A model of working capital with idiosyncratic production risk and firm failure

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Outline of the talk

- Introduction
- Model
- Stationary states
- Dynamic version of model
- Conclusions

Introduction

- Part of a research program for modelling banking systems
- A simple model with banks and risky assets
- Where a fraction of the firms fail
- Banks need to hold loan loss reserves
- Need risk averse firm managers
  - So less than half the firms fail

The model: Firms

- Firm managers are like everyone else except
  - Firm profits enter their utility functions

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t^i + B h_t^i + G(\pi_t^k) \right]$$

– subject to

$$\pi_t^k = \lambda_t \varphi_t^k \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta} - r_t k_t^k - r_t^f w_t h_t^k$$

– $\varphi_t^k \in \left[\varphi^l, \varphi^u\right]$ with a uniform distribution

– r_t^f is the interest on working capital to pay the wage bill

- Given that the utility function is separable, firms managers max

$$E_0 \sum_{t=0}^{\infty} \beta^t G(\pi_t^k)$$

subject to the budget constraint

.The model: Firms

- With a uniform distriubtion, the expected utility maximization problem can be written

$$\max_{k_t^k, h_t^k} \frac{1}{\varphi^u - \varphi^l} \int_{\varphi^l}^{\varphi^u} G(\lambda_t \varphi_t^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta} - r_t k_t^k - r_t^f w_t h_t^k) dj.$$

- We use a $G(\cdot)$ with constant absolute risk aversion

$$G(x) = \eta \left(1 - \exp\left(-\alpha x\right)\right)$$

- α is the coefficient of absolute risk aversion
- Solving the max problem is ugly

.The model: Firms

- FOCs give (after simplification)

$$\frac{r_t k_t^k}{\theta} = \frac{r_t^f w_t h_t^k}{(1-\theta)}$$

$$TC_t = y_t^l \frac{\frac{\varphi^u}{\varphi^l} - e^{\alpha\left(\varphi^u - \varphi^l\right) y_t}}{1 - e^{\alpha\left(\varphi^u - \varphi^l\right) y_t}} + \frac{1}{\alpha}$$

$$TL_t = \frac{\left(TC_t - \varphi^l y_t\right)^2}{2\left(\varphi^u - \varphi^l\right) y_t}$$

$$D_t = \frac{\left(\varphi^u y_t - TC_t\right)^2}{2\left(\varphi^u - \varphi^l\right) y_t}$$

.The model: Financial intermediaries (banks)

- Take deposits (in money) from households
- Lend money to firms for working capital
- Competitive
  - Zero profit condition

$$r_t^d N_t = r_t^f P_t w_t H_t - P_t T L_t$$

  - Equilibrium condition for capital market

$$N_t + (1-\rho)(g_t - 1) M_{t-1} = P_t w_t H_t$$

  - $(1-\rho)(g_t - 1) M_{t-1}$ is the fraction of money growth that goes to the financial system

.The model: Households

- Representative household maximizes

$$\sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) + \frac{h_t}{h_0} A \ln(1-h_0) \right]$$

  - h_t / h_0 is the probability that this family will be required to supply h_0 units of labor
  - subject to the budget constraint

$$\frac{m_t}{P_t} + k_{t+1} = w_t h_t + r_t k_t + (1-\delta) k_t + d_t + \frac{r_t^d n_t}{P_t}$$

  - and the cash-in-advance constraint

$$P_t c_t = m_{t-1} + \rho (g_t - 1) M_{t-1} - n_t$$

  - d_t is the lump sum dividend payment to the household from the firms

.The model: Households

- FOCs are

$$\begin{aligned} \frac{1}{w_t} &= E_t \frac{\beta}{w_{t+1}} (r_{t+1} + (1-\delta)) \\ w_t &= -B r_t^d c_t \\ \frac{1}{r_t^d} &= \beta E_t \frac{P_t c_t}{P_{t+1} c_{t+1}} \end{aligned}$$

