Idiosyncratic measurement error in a simple RBC model

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1 Introduction

In a world where expectations matter, both the kind and quantity of information that households have available and how they process that information to form their expectations matter in determining the equilibrium. In Lucas's famous paper [6], individuals receive information that is a mixture of private and aggregate signals in period t and they are not able to completely separate the two period t signals. Expectations are rational and all past, period $t-1$ and before, information is available for estimating the projection equations. The difficulties in separating private from aggregate signals can result in equilibrium paths with a Phillips curve like response to monetary shocks.

Following on Lucas (and on Phelps [9] before him), a substantial literature has developed on inperfect information and macroeconomics. One branch of this literature has concentrated on the effects of measurement error, especially in the publication of aggregate data. Collard and Dellas [2], Collard, Dellas and Smets [3], and Lorenzoni [5] are some recent examples of work in dynamic macroeconomics associated with imperfect information. In these papers, the imperfect information is a common measurement error in public information (although Lorenzoni includes correct private information about idiosyncratic production).

What is different about this paper from those mentioned above is that here the measurement error is idiosyncratic: the measurement error that affects each household's observation of the data is an iid drawing from the error distribution. The mean of the measurement error is zero in each period so that on average, household information is correct. However, the measurement error adds additional variance to the variables observed by the households.

The additional variance matters because households make OLS forecasts of the expectational variables they need and use a long series of their own observations to estimate the coefficients of a linear projection model for each variable. As is well known, the existence of measurement error introduces bias into the estimation.

2 The model

The model used in this paper is a slight variant on Hansen's [4] RBC model with divisible labor. The basic form of the model is the same, but instead of assuming that households have rational expectations, households use OLS forecasting models to get the expected values for the period $t+1$ variables they need for their period t decisions. What makes the model special is that households observe aggregate data with measurement error. As is well known, measurement error implies that there is a bias in an OLS forecasting model.

There are a unit mass of households, each of whom maximizes the utility function

$$E_t^j \sum_{i=0}^{\infty} \beta^i \left(\ln c_{t+i}^j + A \ln \left(1 - h_{t+i}^j \right) \right)$$

subject to the budget constraint

$$k_{t+1}^j = w_t h_t^j + (r_t + 1 - \delta) k_t^j - c_t^j + \chi_t^j,$$

where χ_t^j is a lump sum insurance transfer or payment that makes $k_{t+1}^j = K_{t+1}$ for all j. Production is competitive and the production function is Cobb-Douglas.

The Hansen RBC model can be written in terms of the following five equations (see McCandless [8], Section 6.1):

$$\begin{aligned} \frac{1}{\beta c_t^j} &= E_t \frac{r_{t+1}^j + 1 - \delta}{c_{t+1}^j}, \\ (1-\theta) \frac{Y_t}{H_t} &= \frac{A C_t}{1 - H_t}, \\ K_{t+1} + C_t &= Y_t + (1-\delta) K_t, \\ Y_t &= \lambda_t K_t^{\theta} H_t^{1-\theta}, \\ r_t &= \theta \frac{Y_t}{K_t}. \end{aligned}$$

Here, Y_t is aggregate output, c_t^j is household j's consumption, h_t^j is household j's labor supply, K_t is total capital, r_t is the rental rate on capital and λ_t is the technology shock. The lump sum taxes and transfers χ_t are made at the end of the period so that every family begins the next period with the same capital stock. Parameters are the discount rate, β, the depreciation rate, δ, the coefficient on log leisure in the utility function, A, and the fraction of income that goes to capital, θ. The first two equations are the first order conditions. The first equation needs to be household specific since it contains household j's forecasts of r_{t+1}^j and c_{t+1}^j which depend on household j's observations of the aggregate variables and the coefficients that household j is using in its OLS forecasting equation. The third equation is the aggregate budget constraint[1], the fourth is the production function and the fifth equation gives the conditions of the competitive factor market in capital.

2.1 Expectation formation via OLS learning in a stochastic stationary state

Instead of the more customary rational expectation, we assume that each household j forecasts future consumption and interest rates using the OLS equations,

$$c_{t+1}^{j} = \varphi_{11} k_{t+1}^{j} + \varphi_{12} y_{t}^{j}$$

and

$$r_{t+1}^{j} = \varphi_{21} k_{t+1}^{j} + \varphi_{22} y_{t}^{j}$$

where the households see each of the aggregate variables in the economy with a measurement error, $\varepsilon_t^{j,x}$, with respect to the aggregate value of that variable. The values of the exogenous variables in the OLS equation that household j sees are

$$k_{t+1}^{j} = K_{t+1} + \varepsilon_t^{j,k}$$

and

$$y_t^j = Y_t + \varepsilon_t^{j,y}.$$

The individual shocks $\varepsilon_t^{j,k}$ and $\varepsilon_t^{j,y}$ are independent across time and across households and have means of zero and variances of $\sigma_x^2 \geq 0$. The values of the variances of the measurement error are not known to the households. Combining these definitions gives a representation for household j's forecast of time $t+1$ consumption and interest rates of

$$\begin{aligned} c_{t+1}^{j} &= \varphi_{11}\left(K_{t+1} + \varepsilon_t^{j,k}\right) + \varphi_{12}\left(Y_t + \varepsilon_t^{j,y}\right), \\ r_{t+1}^{j} &= \varphi_{21}\left(K_{t+1} + \varepsilon_t^{j,k}\right) + \varphi_{22}\left(Y_t + \varepsilon_t^{j,y}\right). \end{aligned}$$

