$

Equilibrium conditions

- Aggregate resource constraint

$$\lambda_t K_t^{\theta} H_t^{1-\theta} = C_t + K_{t+1} - (1-\delta)\, K_t + X_t$$

-

$$\begin{aligned} C_t &= c_t^i \\ M_t &= m_t^i \\ B_t &= b_t^i \\ H_t &= h_t^i \end{aligned}$$

$$K_{t+1} = k_{t+1}^i$$

- the money supply rule

$$M_t = g_t M_{t-1}$$

Stationary state

- In SS: $\overline{P}^* = 1$

- Rest of model

$$\begin{aligned}
\pi &= \beta(1+\overline{r}^f)\frac{e_{t+1}}{e_t} \\
\frac{1}{\beta} &= (\overline{r}+(1-\delta)) \\
-B\pi\overline{C} &= \beta\overline{w} \qquad \overline{C} = \overline{M/P}
\end{aligned}$$

$$\begin{aligned}
\overline{M/P} + \frac{e_t\overline{B}}{P_t} &= \overline{w}\overline{H} + (\overline{r}-\delta)\,\overline{K} + \frac{e_t(1+\overline{r}^f)\overline{B}}{P_t} \\
\overline{w} &= (1-\theta)\,\overline{K}^{\theta}\overline{H}^{-\theta} \qquad \overline{r} = \theta\overline{K}^{\theta-1}\overline{H}^{1-\theta} \\
\overline{r}^f\overline{B} &= \overline{X} \qquad \overline{r}^f = r^* - a\overline{B} \qquad \frac{e_t}{P_t} = 1 \\
M_t &= \overline{g}M_{t-1}
\end{aligned}$$

Stationary state

	$\overline{C}$	$\overline{K}$	$\overline{B}$	$\overline{H}$	$\overline{Y}$	$\overline{X}$
$r^* = .03$ $\overline{g} = 1$	.9095	12.2667	1.9899	.3229	1.1960	.0201
$r^* = .03$ $\overline{g} = 1.19$	.7643	10.2639	1.9899	.2702	1.0008	.0201
$r^* = .00$ $\overline{g} = 1$	.9095	12.6847	−1.0101	.3339	1.2368	−.0102
$r^* = .00$ $\overline{g} = 1.19$	.7643	10.6819	−1.0101	.2812	1.0415	−.0102

Log-linear version of the model

$$\begin{aligned}
0 &= \widetilde{e}_t - E_t\widetilde{e}_{t+1} - E_t\widetilde{P}_{t+1} + E_t\widetilde{P}_{t+2} - E_t\widetilde{C}_{t+1} + E_t\widetilde{C}_{t+2} - \beta\overline{r}^f\widetilde{r}^f_t \\
0 &= P_t - 2E_tP_{t+1} + E_tP_{t+2} - E_tC_{t+1} + E_tC_{t+2} \\
&\quad -E_t\kappa KK_t + (1+\beta)\,E_t\kappa\overline{K}K_{t+1} - \beta E_t\kappa\overline{K}K_{t+2} - \beta E_t\overline{r}r_{t+1} \\
0 &= \widetilde{w}_t + \widetilde{P}_t - E_t\widetilde{P}_{t+1} - E_t\widetilde{C}_{t+1} \\
0 &= \widetilde{P}_t + \widetilde{C}_t - \widetilde{M}_t \\
0 &= \overline{M/P}\widetilde{M}_t - \left[\overline{M/P} - \overline{B}\overline{r}^f\right]\widetilde{P}_t + \overline{B}\widetilde{B}_t + \overline{K}\widetilde{K}_{t+1} - \overline{w}\overline{H}\widetilde{w}_t - \overline{w}\overline{H}\widetilde{H}_t \\
&\quad -\overline{r}\overline{K}\widetilde{r}_t - [\overline{r}+(1-\delta)]\,\overline{K}\widetilde{K}_t - \overline{B}\overline{r}^f\widetilde{e}_t - \overline{B}\overline{r}^f\widetilde{r}^f_{t-1} - \overline{B}\left(1+\overline{r}^f\right)\widetilde{B}_{t-1}
\end{aligned}$$

widthheighteqnarray* $0 = \widetilde{w}_t - \widetilde{\lambda}_t - \theta\widetilde{K}_t + \theta\widetilde{H}_t$

$0 = \widetilde{r}_t - \widetilde{\lambda}_t + (1-\theta)\,\widetilde{K}_t - (1-\theta)\,\widetilde{H}_t$