  where

$$B \equiv \frac{A \ln(1-h_0)}{h_0}$$

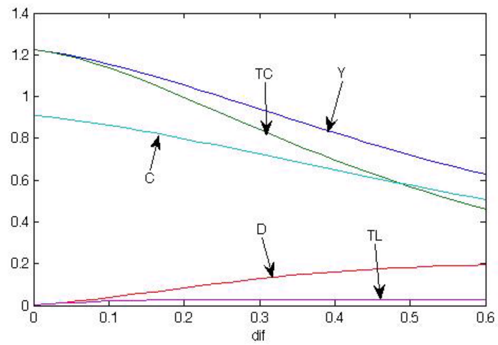


Figure 1: Stationary state values for $\overline{g} = 1$

The model: Equilibrium conditions

- Since all households are alike

$$\begin{aligned} H_t &= h_t \\ M_t &= m_t \\ K_t &= k_t \\ D_t &= d_t \\ C_t &= c_t \\ N_t &= n_t \end{aligned}$$

$$Y_t = y_t$$

- Market clearing in the working capital markets

$$N_t + (1-\rho)(g_t - 1) M_{t-1} = P_t w_t H_t$$

Stationary states, g=1, alpha = 4
Stationary states, g=1, alpha = 4
Stationary states, g=1: Output and alpha (ARA coefficient)
Stationary states, g=1: Firm failure and alpha (ARA coefficient)
Dynamic version of the model

- Log-linearization of the model (around stationary state)
- Use method of undetermined coefficients (a la Uhlig) to solve
- Find linear policy functions of the form

$$\begin{aligned} x_t &= P x_{t-1} + Q z_t \\ y_t &= R x_{t-1} + S z_t \end{aligned}$$

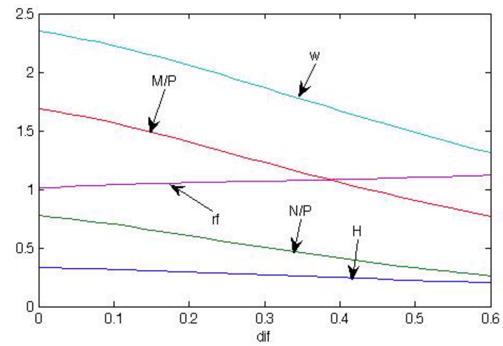


Figure 2: More stationary state values for $\overline{g} = 1$

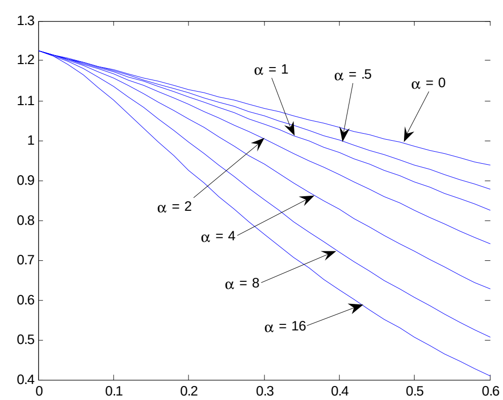


Figure 3: Stationary state output and risk aversion

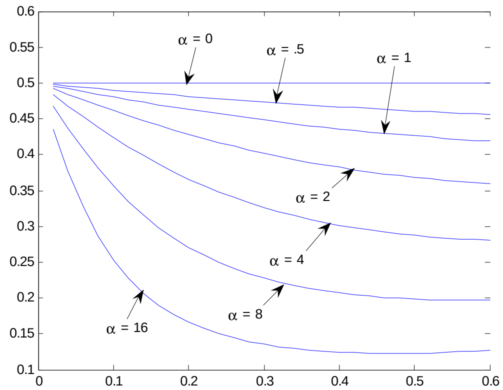


Figure 4: Fraction of firms that fail in the stationary state

where $x_t = \left[\widetilde{K}_{t+1}, \widetilde{M}_t, \widetilde{P}_t\right]'$, $y_t = \left[\widetilde{r}_t, \widetilde{w}_t, \widetilde{Y}_t, \widetilde{C}_t, \widetilde{H}_t, \widetilde{N}_t, , \widetilde{TC}_t, \widetilde{r}_t^d, \widetilde{r}_t^f\right]'$, and $z_t = \left[\widetilde{\lambda}_t, \widetilde{g}_t\right]'$

- using

$$
\begin{aligned}
0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t, \\
0 &= E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right], \\
z_{t+1} &= Nz_t + \varepsilon_{t+1},
\end{aligned}
$$

.