Estimating the parameters of these forecasting equations with OLS, household j uses the data c_s^j, r_s^j k_s^j, and y_{s-1}^j that it has observed for all dates $s \leq t$. Define

$$X = \begin{bmatrix} k_s^j & y_{s-1}^j \end{bmatrix} = \begin{bmatrix} \left(K_s + \varepsilon_s^{j,k}\right) & \left(Y_{s-1} + \varepsilon_{s-1}^{j,y}\right) \end{bmatrix}$$

and

$$Y = \begin{bmatrix} c_s^j & r_s^j \end{bmatrix} = \begin{bmatrix} \left(C_s + \varepsilon_s^{j,c}\right) & \left(r_s + \varepsilon_s^{j,r}\right) \end{bmatrix}.$$

The parameters

$$\Phi = \begin{bmatrix} \varphi_{11} & \varphi_{21} \\ \varphi_{12} & \varphi_{22} \end{bmatrix}$$

[1] The χ_t^j drop out because $\int_0^1 \chi_t^j dj = 0$.

are found from

$$\Phi = \left(X'X\right)^{-1} X'Y.$$

This equation can be written as

$$\begin{aligned}
\left(X'X\right)^{-1} X'Y &= \left(\begin{bmatrix} \left(K_s + \varepsilon_s^{j,k}\right) \\ \left(Y_{s-1} + \varepsilon_{s-1}^{j,y}\right) \end{bmatrix} \begin{bmatrix} \left(K_s + \varepsilon_s^{j,k}\right) & \left(Y_{s-1} + \varepsilon_{s-1}^{j,y}\right) \end{bmatrix} \right)^{-1} \times \\
& \begin{bmatrix} \left(K_s + \varepsilon_s^{j,k}\right) \\ \left(Y_{s-1} + \varepsilon_{s-1}^{j,y}\right) \end{bmatrix} \begin{bmatrix} \left(C_s + \varepsilon_s^{j,c}\right) & \left(r_s + \varepsilon_s^{j,r}\right) \end{bmatrix},
\end{aligned}$$

or as

$$\begin{aligned}
\left(X'X\right)^{-1} X'Y &= \begin{bmatrix} E\left(K_s + \varepsilon_s^{j,k}\right)^2 & E\left(K_s + \varepsilon_s^{j,k}\right)\left(Y_{s-1} + \varepsilon_{s-1}^{j,y}\right) \\ E\left(K_s + \varepsilon_s^{j,k}\right)\left(Y_{s-1} + \varepsilon_{s-1}^{j,y}\right) & E\left(Y_{s-1} + \varepsilon_{s-1}^{j,y}\right)^2 \end{bmatrix}^{-1} \times \\
& \begin{bmatrix} E\left(K_s + \varepsilon_s^{j,k}\right)\left(C_s + \varepsilon_s^{j,c}\right) & E\left(K_s + \varepsilon_s^{j,k}\right)\left(r_s + \varepsilon_s^{j,r}\right) \\ E\left(Y_{s-1} + \varepsilon_{s-1}^{j,y}\right)\left(C_s + \varepsilon_s^{j,c}\right) & E\left(Y_{s-1} + \varepsilon_{s-1}^{j,y}\right)\left(r_s + \varepsilon_s^{j,r}\right) \end{bmatrix}.
\end{aligned}$$

Since the error terms are independent across time and households, the above expression can be simplified to

$$\begin{aligned}
\Phi &= \left(X'X\right)^{-1} X'Y \\
&= \begin{bmatrix} K_s^2 + \sigma_{\varepsilon_k}^2 & K_s Y_{s-1} \\ K_s Y_{s-1} & Y_{s-1}^2 + \sigma_{\varepsilon_y}^2 \end{bmatrix}^{-1} \begin{bmatrix} K_s C_s & K_s r_s \\ Y_{s-1} C_s & Y_{s-1} r_s \end{bmatrix}.
\end{aligned}$$

