Fixed exchange rate policy, external shocks, inflation, and functional income distribution in Argentina

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\(^1\)The opinions expressed in this paper are the author’s and do not necessarily reflect those of the Central Bank of Argentina.
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"There is, perhaps, something a little perplexing in the apparent asymmetry between Inflation and Deflation. For whilst a deflation of effective demand below the level required for full employment will diminish employment as well as prices, an inflation of it above this level will merely affect prices". John Maynard Keynes, *The General Theory...*, Ch. 20.

1. Introduction

After a long deflationary recession, a severe economic and political crisis in Argentina finally put an end to its peculiar form of fixed exchange rate policy (a currency board known as Convertibility) in December, 2001, that had been in place since April 1991. The currency crisis brought about a very significant overshooting of the real peso depreciation, the correction of which was halted by the approximate fixing of the exchange rate at 2.9 pesos per dollar (plus noise). This paper presents a Small Open Economy (SOE) perfect foresight model that seems to be able to represent quite well several important aspects of the Argentine economy both during the deflationary phase of Convertibility and during the inflationary post-Convertibility period. The model is useful in explaining the behavior of the tradable and nontradable sectors during the two periods as well as the effects of the respective fixed (or quasi-fixed) exchange rate policies on income distribution by income class.

Households and firms in the non-tradable sector are assumed to be monopolistically competitive and to have 'rule of thumb' wage and price adjustment equations by which they only attain their optimal ('potential') wage and price in the steady state. This way of representing nominal rigidities, instead of the usual Calvo (1983) or cost of adjustment function expedients, helps to keep the dynamical system of low dimension without significantly sacrificing realism. Hence, we are able to illustrate the dynamics graphically in simple diagrams. For simplicity, tradable sector firms are assumed to be competitive and to have flexible prices.

We argue that the depressive second phase of the Convertibility period, where adverse external shocks (in conjunction with the exchange rate regime) made the peso overvalued in real terms, as well as the relatively high inflationary pressures the economy has been experiencing in the post-Convertibility period can be explained with the simple nonlinear perfect foresight model we develop with asymmetric speeds of adjustments for nontradable price and nominal wage adjustments, respectively, to positive versus negative gaps between 'effective' and 'potential' demands in these markets. The speeds of adjustment are assumed to be considerably lower in deflationary regions than in inflationary regions. Hence, we obtain a dynamical system in the nominal wage and the nontradable goods price indexes that has four regions with different laws of motion that differ only on the speeds of adjustment for the two dynamic equations.

There is abundant evidence on the asymmetry between price increases and reductions, not only in particular markets such as agricultural products and gaso-

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line, but in a wide array of markets. Peltzman (2000), who studies more than 240 markets, finds that output prices tend to respond faster to input increases than to decreases in more than two thirds of the markets examined, independently of whether it is producer goods markets or consumer goods markets. Such asymmetries are not only pervasive, but they are also substantial and durable. He finds that, on average, the immediate response to a positive cost shock is at least twice as large as the response to a negative shock, and that such difference is sustained for at least five to eight months.

Various researchers have tried to find theoretical foundations for such asymmetries. Ball and Mankiw (1994) used a menu-cost model in which there is positive trend inflation to show that shocks that raise firms’ desired prices trigger larger price responses (that defend them from the reduction in their relative price due to trend inflation) than shocks that lower desired prices. Bhaskar (2002), uses a menu cost model without trend inflation to explain why firms are more likely to adjust prices upward than downward. In a model that has greater elasticity of substitution within industries than between industries, such behavior arises as a consequence of the desire by firms to keep their industry prices as high as is sustainable. In his model, while negative shocks reduce output, positive shocks increase prices. This is precisely what Tobin (1972) suggested was a consequence of aggregate demand shocks, not only for prices but also in wage setting. Also, Helm (2006) builds a dynamic oligopoly model that uses a particular convex adjustment cost function. He finds that higher levels of market power are associated with less asymmetry, and faster speeds of adjustment with smaller adjustments. Finally, Pfann and Palm (1992) investigate optimal labor demand schedules for workers of firms that operate under uncertainty and face asymmetric costs of adjusting their workforce. Using time series data for the Netherlands and U.K. manufacturing sectors and a GMM methodology, they find that asymmetric adjustment costs play an important role in the explanation of the unbalanced behavior of labor demand between expansionary and recessionary phases of the business cycle.

In an open economy setting, various researchers have tested for asymmetric real exchange rate (RER) adjustments. Leon and Najarian (2003), for example, estimate and evaluate different classes of regime switching models for a range of advanced and developing economies, including a time-varying threshold autoregressive model (TVTAR), which allows for asymmetrical adjustments when RERs deviate from forecasts, a smooth transition autoregression model (STAR), which allows for asymmetric adjustments, as well as Markov switching models. They conclude that the adjustment dynamics of RERs is not symmetric and that the asymmetry differs across countries. They find that a three-regime STAR model with asymmetric speeds of adjustment between regimes performs best. Similarly, Liew (2004) finds that the US dollar based real exchange rates of Indonesia, Philippines, Singapore and Thailand exhibit a Logistic Smooth Transition Autorregresive (LSTAR)-type nonlinearity, implying that the real exchange rates of these countries have asymmetrical responses towards appreciations and depreciations. (See also Akram, Eitrheim, and Sarno (2005).)

We take both strands of empirical evidence on asymmetric behavior as support for our model assumption of asymmetric RER adjustments. In our case this asymmetry is generated in the price/wage adjustments of monopolistic competi-
tors towards their (profit or utility) optimizing levels within the context of a fixed (or quasi-fixed) exchange rate policy during periods in which large external shocks generate large RER misalignments.

We take as the main external shocks during both periods the international real value of the dollar and the external terms of trade. They are modeled as unexpected, permanent or transitory shocks. Garegnani and Escudé (2007) show that these two variables can account quite well for Argentina’s real exchange rate misalignments over the past 30 years. They estimate an equilibrium correction model that shows that the U.S.A.’s multilateral real exchange rate (MRER) (measured by the Federal Reserve’s Real Broad Dollar Index) and Argentina’s external terms of trade are good long and short run determinants of Argentina’s MRER. These two shocks play an important role in the present paper’s model.

Although having unexpected shocks in a perfect foresight model may seem awkward, we follow Blanchard and Fischer (1989) (footnote 43 to chapter 2) in arguing that it is a simple way of avoiding the complications of an explicitly stochastic framework by assuming that the surprise event was considered so unlikely as to not be taken into account until it actually occurs. Once it occurs, however, there are unanimous expectations with respect to its duration. We also follow Turnovsky (2000) and Schubert and Turnovsky (2002) (see also Sen and Turnovsky (1989a, 1989b, 1990)) in gauging the steady state effects of transitory shocks in a perfect foresight model with perfect capital markets and infinitely lived agents. We show that the model presents hysteresis, i.e., transitory external or policy shocks have permanent effects.

2. The model
2.1. Firms
There are three output sectors: exportable goods ($X$), importable goods ($M$), and non-tradable goods ($N$). All output is non-storable. In each of the tradable sectors there is a representative firm that operates under perfect competition. In the non-tradable sector there is a continuum of monopolistic competitors that gradually adjust prices to their optimal (‘potential’) levels (in a way that is defined below) and sell whatever quantity is demanded. In the model’s steady state, however, these firms will also be maximizing profits in a way similar to the tradable sectors (except that they will have a mark-up on marginal cost). The only variable input is labor, which is mobile across sectors and immobile internationally. All firms are wage takers and use a constant elasticity of substitution (CES) aggregate of all labor types defined by

$$L = \left( \int_0^1 (L_h)^{\psi-1} dh \right)^{\psi^{-1}}, \quad \psi > 1,$$

where $\psi$ is the elasticity of substitution between labor types. As we see in the next section, this expression represents the technology used by a representative labor aggregator under perfect competition. Capital is bolted down and does not depreciate, but generates diminishing returns in each sector. The production functions are:

$$Q_j = F_j(L_j), \quad F_j' > 0, \quad F_j'' < 0, \quad j = X, M, N.$$

All non-tradable firms use the same production function.
2.1.1 Tradable sector firms

In the model’s long run there is no entry, so profits can be positive. Nominal profits in the exportable and importable sectors are:

\[
\begin{align*}
\Pi_X & \equiv (1-t_X)SP^*_X Q_X - WL_X \\
\Pi_M & \equiv SP^*_M Q_M - WL_M
\end{align*}
\]

where \(S\) is the nominal exchange rate, \(W\) is the nominal wage rate, \(P^*_X\) and \(P^*_M\) are the foreign currency prices of the respective goods, and \(t_X\) is an ad valorem uniform export tax rate. Let

\[
\phi \equiv \frac{P^*_X}{P^*_M} = P^*_X, \quad \phi_X \equiv (1-t_X)\phi
\]

be the terms of trade and the export tax adjusted terms of trade, respectively. For simplicity, we assume that the import price is unity, so \(\phi\) is also the export price. Define the real exchange rate (RER) and the product wage in the nontradables sector \(w\) as:

\[
\begin{align*}
e & \equiv \frac{SP^*_M}{P^*_N} = \frac{S}{P^*_N}, \\
w & \equiv \frac{W}{P^*_N}.
\end{align*}
\]

Hence, real profits in the tradable sectors (in terms of non-tradable goods) are the following:

\[
\begin{align*}
\Pi_X/P_N & \equiv \phi_X e Q_X - wL_X \\
\Pi_M/P_N & \equiv e Q_M - wL_M,
\end{align*}
\]

Profit maximization in the tradable sectors yield their labor demands:

\[
\begin{align*}
L_X \left( \frac{w}{e\phi_X} \right) & \equiv (F'_X)^{-1} \left( \frac{w}{e\phi_X} \right) \\
L_M \left( \frac{w}{e} \right) & \equiv (F'_M)^{-1} \left( \frac{w}{e} \right)
\end{align*}
\]

\(L_j' < 0, \ j = X, M.\)

The corresponding output supplies are the following:

\[
\begin{align*}
Q_X \left( \frac{w}{e\phi_X} \right) & \equiv F_X \left( (F'_X)^{-1} \left( \frac{w}{e\phi_X} \right) \right) \\
Q_M \left( \frac{w}{e} \right) & \equiv F_M \left( (F'_M)^{-1} \left( \frac{w}{e} \right) \right)
\end{align*}
\]

\(Q'_j < 0, \ j = X, M.\)

For convenience, let us also define aggregate tradable sector labor demand and export tax corrected tradable output supply as:

\[
\begin{align*}
L_T \left( \frac{w}{e}; \phi_X \right) & \equiv L_X \left( \frac{w}{e\phi_X} \right) + L_M \left( \frac{w}{e} \right) \\
Q_T \left( \frac{w}{e}; \phi_X \right) & \equiv \phi_X Q_X \left( \frac{w}{e\phi_X} \right) + Q_M \left( \frac{w}{e} \right).
\end{align*}
\]
When we use the tradables output supply in the balance of payments below we also use the expression where exportable output is valued at the external price (i.e., the export tax is not subtracted):

$$Q_{TX} \left( \frac{w}{\ell}; \phi, \phi_X \right) \equiv \phi Q_X \left( \frac{w}{c\phi_X} \right) + Q_M \left( \frac{w}{\ell} \right).$$  \tag{7}$$

2.1.2 Nontradable sector firms

In the non-tradable sector there is a profit maximizing, perfectly competitive, representative producer (or aggregator) that converts the intermediate output of individual monopolistically competitive producers into a final non-tradable good using a CES production function:

$$Q_N = \left( \int_0^1 (Q_{N,i})^{(\nu-1)/\sigma} \, di \right)^{\nu/(\nu-1)} , \quad \nu > 1,$$

where $\nu$ the elasticity of substitution between varieties of non-tradable goods. It chooses its demand for individual outputs $Q_{N,i}$ to maximize profit:

$$Q_N P_N - \int_0^1 P_{N,i} Q_{N,i} \, di$$

subject to to its production function. This gives its demand function for individual intermediate non-tradables:

$$P_{N,i} = P_N \left( \frac{Q_{N,i}}{Q_N} \right)^{-\frac{1}{\nu}}.$$  \tag{10}$$