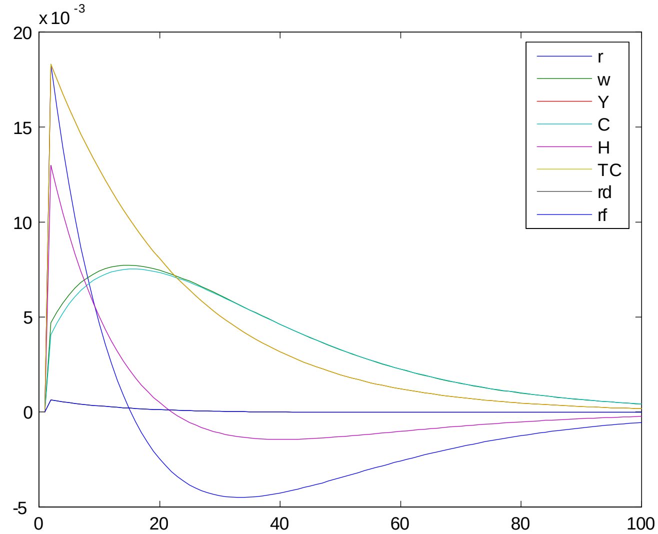
$$
\begin{aligned}
0 &= \widetilde{w}_t - E_t\widetilde{w}_{t+1} + \beta\overline{r}E_t\widetilde{r}_{t+1}, \\
0 &= \widetilde{r}_t^d - \widetilde{w}_t + \widetilde{C}_t, \\
0 &= \widetilde{w}_t + \widetilde{P}_t - E_t\widetilde{P}_{t+1} - E_t\widetilde{C}_{t+1}, \\
0 &= \overline{M/P}\widetilde{M}_t + \left[\overline{r}^n\overline{N/P} - \overline{M/P}\right]\widetilde{P}_t + \overline{K}\widetilde{K}_{t+1} - \overline{w}\overline{H}(\widetilde{w}_t + \widetilde{H}_t) \\
&\quad -\overline{r}\overline{K}\widetilde{r}_t - (\overline{r} + 1 - \delta)\overline{K}\widetilde{K}_t - \overline{D}\widetilde{D}_t - \overline{r}^d\overline{N/P}\widetilde{N}_t - \overline{r}^d\overline{N/P}\widetilde{r}_t^d,
\end{aligned}
$$

widthheighteqnarray* $0 = \overline{C}\left(\widetilde{P}_t + \widetilde{C}_t\right) - (1 + \rho\overline{g} - \rho)\frac{\overline{M/P}}{\overline{g}}\widetilde{M}_{t-1} - \rho\overline{M/P}\widetilde{g}_t + \overline{N/P}\widetilde{N}_t,$

$0 = \overline{TC}\widetilde{TC}_t$

$$
-\left(\frac{\varphi^l\overline{Y}\left(\frac{\varphi^u}{\varphi^l} - e^{\alpha\left(\varphi^u - \varphi^l\right)\overline{Y}}\right)}{\left(1 - e^{\alpha\left(\varphi^u - \varphi^l\right)\overline{Y}}\right)} + \frac{e^{\alpha\left(\varphi^u - \varphi^l\right)\overline{Y}}\alpha\left(\varphi^u - \varphi^l\right)^2\overline{Y}^2}{\left(1 - e^{\alpha\left(\varphi^u - \varphi^l\right)\overline{Y}}\right)^2}\right)\widetilde{Y}_t,
$$