With a long enough time series[2], all households end up with the same values for Φ. Since the $X'X$ matrix is only 2×2, it can be inverted exactly and after a bit of substitution, along with the assumption that the variances are proportionally the same for all variables, that $\sigma_{\varepsilon_k}^2 = K^2 \sigma_\varepsilon^2$ and $\sigma_{\varepsilon_y}^2 = Y^2 \sigma_\varepsilon^2$, one gets stationary state parameters for the forecasting equations of

$$\Phi = \begin{bmatrix} \frac{\overline{C}}{(2+\sigma_\varepsilon^2)\overline{K}} & \frac{\overline{r}}{(2+\sigma_\varepsilon^2)\overline{K}} \\ \frac{\overline{C}}{(2+\sigma_\varepsilon^2)\overline{Y}} & \frac{\overline{r}}{(2+\sigma_\varepsilon^2)\overline{Y}} \end{bmatrix}.$$

The first equation of the model,

$$\frac{1}{\beta c_t^j} = E_t \frac{r_{t+1}^j + 1 - \delta}{c_{t+1}^j},$$

does not aggregate simply. To write out the individual decision process, we

[2] That is, asymptotically.

replace c_{t+1}^j and r_{t+1}^j with their forecasting equations and get

$$\begin{aligned}
\frac{1}{\beta c_t^j} &= E_t \frac{r_{t+1}^j + 1 - \delta}{c_{t+1}^j}, \\
&= \frac{\varphi_{21} k_{t+1}^j + \varphi_{22} y_t^j + 1 - \delta}{\varphi_{11} k_{t+1}^j + \varphi_{12} y_t^j} \\
\frac{1}{\beta c_t^j} &= \frac{\varphi_{21}\left(K_{t+1} + \varepsilon_t^{j,k}\right) + \varphi_{22}\left(Y_t + \varepsilon_t^{j,y}\right) + 1 - \delta}{\varphi_{11}\left(K_{t+1} + \varepsilon_t^{j,k}\right) + \varphi_{12}\left(Y_t + \varepsilon_t^{j,y}\right)}.
\end{aligned}$$

Both sides of the last equation can be inverted and we get

$$\beta c_t^j = \frac{\varphi_{11}\left(K_{t+1} + \varepsilon_t^{j,k}\right) + \varphi_{12}\left(Y_t + \varepsilon_t^{j,y}\right)}{\varphi_{21}\left(K_{t+1} + \varepsilon_t^{j,k}\right) + \varphi_{22}\left(Y_t + \varepsilon_t^{j,y}\right) + 1 - \delta}.$$

Aggregating[3] the left hand side of the above equation gives simply βC_t, but the right hand side is very difficult to aggregate. Since we cannot do it directly, we take a second order Taylor expansion of the right hand side and integrate that. Integrating the Taylor expansion gives the equation

$$\begin{aligned}
\beta C_t = & \frac{EC_{t+1}}{Er_{t+1} + 1 - \delta} \\
& + \varphi_{21} \frac{\varphi_{21} EC_{t+1} - \varphi_{11}\left(Er_{t+1} + 1 - \delta\right)}{\left(Er_{t+1} + 1 - \delta\right)^3} \sigma_{\varepsilon_k}^2 \\
& + \varphi_{22} \frac{\varphi_{22} EC_{t+1} - \varphi_{12}\left(Er_{t+1} + 1 - \delta\right)}{\left(Er_{t+1} + 1 - \delta\right)^3} \sigma_{\varepsilon_y}^2.
\end{aligned}$$

Replacing the coefficients of the OLS equation for what they are equal, assuming that the variances of the two shocks have the same value, and simplifying, one gets

$$\beta \overline{C} = \frac{\frac{2}{2+\sigma_\varepsilon^2} \overline{C}}{\frac{2}{2+\sigma_\varepsilon^2} \overline{r} + 1 - \delta} - 2 \frac{\frac{\overline{C}\,\overline{r}}{(2+\sigma_\varepsilon^2)^2}(1-\delta)}{\left(\frac{2}{2+\sigma_\varepsilon^2} \overline{r} + 1 - \delta\right)^3} \sigma_\varepsilon^2$$

Cancelling out $\overline{C}$, gives an implicit function for the stationary state value of $\overline{r}$ in terms of the parameters of the model and the stationary state values of $\overline{Y}$ and $\overline{K}$,

$$\beta = \frac{\frac{2}{2+\sigma_\varepsilon^2}}{\frac{2}{2+\sigma_\varepsilon^2} \overline{r} + 1 - \delta} - 2 \frac{\frac{\overline{r}}{(2+\sigma_\varepsilon^2)^2}(1-\delta)}{\left(\frac{2}{2+\sigma_\varepsilon^2} \overline{r} + 1 - \delta\right)^3} \sigma_\varepsilon^2 \qquad (1)$$

Notice that if $\sigma_\varepsilon^2 = 0$, this equation becomes the standard expression for this first order condition.

[3] Where $\beta C_t = \beta \int_0^1 c_t^j dj$.