Since there is free entry in the production of the final non-tradable good, profits must be zero. Inserting (10) in (8) and in (9), yields the non-tradables price index and the zero profit condition, respectively:

$$P_N = \left( \int_0^1 (P_{N,i})^{1-\nu} \, di \right)^{\frac{1}{\nu}}.$$  

$$\int_0^1 P_{N,i} Q_{N,i} \, di = Q_N P_N.$$  

Each intermediate non-tradable good producer sets its price according to a ‘rule of thumb’ that gradually leads it to its optimal price. Hence, it is a quantity taker. In the steady state, each non-tradable firm $i$ chooses its price $P_{N,i}$ to maximize its ‘potential’ profit:

$$\Pi_{N,i} = Q_{N,i} P_{N,i} - W L_{N,i}$$

(where $L_{N,i}$ represents the amount of labor (bundled as in (1)) demanded by nontradable firm $i$) subject to its production function $Q_{N,i} = F_N(L_{N,i})$ and its demand function (10). It takes the prices set by the other monopolistic competitors and the aggregate nontradable output as given. Hence, it sets $P_{N,i}$ to maximize:

$$Q_N (P_N)^\nu (P_{N,i})^{1-\nu} - WF_{N}^{-1} \left( Q_N (P_N)^{\nu} (P_{N,i})^{-\nu} \right),$$
which gives its optimal price as a markup over marginal cost:

\[ P_{N;i} = \frac{\nu}{\nu - 1} F'_N \left( \frac{1}{F_N^{-1}(Q_N)} \right) W. \]

Since all producers face the same problem, they all set the same price, so we now drop the subindex and write the resulting expression as:

\[ F'_N \left( F_N^{-1}(Q_N) \right) = \mu_F w \quad \left( \mu_F \equiv \frac{\nu}{\nu - 1} \right) \]

where \( \mu_F \) is the mark-up in the non-tradable sector. Hence, defining non-tradable sector labor demand as:

\[ L_N = F_N^{-1}(Q_N), \]

‘potential’ (and hence, only operative for the model’s steady state) labor demand and output supply in the non-tradable sector are:

\[
\begin{align*}
L_N(w) &\equiv (F'_N)^{-1}(\mu_F w), & L'_N(w) < 0 \tag{11} \\
Q_N(w) &\equiv F_N \left( (F'_N)^{-1}(\mu_F w) \right), & Q'_N(w) < 0.
\end{align*}
\]

Off the steady state the ‘effective’ output of (each variety of) non-tradables \( \bar{Q}_N \) is determined by demand, given that there is price setting, even if it is not optimal. But we must turn to household decisions to determine what their ‘effective’ demand for non-tradable goods is.

### 2.2. Households

There are various classes of households. On the one hand, there are ‘poor’ households (e.g., retired or structurally unemployed) that do not participate in the productive sector, do not save, and to simplify only consume nontradable goods. Their sole income is a government grant \( g \) which is the only government expenditure aside from interest payments. On the other hand, there are ‘active’ (or ‘participating’) households that do participate in production and make inter-temporal decisions. There are two large classes of ‘active’ households, according to the source of their income: entrepreneurs (‘capitalists’, \( K \)) and workers (\( L \)), although we will also consider subclasses like exportable, importable, and non-tradable sector entrepreneurs. The entrepreneurs own all the firms and receive all the profits. For simplicity, they do not receive a specific income for their entrepreneurial activities (aside from profits).

Working households work for a wage. They (or their unions) are assumed to be wage setters that follow a simple ‘rule of thumb’ for determining wage adjustments. Hence, they accept whatever quantity of labor is demanded at the wage rate they set. We show below that their ‘rule of thumb’ is such that in the steady state they set the optimal wage (the ‘potential’ wage). If \( h \) is a working household, it consumes exportable, importable, non-tradable goods, and leisure, according to the following inter-temporal utility function:

\[
\int_0^\infty \left\{ \log \left( C_X^{\theta_1} C_M^{\theta_2} C_N^{1-\theta} \right) - v(L_h) \right\} e^{-\beta s} ds, \tag{12}
\]
where $\theta_1, \theta_2 \in (0, 1)$, $\theta = \theta_1 + \theta_2 < 1$, $\beta < 1$ is the constant inter-temporal discount rate, $v(L_h)$ represents the disutility of labor. If $h$ is an entrepreneurial household, (12) is still valid but the term $v(L_h)$ disappears. We have avoided introducing the subscript $h$ on the other variables to economize notation, but we will introduce it whenever the difference between working and entrepreneurial households, or between entrepreneurial households of different sectors, is relevant.

Define ‘active’ (i.e., excluding the ‘poor’) household consumption expenditure in terms of general purchasing power as:

$$C = \frac{S\phi_X C_X + SC_M + P_NC_N}{P} = \frac{S}{P}\left[\phi_X C_X + C_M + \frac{C_N}{e}\right],$$

(13)

where

$$P = (S\phi_X)^{\theta_1} S^{\theta_2} P_N^{1-\theta} = P_N^{\phi_X^1 \epsilon^\theta}$$

(14)

is the Cobb-Douglas consumer price level that corresponds to our consumption sub-utility function. Minimizing consumption expenditure (13) subject to a given value of the consumption sub-utility function in (12) yields the following first order conditions:

$$S\phi_X C_X = \theta_1 PC$$

(15)

$$SC_M = \theta_2 PC$$

(16)

$$P_NC_N = (1 - \theta) PC.$$  

(17)

Hence:

$$C^{\theta_1} C^{\theta_2} N^{1-\theta} = \kappa_0 C,$$

$$\kappa_0 \equiv \theta_1^{\theta_1} \theta_2^{\theta_2} (1 - \theta)^{(1-\theta)}.$$  

(18)

Since we can always recover the consumption demands for the three types of goods, we work with total consumption expenditure $C$ whenever possible. The inter-temporal utility is therefore:

$$\int_0^\infty \{\log (\kappa_0 C) - v(L_j)\} e^{-\beta s} ds.$$  

(19)

‘Active’ households can hold money $M$ and a foreign riskless bond $B$ denominated in foreign currency. We assume that there are transaction costs $\tau - 1$ that can be reduced by holding money, where gross transaction costs are assumed to be an increasing and convex function of the money to consumption ratio (or inverse of the velocity of money):

$$\tau(M/PC), \quad \tau > 1, \tau' < 0, \tau'' > 0.$$  

Furthermore, the resources used up in transactions are assumed to be the same goods as those purchased with the use of money. Household $h$’s real budget constraint is the following:

$$\frac{\dot{M}}{S} + \dot{B} = \frac{P}{S} [y_h^\sigma - \tau(M/PC)C] + \sigma B,$$  

(20)
where a dot over a variable represents the time derivative and $y_h^*\tau$ is the real income of household $h$ net of taxes (and including subsidies if they exist). Defining $m \equiv M/S$, and $a \equiv m + B$ we can write the budget constraint as:

$$\dot{a} = \dot{m} + \dot{B} = (P/S) [y_h^* - \tau (mS/PC) C] + i^* (a - m). \quad (21)$$

In the case of workers (who are not taxed), the aggregate real income is $y_L^* = (W/P) \tilde{L}$, where $\tilde{L}$ is ‘effective’ employment. Since workers are wage setters, ‘effective’ employment is simply whatever labor demand there is at the wage set. In the case of entrepreneurial households of tradable sector $j$, $y_{K,j}^* = \Pi_j/P - tS/P$ ($j = X, M$), where $t$ is a lump sum tax on entrepreneurial households (which for notational convenience we have expressed in terms of foreign currency). And in the case of entrepreneurial households of the nontradable sector $y_{K,N}^* = \Pi_N/P - tS/P$, where $\Pi_N$ are ‘effective’ profits, to be defined in section 5 below.

As in the case of firms, a perfectly competitive representative labor aggregator converts the individual labor supplies of these monopolistically competitive households into a labor bundle that all firms use as input to production. It chooses its demand for individual labor types $L_h$ as to maximize profit:

$$LW - \int_0^1 L_hW_h dh$$

subject to its aggregation function (1). In complete analogy to the non-tradable final output firm’s determination of its demand for the output of intermediate goods producing firms, this maximization yields the demand for individual varieties of labor, the aggregate wage index and the zero profit condition:

$$L_h = L \left( \frac{W_h}{W} \right)^{-\psi}, \quad (22)$$

$$W = \left( \int_0^1 W_h^1-\psi dh \right)^{1/\psi},$$

$$\int_0^1 L_hW_h dh = LW.$$  

Household $h$ maximizes utility subject to its budget constraint, its labor demand function (22) (in the case of working households) and a No Ponzi Game condition:

$$\lim_{t \to \infty} ae^{-\gamma t} = 0. \quad (23)$$

Hence, the current value Hamiltonian function for a working household is:

$$H = \log (\kappa_0 C) - \psi (LW^\psi W_h^\psi)$$

$$+ \zeta \left\{ \frac{1}{S} \left[ LW^\psi W_h^1-\psi - \tau (mS/PC) PC \right] + i^* (a - m) \right\}.$$ 

Here, the Lagrange multiplier $\zeta$ represents the marginal utility of an extra unit of foreign purchasing power. Household $h$ uses $C$, $m$ and (in the case of a working
household and only for their ‘potential’ nominal wage) $W_h$ as control variables. The ‘effective’ wage rate will be determined by a ‘rule of thumb’. The necessary (and in our case sufficient) conditions for inter-temporal maximization are the first order conditions:

$$H_C = 0, \quad H_m = 0, \quad H_{W_h} = 0, \quad \ddot{\zeta} - \zeta \beta = -H_a,$$

and the transversality condition:

$$\lim_{t \to \infty} a \zeta e^{-\beta t} = 0.$$

(24)

The first order conditions are:

$$C^{-1}S \frac{P}{\hat{P}} = \zeta \left[ \tau \left( \frac{mS}{PC} \right) - \left( \frac{mS}{PC} \right)' \frac{mS}{PC} \right]$$

(25)

$$-\tau' \left( \frac{mS}{PC} \right) = i^*$$

(26)

$$W_h = \mu_H v'(L) S \zeta^{-1} \left( \mu_H \equiv \frac{\psi}{\psi - 1} \right)$$

(27)

$$\ddot{\zeta} = \beta - i^*.$$  

(28)

(27) gives the ‘potential’, or optimal, wage rate as a markup over the marginal rate of substitution of leisure for purchasing power. This first order condition defines working households’ ‘potential’ labor supply:

$$L_S = (v')^{-1} \left( \frac{\zeta L}{\mu_H} \frac{W}{S} \right) \equiv L_S \left( \frac{\zeta L}{\mu_H} \frac{w}{e} \right)$$

(29)

Working households always work as much as is demanded at the wage rate they set, and, because of their ‘rule of thumb’ wage adjustment, outside the steady state they are usually off of their ‘potential’ labor supply function. Also, during the transition to the steady state, (21) must be used in the Hamiltonian function with the ‘effective’ wage income we define below (and which is not a decision variable in the optimization). Hence, (27) disappears for the dynamics of the model but is an important benchmark, valid for the steady state (where ‘effective’ and ‘potential’ concepts coincide). In the case of entrepreneurial households, the Hamiltonian must be modified accordingly, and the resulting first order conditions are (25), (26), and (28).

For simplicity we assume perfect capital mobility. Hence, (28) implies that in order to have an interior solution the foreign nominal riskless interest rate must be equal to the inter-temporal discount rate: $i^* = \beta$. Throughout this paper we assume that $i^*$ and $\beta$ are constant and equal. Therefore, the marginal utility of an

3The simplifying assumption of perfect capital mobility hence has the cost of not being able to consider foreign interest rate shocks (without introducing modifications to the model, such as a time-varying $\beta$). The model may be generalised for non-perfect capital mobility through the introduction of a risk (or liquidity) premium on loans from abroad at the cost of considerable complications.
extra unit of foreign purchasing power $\zeta$ is almost always constant but may jump to a new level whenever there is an unexpected shock, as we see below.