$0 = \widetilde{TC}_t - \widetilde{r}_t - \widetilde{K}_t,$



widthheighteqnarray* $0 = \widetilde{TC}_t - \widetilde{r}_t^f - \widetilde{w}_t - \widetilde{H}_t,$

$$0 = \overline{TL}\widetilde{TL}_t - \frac{\overline{TC}\left(\overline{TC}-\varphi^l\overline{Y}\right)}{(\varphi^u-\varphi^l)\overline{Y}}\widetilde{TC}_t + \frac{\overline{TC}^2-\left(\varphi^l\overline{Y}\right)^2}{2(\varphi^u-\varphi^l)\overline{Y}}\widetilde{Y}_t,$$
$$0 = \overline{D}\widetilde{D}_t + \frac{\overline{TC}\left(\varphi^u\overline{Y}-\overline{TC}\right)}{(\varphi^u-\varphi^l)\overline{Y}}\widetilde{TC}_t - \frac{\left(\varphi^u\overline{Y}\right)^2-\overline{TC}^2}{2(\varphi^u-\varphi^l)\overline{Y}}\widetilde{Y}_t$$
$$0 = \widetilde{Y}_t - \widetilde{\lambda}_t - \theta\widetilde{K}_t - (1-\theta)\widetilde{H}_t,$$
$$0 = \overline{r}^f\widetilde{r}_t^f - \frac{\overline{r}^d\overline{N/P}}{\overline{w}\overline{H}}\left(\widetilde{r}_t^d + \widetilde{N}_t - \widetilde{P}_t\right) - \frac{\overline{TL}}{\overline{w}\overline{H}}\widetilde{TL}_t + \overline{r}^f\left(\widetilde{w}_t + \widetilde{H}_t\right),$$
$$0 = \overline{N/P}\widetilde{N}_t - \overline{w}\overline{H}\left(\widetilde{w}_t + \widetilde{H}_t\right) - \left[(1-\rho)\left(1-\frac{1}{\overline{g}}\right)\overline{M/P} + \overline{N/P}\right]\widetilde{P}_t$$
$$+ (1-\rho)\left(1-\frac{1}{\overline{g}}\right)\overline{M/P}\widetilde{M}_{t-1} + (1-\rho)\overline{M/P}\widetilde{g}_t,$$
$$0 = \widetilde{M}_t - \widetilde{g}_t - \widetilde{M}_{t-1}.$$

Impulse response functions (tech shock)

- with $dif = .001$

Impulse response functions (tech shock)

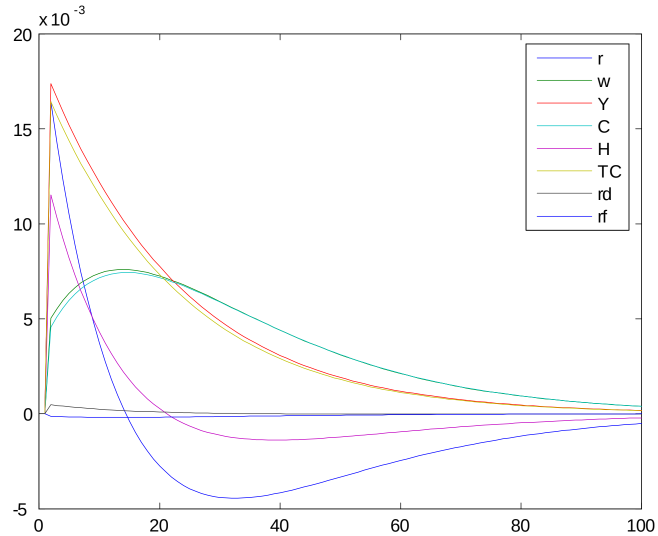
- with $\alpha = .5, dif = .6$

Impulse response functions (tech shock)

- with $\alpha = 4, dif = .6$

Impulse response functions (money shock)