$\overline{K} = 12.6695,$
$\overline{Y} = 1.2353,$
$\overline{C} = .9186,$
$\overline{H} = .3335,$
$\overline{r} = .0351.$

Table 1: Stationary state values without measurement error

2.2 The stochastic stationary state

The five equations for the stationary state are Equation 1 and the four equations

$$\begin{aligned} (1-\theta)\frac{\overline{Y}}{\overline{H}} &= \frac{A\overline{C}}{1-\overline{H}}, \\ \overline{C} &= \overline{Y} - \delta\overline{K}, \\ \overline{Y} &= \overline{K}^{\theta}\overline{H}^{1-\theta}, \qquad (2) \\ \overline{r} &= \theta\frac{\overline{Y}}{\overline{K}}. \qquad (3) \end{aligned}$$

Hansen's example economy used the parameter values $\beta = .99$, $\delta = .025$, $\theta = .36$, and $A = 1.72$. In the stationary state there are no technology shocks and $\lambda_t = 1$. Solving for the stationary state with zero measurement error, $\sigma_\varepsilon^2 = 0$, the stationary state values for the aggregate variables are the standard ones and these are shown in Table 1.

The calculated[4] stationary states for the same economy but with measurement error are shown in Figures 1 and 2. Larger variances of the measurement error, σ_ε^2, result in stationary states with larger values of $\overline{K}$, $\overline{Y}$, $\overline{C}$, and $\overline{H}$ and lower values for $\overline{r}$. Notice that in Figure 2, as the measurement error increases the expected values for EC_{t+1} and Er_{t+1} are progressively smaller than the realized stationary state values for those variables. This is the bias introduced by the idiosyncratic measurement error.

It is well known in econometrics that the kind of measurement error that we posit here generates a downward bias in the coefficients of OLS estimates. The bias results in a first order equation , Equation 1, where the new second term on the right hand side is negative and, to get the equation to balance, the equilibrium interest rate must be lower with measurement error than without. The rental equation (Equation 3) must hold so capital must rise relative to output and since the coefficient on capital in the production function (θ in Equation 2) is less than 1, the new equilibrium occurs with both capital and output higher than in the no measurement error case.

It may seem strange that consumption and output are higher in an economy where individuals have noisy information about the state of the economy than when they have correct information. However, there are some cases where policy

[4] All calculations shown here are from Matlab.

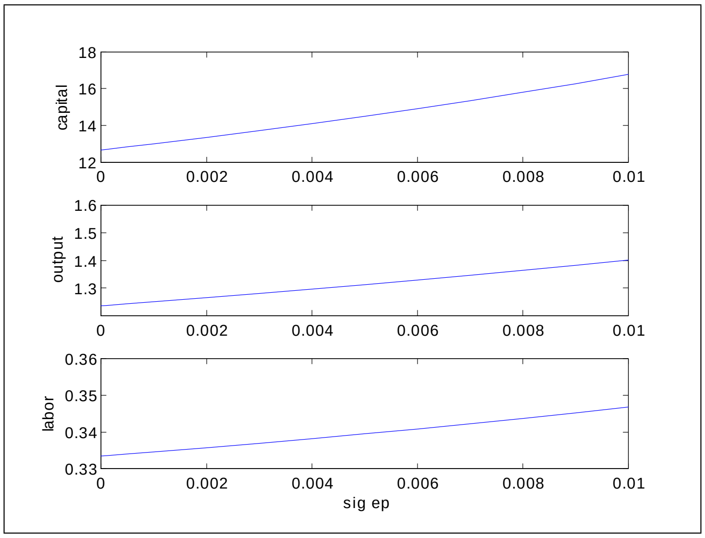

Figure 1: Stationary state values for $\overline{K}$, $\overline{Y}$, and $\overline{H}$ as a function of the measurement error

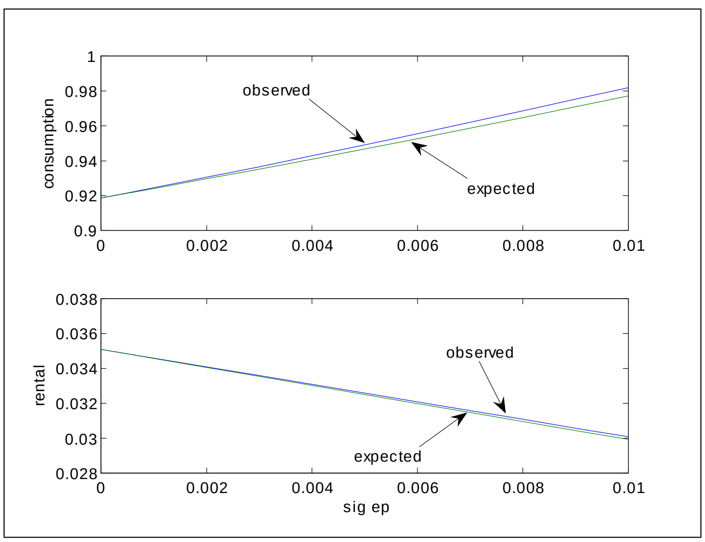

Figure 2: Stationary state values for $\overline{C}$, $\overline{r}$, EC_{t+1}, and Er_{t+1} as function of measurement error

makers prefer not to give the public good information about what they are doing and therefore worsening the ability of the public to predict future economic variables. In monetary and exchange rate policy this is called constructive ambiguity. In the model presented here, economies with greater "constructive ambiguity" have higher output, consumption, and utility (not shown) than economies with full information.

