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Macroeconomics II
Money in the Utility function

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1 Money in the Utility function

Money in the Utility function

- Alternative way to add money to model
- Utility approximates benefits from using money in transactions
- Use sub-utility of the form

$$u(c_t^i, \frac{m_t^i}{P_t}, l_t^i) = u(c_t^i, \frac{m_t^i}{P_t}, 1 - h_t^i),$$

Households maximize

- Households choose sequences of $\{c_t^i, m_t^i, k_{t+1}^i, h_t^i\}_{t=0}^{\infty}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, \frac{m_t^i}{P_t}, 1 - h_t^i),$$

subject to the sequence of period t budget constraints,

$$c_t^i + k_{t+1}^i + \frac{m_t^i}{P_t} = w_t h_t^i + r_t k_t^i + (1-\delta) k_t^i + \frac{m_{t-1}^i}{P_t} + (g_t - 1)\frac{M_{t-1}}{P_t}$$

- $(g_t - 1)M_{t-1}$ is a lump-sum transfer of money
- we use

$$u(c_t^i, \frac{m_t^i}{P_t}, 1 - h_t^i) = \ln c_t^i + D \ln\left(\frac{m_t^i}{P_t}\right) + B h_t^i$$

Household's FOCs

- First order conditions for the household are

$$\begin{aligned}
\frac{1}{c_t^i} &= \beta E_t \frac{P_t}{c_{t+1}^i P_{t+1}} + \frac{DP_t}{m_t^i} \\
\frac{1}{c_t^i} &= \beta E_t \frac{1}{c_{t+1}^i} \left[r_{t+1} + (1-\delta) \right] \\
\frac{1}{c_t^i} &= -\frac{B}{w_t}
\end{aligned}$$

- another equation from teh budget constraint

$$c_t^i + k_{t+1}^i + \frac{m_t^i}{P_t} = w_t h_t^i + r_t k_t^i + (1-\delta)k_t^i + \frac{m_{t-1}^i}{P_t} + (g_t - 1)\frac{M_{t-1}}{P_t}$$

Prices and money

- Aggregate version of the first FOC is

$$\frac{1}{P_t C_t} = \beta E_t \frac{1}{C_{t+1} P_{t+1}} + D \frac{1}{M_t}$$

- But

$$E_t \frac{1}{P_{t+1} C_{t+1}} = \beta E_t \frac{1}{C_{t+2} P_{t+2}} + D \frac{1}{M_{t+1}}$$

- substituting this into the first FOC gives

$$\begin{aligned}
\frac{1}{P_t C_t} &= \beta E_t \left[\beta E_t \frac{1}{C_{t+2} P_{t+2}} + D \frac{1}{M_{t+1}} \right] + D \frac{1}{M_t} \\
&= \beta^2 E_t \frac{1}{C_{t+2} P_{t+2}} + \beta E_t D \frac{1}{M_{t+1}} + D \frac{1}{M_t}
\end{aligned}$$

- Repeated substitutions give

$$\frac{1}{P_t} = D C_t \sum_{j=0}^{\infty} \beta^j E_t \frac{1}{M_{t+j}}$$

Prices and money

- The money growth rule is

$$M_{t+1} = g_{t+1} M_t$$

- prices in terms of a sequence of growth rates of money are

$$\frac{1}{P_t} = \frac{D C_t}{M_t} \sum_{j=0}^{\infty} \beta^j E_t \prod_{k=1}^{j} \frac{1}{g_{t+k}}$$

- Prices are forward looking and depend on the entire expected future growth rates of money
- We assume money growth follows the process

$$\ln g_t = (1-\pi)\ln \overline{g} + \pi \ln g_{t-1} + \varepsilon_t^g$$

The production sector

- Production is standard and competitive
- The production function is

$$Y_t = \lambda_t K_t^{\theta} H_t^{1-\theta},$$

with costs of

$$costs = w_t H_t + r_t K_t.$$

- Competitive factor market conditions are

$$r_t = \theta \lambda_t K_t^{\theta-1} H_t^{1-\theta},$$

and

$$w_t = (1-\theta)\,\lambda_t K_t^{\theta} H_t^{-\theta}.$$

The stochastic shocks for technology, λ_t follow the process

$$\ln \lambda_t = \gamma \ln \lambda_{t-1} + \varepsilon_t^{\lambda},$$

Equilibrium conditions

- Given that there is a unit mass of identical agents, aggregate variables are

$$\begin{aligned} C_t &= c_t^i \\ M_t &= m_t^i \\ H_t &= h_t^i \end{aligned}$$

and

$$K_t = k_t^i$$

Stationary states

- Stationary state version of the FOCs are

$$\begin{aligned} \frac{1}{\overline{C}} &= \beta \frac{P_t}{\overline{C} P_{t+1}} + D \frac{P_t}{M_t} \\ \overline{r} &= \frac{1}{\beta} - (1-\delta) \\ \overline{C} &= -\frac{\overline{w}}{B} \end{aligned}$$