(26) may be inverted to yield the money demand function for any ‘active’ household:

$$m = (-\tau')^{-1} (i^*) P \frac{S}{C} \equiv \ell (i^*) P \frac{S}{C}, \quad \ell' < 0. \quad (30)$$

The assumption that $i^*$ is constant carries over to the money/consumption ratio. Inserting the last expression in (19) gives constant transaction costs per unit of consumption:

$$\tau (\ell (i^*)) \equiv \tau.$$

Hence, the household’s budget constraint (21) can also be written as:

$$\dot{a} = (P/S) \{y_k^0 - \tau^* C\} + i^* a, \quad (31)$$

where we defined a term which includes the goods used up in transaction costs as well as the opportunity cost of holding money:

$$\tau^* \equiv \tau (\ell (i^*)) + i^* \ell (i^*).$$

Also, let us denominate $\varphi(mS/PC)$ the function that appears in (25) and gives the total effect on expenditure (i.e., including transaction cost related expenditures) of a marginal increase in consumption:

$$\varphi(mS/PC) \equiv \tau ((mS/PC)) - (mS/PC)\tau'((mS/PC)).$$

This too is constant:

$$\varphi(\ell (i^*)) \equiv \varphi.$$

Hence, the first order condition (25) can be written more succinctly as:

$$P \frac{S}{C_k} = (\varphi \zeta_k)^{-1}, \quad (k = K, L). \quad (32)$$

We have here introduced the subscript $k$ as a reminder that the consumption levels of different ‘active’ income classes can be quite different, according (inversely) to their respective marginal utilities of real income. Note also that in this simple log utility framework (30) and (32) imply that $m$ is also almost always constant (not only $mS/PC$, which is always constant since we have excluded jumps in $i^*$). This implies that whenever we say below that the government (or Central Bank) devalues, it must do so by a one time money injection that keeps $m \equiv M/S$ constant.

We can now write the flow budget constraint (31) of the representative household of class $k$ as:

$$\dot{a}_k = (P/S) y_k^0 - (\tau^*/\varphi) \zeta_k^{-1} + i^* a_k. \quad (33)$$

Let 1 be the quantity (measure) of working households and $\eta < 1$ the quantity of entrepreneurial households. Then the aggregate consumption of ‘active’ households is:

$$C = C_K + C_L = \int_0^\zeta C_h \, dh' + \int_1^1 C_h \, dh = (S/P) (\zeta \varphi)^{-1}, \quad (34)$$

where we defined an aggregate of the marginal utilities of the foreign purchasing power of income:

$$\zeta^{-1} \equiv \eta \zeta_K^{-1} + \zeta_L^{-1}.$$
2.3. The public sector and the balance of payments

We assume the Government (including the Central Bank) only spends on its grant to the ‘poor’ and interest on its net foreign debt, collects a constant lump-sum tax \( t \) on entrepreneurial households, uses foreign debt financing, and (particularly when there is a positive terms of trade shock) may impose an export tax \( t_X \). The government’s flow budget constraint is:

\[
\dot{D} - \dot{R} + \dot{m} = \frac{P_N}{S} g - \eta t - t_X \phi(Q_X - \tau C_X) + i^*(D - R). \tag{35}
\]

where \( D \) is the public foreign debt and \( R \) is the Central Bank’s international reserves. The government is assumed to be always intertemporally solvent. Hence, it satisfies a No Ponzi Game condition:

\[
\lim_{t \to \infty} (D - R) e^{-i^* t} = 0. \tag{36}
\]

The Central Bank is assumed to have a fixed exchange rate policy that keeps \( S \) constant by purchasing (selling) excess supply (demand) of foreign exchange.

Using (34), (2), and (15), and noting that \( m \) is (almost always) constant, (35) becomes:

\[
\dot{D} - \dot{R} = \frac{g}{e} - \eta t - t_X \phi(Q_X \left( \frac{w}{e\phi_X} \right) + \frac{\tau \theta_1}{\varphi} \frac{t_X}{1 - t_X} \zeta^{-1} + i^*(D - R)
\]

Consolidating the budget constraints of households (21) and the government (35), and using the definition of \( \phi_X \) (2), we obtain the balance of payments equation:

\[
\begin{align*}
\dot{N} &= \phi(Q_X - \tau C_X) - (\tau C_M - Q_M) + i^* N \\
&= Q_{TX} \left( \frac{w}{e}; \phi, \phi_X \right) - \left( \theta + \frac{t_X}{1 - t_X} \theta_1 \right) \frac{\tau}{\varphi} \zeta^{-1} + i^* N;
\end{align*}
\tag{37}
\]

where we defined the SOE’s net international position:

\[
N = B + R - D,
\]

used the identity

\[
\frac{\theta_1}{1 - t_X} + \theta_2 = \theta + \frac{t_X}{1 - t_X} \theta_1,
\]

and for the second equality used (7), (15), (16), and (34). Furthermore, we assume that the two terms in parenthesis (exports and imports, respectively) in the first equality are always positive. Note that the export tax appears in the balance of payments equation because the consumption of exportables must be valued at international prices (which are higher than domestic prices when there is an export tax).

2.4. ‘Effective’ demand, wage and non-tradable price setting, and the dynamical system

According to (17), (14), and (34), aggregate ‘active’ household demand for non-tradable goods is:

\[
C_N = (1 - \theta) PC / P_N = (1 - \theta) (\zeta \varphi)^{-1} e.
\]
Hence, to satisfy demand at the posted aggregate price $P_N$, the non-tradable sector must produce to satisfy the following ‘effective demand’:

$$\frac{\tau}{\varphi}(1 - \theta)e\zeta^{-1} + g \equiv \tilde{Q}_N \left( e\zeta^{-1}; g \right), \tag{38}$$

which includes the resources used up in transaction costs $(\tau - 1)$ and the demand by ‘poor’ households $(g)$. Also, ‘effective’ labor demand and employment in the non-tradable sector is derived from output supply as:

$$F_N^{-1} \left( \tilde{Q}_N \right) = F_N^{-1} \left( \frac{\tau}{\varphi}(1 - \theta)e\zeta^{-1} + g \right) \equiv \tilde{L}_N \left( e\zeta^{-1}; g \right). \tag{39}$$

Hence, we can now define the ‘effective’ profit income in the non-tradable sector (in terms of non-tradables) as:

$$\Pi_N / P_N \equiv \tilde{Q}_N - w\tilde{L}_N.$$

Note that using (6), (3), (4), and (38) we can define ‘effective’ aggregate demand (and output) in foreign currency as:

$$Q_T \left( \frac{W}{S}; \phi_X \right) + \frac{P_N}{S} \tilde{Q}_N \left( \frac{S}{P_N}\zeta^{-1}; g \right).$$

However, we will not use this concept below.

Non-tradable price and wage setters are assumed to gradually adjust prices and wages, respectively, at constant and exogenous speeds $\alpha$ and $\beta$, towards their ‘potential’ (or optimal) prices and wages, respectively, i.e. those that maximize profits and utilities. Hence, under our assumption of a fixed exchange rate at, say, $\underline{S}$, the laws of motion of $P_N$ and $W$ are given by:

$$\pi_N \equiv \frac{d \log P_N}{dt} = \alpha \left[ \tilde{Q}_N \left( \frac{S}{P_N}\zeta^{-1}; g \right) - Q_N \left( \frac{W}{P_N} \right) \right], \tag{40}$$

$$\pi_W \equiv \frac{d \log W}{dt} = \beta \left[ L_T \left( \frac{W}{S}; \phi_X \right) + \tilde{L}_N \left( \frac{S}{P_N}\zeta^{-1}; g \right) - L_S \left( \frac{1}{\mu_H}\zeta^{-1} \frac{W}{S} \right) \right]. \tag{41}$$

Nontradable inflation varies proportionally to the ‘output gap’ in this sector, where we define ‘output gap’ as the excess ‘effective’ demand with respect to ‘potential’ output. In complete analogy, wage inflation varies proportionally to the ‘employment gap’, where this is defined as the excess ‘effective’ labor demand with respect to ‘potential’ employment. We analyze this dynamical system below. Although the exogenous speeds of adjustment beg to be explained theoretically, we make no apology for not doing so and base our use of this model on its simplicity, communicability, and its ability to represent complex business cycle and income distribution issues.\footnote{A micro-founded version of price and wage stickiness for the present model can be derived along New Keynesian lines as in Escudé (2004, 2006). It yields a four dimensional dynamical system in the variables $\pi_N, \pi_W, e$, and $w$ that have the same steady state as the system we use here (along with integral equations for the inter-temporal budget constraints of the SOE and...}
Calvo (1983) setup (where there are agents who receive optimal price-setting signals while the rest do not). All price or wage setters follow the same simple ‘rule of thumb’ and (even though we assume here there is perfect foresight) only need to know the information that allows them to adjust their price/wage: the aggregate ‘effective’ demands for their type of output or work and the ‘potential’ output or labor supply that pertains to their individual profit/utility maximization. Hence, the same basic framework could be used in a model with less than perfect foresight.

Also, notice that our ‘potential’ output and employment levels differ from the ‘natural’ concepts, which reflect the absence of nominal rigidities (as here) but also whatever current shocks impinge on the economy (see Woodford (2003)). In contrast, here there is no discrepancy between ‘potential’ and ‘steady state’ levels. Hence, our ‘gaps’ are with respect to the steady state levels.

2.5. Fixed exchange rate dynamics

The complete dynamic system includes (40), (41), and various other equations such as the flow budget constraints and the corresponding intertemporal budget constraints of households, the government, and the SOE, which play a crucial role in the determination of the equilibrium levels of the marginal utilities of real income $\zeta_k$ for the various ‘active’ income classes or subclasses ($k = L, K, K_x, K_M, K_N$). However, we first concentrate on the two dynamical equations, assuming given (equilibrium) values of $\zeta$ and $\zeta_L$. After linearizing around the steady state, the (log-log) slope of the price stability line ($\pi_N = 0$) and the wage stability line ($\pi_W = 0$) in the right panels of Figures 1 and 2 are the following:

$$\frac{W}{P_N} \frac{dP_N}{dW} \big|_{\pi_N=0} = \frac{W}{P_N} \frac{Q'_N}{Q_N} - \frac{S}{P_N} \zeta^{-1} Q'_N \in (0, 1), \tag{42}$$

$$\frac{W}{P_N} \frac{dP_N}{dW} \big|_{\pi_W=0} = -\frac{L'_T}{\frac{S}{P_N} \zeta^{-1} L'_N} - \frac{L'_T}{\frac{S}{P_N} \zeta^{-1} L'_N} < 0. \tag{43}$$

While the first has a positive slope that is less than one, the second has a negative slope.

In the Appendix we prove that under our general assumptions a steady state for the dynamical system (40)-(41) is locally asymptotically stable. Hence, any path workers). The first two equations in that system are forward looking Phillips curves, with the time derivatives of $\pi_N$ and $\pi_W$ on the left hand side. The remaining two equations are simply the identities that give the time derivatives of $e$ and $w$, respectively, in terms of $\pi_N$ and $\pi_W$. Although this system has a more usual microfoundation, it is not necessarily better than the one we develop here. In particular, if one uses the Calvo (1983) model, the assumption that a constant fraction of price or wage setters cannot optimize and merely maintain a constant price is every bit ‘ad hoc’ as our constant speeds of price or wage adjustments. However it yields a dynamical system of twice as many dimensions. We have opted for a simpler model that is suitable for our present needs, renders considerable price and wage level inertia, and is simple enough to use for the analysis of income and wealth distribution. Obviously, it can only be used in a context of moderate inflation, as one would expect if there is a fixed exchange rate policy. If, however, the government were to try to further retard the rate of real appreciation by gradually depreciating the currency the model would have to be modified. A constant crawling peg policy can be easily accommodated.
that starts in a vicinity of the steady state gets progressively closer to it. Note that neither the steady state nor the no-inflation lines depend on the exogenous speeds of adjustment $\alpha$ and $\beta$. However, the paths to the steady state starting from any initial point do depend on $\alpha$ and $\beta$. In particular, whether the paths spiral towards equilibrium (a focus) or get there directly (a node) depends on the relative speeds of adjustment for prices and wages ($\beta/\alpha$). As we prove in the Appendix, whenever $\beta/\alpha$ is in a neighborhood of a certain positive value, there is spiraling and whenever it is outside there is no spiraling. In the former case (a focus), the paths towards the steady state successively cut through both inflationary and deflationary regions. For concreteness we base our exposition on the case of a node. Figure 1 depicts the the case of a node in which $\beta/\alpha$ has a low value, i.e., in which price setters adjust their prices relatively fast in comparison to wage setters. Note that the slope of the ray from the origin to the steady state is the inverse of the steady state product wage in the nontradable sector. Starting from point A at which the product wage in the non-tradable sector is below its steady state value and there is non-tradable price stability but positive wage inflation, there is a (possibly long) inflationary equilibrium path that eventually converges to the steady state C. During the transition, the product wage in the nontradable sector falls to its equilibrium level $w^*$. The quadrant on the left shows that the nontradable inflation makes the real exchange rate decrease from $e_0$ to its equilibrium level $e^*$.