3 The dynamic version of the model

The dynamics of the model are analyzed using a log-linear approximation of the original model. The log-linear version of the basic model is

$$\begin{aligned}
0 &= \widetilde{c}_t^j - E_t\widetilde{c}_{t+1}^j + \beta\overline{r}E_t\widetilde{r}_{t+1}^j, \\
0 &= \widetilde{Y}_t - \frac{\widetilde{h}_t^j}{1-\overline{H}} - \widetilde{c}_t^j, \\
0 &= \overline{Y}\widetilde{Y}_t - \overline{C}\widetilde{C}_t + \overline{K}\left[(1-\delta)\widetilde{K}_t - \widetilde{K}_{t+1}\right], \\
0 &= \widetilde{\lambda}_t + \theta\widetilde{K}_t + (1+\theta)\widetilde{H}_t - \widetilde{Y}_t, \\
0 &= \widetilde{Y}_t - \widetilde{K}_t - \widetilde{r}_t,
\end{aligned}$$

and the technology shock follows the stochastic process

$$\widetilde{\lambda}_t = \gamma\widetilde{\lambda}_{t-1} + \widetilde{\varepsilon}_t^\lambda. \tag{4}$$

Because of measurement error, household j observes each log-linear aggregate variable $\widetilde{X}_t$ as

$$\widetilde{X}_t^j = \widetilde{X}_t + \mu_t^{X,j},$$

where $\widetilde{X}_t$ are the realized values and the errors $\mu_t^{X,j}$ are iid across households and across time. This characterization of the shocks comes from the linear approximation of the individual variables and from measurement error that depends on the stationary state size of the variable. We define

$$X_t^j = \overline{X}e^{\widetilde{X}_t^j} \approx \overline{X}\left(1+\widetilde{X}_t^j\right) = \overline{X}\left(1+\widetilde{X}_t\right) + \overline{X}\mu_t^{X,j}.$$

This can be simplified to

$$\overline{X}\widetilde{X}_t^j = \overline{X}\widetilde{X}_t + \overline{X}\mu_t^{x,j},$$

or

$$\widetilde{X}_t^j = \widetilde{X}_t + \mu_t^{X,j}.$$

Although each variable has an iid shock, we assume that the variance of measurement error in the log-linear version of the model is the same for all variables, σ_μ^2.

Households forecast $E_t\widetilde{c}^j_{t+1}$ and $E_t\widetilde{r}^j_{t+1}$ using the OLS equation

$$\begin{bmatrix} E_t\widetilde{C}_{t+1} & E_t\widetilde{r}_{t+1} \end{bmatrix} = \begin{bmatrix} \varphi(t-1)_{11} & \varphi(t-1)_{21} \\ \varphi(t-1)_{12} & \varphi(t-1)_{22} \end{bmatrix} \begin{bmatrix} \widetilde{K}^j_{t+1} & \widetilde{Y}^j_t \end{bmatrix},$$

where $\varphi(t-1)_{ik}$ denotes the time $t-1$ estimation of the coefficient of variable k in equation i. In each period t the estimation is updated at the end of the period using a recursive OLS algorithm[5] with the observations generated in period t. The updated coefficients are used for forecasting during period $t+1$. Since the measurement error is iid, all households end up (asymtotically, at least) with the same values for the coefficients of the forecasting equation.

As with the stationary states, measurement error causes the coefficients of the OLS estimation to be biased. Using OLS based on its available data, the households estimate

$$\Phi = \begin{bmatrix} \varphi(t-1)_{11} & \varphi(t-1)_{21} \\ \varphi(t-1)_{12} & \varphi(t-1)_{22} \end{bmatrix}$$

as

$$\Phi = \left(\left(X^j\right)' X^j \right)^{-1} \left(X^j\right)' Y^j \tag{5}$$

with $X^j = \begin{bmatrix} \left\{\widetilde{K}^j_t\right\} & \left\{\widetilde{Y}^j_{t-1}\right\} \end{bmatrix}$ and $Y^j = \begin{bmatrix} \left\{\widetilde{C}_t\right\} & \left\{\widetilde{r}_t\right\} \end{bmatrix}$. Because the measurement errors are iid, equation 5 is equal to