- The stationary state budget constraint is simply

$$\overline{Y} = \overline{C} + \delta\overline{K}$$

- The factor market conditions in a stationary state are

$$\begin{aligned}\overline{r} &= \theta\overline{K}^{\theta-1}\overline{H}^{1-\theta} \\ \overline{w} &= (1-\theta)\,\overline{K}^{\theta}\overline{H}^{-\theta}\end{aligned}$$

- The production function is

$$\overline{Y} = \overline{K}^{\theta}\overline{H}^{1-\theta}$$

Stationary states

- The rental on capital is

$$\overline{r} = \frac{1}{\beta} - (1-\delta)$$

- From the factor market conditions, get

$$\overline{w} = (1-\theta)\left[\frac{\theta}{\overline{r}}\right]^{\frac{\theta}{1-\theta}}$$

- with this $\overline{w}$, get

$$\overline{C} = -\frac{\overline{w}}{B}$$

- The first first order condition can be written as

$$1 = \frac{\beta}{g} + D\frac{\overline{C}}{\overline{M/P}}$$

  to give

$$\overline{M/P} = D\frac{g\overline{C}}{g-\beta}$$

Stationary states

- Use the factor market conditions to get $\overline{H}$ as a function of $\overline{K}$, put this into the production function to get

$$\overline{Y} = \overline{K}\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{1-\theta}$$

- Use the stationary state household budget constraint to get

$$\overline{K} = \frac{\overline{C}}{\left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]^{1-\theta} - \delta}$$

- Use the above equation to get output
- Using the factor market conditions, get

$$\overline{H} = \left[\frac{\overline{r}(1-\theta)}{\overline{w}\theta}\right]\overline{K}$$

Stationary states

- For the standard economy with $\beta = .99$, $\delta = .025$, $\theta = .36, B = -2.5805$, and set $D = .01$ to get same money holdings as in the cia model when $\overline{g} = 1$.

Variable	*Stationary state value*
$\overline{r}$	.035101
$\overline{w}$	2.3706
$\overline{C}$	.9187
$\overline{K}$	12.6707
$\overline{H}$	.3335
$\overline{Y}$	1.2354
$\overline{M/P}$	$\frac{.009187\overline{g}}{\overline{g}-.99}$

Log-linear version of the model

$$\begin{aligned}
0 &= \widetilde{C}_t + \left[\frac{\beta}{\overline{g}} + \frac{D\overline{C}}{\overline{M/P}}\right]\widetilde{P}_t - \frac{\beta}{\overline{g}}E_t\widetilde{C}_{t+1} - \frac{\beta}{\overline{g}}E_t\widetilde{P}_{t+1} - \frac{D\overline{C}}{\overline{M/P}}\widetilde{M}_t, \\
0 &= \widetilde{w}_t + \beta\overline{r}E_t\widetilde{r}_{t+1} - \beta\left[\overline{r} + (1-\delta)\right]E_t\widetilde{w}_{t+1}, \\
0 &= \widetilde{w}_t - \widetilde{C}_t, \\
0 &= \overline{C}\widetilde{C}_t + \overline{K}\widetilde{K}_{t+1} - \overline{w}\overline{H}\widetilde{w}_t - \overline{w}\overline{H}\widetilde{H}_t - \overline{r}\overline{K}\widetilde{r}_t - \left[\overline{r} + (1-\delta)\right]\overline{K}\widetilde{K}_t, \\
0 &= \widetilde{Y}_t - \widetilde{\lambda}_t - \theta\widetilde{K}_t - (1-\theta)\widetilde{H}_t, \\
0 &= \widetilde{r}_t - \widetilde{\lambda}_t - (\theta-1)\widetilde{K}_t - (1-\theta)\widetilde{H}_t, \\
0 &= \widetilde{w}_t - \widetilde{\lambda}_t - \theta\widetilde{K}_t + \theta\widetilde{H}_t, \\
0 &= \widetilde{M}_t - \widetilde{g}_t - \widetilde{M}_{t-1}.
\end{aligned}$$