At any of the points in the path A-B-C there is real exchange rate misalignment, with the RER greater than the steady state level at C, implying an undervalued peso. Without Central Bank intervention in the foreign exchange market, the nominal exchange rate would quickly fall to reduce or eliminate the misalignment, as well as the associated inflationary process. For example, assume that when the economy is at B the Central Bank lets the currency appreciate (a reduction in $\overline{S}$). Then the no wage inflation line shifts to the left and the no nontradables inflation
line shifts to the right. If the appreciation is of the appropriate magnitude, the new steady state for $P_N$ and $W$ gets close to B. Hence, instead of following the inflationary path that leads to C, a (possibly much) shorter path is necessary. In the left panel we see that the nominal appreciation shifts the hyperbola to the right, and the RER rate immediately becomes equal to $e^*$. If the nominal appreciation were to take the steady state to B exactly there would be no further dynamics. Of course, in general the one-time nominal appreciation cannot take the economy exactly to the steady state but nevertheless the path to the steady state is (possibly considerably) shortened.

We now address what happens in the non-tradable and tradable sectors along the inflationary path that leads from A to C. Since the real exchange rate is falling throughout this path, (38) implies that non-tradable output $\tilde{Q}_N$ is also falling from a level that is above the steady state level. Hence, employment in the non-tradable sector $\tilde{L}_N$ is also above the steady state level and falling. Furthermore, the product wage in the tradable sectors ($w/e = W/\tilde{S}$) increases throughout the path. Hence, tradable sector output decreases towards the steady state level from levels that are above it, and the same happens with employment in this sector. The economy is overheated in both sectors (in comparison to ‘potential’), which is why there is inflationary pressure on prices and wages.

Let us now consider the balance of payments along the dynamic path. According to (15)-(16) and (34), the consumption of tradables is simply:

$$\phi_X C_X = \theta_1 (\zeta \varphi)^{-1},$$

$$C_M = \theta_2 (\zeta \varphi)^{-1}.$$

The consumption of exportables and importables remain constant, whereas the outputs of both tradables decrease toward their long run values. Hence, exports decrease and imports increase, with a trade balance that is above the steady state level during this path and falls towards the steady state level.

Consider now what happens with the real wage along the transition. The real wage is:

$$\omega \equiv \frac{W}{P} = \frac{W}{(\phi_X S)^{\theta_1} S^{\theta_2} P^{1-\theta}} = \frac{W}{\phi_X^{\theta_1} e^{\theta}}. \quad (44)$$

There are two variants in the low $\beta/\alpha$ case we are analyzing, since any line along which the real wage is constant has a positive (log-log) slope of

$$\left. \frac{W}{P} \right|_{\omega = \omega_0} = \frac{1}{1 - \theta}$$

which may be higher or lower than the line towards which the equilibrium path tends asymptotically (which is the eigenvector associated to the least negative eigenvalue, the slope of which depends on $\beta/\alpha$. See the Appendix.). If the eigenvector has a slope that is lower than $(1-\theta)^{-1}$ the real wage increases monotonically towards this level. (This is the case we depicted in Figure 1, where the dominant eigenvector (labeled DE) has a slope that is lower than 1 and, hence, lower than $(1-\theta)^{-1}$). On the other hand, if the opposite situation occurs, the real wage increases towards the steady state, overshoots this level and then gradually declines towards the long run level. Workers’ real income can in general be below or above
the steady state value during the transition (or most of the transition), according to whether or not the lower than steady state real wage predominates over the higher than steady state employment level. But if labor demand in the tradable sectors and effective employment in the nontradable sector are inelastic, which is the usual case, workers’ real income is lower than in the steady state throughout the whole path in the first variant and throughout most of the path in the second.

The case in which $\beta/\alpha$ has a high value (i.e., in which wage setters adjust relatively fast in comparison to price setters) is very similar to the case we analyzed in detail. The difference is that the eigenvector line towards which the path tends has a negative slope, as Figure 2 shows. Hence, any path that starts from the no-inflation line (as A) eventually reaches the price inflation and wage deflation region before converging to the steady state. In this case, the RER also converges downwards monotonically to its steady state level but the product wage in the non-tradable sector overshoots. The real wage also necessarily overshoots in this case.

**Figure 2**

High $\beta/\alpha$

In the case of a focus, there is also a long inflationary period after which the economy goes into the price inflation and wage deflation region. However, before converging to the steady state the economy goes through a wage-price spiral that takes it successively to each of the four inflation/deflation regions, repeatedly.

2.6. The steady state

2.6.1 The nontradable and labor market clearing conditions

The non-tradable and labor market equilibrium conditions in terms of the real exchange rate $e$ and the product wage in the non-tradable sector $w$ are:

\[
\frac{\tau}{\phi}(1 - \theta)e\zeta^{-1} + g = Q_N(w),
\]

(45)
\begin{equation}
(F_N)^{-1}\left(\frac{\tau}{\varphi}(1-\theta)e\zeta^{-1}+g\right) = L_S\left(\frac{1}{\mu_H\zeta^{-1}}\frac{w}{e}\right) - L_T\left(\frac{w}{e} : \phi_X\right).
\end{equation}

For convenience, we use the first of these equations and (11) to express the second equation as:

\begin{equation}
L_N(w) = L_S\left(\frac{1}{\mu_H\zeta^{-1}}\frac{w}{e}\right) - L_T\left(\frac{w}{e} : \phi_X\right).
\end{equation}

Figure 3

Labor market

Let us temporarily consider these two equations in isolation for given values of the exogenous parameters and for the equilibrium marginal utilities of real income. In none of the exercises we perform will \(\phi_X\) vary, since any change in \(\phi\) will be compensated by a change in \(t_X\). Hence, in the following figures we disregard the influence of \(\phi_X\). Figure 3 shows the labor market equilibrium in the steady state. Labor supply increases with \(w/e\) and we assume it has a saturation point at \(L\). An increase in the inverse marginal utility of real income for workers \(\zeta_L^{-1}\) makes the labor supply line shift to the left (which is what the negative sign indicates) with the same saturation point. Labor demand from the tradable sectors is decreasing in \(w/e\), tending to zero as \(w/e\) tends to infinity. The available labor supply for the non-tradable sector is the horizontal distance between the two curves. There is a level of \(w/e\) which is so low that there is no labor left over for the non-tradable sector. We denote this level \((w/e)_{\text{min}}\). It is obviously increasing in \(\zeta_L^{-1}\). Hence, at this level of \(w/e\) the non-tradable output is zero. On the other hand, when \(w/e\) tends to infinity, the labor used up in the non-tradable sector tends to the maximum labor supply possible \(\bar{L}\), and there is no labor available for the tradable sectors.

Figure 4 shows both the labor market clearing (L) and the non-tradable market clearing (N) lines for given the values of \(g\), \(\zeta\), and \(\zeta_L\). It is evident that if there exists a point of intersection it is necessarily unique. But it is important to bear in mind that \(\zeta\) and \(\zeta_L\) are endogenous variables in our model, intimately related to the wealth of entrepreneurs and workers. Although they are almost always constant,
they jump to a new level whenever there is an exogenous unexpected shock that has wealth effects. For the moment we just consider the effects of these variables and the exogenous parameter $g$ on the steady state values of $e$ and $w$.

Figure 4

Steady State

A permanent increase in $g$ shifts the N line to the left and hence reduces both $e$ and $w$, while increasing $w/e$. Hence the nontradable sector expands and the tradable sectors contract. An increase in the inverse aggregate marginal utility of income $\zeta$, which increases private demand for non-tradable goods, has the same effect. On the other hand, an increase in the inverse marginal utility of income for working households $\zeta_L^{-1}$, by reducing the wage set by working households (or their unions) and hence increasing their supply of labor, shifts the L line to the right and has the effect of reducing $e$ and increasing $w$.

2.6.2 The steady state

To obtain the steady state values of $w$, $e$, $\zeta^{-1}$, and $\zeta_L^{-1}$ (or equivalently $W$, $P_N$, $\zeta^{-1}$, and $\zeta_L^{-1}$; given a value for $S$) we must consider not only equations (45) and (46) but also the intertemporal balance of payments and the aggregate of workers’ budget constraints calculated at $t = 0$:

\begin{equation}
N_0 + \int_0^{\infty} Q_{TX} \left( \frac{w}{e}; \phi; \phi_X \right) e^{-r^* t} dt = \frac{\tau \zeta^{-1}}{\varphi} \int_0^{\infty} \left( \theta + \frac{t_X}{1 - t_X} \theta_1 \right) e^{-r^* t} dt, \quad (47)
\end{equation}

\begin{equation}
a_{L,0} + \int_0^{\infty} \frac{w}{e} \left[ L_T \left( \frac{w}{e}; \phi_X \right) + \tilde{L}_N \left( e \zeta^{-1}; g \right) \right] e^{-r^* t} dt = \frac{\tau^* \zeta_L^{-1}}{\varphi^* \tau^* \zeta_L^{-1}}. \quad (48)
\end{equation}

(47) states that the initial international investment position of the SOE ($N_0$, which is invariant to shocks here since we disregard debt reductions) plus the net present
value of tradable output must equal the net present value of tradable consumption. Given whatever expectations there may be for the future evolution of the real exchange rate, the product wage in the nontradable sector, the terms of trade, and the export tax, one may think of this equation as giving the equilibrium level of the aggregate inverse marginal utility of real income $\zeta^{-1}$ (of course, within the four equation system). (48) states that the initial financial wealth of working households $a_{L,0}$ plus the net present value of their labor income must equal the net present value of their consumption. One may think of this equation as giving the equilibrium level of the inverse marginal utility of real income for working households $\zeta_L^{-1}$.

Because the values of the integrals on the left hand sides of the equalities in the last two equations depend on the initial values of $w$ and $e$ (or, equivalently, $W$, $P_N$, and $S$), the steady state values of $w$ and $e$ depend on these initial values. And the equilibrium values of $\zeta^{-1}$ and $\zeta_L^{-1}$ must reflect all the information the agents have concerning the future. When new information is available, these variables jump to their new equilibrium values and stay there until new information that affects long run solvency arrives. In particular, if there is new information concerning a future event, i.e., one that takes place at $t = T > 0$, they must jump at $t = 0$ and remain in that level before and after $t = T$ if no new information arrives (see Schubert and Turnovsky (2002)).

Let us for the moment assume that there is no terms of trade shock (and hence $\phi_X = \phi$), so that the r.h.s. of (47) becomes $\theta \tau \zeta^{-1}/(i^* \varphi)$. Using the modified (47) to eliminate $\zeta^{-1}$, as well as (39), (45) and (48) become:

$$\frac{1 - \theta}{\theta} i^* e \left[ N_0 + \int_0^\infty Q_{TX} \left( \frac{w}{e}; S \right) e^{-i^* t} dt \right] + g = Q_N \left( w \right). \quad (49)$$

$$a_{L,0} + \int_0^\infty \frac{w}{e} \{ L_T \left( \frac{w}{e}; S \right) \} \text{e}^{-i^* t} dt \text{e}^{-i^* t} dt$$

$$+ F_N^{-1} \left( \frac{1 - \theta}{\theta} i^* \right) \left[ N_0 + \int_0^\infty Q_{TX} \left( \frac{w}{e}; S \right) e^{-i^* t} dt \right] e + g \right) e^{-i^* t} dt \right) \text{e}^{-i^* t} dt}$$

$$= \frac{\tau^*}{\varphi i^*} \zeta_L^{-1}.$$
does not reflect this). And using the last equation to eliminate $\zeta_L^{-1}$, (46) becomes:

$$L_S \left( a_{L,0} + \int_0^\infty \frac{w}{e} \left( L_T \left( \frac{w}{e}; S \right) + F_N^{-1} \left( \frac{1-g}{g} i^* \left( N_0 + \int_0^\infty Q_{TX} \left( \frac{w}{e}; S \right) e^{-i^* t} dt \right) e + g \right) e^{-i^* t} dt \right) \right)$$

$$-L_T \left( \frac{w}{e} \right) = L_N (w).$$

Equations (49) and (50) determine the steady state levels of $w$ and $e$, given (forever) constant values for $g$ and $S$. Hence, we can obtain a modified form for Figure 4 (see Figure 5) where both curves depend on the value of $g$ and $S$ (and do not now depend on $\zeta^{-1}$ or $\zeta_L^{-1}$). They also depend on the initial values of $W$ and $P_N$, but we want to stress the dependence on $S$ because this is a policy variable.