$$\Phi = \left((X)' X + \begin{bmatrix} \sigma^2_{\mu^K} & 0 \\ 0 & \sigma^2_{\mu^Y} \end{bmatrix} \right)^{-1} (X)' Y$$

or

$$\Phi = \left(\begin{bmatrix} var\widetilde{K} & cov\widetilde{K}\widetilde{Y} \\ cov\widetilde{K}\widetilde{Y} & var\widetilde{Y} \end{bmatrix} + \begin{bmatrix} \sigma^2_{\mu^K} & 0 \\ 0 & \sigma^2_{\mu^Y} \end{bmatrix} \right)^{-1} \begin{bmatrix} cov\widetilde{K}\widetilde{C} & cov\widetilde{K}\widetilde{r} \\ cov\widetilde{Y}\widetilde{C} & cov\widetilde{Y}\widetilde{r} \end{bmatrix}. \tag{6}$$

Since the measurement errors have the same variance for every household, all households have the same value for the additional variance, $\sigma^2_{\mu^K}$ and $\sigma^2_{\mu^Y}$, in their OLS equations and all get the same Φ.

[5] The equation for the recursive OLS parameter estimate at date t is

$$\varphi_t = \varphi_{t-1} + \frac{P_{t-1}x'_t}{1 + x_t P_{t-1} x'_t} \left(y_t - x_t\varphi_{t-1}\right)$$

with the updating rule for P_t is

$$P_t = \left[I - \frac{P_{t-1}x'_t}{1 + x_t P_{t-1} x'_t} x_t \right] P_{t-1}.$$

The version of recursive estimation that is used in the paper has decreasing gain so all data points are given the same weight.

3.1 Solving the dynamic learning model

To solve the learning version of the Hansen model, one needs to write out it in a state space format. The variables to be determined at date t are

$$x_t = \begin{bmatrix} K_{t+1} \\ H_t \\ Y_t \\ C_t \\ r_t \\ E_t\widetilde{C}_{t+1} \\ E_t\widetilde{r}_{t+1} \\ \widetilde{\lambda}_t \end{bmatrix}.$$

The model is comprised of two parts. The state space version of the log-linear model, written as

$$A_t\left(\Phi_{t-1}\right)x_t = B_t\left(\Phi_{t-1}\right)x_{t-1} + C\varepsilon_t,$$

and the updating equation for the parameters of the least squares forecasting equations, which can be given as

$$\begin{bmatrix} \Phi_t \\ P_t \end{bmatrix} = G\left(\begin{bmatrix} \Phi_{t-1} \\ P_{t-1} \end{bmatrix}, x_t\right).$$

Data from period t is used to update both the parameters in the forecasting equation and the variance [6] The matrix A is written as $A_t\left(\Phi_{t-1}\right)$ to remind us that it has parameters in the least squares updating equation that were found using the data up to period $t-1$. If the matrix $A_t\left(\varphi_{t-1}\right)$ is invertable, then the state space model can be solved directly in each period as

$$x_t = \left[A_t\left(\Phi_{t-1}\right)\right]^{-1}B_t\left(\Phi_{t-1}\right)x_{t-1} + \left[A_t\left(\Phi_{t-1}\right)\right]^{-1}C\varepsilon_t.$$

[6] The equation for the recursive OLS parameter estimate at date t is

$$\varphi_t = \varphi_{t-1} + \frac{P_{t-1}x_t'}{1 + x_tP_{t-1}x_t'}\left(y_t - x_t\varphi_{t-1}\right)$$

with the updating rule for P_t is

$$P_t = \left[I - \frac{P_{t-1}x_t'}{1 + x_tP_{t-1}x_t'}x_t\right]P_{t-1}.$$

This is the version with decreasing gain (where all data points are given the same weight).

The matrices A_t, B_t, and C_t for the log-linear version of the Hansen economy are

$$A_t = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -1 & \beta\overline{r} & 0 \\ 0 & -\frac{1}{1-\overline{H}} & 1 & -1 & 0 & 0 & 0 & 0 \\ -\overline{K} & 0 & \overline{Y} & -\overline{C} & 0 & 0 & 0 & 0 \\ 0 & 1+\theta & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ -\varphi_{11}^{1}(t-1) & 0 & -\varphi_{12}^{1}(t-1) & 0 & 0 & 1 & 0 & 0 \\ -\varphi_{21}^{1}(t-1) & 0 & -\varphi_{22}^{1}(t-1) & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B_t = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(1-\delta)\overline{K} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma \end{bmatrix},$$

and

$$C = \left[\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]'.$$

3.2 Results of dynamic version of model

Because constant gain OLS can be very slow to converge, the economy is run for 40,000 periods with a forgetting factor of .99999 and then run again using the average values for Φ over the last 20,000 periods as a new starting point. The economy is run twice for 40,199 periods beginning with the coefficients found above but without forgetting (with the forgetting factor = 1). In both of these runs, the same normally distributed shocks are applied to the economy except that in period 40,001 of the second running, an additional impulse of .1 is applied to the technology shock. The impulse response function to a technology shock for this economy is found by subtracting the last 199 observations of the first run without forgetting from the second run.

