

The importance of good public information

A model of learning with idiosyncratic measurement error

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The importance of information

- In economics, general assumption that more precise information is better
 - lets decision makers make better decisions (improve utility, profits, etc.)
 - this holds true for regulation of financial assets
 - lots of regulation is about what information must be provided
 - who can use what information (i.e. prohibitions of insider trading)
- Hirshleifer (AER 1971) distinguishes between
 - foreknowledge
 - discovery
 - and concentrates on the value of foreknowledge in exchange
- Differentiates between private (foreknowledge) and public knowledge
- finds that the social value (economic) of sure public information is zero
 - because since everyone has it and has no way to exploit it
 - people are identical
- private information has only limited value (limited by wealth)

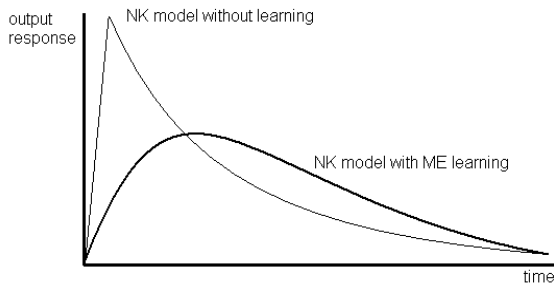
Lucas' model with imperfect information

- Robert Lucas (1975), JPE, An equilibrium model of the business cycle
- an islands model with different information on each island
- people can't distinguish perfectly between aggregate and individual signals
- so they respond to some aggregate price signal as if private
- Results: monetary injections generate a Phillips curve

- Learning (OLS learning)
 - Marcet and Sargent JET (1989): Learning and convergence to RE
 - relatively weak convergence result (in a neighborhood of RE equilibrium)
 - experience shows that convergence is more robust
 - Evans and Honkapohja, Princeton UP(2001), Learning and Expectations in Macroeconomics
 - major reference on this topic
- OLS learning in linear models (M&S)
- Some newer themes: escape dynamics: Noah Williams (2009)
 - when and how often economies with learning will move away from RE equilibrium
- The importance of forgetting
- Concept of self confirming equilibrium
 - with learning inside a model
 - an equilibrium where the projected values are (in distribution) correct
 - the particular equilibrium is a function of the type of learning

Measurement error and learning

- A literature on the importance of measurement error in published data
 - data revisions reflect the errors of the early reports
- Collard and Dellas (2004) and Smets (2009)
 - use learning inside a New Keynesian model
 - data used for learning has measurement errors
 - same errors for everyone (since it is the published information)
 - These models get slower response of prices and output to money injections



Idea of this paper

- Start with a very simple model (Hansen's RBC model)
- have households learn about aggregate values of variables with measurement error
- this measurement noise is idiosyncratic
- Each household estimates forecasting equations from noisy data
- Forecasting parameters are biased (well known fact about ME)
- Find Stationary state with idiosyncratic noise
- find dynamic model with idiosyncratic noise
- Explain how public information can return to RE equilibrium

The model (Hansen)

- Households max

$$E_t^j \sum_{i=0}^{\infty} \beta^i \left(\ln c_{t+i}^j + A \ln (1 - h_{t+i}^j) \right)$$

subject to

$$k_{t+1}^j = w_t h_t^j + (r_t + 1 - \delta) k_t^j - c_t^j + \chi_t^j,$$

- Production is competitive and the production function is Cobb-Douglas

$$Y_t = \lambda_t K_t^\theta H_t^{1-\theta}$$

- factor markets result in

$$r_t = \theta \frac{Y_t}{K_t}$$

The full model

- The Hansen RBC model can be written in terms of the following five equations :

$$\begin{aligned}\frac{1}{\beta c_t^j} &= E_t \frac{r_{t+1}^j + 1 - \delta}{c_{t+1}^j}, \\ (1 - \theta) \frac{Y_t}{H_t} &= \frac{AC_t}{1 - H_t}, \\ K_{t+1} + C_t &= Y_t + (1 - \delta)K_t, \\ Y_t &= \lambda_t K_t^\theta H_t^{1-\theta}, \\ r_t &= \theta \frac{Y_t}{K_t}.\end{aligned}$$