Log-linear version of the model

- the three "state" variables, $x_t = \left[\widetilde{K}_{t+1}, \widetilde{M}_t, \widetilde{P}_t\right]$,
- the five "jump" variables, $y_t = \left[\widetilde{r}_t, \widetilde{w}_t, \widetilde{C}_t, \widetilde{Y}_t, \widetilde{H}_t\right]$
- the stochastic variables $z_t = \left[\widetilde{\lambda}_t, \widetilde{g}_t\right]$

- Solve the model as

$$
\begin{aligned}
0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t, \\
0 &= E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right], \\
z_{t+1} &= Nz_t + \varepsilon_{t+1},
\end{aligned}
$$

- to get

$$x_{t+1} = Px_t + Qz_t,$$

and

$$y_t = Rx_t + Sz_t.$$

Linear policy function

- The solution to the model are linear policy functions

$$x_{t+1} = Px_t + Qz_t$$

and

$$y_t = Rx_t + Sz_t$$

- where

$$P = \begin{bmatrix} 0.9418 & 0 & 0 \\ 0 & 1 & 0 \\ -0.5316 & 1 & 0 \end{bmatrix},$$

$$Q = \begin{bmatrix} 0.1552 & 0 \\ 0 & 1 \\ -0.4703 & 1.6648 \end{bmatrix},$$

Linear policy function

- and

$$R = \begin{bmatrix} -0.9450 & 0 & 0 \\ 0.5316 & 0 & 0 \\ 0.5316 & 0 & 0 \\ 0.0550 & 0 & 0 \\ -0.4766 & 0 & 0 \end{bmatrix},$$

and

$$S = \begin{bmatrix} 1.9417 & 0 \\ 0.4703 & 0 \\ 0.4703 & 0 \\ 1.9417 & 0 \\ 1.4715 & 0 \end{bmatrix}.$$

- Notice the coefficient on prices of a money growth shock

Money and prices

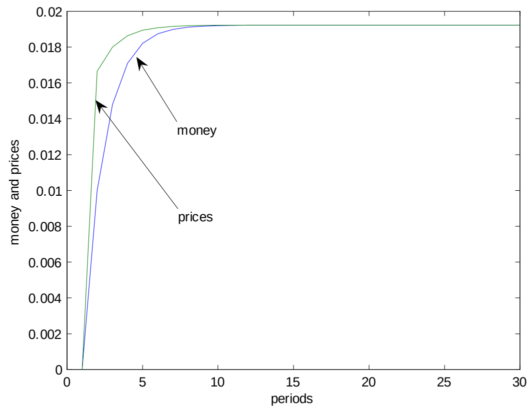

Figure 1: Response of money and prices to money growth shock

- Recall that prices are forward looking in this model
- The response of money and prices to a money growth shock are

Real variables and a technology shock

- These are the same as the responses in Cooley-Hansen

Seigniorage

- Government budget constaint is

$$g_t = \widehat{g}_t \overline{g} = \frac{M_t - M_{t-1}}{P_t}$$

- let $\overline{g}$ be the average government deficit financed with money issue and the random variable $\widehat{g}_t$ follows the process

$$\ln \widehat{g}_t = \pi \ln \widehat{g}_{t-1} + \varepsilon_t^g$$

  where $\varepsilon_t^g \sim N(0, \sigma_g)$, with σ_g as the standard error.

- Define φ_t as the gross growth rate of money that is required to finance the budget deficit in period t.
- Then,

$$M_t = \varphi_t \, M_{t-1}$$

  and

$$g_t = \widehat{g}_t \overline{g} = \frac{(\varphi_t \; - 1) \, M_{t-1}}{P_t} = \frac{(\varphi_t \; - 1)}{\varphi_t} \frac{M_t}{P_t}$$