When there are unexpected terms of trade shocks the above equations only need slight modifications. For example, with a permanent terms of trade shock, the r.h.s. of (47) becomes

$$\frac{\tau \zeta^{-1}}{i^* \varphi} \left( \theta + \frac{t_X}{1-t_X} \theta_1 \right) = \frac{\tau \zeta^{-1}}{i^* \varphi} \left( \theta + \left( \frac{\phi}{\phi_X} - 1 \right) \theta_1 \right),$$

where the equality is based on the second identity in (2). And similarly, with a transitory terms of trade shock that lasts until $t = T$, the r.h.s. of (47) becomes:

$$\frac{\theta \tau \zeta^{-1}}{i^* \varphi} + \theta_1 \left( \frac{\phi}{\phi_X} - 1 \right) \int_0^T e^{-i^* t} dt = \frac{\theta \tau \zeta^{-1}}{i^* \varphi} + \theta_1 \left( \frac{\phi}{\phi_X} - 1 \right) \left( 1 - e^{-i^* T} \right).$$

In any of these cases the N and L curves depend additionally on $\phi$.

---

5Note that we can rewrite these equations (as well as the following ones) using the nominal variables $W$, $P_N$, and $S$. We use the real variables here merely to obtain Figure 5, which has the real variables in the axes.
Returning to the general case in which there can be a permanent or transitory terms of trade shock, we can also use the flow balance of payments steady state equilibrium ($N = 0$) and the flow workers’ budget constraint steady state equilibrium ($a_L = 0$) to obtain the steady state values of $N$ and $a_L$:

$$0 = Q_{TX} \left( \frac{w}{e} ; \phi, \phi_X \right) - \frac{\tau}{\varphi} \left( \theta + \frac{t_X}{1 - t_X} \theta_1 \right) \zeta^{-1} + i^* N,$$

$$0 = \frac{w}{e} \left[ L_T \left( \frac{w}{e} ; \phi_X \right) + N \left( e \zeta^{-1} ; g \right) \right] - \frac{\tau^*}{\varphi} \zeta_L^{-1} + i^* a_L,$$

Furthermore, we assume that the government is always solvent, which means that it adjusts the level of $t$ so as to comply with its intertemporal budget constraint every time there is a surprise shock (and possibly a policy response, such as establishing or increasing $t_X$). The government’s intertemporal and flow budget constraints, respectively, determine the steady state levels of $t$, and $D - R$:

$$D_0 - R_0 + \int_0^\infty \frac{g}{e} e^{-i^* t} dt + \frac{\tau \theta_1}{\varphi} \zeta^{-1} \int_0^\infty \frac{t_X}{1 - t_X} e^{-i^* t} dt$$

$$= \int_0^\infty t_X \phi Q_X \left( \frac{w}{e \phi_X} \right) + \eta t \left[ e^{-i^* t} dt. \right]$$

$$0 = \frac{g}{e} - \eta t - t_X \phi Q_X \left( \frac{w}{e \phi_X} \right) + \frac{\tau \theta_1}{\varphi} \left( \frac{t_X}{1 - t_X} \zeta^{-1} + i^* (D - R). \right)$$

(51) states that the government’s initial net debt plus the net present value of its subsidy to the ‘poor’ and the pricing effect of the export tax (if it exists), must equal the net present value of future export tax and lump sum tax revenues.
We can now give a better account of the comparative statics exercises of the previous subsection. Consider a permanent increase in $g$. Given the initial values of the variables, the present value of working households’ incomes increases due to an increased employment in the nontradable sector, making $\zeta L^{-1}$ increase. This decreases labor supply and, hence, shifts the $L$ line to the right. The permanent increase in $g$ also shifts the $N$ line to the left. Therefore, the steady state RER falls, and the product wage in the tradable sectors $w/e$ increases, generating the reduction in the tradable sector employment and output levels required to accommodate the increased demand for nontradable sector employment and output. Similarly, a higher initial level of $S$ has the same effects on the steady state values of $e$ and $w/e$. In particular, the government has the ability to permanently affect the income and wealth distribution among income classes by choosing the value of $S$. The higher it is set above its long run equilibrium level the more the income distribution by income class tilts towards entrepreneurs and against workers, and the more intensive is the inflationary process that leads to the new long run equilibrium levels for $W$ and $P_N$.

Notice that the model displays path dependence (or hysteresis), since temporary changes in $\phi$, for example, affect the whole future path of the endogenous variables, including the steady state values.

3. An application to Argentina’s recent past
3.1 The deflationary phase of the Convertibility period
The exchange rate policy of quasi-fixing the nominal exchange rate within a small band of 2.9 pesos per dollar began around the third quarter of 2002, after a deep triple crisis during 2001 and the first half of 2002 that marked the end of 10 years of the fixed exchange rate regime that was known as Convertibility. Since the latter was an explicitly fixed exchange rate regime we can apply the present model for the analysis of the Convertibility period. Argentina faced several external shocks in the second half of the 90s, but the most relevant ones were the strong dollar shock and a much more transitory terms of trade shock. Figure 6 shows the U.S.A.’s multilateral RER (MRER) as measured by the Federal Reserve’s Real Broad Dollar Index. It shows the impressive strengthening of the dollar during 1995-2001. The worst episode in this process was clearly Brazil’s devaluation in early 1999, but the dollar had been getting stronger since mid-1995, so the fixed exchange to the dollar implied a loss of competitiveness with the rest of the currencies relevant to Argentina’s foreign trade (and accounting for 85% of that trade if we include Brazil). Figure 7 shows the effects of these shocks on Argentina’s MRER. (Here an increase implies a real depreciation.) Figure 8 shows that there was also an adverse terms of trade shock during 1999. However, it was short-lived.6

In the model presented here we have so far abstained from explicitly accounting for the multilateral nature of the real exchange rate. Assume now that Argentina’s nominal exchange rate is a geometric average of its nominal exchange rate with the U.S. dollar and its nominal exchange rate with the Brazilian real (which is here assumed to represent all currencies other than the dollar). Then the MRER can

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6Of course, there were also important financial shocks starting with the Russian crisis in mid-1998. However, our model cannot reflect such shocks.
be represented by:

\[ e = \frac{\overline{S}_U S \overline{S}_{BR}^{1-\kappa}}{P_N} \]

where \( \overline{S}_{US} \) is the constant nominal exchange rate against the dollar. The strong dollar shock can hence be represented in stylized form by a one time unexpected fall in \( S_{BR} \) (pesos per real) that makes \( e \) also fall.\(^7\) Note that since there were no export taxes during the Convertibility period, \( \phi_X = \phi \).

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\(^7\)For a more detailed analysis of the strong dollar shock see Escudé (2004) and for an empirical estimation of Argentina’s equilibrium RER see Garegnani and Escudé (2005), who develop an Equilibrium Correction Model to explain Argentina’s multilateral real exchange rate (MRER) and its misalignment. They find a cointegrating relation between this variable and 1) Argentina’s terms of trade, and 2) the U.S. MRER, as measured by the Federal Reserve’s Real Broad Dollar index. According to this study, Argentina’s MRER was in long run equilibrium in the second quarter of 1996.
One of the virtues of the model we use in this paper is that the speeds of adjustment $\alpha$ and $\beta$ can have different values for deflationary adjustments than for inflationary adjustments. Let us call the speeds of downward adjustment $\underline{\alpha}$ and $\underline{\beta}$, with $\underline{\alpha}<\alpha$ and $\underline{\beta}<\beta$. Then our dynamical system has four different laws of motion, one for each of the four inflation/deflation regions. These regions may be characterized by $(\alpha,\beta)$, $(\underline{\alpha},\underline{\beta})$, $(\underline{\alpha},\overline{\beta})$, and $(\overline{\alpha},\beta)$. Every time the path for $P_N$ and $W$ cuts through one of the no-nontradable-inflation or no-wage inflation lines, there is a switch to a law of motion that only differs from the previous one in the size of one of the speeds of adjustment. Although the inflation rate paths are everywhere continuous, they have kinks at zero. The four laws of motion are formally the same except for the size of the speeds of adjustment.\(^8\)

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\(^8\)Note that Picard’s theorem on existence and uniqueness of solutions for ordinary differential equations does not require a differentiable r.h.s. but merely that a Lipschitz condition be satisfied. In the Appendix we prove that the assumption $\underline{\alpha}<\alpha$, $\underline{\beta}<\beta$, does not invalidate the Lipschitz condition requirement as long as the gap functions are differentiable (and hence satisfy a Lipschitz condition), as is the case with any well-behaved utility and production functions.
We can use Figure 1 to illustrate the impact of the two simultaneous and unexpected shocks in terms of the dynamical system. We take the low $\beta/\alpha$ case for convenience. Assume that in, say, 1996 the economy was in a long run equilibrium in C. The unexpected and transitory adverse external shocks shifted the equilibrium point to somewhere southwest of C, so that C became a point in the double deflation region of the new configuration. With a fixed exchange rate (both before and after the strong dollar shock), the economy was in desperate need of price and wage deflation, so it started treading down a deflationary path for non-tradables and wages that could "eventually" lead to the new steady state at a point like B (this path is not shown in Figure 1 but would be symmetric to the one that is shown, approaching the dominant eigenvector asymptotically from below). However, the speeds of downward adjustment ($\underline{\alpha}$ and $\underline{\beta}$) are dreadfully low. In fact, the wages and nontradables deflation rates were almost nil (Figures 9 and 12), while unemployment and underemployment grew to catastrophic levels (Figure 11). The RER was supposed to rise to its steady state level. Hence, the non-tradable output $\tilde{Q}_N$ and employment $\tilde{L}_N$ were also supposed to gradually rise to their 'potential' levels. Also, the product wage in the tradable sectors ($w/e = W/S$) was supposed to fall throughout the path, making tradable sector output and employment increase towards their ‘potential’ levels. Since the economy was in recession in both sectors, deflationary pressure on prices and wages were generated. However, the speeds of adjustment $\underline{\alpha}$ and $\underline{\beta}$ were very close to zero.
The government tried to finance its loss of revenues with mounting debt, but the issue of the continuation or not of the Convertibility regime became highly political, with no political party with chances of winning the 1999 elections willing to bear the high political cost of a regime change since it was widely known that the economic consequences would be disastrous (as the "commitment" to the currency board had intended), mainly due to its consequences on the financial system, which was exposed to such large currency mismatches of its debtors that a significant change of relative prices against nontradables would impair its solvency. And the deflationary adjustment was so slow that it became clear to most economic agents that there were political limits to the feasibility of waiting until the reversion of the adverse shocks. Indeed, the amazing fact that begs for an explanation is why the country’s leadership allowed such high social costs to be incurred without
changing the regime and only waited until the capital markets made the decisions that triggered the triple crisis (debt, banking, and currency). After almost two years of a new administration based on a new coalition, with no sign of economic recovery, the markets decided to stop financing the government. This led to a deep political crisis that made the president resign from office, and spawned a succession of short lived presidents in a lapse of 30 days, one of which declared the default on the public debt. It should be clear that we do not model the collapse of the regime here but merely the dynamics before the collapse and after.

Figure 11

Argentina: Unemployment and Underemployment

Figure 12

Nontradable Prices (CPI Services) vs. Wages
12 month m.a.
3.2. The "high real exchange rate" policy in the post-Convertibility period
The debt crisis accelerated the banking and currency crises, generating a large outflow of capital. When it became clear that Central Bank reserves would soon be depleted, the government finally decided to devalue, first moderately (taking the exchange rate to 1.4 pesos per dollar), and subsequently letting the exchange rate skyrocket. After a strong overshooting (that made the exchange rate reach almost 4 pesos per dollar) there was some correction but the government eventually decided to place a floor on the nominal exchange in the vicinity of 2.9 pesos per dollar (Figure 13). With a (correct) market perception of real undervaluation of
the peso, this became almost equivalent to fixing the nominal exchange rate once again (with the exception of a small non-explicit band of fluctuation and some later upward corrections that raised the exchange rate to 3.1 pesos per dollar by 2007). Figure 10 shows that the foreign currency value of the nontradable sectors’ GDP was severely contracted.