Forecasting equations (1)

- Households estimate the forecasting equations

$$c_{t+1}^j = \varphi_{11} k_{t+1}^j + \varphi_{12} y_t^j$$

and

$$r_{t+1}^j = \varphi_{21} k_{t+1}^j + \varphi_{22} y_t^j$$

- estimated with the noisy data that they have
- Household j has the data history comprised of

$$k_{t+1}^j = K_{t+1} + \varepsilon_t^{j,k}$$

and

$$y_t^j = Y_t + \varepsilon_t^{j,y}.$$

Forecasting equations (2)

- Define

$$X = \begin{bmatrix} k_s^j & y_{s-1}^j \end{bmatrix} = \begin{bmatrix} (K_s + \varepsilon_s^{j,k}) & (Y_{s-1} + \varepsilon_{s-1}^{j,y}) \end{bmatrix}$$

- and

$$Y = \begin{bmatrix} c_s^j & r_s^j \end{bmatrix} = \begin{bmatrix} (C_s + \varepsilon_s^{j,c}) & (r_s + \varepsilon_s^{j,r}) \end{bmatrix}.$$

- The parameters for the forecasting model are found from

$$\Phi = \begin{bmatrix} \varphi_{11} & \varphi_{21} \\ \varphi_{12} & \varphi_{22} \end{bmatrix}$$

- are found from

$$\Phi = (X'X)^{-1} X'Y.$$

Forecasting equations (3)

- This equation can be written as

$$\begin{aligned} & (X'X)^{-1} X'Y \\ = & \left(\begin{bmatrix} (K_s + \varepsilon_s^{j,k}) \\ (Y_{s-1} + \varepsilon_{s-1}^{j,y}) \end{bmatrix} \begin{bmatrix} (K_s + \varepsilon_s^{j,k}) & (Y_{s-1} + \varepsilon_{s-1}^{j,y}) \end{bmatrix} \right)^{-1} \\ & \times \begin{bmatrix} (K_s + \varepsilon_s^{j,k}) \\ (Y_{s-1} + \varepsilon_{s-1}^{j,y}) \end{bmatrix} \begin{bmatrix} (C_s + \varepsilon_s^{j,c}) & (r_s + \varepsilon_s^{j,r}) \end{bmatrix}, \end{aligned}$$

- Because the shocks are independent, this becomes

$$\begin{aligned} \Phi & = (X'X)^{-1} X'Y \\ & = \begin{bmatrix} K_s^2 + \sigma_{\varepsilon_k}^2 & K_s Y_{s-1} \\ K_s Y_{s-1} & Y_{s-1}^2 + \sigma_{\varepsilon_y}^2 \end{bmatrix}^{-1} \begin{bmatrix} K_s C_s & K_s r_s \\ Y_{s-1} C_s & Y_{s-1} r_s \end{bmatrix}. \end{aligned}$$

Forecasting equations (4)

- Since the $X'X$ matrix is only 2×2 , it can be inverted exactly and after a bit of substitution, along with the assumption that the variances are proportionally the same for all variables, that $\sigma_{\varepsilon_k}^2 = K^2\sigma_{\varepsilon}^2$ and $\sigma_{\varepsilon_y}^2 = Y^2\sigma_{\varepsilon}^2$, one gets stationary state parameters for the forecasting equations of

$$\Phi = \begin{bmatrix} \frac{\bar{C}}{(2+\sigma_{\varepsilon}^2)\bar{K}} & \frac{\bar{r}}{(2+\sigma_{\varepsilon}^2)\bar{K}} \\ \frac{\bar{C}}{(2+\sigma_{\varepsilon}^2)\bar{Y}} & \frac{\bar{r}}{(2+\sigma_{\varepsilon}^2)\bar{Y}} \end{bmatrix}.$$