$0 = \overline{B}\widetilde{B}_t - \left(1+\overline{r}^f\right)\overline{B}\widetilde{B}_{t-1} - \overline{r}^f\overline{B}\widetilde{r}^f_{t-1} - \overline{X}\widetilde{P}^*_t - \overline{X}\widetilde{X}_t$

$0 = \overline{r}^f\widetilde{r}^f_t + a\overline{B}\widetilde{B}_t$

$0 = \widetilde{e}_t - \widetilde{P}_t + \widetilde{P}^*_t$

$0 = \widetilde{M}_t - \widetilde{g}_t - \widetilde{M}_{t-1}$

Log-linear version

- $x_t = \left[\widetilde{K}_{t+1}, \widetilde{M}_t, \widetilde{P}_t, \widetilde{B}_t, \widetilde{r}_t^f\right]'$
- $y_t = \left[\widetilde{C}_t, \widetilde{r}_t, \widetilde{w}_t, \widetilde{H}_t, \widetilde{e}_t, \widetilde{X}_t\right]'$
- $z_t = \left[\widetilde{\lambda}_t, \widetilde{g}_t, \widetilde{P}_t^*\right]'$
- We can write the system as

$$\begin{aligned} 0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t \\ 0 &= E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right] \\ z_{t+1} &= Nz_t + \varepsilon_{t+1} \end{aligned}$$

- Solve for

$$\begin{aligned} x_{t+1} &= Px_t + Qz_t, \quad \text{and} \\ y_t &= Rx_t + Sz_t. \end{aligned}$$

- For the economy with $\overline{g} = 1$ and $r^* = .03$

.

$$P = \begin{bmatrix} 0.9852 & 0 & 0 & 0.0102 & 0.0001 \\ 0 & 1 & 0 & 0 & 0 \\ -0.3241 & 0 & 0 & -0.0919 & -0.0009 \\ 0.0436 & 0 & 0 & 0.8068 & 0.0081 \\ -0.0859 & 0 & 0 & -1.5894 & -0.0159 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.0586 & 0.0031 & -0.0735 \\ 0 & 1 & 0 \\ -0.7477 & 1.4201 & 0.4768 \\ 0.1674 & 0.1408 & 1.1701 \\ -0.3299 & -0.2774 & -2.3052 \end{bmatrix}$$

height R= $\begin{bmatrix} 0.3241 & 0 & 0 & 0.0919 & 0.0009 \\ -0.5761 & 0 & 0 & -0.1634 & -0.0016 \\ 0.3241 & 0 & 0 & 0.0919 & 0.0009 \\ 0.0998 & 0 & 0 & -0.2552 & -0.0026 \\ -0.3241 & 1 & 0 & -0.0919 & -0.0009 \\ 4.3162 & 0 & 0 & -20.1257 & -0.2013 \end{bmatrix}$

$$S = \begin{bmatrix} 0.7477 & -0.4201 & -0.4768 \\ 1.4485 & -0.0532 & 0.8476 \\ 0.7477 & -0.4201 & -0.4768 \\ 0.7009 & -0.0832 & 1.3244 \\ -0.7477 & 1.4201 & -0.5232 \\ 16.5774 & 13.9391 & 114.8437 \end{bmatrix}$$

Impulse response functions for a technology shock

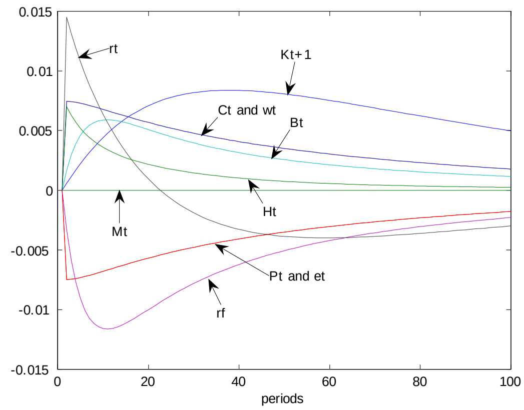

Figure 3: Response functions to a techology shock, $\overline{g} = 1$, $r^* = .03$

