Macroeconomia II
Working Capital (3)
Taylor rules and stability

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Stability under a Taylor rule

- Taylor rule can change dynamics of model
- money is not a stochastic process but follows rule
- This rule change dynamics
- Question: Does this rule make the model unstable?
- What does unstable mean?
  - Explosive
    - Not enough stable eigenvalues for the system
    - No non-explosive equilibria
    - These can't be equilibrium if on real variables (transversality conditions)
  - Multiple equilibria (at least 2 and linear combinations)
    - Too many stable eigenvalues (more than expectational equations)
    - At least two sets of eigenvalues that give stable equilibrium
    - Linear combinations of these also work ⇒ infinite equilibria
  - Unique equilibria
    - number of stable eigenvalues equals number of expectational equations

Taylor rules and the data

- What data is the Taylor rule using

– current data (as before)

$$r_t^f = a\left(Y_t - \overline{Y}\right) + b\left(\pi_t - \overline{\pi}\right) + \overline{r}^f$$

– past data (only lagged inflation and output available)

$$r_t^f = a\left(Y_{t-1} - \overline{Y}\right) + b\left(\pi_{t-1} - \overline{\pi}\right) + \overline{r}^f$$

– future data (use rational expectations forecasts of variables)

$$r_t^f = a\left(E_t Y_{t+1} - \overline{Y}\right) + b\left(E_t \pi_{t+1} - \overline{\pi}\right) + \overline{r}^f$$

- What choices of a and b result in unique equilibria under each rule

Finding the unique solution parameter sets

- Let $a \in [-1.5, 1.5]$ and $b \in [-1.5, 1.5]$
- For each pair of parameter values
  – set up the model with these values in the Taylor rule
  – Solve the matrix quadratic equation using generalized eigenvalue methods
  – This means finding the generalized eigenvalues of $\begin{bmatrix} B & C \\ I & \overrightarrow{0} \end{bmatrix}$ and $\begin{bmatrix} A & \overrightarrow{0} \\ \overrightarrow{0} & I \end{bmatrix}$ from the equation

$$\begin{bmatrix} B & C \\ I & \overrightarrow{0} \end{bmatrix} \begin{bmatrix} X^{11} & X^{21} \\ X^{12} & X^{22} \end{bmatrix} = \begin{bmatrix} A & \overrightarrow{0} \\ \overrightarrow{0} & I \end{bmatrix} \begin{bmatrix} X^{11} & X^{21} \\ X^{12} & X^{22} \end{bmatrix} \begin{bmatrix} \Delta^1 & \overrightarrow{0} \\ \overrightarrow{0} & \Delta^2 \end{bmatrix}$$

  – count the number of eigenvalues less than or equal to one (call numeig)

Finding the unique solution parameter sets

- – If numeig equals the number of expectational equation, the solution is unique
  – If numeig is greater than the number of expectational equation, there are multiple non-explosive solutions
  – If numeig is less than the number of expectational equation, only explosive solutions exist
- Make a map on the $a - b$ plane separating those with unique or multiple (infinite) solutions