Applying the forecasting equations to the model (1)

- The first equation of the model is

$$\frac{1}{\beta c_t^j} = E_t \frac{r_{t+1}^j + 1 - \delta}{c_{t+1}^j}$$

- Apply the equations for forecasting to get

$$\begin{aligned} \frac{1}{\beta c_t^j} &= E_t \frac{r_{t+1}^j + 1 - \delta}{c_{t+1}^j}, \\ &= \frac{\varphi_{21} k_{t+1}^j + \varphi_{22} y_t^j + 1 - \delta}{\varphi_{11} k_{t+1}^j + \varphi_{12} y_t^j} \\ \frac{1}{\beta c_t^j} &= \frac{\varphi_{21} (K_{t+1} + \varepsilon_t^{j,k}) + \varphi_{22} (Y_t + \varepsilon_t^{j,y}) + 1 - \delta}{\varphi_{11} (K_{t+1} + \varepsilon_t^{j,k}) + \varphi_{12} (Y_t + \varepsilon_t^{j,y})}. \end{aligned}$$

Applying the forecasting equations to the model (2)

- Both sides of the last equation can be inverted and we get

$$\beta c_t^j = \frac{\varphi_{11} (K_{t+1} + \varepsilon_t^{j,k}) + \varphi_{12} (Y_t + \varepsilon_t^{j,y})}{\varphi_{21} (K_{t+1} + \varepsilon_t^{j,k}) + \varphi_{22} (Y_t + \varepsilon_t^{j,y}) + 1 - \delta}.$$

- aggregating the LHS is easy and gives βC_t
- Aggregating the RHS is quite difficult and is done by approximation

Applying the forecasting equations to the model (3)

- Taking a second order Taylor approximation of the RHS gives

$$\begin{aligned}\beta C_t &= \frac{EC_{t+1}}{Er_{t+1} + 1 - \delta} \\ &+ \varphi_{21} \frac{\varphi_{21} EC_{t+1} - \varphi_{11} (Er_{t+1} + 1 - \delta)}{(Er_{t+1} + 1 - \delta)^3} \sigma_{\varepsilon_k}^2 \\ &+ \varphi_{22} \frac{\varphi_{22} EC_{t+1} - \varphi_{12} (Er_{t+1} + 1 - \delta)}{(Er_{t+1} + 1 - \delta)^3} \sigma_{\varepsilon_y}^2.\end{aligned}$$

replacing the parameters of the forecasting equations with what they equal gives

$$\beta \bar{C} = \frac{\frac{2}{2+\sigma_\varepsilon^2} \bar{C}}{\frac{2}{2+\sigma_\varepsilon^2} \bar{r} + 1 - \delta} - 2 \frac{\frac{\bar{C}\bar{r}}{(2+\sigma_\varepsilon^2)^2} (1 - \delta)}{\left(\frac{2}{2+\sigma_\varepsilon^2} \bar{r} + 1 - \delta\right)^3} \sigma_\varepsilon^2$$

Applying the forecasting equations to the model (4)

- Cancelling out \bar{C} , gives

$$\beta = \frac{\frac{2}{2+\sigma_\varepsilon^2}}{\frac{2}{2+\sigma_\varepsilon^2}\bar{r} + 1 - \delta} - 2 \frac{\frac{\bar{r}}{(2+\sigma_\varepsilon^2)^2} (1 - \delta)}{\left(\frac{2}{2+\sigma_\varepsilon^2}\bar{r} + 1 - \delta\right)^3} \sigma_\varepsilon^2$$

- Notice that if $\sigma_\varepsilon^2 = 0$, this equation becomes the standard expression for this first order condition.