- $\overline{g} = 1$ and $r^* = .03$

Impulse response functions for a technology shock

- $\overline{g} = 1$ and $r^* = .00$

Impulse response functions for a money growth shock

- $\overline{g} = 1$ and $r^* = .03$

Impulse response functions for a money growth shock

- $\overline{g} = 1$ and $r^* = .00$

Impulse response functions for a foreign price shock

- $\overline{g} = 1$ and $r^* = .03$

Impulse response functions for a foreign price shock

- $\overline{g} = 1$ and $r^* = .00$

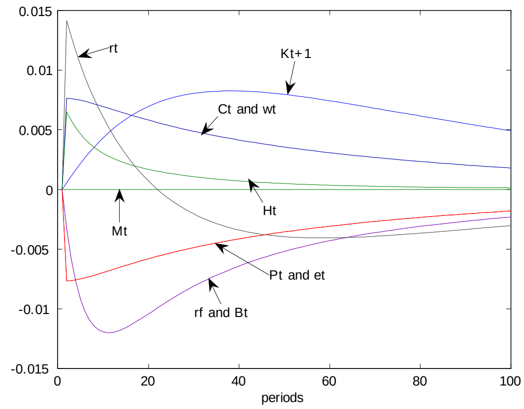

Figure 4: Response functions to a techology shock, $\overline{g} = 1$, $r^* = .00$

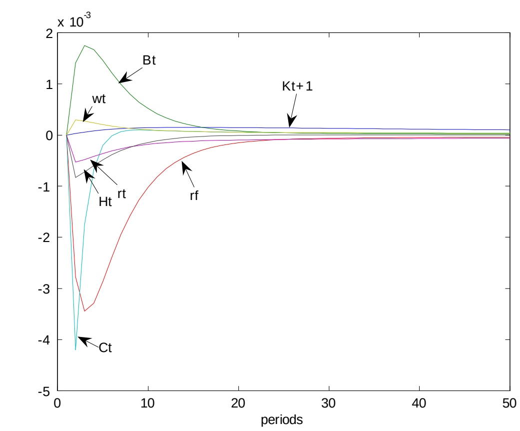

Figure 5: Response functions to monetary shock, $\overline{g} = 1$, $r^* = .03$

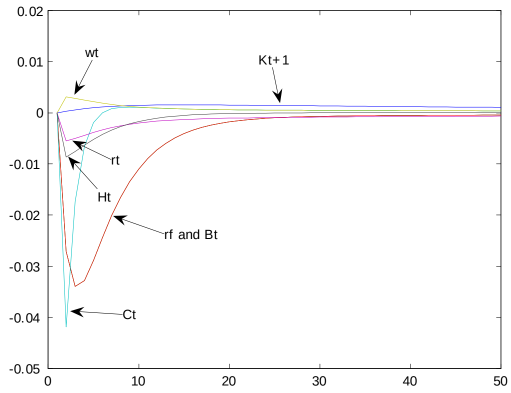

Figure 6: Response functions to monetary shock, $\overline{g} = 1$, $r^* = .00$

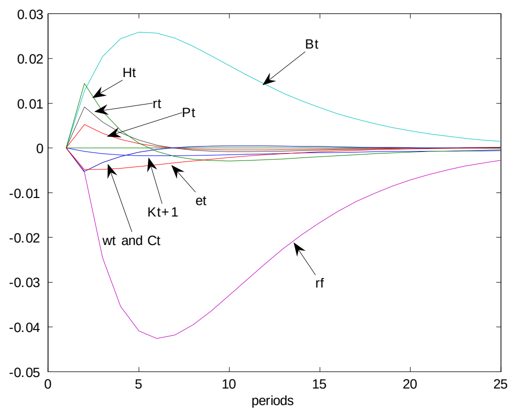

Figure 7: Response functions to foreign price shock, $\overline{g} = 1$, $r^* = .03$

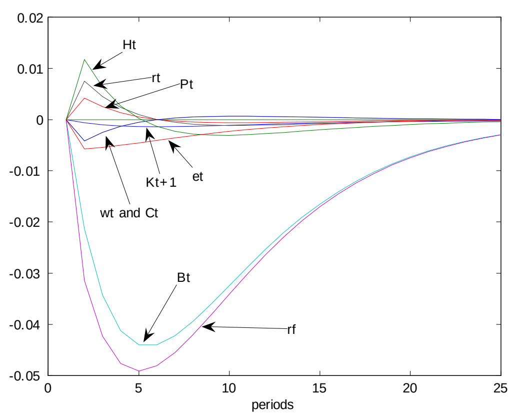

Figure 8: Response functions to foreign price shock, $\overline{g} = 1$, $r^{*} = .00$