- with $dif = .001, \rho = 0$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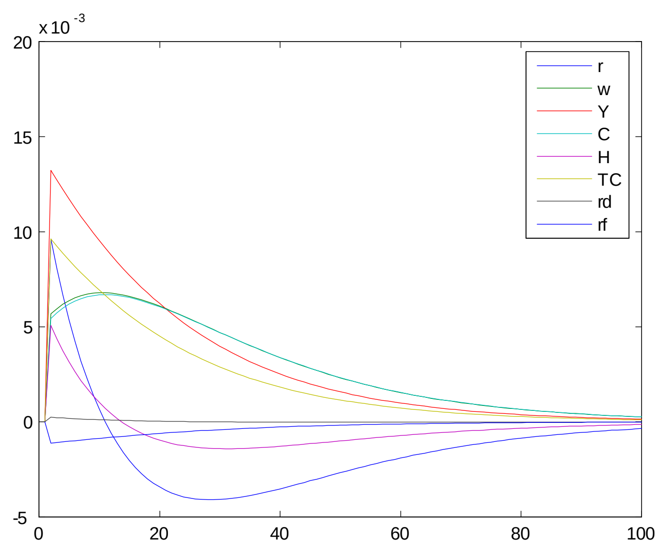
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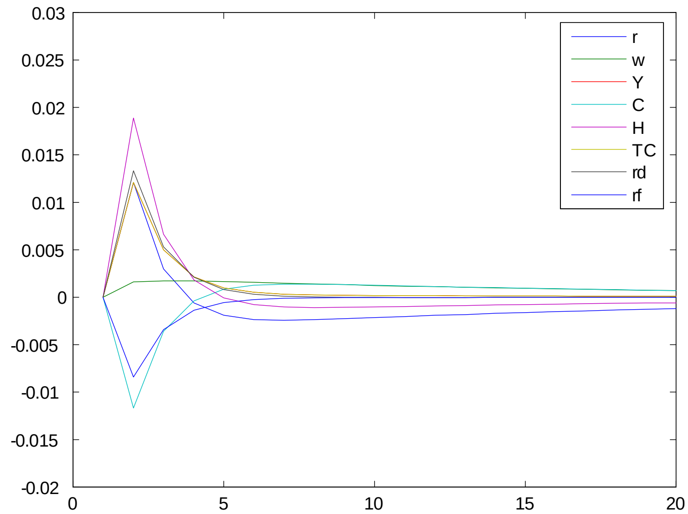

Impulse response functions (money shock)

- with $\alpha = .5, dif = .6, \rho = 0$

Impulse response functions (money shock)

- with $\alpha = 4, dif = .6, \rho = 0$

Impulse response functions (money shock)

- with $dif = .001, g = 1.2, \rho = 0$

Impulse response functions (money shock)

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Impulse response functions (money shock)

- with $\alpha = 4, dif = .6, g = 1.2, \rho = 0$

Impulse response functions (money shock)

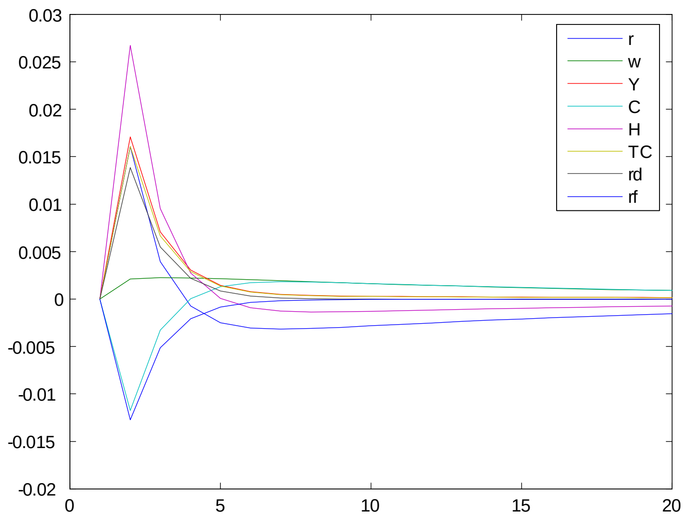
- with $\alpha = .5, dif = .001, g = 1.2, \rho = 1$

Impulse response functions (money shock)

- with $\alpha = .5, dif = .6, g = 1.2, \rho = 1$

Impulse response functions (money shock)