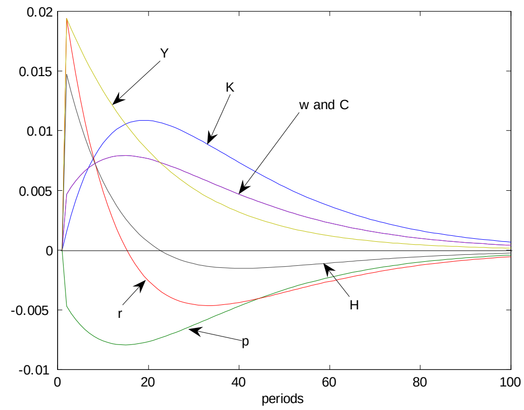

Figure 2: Responses to a .01 impulse in technology

Seigniorage

- Households do not receive direct transfers from government
- Household budget constraint is

$$c_t^i + k_{t+1}^i + \frac{m_t^i}{P_t} = w_t h_t^i + r_t k_t^i + (1-\delta) k_t^i + \frac{m_{t-1}^i}{P_t}$$

Full model is

$$\begin{aligned}
\frac{1}{C_t} &= \beta E_t \frac{P_t}{C_{t+1} P_{t+1}} + \frac{DP_t}{M_t}, \\
\frac{1}{w_t} &= \beta E_t \frac{1}{w_{t+1}} \left[r_{t+1} + (1-\delta) \right], \\
w_t &= -BC_t, \\
C_t + K_{t+1} + \frac{M_t}{P_t} &= w_t H_t + r_t K_t + (1-\delta) K_t + \frac{M_{t-1}}{P_t}, \\
Y_t &= \lambda_t K_t^{\theta} H_t^{1-\theta}, \\
r_t &= \theta \lambda_t K_t^{\theta-1} H_t^{1-\theta}, \\
w_t &= (1-\theta)\, \lambda_t K_t^{\theta} H_t^{-\theta}, \\
\widehat{g}_t \overline{g} &= \frac{M_t - M_{t-1}}{P_t}.
\end{aligned}$$

Stationary states

- Notice how output changes to cover seigniorage

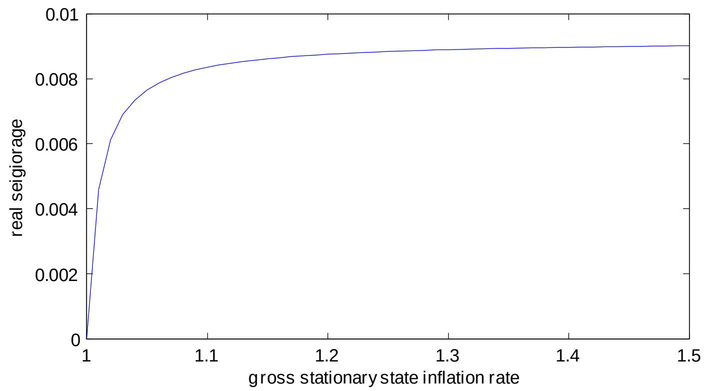

Figure 3: Real seigniorage

Annual inflation	0%	10%	100%	400%
Corresponding $\overline{\varphi}$	1	1.024	1.19	1.41
rental	0.0351	0.0351	0.0351	0.0351
wages	2.3706	2.3706	2.3706	2.3706
consumption	0.9187	0.9187	0.9187	0.9187
real balances $=\overline{M/P}$	0.9187	0.2767	0.0547	0.0308
output	1.2354	1.2441	1.2472	1.2475
capital	12.6707	12.7601	12.7910	12.7943
hours worked	0.3335	0.3359	0.3367	0.3368
seigniorage $=\overline{g}$	0	0.0065	0.0087	0.0090

Stationary states (Bailey curve)

- Relation between real seigniorage and inflation rate

Log-linearization

- Only two equations need changes

$$\begin{aligned} 0 \;=\;& \overline{C}\widetilde{C}_t + \overline{K}\widetilde{K}_{t+1} + \overline{M/P}\widetilde{M}_t - \overline{M/P}\left(1 - \frac{1}{\overline{\varphi}}\right)\widetilde{P}_t \\ & - \overline{w}\overline{H}\widetilde{w}_t - \overline{w}\overline{H}\widetilde{H}_t - \overline{r}\overline{K}\widetilde{r}_t - \left[\overline{r} + (1-\delta)\right]\overline{K}\widetilde{K}_t - \frac{\overline{M/P}}{\overline{\varphi}}\widetilde{M}_{t-1} \end{aligned}$$

- the seigniorage equation

$$0 = \overline{g}\widetilde{g}_t + \overline{M/P}\left(1 - \frac{1}{\overline{\varphi}}\right)\widetilde{P}_t - \overline{M/P}\widetilde{M}_t + \frac{\overline{M/P}}{\overline{\varphi}}\widetilde{M}_{t-1}$$

- where $\widetilde{g}_t \equiv \ln \widehat{g}_t - \ln 1 = \ln \widehat{g}_t$ is the log of the shock to the government deficit that is financed by seigniorage.