In terms of the present model, we can think of the decision of foreign creditors to discontinue the financing of the public debt as being an unexpected shock that aborted the defective deflationary adjustment and forced a change of policy. However, this time there were also favorable unexpected and transitory shocks, opposite to those that mortally wounded the Convertibility regime: a favorable terms of trade shock (Figure 8) and the beginning of a new phase of dollar depreciation (Figure 6). The policy response was basically the huge devaluation with overshooting and a quasi-fixing of the nominal exchange rate, and a simultaneous imposition of an export tax that roughly shielded the domestic price of exportables from the impact of the terms of trade shock.

For convenience, we analyze the effects on $\zeta^{-1}$ and $\zeta_L^{-1}$ of this complex shock by separating it into two parts: 1) the devaluation with overshooting and fixing of the exchange rate, 2) an unexpected temporary and favorable terms of trade shock accompanied by an export tax that shields the domestic price of exportables as long as the terms of trade shock lasts. For simplicity we merge the increase in the multilateral RER due to the depreciation of the dollar with 1) since the (sudden) devaluation far exceeded the effects of the (gradual) dollar depreciation.

### 3.2.1 A devaluation with overshooting and fixing of the nominal exchange rate

Assume an unexpected devaluation occurs at $t = 0$, when the economy is off the steady state and has an overvalued currency. In Figure 1, the economy is initially at a point like B with the steady state at a point southwest of B. The overdevaluation takes the equilibrium point to a point like C, shifting the hyperbola in the left hand quadrant to the left. The actual RER increases but nontradable prices and the nominal wage only respond sluggishly over time. A long inflationary path begins at B that gradually takes the economy to C.

An instant before the devaluation, the intertemporal balance of payments (47) is:

$$N_0 + \int_0^\infty Q_{TX} \left( \frac{W}{S_0}; \frac{W_0}{S_0} \right) e^{-i^* t} dt = \frac{\theta \tau \zeta_0^{-1}}{i^* \varphi},$$

since there is no export tax. The notation within the integral sign indicates the path of tradable output that starts at the initial nominal exchange rate and nominal wage. Immediately after the devaluation, the intertemporal balance of payments becomes:

$$N_0 + \int_0^\infty Q_{TX} \left( \frac{W}{S_1}; \frac{W_0}{S_1} \right) e^{-i^* t} dt = \frac{\theta \tau \zeta_1^{-1}}{i^* \varphi}.$$

---

9Figures 9 and 12 show that there was hardly any jump in the nontradable price level.
While $S$ has jumped to a new level, the nominal wage has not had time to change. The marginal utility of real income, however jumps to a new level that ensures intertemporal solvency for the economy as a whole. Subtracting the first equation from the second yields:

$$
\int_0^\infty \left\{ Q_{TX} \left( \frac{W}{S_1}; \frac{W_0}{S_1} \right) - Q_{TX} \left( \frac{W}{S_0}; \frac{W_0}{S_0} \right) \right\} e^{-i\tau t} dt = \frac{\theta_T}{i^*} \left( \zeta_1^{-1} - \zeta_0^{-1} \right) > 0.
$$

Since the tradable output increases with the devaluation during the whole path (in the low $\beta/\alpha$ case we take as benchmark and during most of the path for the other cases), it is clear that $\zeta^{-1}$ increases, and therefore, so does aggregate consumption.

On the other hand, the initial intertemporal budget constraint of working households (48) is

$$
a_{L,0} + \int_0^\infty \frac{W}{S_0} \left[ L_T \left( \frac{W}{S_0}; \frac{W_0}{S_0} \right) + \bar{L}_N \left( \frac{S_0}{P_N} \zeta_0^{-1}; \frac{S_0}{P_{N,0}} \right) \right] e^{-i\tau t} dt = \frac{\tau^*}{\varphi i^*} \zeta_{L,0}. \tag{52}
$$

After the devaluation it becomes:

$$
a_{L,0} + \int_0^\infty \frac{W}{S_1} \left[ L_T \left( \frac{W}{S_1}; \frac{W_0}{S_1} \right) + \bar{L}_N \left( \frac{S_1}{P_N} \zeta_1^{-1}; \frac{S_1}{P_{N,0}} \right) \right] e^{-i\tau t} dt = \frac{\tau^*}{\varphi i^*} \zeta_{L,1}. \tag{53}
$$

There are clearly two opposite effects on working households’ income. First there is an employment effect. Employment increases in the tradable sectors since the product wage diminishes with the devaluation, and it also increases in the non-tradable sector, both because the devaluation makes nontradable goods relatively cheaper, producing substitution in consumption from tradables to nontradables, and also because it increases aggregate consumption (i.e., increases $\zeta^{-1}$). However, the reduction in the purchasing power of the wage exerts an opposite influence. Assuming that the demand for labor is inelastic in all these sectors, the real wage effect predominates and hence working households’ consumption declines\textsuperscript{10}. Subtracting (52) from (53) yields:

$$
\int_0^\infty \frac{W}{S_1} \left[ L_T \left( \frac{W}{S_1}; \frac{W_0}{S_1} \right) + \bar{L}_N \left( \frac{S_1}{P_N} \zeta_1^{-1}; \frac{S_1}{P_{N,0}} \right) \right] e^{-i\tau t} dt - \int_0^\infty \frac{W}{S_0} \left[ L_T \left( \frac{W}{S_0}; \frac{W_0}{S_0} \right) + \bar{L}_N \left( \frac{S_0}{P_N} \zeta_0^{-1}; \frac{S_0}{P_{N,0}} \right) \right] e^{-i\tau t} dt = \frac{\tau^*}{\varphi i^*} \left( \zeta_{L,1}^{-1} - \zeta_{L,0}^{-1} \right) < 0.
$$

\textsuperscript{10} Castro, Olarreaga, and Saslavsky (2006), for example, estimate the wage elasticity of labor demand in Argentina’s manufacturing sector as 0.3. See also comparable figures for Chile, Colombia and Mexico in Fajnzylber and Maloney (2001), for Russia in Lehmann, and Konings (2001), and for OECD countries in Bruno, Falzoni and Helg (2003).
Hence, even though aggregate consumption increases, working households’ consumption declines. This implies that the increase in entrepreneurial households’ consumption increases so much that it more than offsets the reduction in working households’ consumption. This is a very interesting result. The opposite effects were taking place during the depressive phase of Convertibility. Working households where then, so to speak, "living beyond their means" because, although they were underemployed (we cannot represent unemployment in the present model), their real wages were too high, while entrepreneurs were suffering from low real profits in all sectors. The "high real exchange rate policy" of the post-Convertibility period can in all fairness be called "low real wage policy". It redistributes wealth and consumption capacity from workers to entrepreneurs. This regressive redistribution is greater the more undervalued in real terms is the domestic currency when the new nominal exchange rate is set. It reflects the new wisdom of the old ‘progresista’ economic thought that inspires the new policies.

3.2.2 A temporary and favorable terms of trade shock compensated domestically with an export tax

Although the overdevaluation is the main ingredient of the post-Convertibility era, it is only part of the story . There was also a very impressive positive terms of trade shock (Figure 8) the domestic effects of which were largely compensated with the imposition of an export tax. We simplify by assuming that the export tax exactly shields the domestic price of exportables from being affected by the export price increase. Hence, although \( \phi \) increases from \( \phi_0 \) to \( \phi_1 \), \( \phi_X \equiv \phi(1 - t_X) \) remains constant. We also assume that the government either reduces its lump sum tax on entrepreneurs or produces a one time debt reduction in order to use its new source of revenues.\(^{11}\) Assume that the favorable terms of trade shock occurs at \( t = 0 \) and is (unanimously) expected to last until \( t = T \). We analyze the terms of trade shock assuming that the new and increased nominal exchange rate \( S_1 \) was imposed an instant earlier. Hence, the pre-devaluation net international position \( (N_0) \) has not had time to change. The marginal utilities, however, had jumped an instant earlier.

The effect on aggregate wealth  The initial (post-devaluation and pre terms of trade shock) intertemporal balance of payments (47) is:

\[
N_0 + \int_0^\infty Q_T X \left( \frac{W}{S_1}, \frac{W_0}{S_1}, \phi_0, \phi_X \right) e^{-it} dt = \frac{\theta \tau \zeta^{-1}}{i \varphi}, \tag{54}
\]

\(^{11}\)We could alternatively (or additionally) assume there is an increase in \( g \), but this would generate an additional effect on the steady state so we prefer to leave it constant.
since initially there is no export tax. After the shock, the intertemporal balance of payments becomes:

\[ N_0 + \int_0^T Q_{TX} \left( \frac{W}{S_1}; \frac{W_0}{S_1}, \phi_1, \phi_X \right) e^{-\iota t} dt \]

\[ + \int_T^\infty Q_{TX} \left( \frac{W}{S_1}; \frac{W_0}{S_1}, \phi_0, \phi_X \right) e^{-\iota t} dt \]

\[ = \frac{\tau \zeta_2}{\iota^* \varphi} \left\{ \int_0^T \left( \theta + \left( \frac{\phi}{\phi_X} - 1 \right) \theta_1 \right) e^{-\iota t} dt + \theta \int_T^\infty e^{-\iota t} dt \right\} \]

\[ = \frac{\tau \zeta_2}{\iota^* \varphi} \left\{ \left( \theta + \left( \frac{\phi}{\phi_X} - 1 \right) \theta_1 \right) \left( 1 - e^{-\iota T} \right) + \theta e^{-\iota T} \right\} \]

Subtracting (54) from (55) and rearranging yields:

\[ \frac{\theta_T}{\iota^* \varphi} \left( \zeta_2^{-1} - \zeta_1^{-1} \right) \]

\[ = \int_0^T \left\{ Q_{TX} \left( \frac{W}{S_1}; \frac{W_0}{S_1}, \phi_1, \phi_X \right) - Q_{TX} \left( \frac{W}{S_1}; \frac{W_0}{S_1}, \phi_0, \phi_X \right) \right\} e^{-\iota t} dt \]

\[ \frac{1 + \left( \frac{\phi}{\phi_X} - 1 \right) \theta_1}{\theta} \left( 1 - e^{-\iota T} \right) \]

\[ = \frac{\theta_T}{\iota^* \varphi} \left\{ \left( \frac{\phi}{\phi_X} - 1 \right) \theta_1 \left( 1 - e^{-\iota T} \right) \right\} \]

The effect of the shock on \( \zeta^{-1} \) is hence positive as long as the export repriceing effect (that converts the domestic price of exports to the international price) is not so high as to compensate for the increase in tradable output during the transition. We (realistically) assume that this is the case. Hence, the export-tax compensated terms of trade shock increases aggregate consumption (with respect to the already increased level generated by the devaluation).

Note that if the export tax were not imposed, tradable output at international prices \( Q_{TX} \) would rise not only due to the pure price effect but also due to the increase in exportable output through the reduction in the product wage in the exportable sector. Instead of (56), we would have a greater increase in \( \zeta^{-1} \):

\[ \frac{\theta_T}{\iota^* \varphi} \left( \zeta_2^{-1} - \zeta_1^{-1} \right) \]

\[ = \int_0^T \left\{ Q_{TX} \left( \frac{W}{S_1}; \frac{W_0}{S_1}, \phi_1 \right) - Q_{TX} \left( \frac{W}{S_1}; \frac{W_0}{S_1}, \phi_0 \right) \right\} e^{-\iota t} dt \]
The effect on working households’ aggregate wealth  The initial intertemporal budget constraint of working households is

\[ a_{L,0} + \int_{0}^{\infty} \frac{W}{S_1} \left[ L_T \left( \frac{W}{S_1} \phi_X \right) + \tilde{L}_N \left( \frac{S_1}{P_N} \zeta_1^{-1}; \frac{S_1}{P_{N0}} \right) \right] e^{-\gamma t} dt = \frac{\tau^*}{\phi^* \zeta_1 L_{1,1}}. \]

After the shock it becomes:

\[ a_{L,0} + \int_{0}^{\infty} \frac{W}{S_1} \left[ L_T \left( \frac{W}{S_1} \phi_X \right) + \tilde{L}_N \left( \frac{S_1}{P_N} \zeta_2^{-1}; \frac{S_1}{P_{N0}} \right) \right] e^{-\gamma t} dt = \frac{\tau^*}{\phi^* \zeta_2 L_{1,2}}. \]  \( (57) \)

Note that tradable sector employment is not directly affected by the terms of trade shock due to the imposition of the export tax (although the present value of the complete path of tradable sector income may vary due to the displacement of the steady state as a consequence of the temporary shock). However, employment in the nontradable sector is favorably affected due to the increase in aggregate consumption through the rise in \( \zeta^{-1} \). Hence, there appears to be some sharing of the favorable and temporary increase in the terms of trade with working households.