The stationary state (1)

The five equations for the stationary state are

$$\beta = \frac{\frac{2}{2+\sigma_\varepsilon^2}}{\frac{2}{2+\sigma_\varepsilon^2}\bar{r} + 1 - \delta} - 2 \frac{\frac{\bar{r}}{(2+\sigma_\varepsilon^2)^2} (1 - \delta)}{\left(\frac{2}{2+\sigma_\varepsilon^2}\bar{r} + 1 - \delta\right)^3 \sigma_\varepsilon^2}$$
$$(1 - \theta) \frac{\bar{Y}}{\bar{H}} = \frac{A\bar{C}}{1 - \bar{H}},$$
$$\bar{C} = \bar{Y} - \delta\bar{K},$$
$$\bar{Y} = \bar{K}^\theta \bar{H}^{1-\theta},$$
$$\bar{r} = \theta \frac{\bar{Y}}{\bar{K}}.$$

The stationary state (2)

- For the parameter values of $\beta = .99$, $\delta = .025$, $\theta = .36$, and $A = 1.72$
- The stationary state values when $\sigma_{\varepsilon}^2 = 0$ are the usual

$\bar{K} = 12.6695,$
$\bar{Y} = 1.2353,$
$\bar{C} = .9186,$
$\bar{H} = .3335,$
$\bar{r} = .0351.$

The stationary states with idiosyncratic measurement error

- The effects of bias in the forecasting estimations

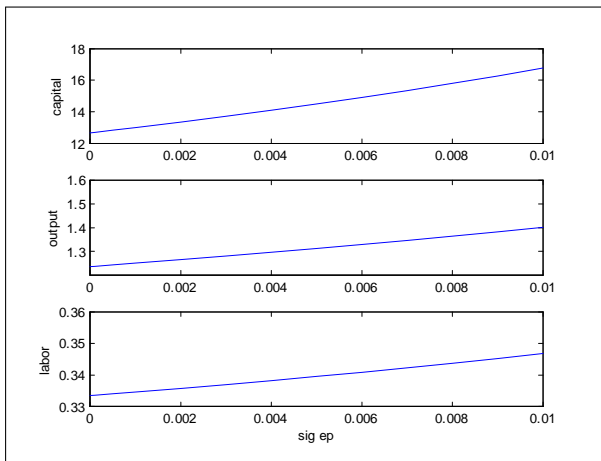


Figure: Stationary state values for \bar{K} , \bar{Y} , and \bar{H} as a function of the measurement error

The stationary states with idiosyncratic measurement error

- The effects of bias in the forecasting estimations

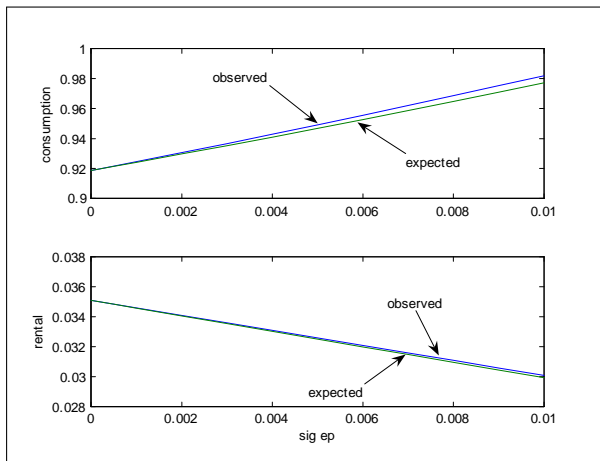


Figure: Stationary state values for \bar{C} , \bar{r} , EC_{t+1} , and Er_{t+1} as function of measurement error

The dynamic version of the model

- The log-linear version of the basic model is

$$0 = \tilde{c}_t^j - E_t \tilde{c}_{t+1}^j + \beta \bar{r} E_t \tilde{r}_{t+1}^j,$$

$$0 = \tilde{Y}_t - \frac{\tilde{H}_t^j}{1 - \bar{H}} - \tilde{c}_t^j,$$

$$0 = \bar{Y} \tilde{Y}_t - \bar{C} \tilde{C}_t + \bar{K} \left[(1 - \delta) \tilde{K}_t - \tilde{K}_{t+1} \right],$$

$$0 = \tilde{\lambda}_t + \theta \tilde{K}_t + (1 + \theta) \tilde{H}_t - \tilde{Y}_t,$$

$$0 = \tilde{Y}_t - \tilde{K}_t - \tilde{r}_t,$$

- and

$$\tilde{\lambda}_t = \gamma \tilde{\lambda}_{t-1} + \tilde{\varepsilon}_t^\lambda.$$