As was shown in the section on stochastic stationary states, the stochastic stationary state around which we are studying the dynamics depends on the variance of the measurement error. For the example economy shown here, I use a variance of measurement error for each variable X equal to $\sigma_X^2 = .001 \cdot \overline{X}^2$. The stationary state values for the aggregate variables in the example economy with this variance for the measurement error are given in Table 2.

The only source of aggregate movements in the simple Hansen economy is from the stochastic process for technology, from $\widetilde{\varepsilon}_t^{\lambda}$ in equation 4. The stochastic process for technology, $\widetilde{\lambda}_t$, and the stock of capital, $\widetilde{K}_t$, are the state variables for the economy. The coefficient γ in equation 4 determines the persistence of

$\overline{K}$	13.0032
$\overline{H}$	0.3346
$\overline{Y}$	1.2496
$\overline{C}$	0.9245
$\overline{r}$	0.0346

Table 2: Aggregate stationary state values

τ	10	1	.1	.01	.001	.0001
φ_{11}	0.4989	0.4856	0.4295	0.3258	0.2260	0.1978
φ_{12}	0.2892	0.2763	0.2698	0.2053	0.1163	0.0914
φ_{21}	-1.0079	-0.9936	-0.8935	-0.6341	-0.4987	-0.5149
φ_{22}	0.9702	0.9583	0.8572	0.5134	0.2101	0.1088

Table 3: Forecasting equation coefficients

technology shocks in technology and is a constant. The solution of the log-linear version of the model gives each variable $\widetilde{X}_t$, including $\widetilde{K}_{t+1}$, as a linear function

$$\widetilde{X}_t = a_x \widetilde{K}_t + b_x \widetilde{\lambda}_t.$$

Therefore, the variance of every variable in the economy, including $\widetilde{K}_{t+1}$ is determined by the parameters a_x and b_x and the variance of $\widetilde{\lambda}_t$. For example, the variance of $\widetilde{K}_{t+1}$ is

$$\begin{aligned} var\left(\widetilde{K}_{t+1}\right) &= E\left(a_x \widetilde{K}_t + b_x \widetilde{\lambda}_t\right)^2 = a_x^2 var\left(\widetilde{K}_t\right) + b_x^2 var\left(\widetilde{\lambda}_t\right) \\ &= \frac{b_x^2}{1-a_x^2} var\left(\widetilde{\lambda}_t\right) = \frac{b_x^2}{1-a_x^2}\frac{1}{1-\gamma} var\left(\widetilde{\varepsilon}_t^{\lambda}\right) \end{aligned}$$

Given that changes in the measurement error changes the stationary state around which we analyze the economy, it is useful to keep the variance of the measurement error constant and consider a range of variances for the shock to the technology process. Define τ as the ratio of the variance of the shock to technology, $\sigma^2_{\varepsilon^\lambda}$, to the variance of the measurement error, σ^2_{μ}. Since one can consider σ^2_{μ} as noise in the economy relative to a technology signal, $\sigma^2_{\varepsilon^\lambda}$, the ratio $\tau = \sigma^2_{\varepsilon^\lambda}/\sigma^2_{\mu}$ is a kind of signal to noise ratio for the economy.

For an example economy with a variance of measurement error of .001, the coefficients of the forecasting equation for a range of variance for the technology shock from .01 to .00001 are shown in Table 3. The range of the signal to noise ratio is from 10 to .0001. The table indicates that as the signal to noise ratio declines, the trend is for the parameters in the OLS forecasting equation to become smaller in absolute value.

Impulse-response functions generated, as described above, from a .01 impulse to technology are shown in Figure 3. The line marked with λ is the path for technology and is the same in each graph. The order of the variables, from top

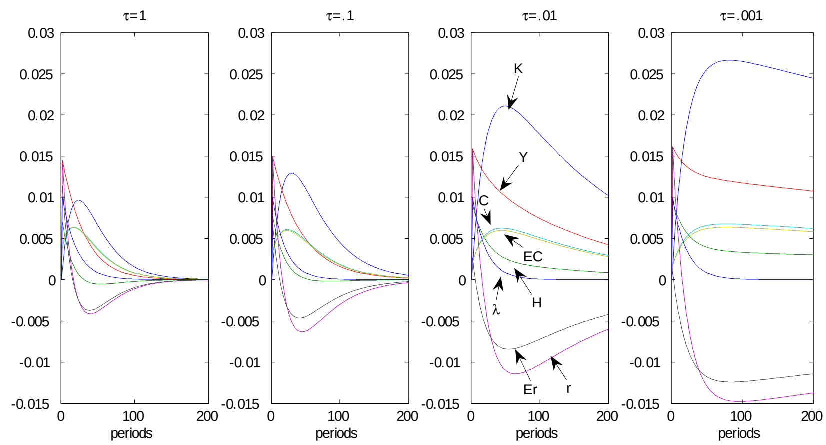

Figure 3: Impulse-response functions

to bottom, is the same in each graph. One very clear result is that as the size of the underlying variance in technology and the signal to noise ratio decline, both the size and the persistence of the responses of the variables in the economy to a technology impulse increase quite dramatically.