Notice that without the export tax, given \( W \) and \( P_N \), employment would be higher both in the exportable sector and in the nontradable sector. The latter, as we have seen above, is due to a higher increase in \( \zeta^{-1} \).

A more complete analysis of the effect of a terms of trade shock would require the explicit evaluation of the present value of workers income taking into account that the steady state is affected by the temporary shock. The increase in \( \zeta^{-1} \) shifts the N line to the left in Figure 5. This implies a reduction in both in \( e = S_1/P_N \) and in \( w/e = W/S_1 \) and hence a reduction in the steady state nominal wage and an increase in the steady state nontradable price. In Figure 1 the steady state would be somewhere to the northwest of C, and hence the path to this new steady state would be (starting from, say, B) northwest of the one depicted, implying a lower real wage along the path. Due to labor demand inelasticity, working households’ real incomes would also decrease if \( \zeta^{-1} \) were to remain constant. However, \( \zeta^{-1} \) increases, so we would need to compute the present value of the complete path of workers’ real income to be more specific. This would require explicitly calculating the effect of the temporary shock on the steady state system of four equations above. We do not delve further into this here. Although there appears to be some sharing of the favorable terms of trade shock with working households through the imposition of an export tax, this is quite independent of the negative effect on working households’ real incomes and wealth of the ‘high real exchange rate/low real wages’ policy. However, the fact that this policy takes place during a period in which the terms of trade are exceptionally high is important for the magnitude of the positive fiscal revenue effect of the export tax. This points to an objective of having very high tax revenues which are easy to enforce or modify, and not shared with the provinces except by discretion (or political favor), as the ulterior motive of the policy.

3.3 Final remarks on the ‘high real exchange' policy
The often repeated story of reducing exportable sectors’ profits on the upside with no equivalent compensation in the downside can go a long way towards explaining
Argentina’s poor relative performance in agriculture and livestock over the long run (in comparison to, say, Australia and New Zealand), precisely the sectors in which the country has clear comparative advantage. However, it would not be completely fair to use this argument for the post-Convertibility period since the ‘high real exchange rate’ policy has a positive effect on these sectors’ aggregate profits (and purchasing power) to the extent that they have nontradable inputs (and to the extent that the entrepreneurs involved live and spend in Argentina). The net effect is difficult to ascertain. However, there are specific subsectors that have also been strongly affected by price controls and/or energy rationing, or have a particularly high share of imported inputs in their costs. Price controls have been an integral part of the government’s policies for dealing with the inflationary pressures generated by its foreign exchange policy. The discretion with which the schedule of the export tax is modified and its high level in historical perspective make it prone to conflicts over the distribution of the extraordinary rents generated by the international markets.

The decision to place a floor on the nominal exchange rate when the peso is substantially undervalued, as we have seen, implies heavily taxing workers’ real incomes while generating an extended boom in the tradable sectors that keeps aggregate profits in these sectors above their steady state levels. The exportables sector entrepreneurs do not reap the full benefit of the combined favorable export prices and ‘high real exchange rate’ policy because the introduction of the export tax makes it share its revenues with the government (and some subsectors are affected by inefficient price controls). The importables sector entrepreneurs are the main beneficiaries of the policy, along with the central government, that can rely on this easy to collect tax that is not shared with the provinces to deepen its own political power base (or undermine it, depending on the net political results of the policy). Although the nontradable sector entrepreneurs are discriminated with respect to the tradable sectors through unfavorable relative prices, they also benefit from the low real wages. Notice that the bulk of these sectors are very labor intensive.

The ‘poor’ can benefit or not, according to what the government actually does with its increased revenues. If the government uses these to payoff part of its debt, invest on projects that are not targeted to the ‘poor’, subsidize firms, or reduce taxes that mainly affect entrepreneurs, they do not benefit. Indeed, if we realistically made the ‘poor’ sector expand after the Convertibility crisis, and/or realistically assumed that they consume exportable foodstuff, and/or realistically assumed that they receive their subsidy in money, their per capita real incomes would probably be negatively affected by the mix of policies. Although the government has the potential to use part of its revenues in expanding the subsidy to the ‘poor’ instead of (or in addition to) reducing the taxes on the ‘rich’ entrepreneurs, reducing its debt, or paying out subsidies to entrepreneurs that suffer price controls, the fact that this sector has scant lobbying power makes the gradual inflationary dilution of the subsidies probable. Analyzing these issues, however, would take us far from the intended scope of this paper.

The fact that the ‘low real wages’ policy was geared immediately after a severe fall in real incomes raises questions related both to equity and to social (and political) risk-taking. It is somewhat paradoxical that a Peronist administration should
have implemented this policy, since it seems the antithesis of what Perón did in his
times: redistribute income in favor of workers during a period of prosperity (for
Argentina, due to the high demand for foodstuff in the aftermath of World War II).
While Perón’s economic program was so beneficial to workers that it was unable to
withstand a disruption of the ‘tailwinds’ it was mounted on, the present ‘low real
wage’ policy could have been interpreted as a policy-induced ‘pro-capitalism’ policy
shock, at least in its intentions. However, the apparent inability to deal with the
high inflationary pressures risks undermining the support of the very same sectors
that have mainly benefitted, overall, from the policy.

Whether or not the regressive income distribution effects are a sacrifice that
pays off by helping to put the economy on a sustainable growth path (that could
eventually lead to a less inequitable income distribution on sustainable terms)
depends on the effectiveness of the policy set to generate increased productivity
through innovations and investment on state of the art technologies, as well as
better education and health services. For the first, a good investment climate in
which the prospect of high returns generate the proper incentives is crucial. For
the second, the maintenance of high fiscal revenues that are efficiently invested in
schools and hospitals is fundamental. But the inability to put a lid on inflation can
easily become an obstacle for sustainable growth. The paradox is that inflation
could be easily be reduced by moderating the ‘high real exchange rate’ policy that
is ultimately responsible for it, i.e., by allowing a significant nominal appreciation
of the peso and a moderate cooling down of the economy. It is still time to learn
from the old adage: ‘a stitch, in time, saves nine’.

4. Conclusion
This paper has developed a SOE perfect foresight-perfect capital markets model
that can help to explain various features of Argentina’s recent past, including the
difficulties experienced in the second phase of Convertibility and the recent revival
of the inflationary process. Households and firms in the non-tradable sector are
monopolistically competitive and have ‘rule of thumb’ sticky wage or price adjust-
ment equations while tradable sector firms are competitive and have flexible prices.
Furthermore, the exogenous price and wage speeds of adjustments are highly asym-
metric between inflationary and deflationary regions. We model external shocks
(basically terms of trade and ‘strong dollar’ shocks) as unexpected, with unani-
mously expected termination dates when they are transitory. We argue that the
quasi-fixed exchange rate policy in a context of a significantly undervalued peso is
responsible for the relatively strong inflationary pressures the economy has been ex-
periencing lately. The framework that is developed is also useful for thinking about
several aspects of Argentina’s political economy and income distribution by income
class. In particular, we show that the ‘high real exchange’ policy which fixes the
nominal exchange when the peso is undervalued in real terms generates a policy-
induced inflationary process, a regressive income and wealth distribution between
working and entrepreneurial income classes, and overemployment/overproduction
in both the tradable and nontradable sectors. When there are export taxes that
compensate for the effect of a positive terms of trade shock on the domestic price
of exportables, the government benefits from extra fiscal revenues.
Appendix 1: The mathematics
1) Existence and uniqueness of the solution to the dynamic system with asymmetric speeds of adjustments

Because of the different values that $\alpha$ and $\beta$ have in the inflation and deflation regions, we must check that the sufficient conditions for the existence and uniqueness of the solution paths for $W$ and $P_N$ are verified. We assume that those conditions are verified for the dynamic system in which $\alpha$ and $\beta$ have the same values in both regions and check that the modification makes these conditions still hold. Loosely, we may say that these conditions are that the functions on the r.h.s. of the differential equation satisfy a Lipschitz condition. In the two dimensional case (and using a convenient metric), a function $G(x,y)$ satisfies a Lipschitz condition if for any $(x_1, y_1)$ and $(x_2, y_2)$ in a closed and bounded region there exists a number $N$ such that\footnote{See Elsgoltz (1977).}

$$|G(x_1, y_1) - G(x_2, y_2)| \leq N \{|x_1 - x_2| + |y_1 - y_2|\}. \quad (58)$$

Notice that we can rewrite the dynamical system in a way that makes explicit the asymmetries in the speeds of adjustment:

$$\frac{d \log P_N}{dt} = \max \{\alpha G_N (W, P_N), \alpha G_N (W, P_N)\} \quad (59)$$
$$\frac{d \log W}{dt} = \max \{\beta G_W (W, P_N), \beta G_W (W, P_N)\}. \quad (60)$$

where we defined the nontradable output gap and the employment gap as:

$$G_N (W, P_N; \overline{S}, \zeta^{-1}, g) \equiv \tilde{Q}_N \left( \frac{\overline{S}}{P_N} \zeta^{-1}; g \right) - Q_N \left( \frac{W}{P_N} \right)$$

$$G_W (W, P_N; \overline{S}, \zeta^{-1}, \zeta_L^{-1}, \phi_X, g) \equiv L_T \left( \frac{W}{\overline{S}}; \phi_X \right) + \tilde{L}_N \left( \frac{\overline{S}}{P_N} \zeta^{-1}; g \right) - L_S \left( \frac{1}{\mu H \zeta_L^{-1}} \frac{W}{\overline{S}} \right).$$

Assuming that these gap functions satisfy a Lipschitz condition in the relevant domain, we must prove that so do the functions on the r.h.s. of (59) and (60). Hence, we must prove that the function

$$f_N (W, P_N) \equiv \max \{\alpha G_N (W, P_N), \alpha G_N (W, P_N)\}$$

satisfies a Lipschitz condition, i.e., that for any $(x_1, y_1)$ and $(x_2, y_2)$ in a closed and bounded region there exists a number $N$ such that

$$|f_N (W_1, P_{N1}) - f_N (W_2, P_{N2})| \leq N \{|W_1 - W_2| + |P_{N1} - P_{N2}|\}.$$
In case 1) \( f_N(W, P_N) = \alpha G_N(W, P_N) \) for the two points in question. Hence, we must prove that there exists an \( N \) such that

\[
|\alpha G_N(W_1, P_{N1}) - \alpha G_N(W_2, P_{N2})| \leq N \{|W_1 - W_2| + |P_{N1} - P_{N2}|\}.
\]  

(61)

Because \( G_N \) satisfies a Lipschitz condition, we know from (58) that there exists an \( N_0 \) such that

\[
|G_N(W, P_N) - G_N(W_2, P_{N2})| \leq N_0 \{|W_1 - W_2| + |P_{N1} - P_{N2}|\}.
\]  

(62)

Hence, if we multiply both sides by \( \alpha \) we obtain (61) for \( N = \alpha N_0 \).

In case 2) \( f_N(W, P_N) = \alpha G_N(W, P_N) \) for the two points in question. Hence, we merely have to replacing \( \alpha \) by \( \alpha \) in the reasoning for case 1).

In case 3) \( f_N(W_1, P_{N1}) = \alpha G_N(W_1, P_{N1}) \) and \( f_N(W_2, P_{N2}) = \alpha G_N(W_2, P_{N2}) \).