- Next slide gives matrices for $\overline{\varphi} = 1$ and $\overline{\varphi} = 1.19$

[5cm]
5cm

$$
\begin{aligned}
P &= \begin{bmatrix} 0.9418 & 0 & 0 \\ 0 & 1 & 0 \\ -0.5316 & 1 & 0 \end{bmatrix} \\
Q &= \begin{bmatrix} 0.1552 & 0 \\ 0 & 0 \\ -0.4703 & 0 \end{bmatrix} \\
R &= \begin{bmatrix} -0.9450 & 0 & 0 \\ 0.5316 & 0 & 0 \\ 0.5316 & 0 & 0 \\ 0.0550 & 0 & 0 \\ -0.4766 & 0 & 0 \end{bmatrix} \\
S &= \begin{bmatrix} 1.9417 & 0 \\ 0.4703 & 0 \\ 0.4703 & 0 \\ 1.9417 & 0 \\ 1.4715 & 0 \end{bmatrix}
\end{aligned}
$$

5cm

$$
\begin{aligned}
P &= \begin{bmatrix} 0.9418 & 0 & 0 \\ -0.3243 & 1 & 0 \\ -2.0213 & 1 & 0 \end{bmatrix} \\
Q &= \begin{bmatrix} 0.1547 & -0.0005 \\ -0.9330 & 0.2178 \\ -5.8433 & 0.3644 \end{bmatrix} \\
R &= \begin{bmatrix} -0.9477 & 0 & 0 \\ 0.5331 & 0 & 0 \\ 0.5331 & 0 & 0 \\ 0.0523 & 0 & 0 \\ -0.4807 & 0 & 0 \end{bmatrix} \\
S &= \begin{bmatrix} 1.9359 & 0.0011 \\ 0.4735 & -0.0006 \\ 0.4735 & -0.0006 \\ 1.9359 & 0.0011 \\ 1.4624 & 0.0017 \end{bmatrix}
\end{aligned}
$$

Response of real variables to a seigniorage shock
Response of nominal variables to a technology shock

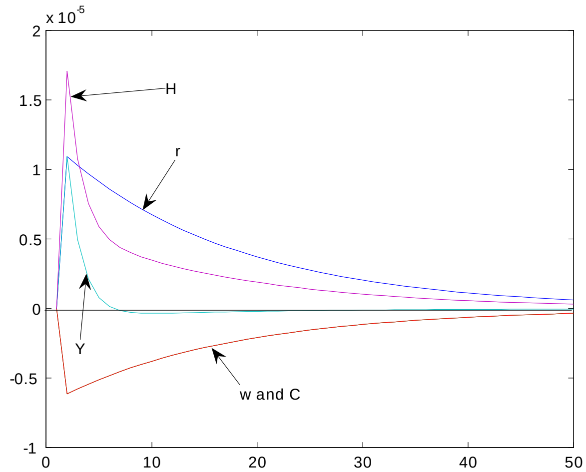

Figure 4: Response to seigniorage shock, $\overline{\varphi} = 1.19$

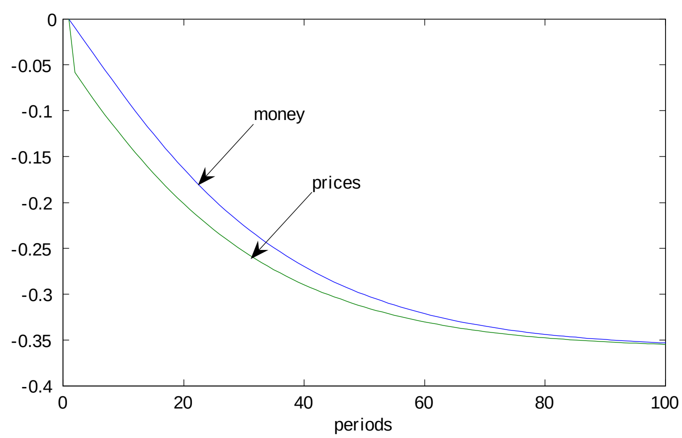

Figure 5: Responses of money and prices to a technology shock when seigniorage is collected in the stationary state, $\overline{\varphi} = 1.19$