4 The value of public information

This model helps make clear the benefits of an honest government data collection and distriubtion office. What happens if a government collects a sample of family observations on the relevant data and distributes the means of these observations to all members of the economy. The government must do this data collection because there is no clear private return for doing so.

To see why the incentives are against private collection of the information, consider the situation of a member of the economy at time t. This member must pay some cost to collect observations from other members of the economy and calculate the average. By doing do, this member will have a better estimate of the values of the exogenous variables in the forecasting regression. However, this data provides no immediate use, either to the individual who collected it or to other members of the economy. Adding one better data point to the long history of exogenous data (or in a recursive OLS, adding one good data point) will not change the values of the estimated parameters. The private collector needs to acquire a long series of survey data and can sell that. Once the series is long enough, the benefits to collecting more go toward zero. Since private benefits come from collecting the data are small and the costs large, it is likely that no private collection will occur.

Add a government to the economy. The only thing that the government does is to survey a sample of the households and ask them for their observations on the variables in X. The government then averages the results of the survey and gives the information to all households.

The mean and variance of the sample taken by the government (given that the original measurement error is normally distributed with mean μ and variance σ_μ^2) tend to μ and σ_μ^2/n as $n \to \infty$ where n is the sample size. For a sufficiently large sample, the government can drive the difference of mean of the sample and the actual mean close to zero. With even a modest sample, the government published data will have a smaller variance than each family's private information.

In each period t, households use the data distributed by the government for their calculations of Φ_t, so the matrix $\begin{bmatrix} \sigma^2_{\mu^K} & 0 \\ 0 & \sigma^2_{\mu^Y} \end{bmatrix}$ is replaced by $\begin{bmatrix} \sigma^2_{\mu^K,g} & 0 \\ 0 & \sigma^2_{\mu^Y,g} \end{bmatrix}$ and as the sample grows, this matrix goes to the limit: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

There are two results of the government's information service. The stationary state values of the aggragate variables of the economy go towards the rational expectations values. This implies that stationary state output, consumption, capital, labor and utility will be smaller because of the data collection. So the economy is worse off in stationary states because of the data collection.

The government's data collection has the effect of raising the signal to noise ratio for all members of the economy. Reducing the signal to noise ration, reducing $\tau = \sigma^2_{\varepsilon^\lambda}/\sigma^2_\mu$, lowers the coefficient on lagged capital in the capital equation, reduces the eigenvalue associated with capital and reduces the persistance of technology shocks. While reducing persistance means that good shocks do not last as long, neither do negative shocks. In addition, the cycles in the economy are much smaller. Figure 4 shows simulated time paths for three values of τ using the same stochastic processes for technology and for measurement errors in each of the three economies. As is shown, very low signal to noise ratios can generate large and long cycles in the economy. If data collection by the government reduces the signal to noise ratio that the households face, it can reduce the amplitude and magnitude of the cycles in the economy.

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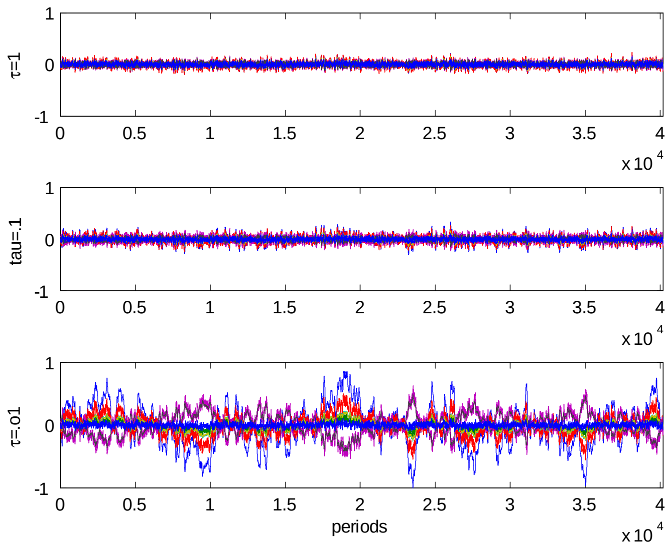

Figure 4: Time paths for economies with $\sigma^2_{\varepsilon^\lambda} = .0001$ and $\sigma^2_\mu = \{.0001, .001, .01\}$

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