- with $\alpha = 4, dif = .6, g = 1.2, \rho = 1$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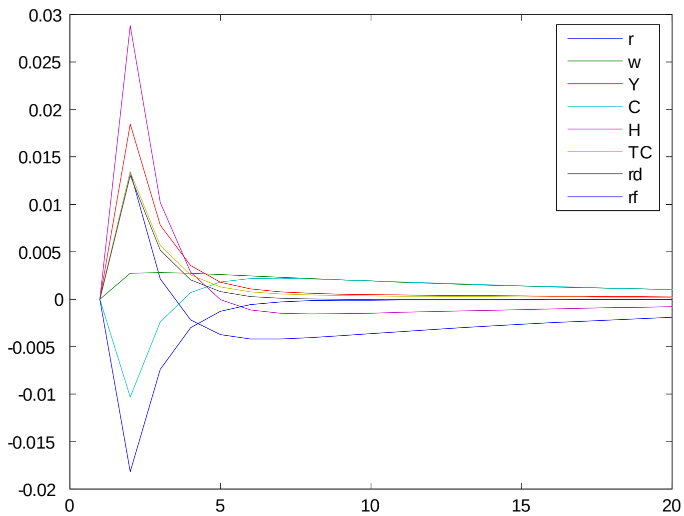
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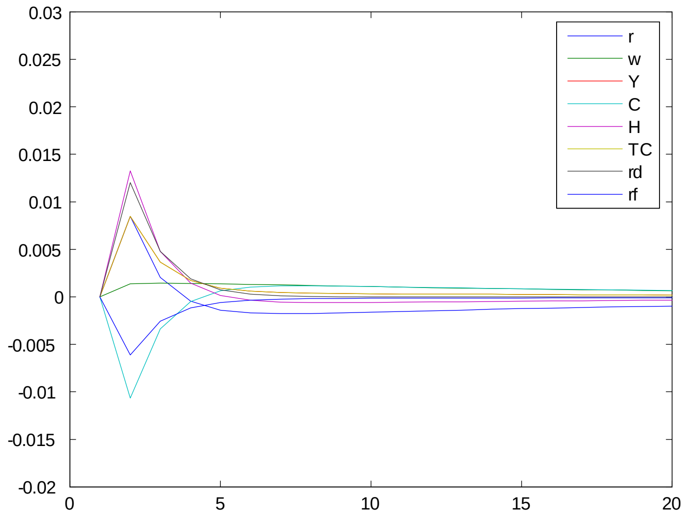
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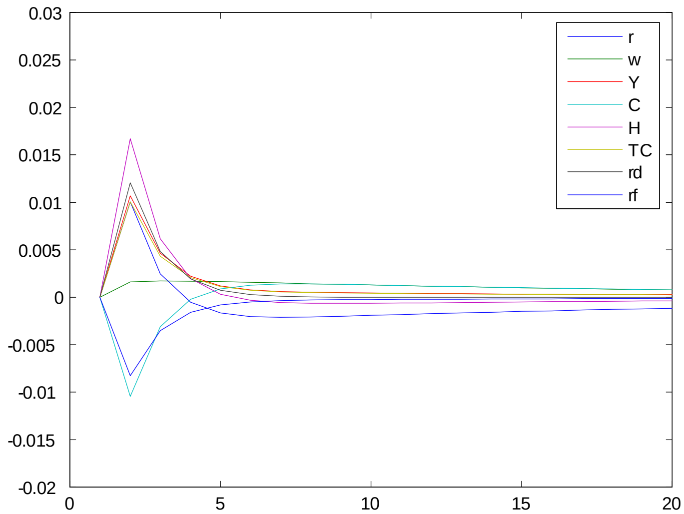
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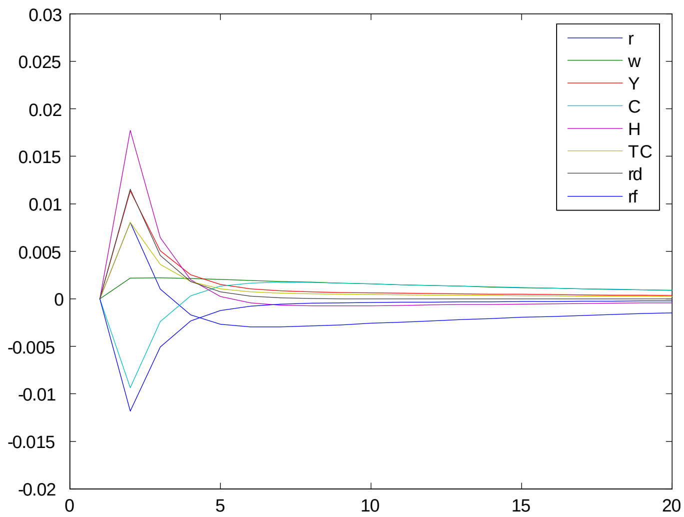