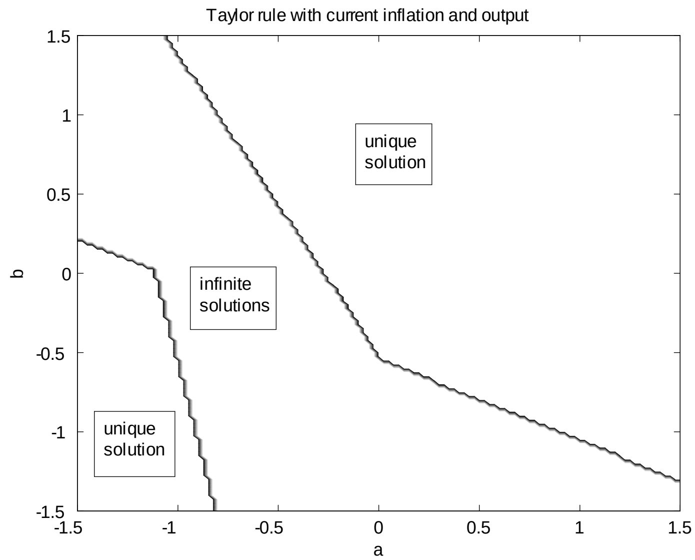

Figure 1: Purely current Taylor rule, $\overline{g}^M = 1.03$

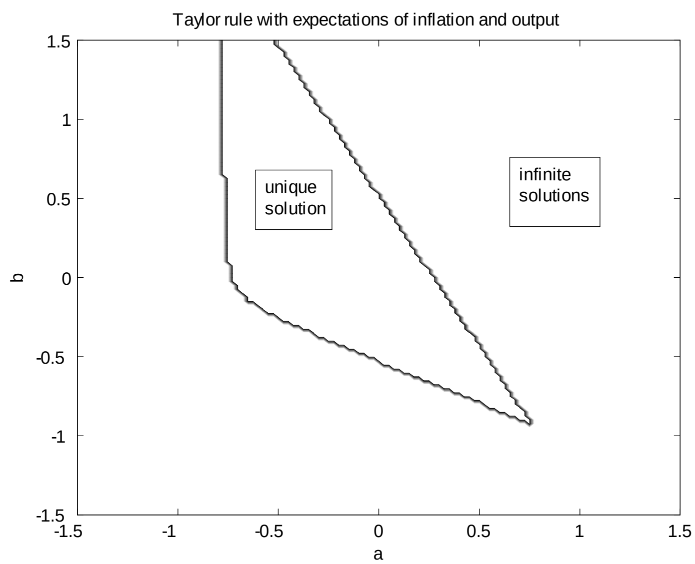

Figure 2: Purely expectational Taylor rule, $\overline{g}^M = 1.03$

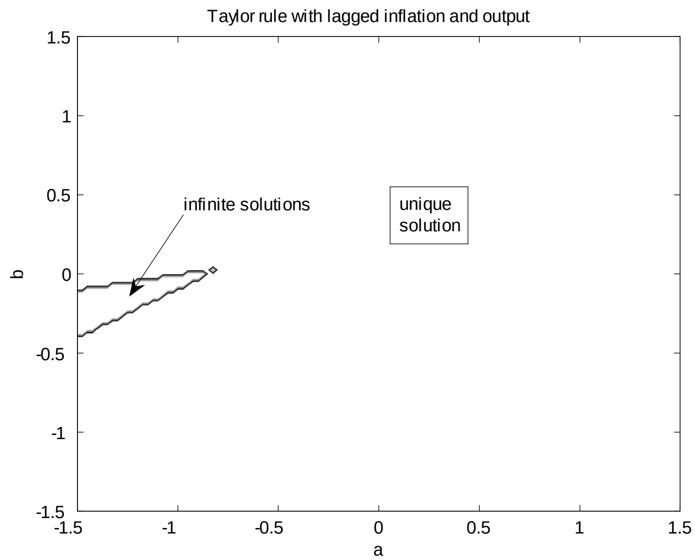

Figure 3: Purely lagged Taylor rule, $\overline{g}^M = 1.03$

- Note: with historical data the state variables need to be expanded to include Y_{t-1} and P_{t-2}

a-b plane for Taylor rule with current data
a-b plane for Taylor rule with expectational data
a-b plane for Taylor rule with historical (time t-1) data
a-b plane for .5 lagged and .5 future pi and Y(t-1)

- It is frequently recommended to use a mix of
  - time t+1 expected values for inflation
  - time t-1 historical inflation
  - time t-1 output (the data that is available)
- The idea is that the historical data should make multiple solutions more unlikely
- In this model: it works

a-b plane for .5 lagged and .5 future pi and Y(t-1)

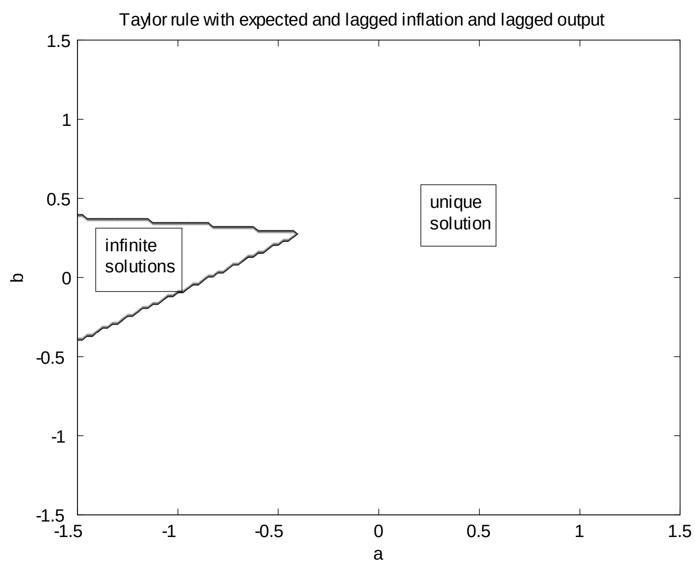

Figure 4: Set of unique solutions for Taylor rule with expected and lagged inflation, $\overline{g}^M = 1.03$