Hence, we need to prove that there exists an \( N \) such that

\[
|\alpha G_N(W_1, P_{N1}) - \alpha G_N(W_2, P_{N2})| \leq N \{|W_1 - W_2| + |P_{N1} - P_{N2}|\}.
\]  

Since

\[
|\alpha G_N(W_1, P_{N1}) - \alpha G_N(W_2, P_{N2})| < |\alpha G_N(W_1, P_{N1}) - \alpha G_N(W_2, P_{N2})|,
\]  

it suffices to prove that there exists an \( N \) such that (61) is verified. So again, multiplying both sides of (62) by \( \alpha \) we obtain (61) for \( N = \alpha N_0 \).

Case 4) is exactly analogous to case 3), so we need not repeat the argument. Finally, the cases where either \( G_N(W_1, P_{N1}) = 0 \) or \( G_N(W_2, P_{N2}) = 0 \) are trivial. Hence, we have proved that \( f_N(W, P_N) \) satisfies a Lipschitz condition.

2) Local asymptotic stability of the steady state

Let us now concentrate on the case of symmetry in the speeds of adjustment. The general case may be thought of as being composed of a configuration of four different dynamical systems (that vary only in the magnitude of one or both of \( \alpha \) and \( \beta \)) where there are four regions, in each of which one of the systems functions. Since in any of these systems there is a unique solution for a given initial condition, this poses no problem.

The linear approximation to system (40)-(41) in a neighborhood of the steady state \((\tilde{P}_N, \tilde{W})\) is:

\[
\begin{bmatrix}
\dot{P}_N \\
\dot{W}
\end{bmatrix} = \begin{bmatrix}
\alpha \left[ \frac{W}{P_N} Q'_N - \frac{S}{P_N} \xi^{-1} \tilde{Q}'_N \right] & -\alpha Q'_N & 0 \\
-\beta \frac{W}{P_N} \tilde{L}'_N & -\beta \frac{W}{\xi} \tilde{L}'_N & -\beta \frac{W}{\mu} \xi^{-1} \tilde{L}'_N
\end{bmatrix} \begin{bmatrix}
P_N - \tilde{P}_N \\
W - \tilde{W}
\end{bmatrix}
\]  

(63)

where all the variables in the (linearized) system matrix \( \Gamma \) are valued at the (steady state) equilibrium. Let us express the system matrix in the simplified form:

\[
\Gamma \equiv \begin{bmatrix}
\alpha A & \alpha B \\
\beta C & \beta D
\end{bmatrix} = \begin{bmatrix}
- & + \\
- & -
\end{bmatrix},
\]

(64)

where the notation is obvious and the second equality specifies the sign pattern of the elements of \( \Gamma \).
The trace and determinant of $\Gamma$ are:

\begin{align*}
\text{trace}(\Gamma) &= \alpha A + \beta D < 0 \\
\det(\Gamma) &= \alpha \beta (AD - BC) > 0.
\end{align*}

Let $\lambda_1$, $\lambda_2$, be the eigenvalues of $\Gamma$, i.e. the roots of the characteristic polynomial:

$$\lambda^2 - \text{trace}(\Gamma)\lambda + \det(\Gamma).$$

Hence:

\begin{align*}
\lambda_1 &= \frac{\alpha A + \beta D + [(\alpha A + \beta D)^2 - 4\alpha \beta (AD - BC)]^{1/2}}{2} \\
\lambda_2 &= \frac{\alpha A + \beta D - [(\alpha A + \beta D)^2 - 4\alpha \beta (AD - BC)]^{1/2}}{2},
\end{align*}

and thus

$$\text{trace}(\Gamma) = \lambda_1 + \lambda_2 < 0$$
$$\det(\Gamma) = \lambda_1 \lambda_2 > 0.$$  

The second inequality implies that the real parts of the two roots have the same sign, and the first implies that they must be negative. Therefore, all paths that start in a neighborhood of the equilibrium converge to it asymptotically. (See Varian (1981)).

3) Characterization of the phase portrait

Whether the two roots are real or complex depends on the sign of the discriminant (the term within the square root in (65)). After simplifying, we have:

$$\text{disc}(\Gamma) = [\alpha A - \beta D]^2 + 4\alpha \beta BC.$$  

This expression may be positive or negative.\(^\text{13}\) If it is positive, the two roots are real and the equilibrium is a stable node, with all paths leading directly to the steady state as depicted in Figures 1 and 2, without spiraling around it. If it is negative the two roots are complex conjugates and the equilibrium is a stable focus with all paths spiraling around and towards it. It is evident that the two exogenous speeds of adjustment $\alpha$ and $\beta$ play a crucial role in determining whether the equilibrium is a node or a focus.

The discriminant may be expressed as

$$\text{disc}(\Gamma) = \alpha^2 [(A - \gamma D)^2 + 4\gamma BC] \equiv \alpha^2 f(\gamma),$$

where we have defined $\gamma \equiv \beta/\alpha$. Its sign depends solely on the sign of $f(\gamma)$. Notice that $f(0) = A^2$, and $f'(\gamma) = 0$ if and only if

$$\gamma = \frac{1}{D} \left( A - 2 \frac{BC}{A} \right) \equiv \gamma > 0.$$  

\(^{13}\)We leave out the degenerate case in which the discriminant is zero.
Also,
\[ f(\gamma) = 4 \frac{BC}{D} \left( A - \frac{BC}{D} \right) < 0. \]

Hence, whenever \( \gamma \) is within an open neighborhood of \( \gamma \) the discriminant is negative and whenever it is outside of this neighborhood the discriminant is positive. Moreover, that open interval is \( (\gamma_1, \gamma_2) \), where \( \gamma_1 = \gamma - \tau \), \( \gamma_1 = \gamma + \tau \), and
\[ \tau \equiv \frac{2}{D^2} \left[ BC \left( BC - AD \right) \right]^{1/2}. \]

Hence, if the ratio of the two speeds of adjustment is sufficiently small (within \((0, \gamma_1)\)) or sufficiently large (within \((\gamma_2, \infty)\)) the equilibrium is a stable node, as in Figures 1 and 2, respectively. However, if the ratio is near \( \gamma \) (within \((\gamma_1, \gamma_2)\)) the equilibrium is a stable focus.

4) The equilibrium paths of \( P_N \) and \( W \), given the steady state levels of \( \zeta_K \) and \( \zeta_L \)

Given \( \mathcal{S} \), let \( \zeta^{-1}, \zeta_L^{-1} \), \( \bar{P}_N \) and \( \bar{W} \) be the unique steady state of the dynamical system. Then the explicit solution paths for \( P_N(t) \) and \( W(t) \) are given by:
\[
\begin{align*}
P_N(t) - \bar{P} & = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \\
W(t) - \bar{W} & = c_1 \frac{\lambda_1 - \alpha A}{\alpha B} e^{\lambda_1 t} + c_2 \frac{\lambda_2 - \alpha A}{\alpha B} e^{\lambda_2 t}
\end{align*}
\]
where in this context \( e \) is the constant 2.718... (not the real exchange rate), and \( c_1 \) and \( c_2 \) are constants that depend on the initial conditions. Also,
\[
\begin{bmatrix}
1 \\
\lambda_1 - \alpha A \\
\lambda_2 - \alpha A
\end{bmatrix}
\]
are the eigenvectors corresponding to \( \lambda_1 \) and \( \lambda_2 \), when they are normalized such that the first element is unity (assuming it is not zero) (see Bellman (1960)). Given any initial point \((W(0), P_N(0)) \equiv (W^0, P_N^0)\), it is readily verified that the constants must be:
\[
\begin{align*}
c_1 & = \frac{\alpha A - \lambda_2}{\lambda_2 - \lambda_1} \left( \bar{P} - P^0 \right) + \frac{\alpha B}{\lambda_2 - \lambda_1} \left( \bar{W} - W^0 \right) \\
c_2 & = \frac{\lambda_1 - \alpha A}{\lambda_2 - \lambda_1} \left( \bar{P} - P^0 \right) - \frac{\alpha B}{\lambda_2 - \lambda_1} \left( \bar{W} - W^0 \right)
\end{align*}
\]

Hence, the equilibrium paths starting from any initial point are given by inserting these expressions in (66) and (67).

Let us take the case with real and distinct roots, which we have seen are both negative. According to (65), \( \lambda_2 < \lambda_1 < 0 \). Hence,
\[
\begin{align*}
P_N(t) - \bar{P} & = \frac{c_1 + c_2 e^{(\lambda_2 - \lambda_1)t}}{c_1 \frac{\lambda_1 - \alpha A}{\alpha B} + c_2 \frac{\lambda_2 - \alpha A}{\alpha B} e^{(\lambda_2 - \lambda_1)t}} \to \frac{\alpha B}{\lambda_1 - \alpha A} \text{ as } t \to \infty.
\end{align*}
\]

Hence, the path of \((W(t), P_N(t))\) tends towards the long run equilibrium point \((\bar{W}, \bar{P}_N)\) by approaching asymptotically the line that represents the (normalized) eigenvector that corresponds to the least negative eigenvalue \( \lambda_1 \) in a \((W, P_N)\) plane (call it the dominant eigenvector). Furthermore, since \( B > 0 \) and \( A < 0 \), the sign of the slope of the dominant eigenvector is the sign of \( \lambda_1 - \alpha A \), which may be positive or negative, as depicted in Figures 1 and 2, respectively.
Appendix 2: The dynamic equilibrium concept used in relation to the literature

The dynamic equilibrium concept we use apparently has its roots in Lange (1944), who used the term ‘disequilibrium’ to refer to situations in which price setting agents in a context of monopolistic competition were selling as much as was demanded at the price set but not maximizing utility or profits. His Appendix shows that he had an explicit dynamical system in mind that had profit and utility maximizing in the steady state. However, a clear mathematical way of modeling monopolistic competition was yet decades away. He states:

The nature of economic equilibrium, as well as disequilibrium, in a monopolistic...market differs from that in a perfectly competitive market. In the latter, disequilibrium consists in excess demand or excess supply. Monopolistic supply, however, is always equal to demand for the good in question....A monopolistic...market is in equilibrium when the quantity sold and bought is such that it maximizes the profit of the monopolist....In this case, there is no tendency to change either price or quantity. Disequilibrium occurs in a monopolistic...market when the quantity sold and bought differs from the equilibrium amount. When a monopolist sells more than the equilibrium amount, he will restrict his supply and raise his price. The reverse happens when he sells less than the equilibrium amount. We shall denote these cases monopolistic underrestriction and overrestriction of supply, respectively....Thus monopolistic underrestriction...perform(s) the function of excess demand under perfect competition, while monopolistic overrestriction...perform(s) the function of excess supply.14

Nikaido (1975) uses a similar concept, also inspired in Lange (1944), although within a basically static Leontief model. He proposes a dynamical system that has some similarity to the one used here but does not study its stability properties. Nikaido and Kobayashi (1978) do study an explicitly dynamic model of wage-price spirals within the Leontief-Sraffa framework, where "capitalists save but do not consume while workers consume but do not save". In that paper, the price adjustment depends on a fixed speed of adjustment towards the Leontief-Sraffa prices while the wage adjustment is asymmetric, with no downward adjustment, and upward wage adjustments that make the rate of wage inflation proportional to price inflation. The present analysis has asymmetric adjustments both for nontradable prices and wages. Also, both speeds of adjustment are exogenous in this paper whereas Nikaido and Kobayashi make the upwards wage speed of adjustment endogenous by making it vary inversely with the real wage.

The framework we use is more directly related to the modern formalizations of monopolistic competition by Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987), although in an open economy dynamic optimizing framework, where the optimal price or wage is only obtained in the steady state because in the interim an ‘ad hoc’, or ‘rule of thumb’, pricing policy is used. In Nikaido and Kobayashi

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14Ibid. p. 35. All the missing parts relate to monopsonistic competition, which we have omitted to make the reading easier.
the price adjustments are related to gaps in profit (with respect to a ‘normal’ rate of profit) and wage adjustments are proportional to price adjustments, whereas our approach is closer to that of Lange (1944) in his Appendix, but with significant differences. Lange (1944) makes price adjustments depend on ‘marginal gain functions’, which give the marginal increase in profit or utility, respectively, upon a change in price, whereas we have adjustments that are proportional to ‘gaps’ that measure the excess of ‘effective’ output and employment with respect to ‘potential’ levels, where the latter are defined as the profit maximizing level of output in the nontradable sector and the utility maximizing level of labor supply, respectively. Hence, although our steady state coincides with that of Lange, the transition differs.

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