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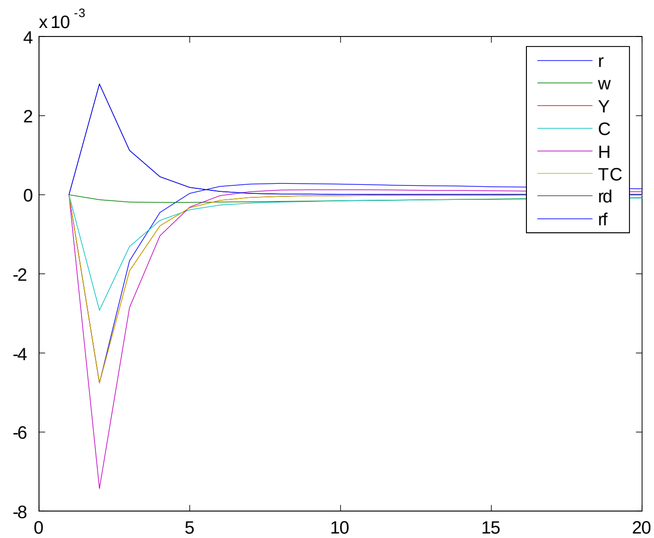

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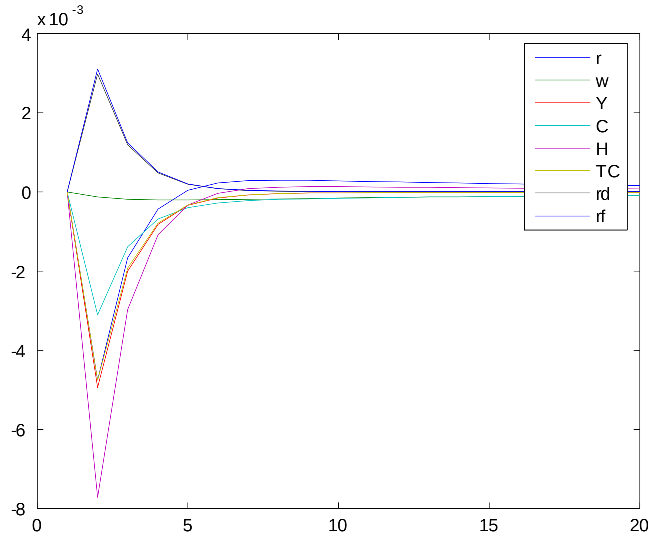
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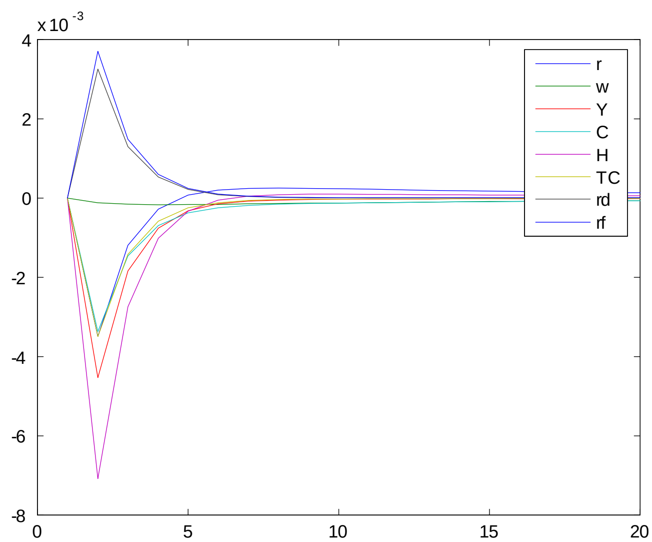
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