The dynamic version of the model

- The expectational variables, $E_t \tilde{c}_{t+1}^j$ and $E_t \tilde{r}_{t+1}^j$ are determined by each family using an OLS model and using \tilde{K}_{t+1}^j and \tilde{Y}_t^j as explanatory variables
- the equation is

$$\begin{bmatrix} E_t \tilde{C}_{t+1} & E_t \tilde{r}_{t+1} \end{bmatrix} = \begin{bmatrix} \varphi(t-1)_{11} & \varphi(t-1)_{21} \\ \varphi(t-1)_{12} & \varphi(t-1)_{22} \end{bmatrix} \begin{bmatrix} \tilde{K}_{t+1}^j & \tilde{Y}_t^j \end{bmatrix},$$

- where the $\varphi(t-1)_{11}$ are estimated using data available up to time t-1
- The OLS estimation is

$$\Phi = \left((X^j)' X^j \right)^{-1} (X^j)' Y^j$$

The dynamic version of the model

- Given the measurement error, a bias is introduced into the estimates
- The bias can be seen in

$$\Phi = \left(\begin{bmatrix} \text{var}\tilde{K} & \text{cov}\tilde{K}\tilde{Y} \\ \text{cov}\tilde{K}\tilde{Y} & \text{var}\tilde{Y} \end{bmatrix} + \begin{bmatrix} \sigma_{\mu^K}^2 & 0 \\ 0 & \sigma_{\mu^Y}^2 \end{bmatrix} \right)^{-1} \\ \times \begin{bmatrix} \text{cov}\tilde{K}\tilde{C} & \text{cov}\tilde{K}\tilde{r} \\ \text{cov}\tilde{Y}\tilde{C} & \text{cov}\tilde{Y}\tilde{r} \end{bmatrix}.$$

The dynamic version of the model (solving the model)

- A state space version of the model is used
- the variables are

$$x_t = \left[\tilde{K}_{t+1} \quad \tilde{H}_t \quad \tilde{Y}_t \quad \tilde{C}_t \quad \tilde{r}_t \quad E_t \tilde{C}_{t+1} \quad E_t \tilde{r}_{t+1} \quad \tilde{\lambda}_t \right]$$

- and the state space formulation of the problem is

$$A_t(\Phi_{t-1}) x_t = B_t(\Phi_{t-1}) x_{t-1} + C \varepsilon_t,$$

- the recursive updating equation for the parameters is

$$\begin{bmatrix} \Phi_t \\ P_t \end{bmatrix} = G \left(\begin{bmatrix} \Phi_{t-1} \\ P_{t-1} \end{bmatrix}, x_t \right).$$

- The equation for the recursive OLS parameter estimate at date t is

$$\varphi_t = \varphi_{t-1} + \frac{P_{t-1}x_t'}{1 + x_t P_{t-1} x_t'} (y_t - x_t \varphi_{t-1})$$

- with the updating rule for P_t is

$$P_t = \left[I - \frac{P_{t-1}x_t'}{1 + x_t P_{t-1} x_t'} x_t \right] P_{t-1}.$$

The dynamic version of the model (solving the model)

- the state space model can be solved directly in each period as

$$x_t = [A_t(\Phi_{t-1})]^{-1} B_t(\Phi_{t-1}) x_{t-1} + [A_t(\Phi_{t-1})]^{-1} C \varepsilon_t.$$

- where

$$A_t = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -1 & \beta \bar{r} & 0 \\ 0 & -\frac{1}{1-H} & 1 & -1 & 0 & 0 & 0 & 0 \\ -\bar{K} & 0 & \bar{Y} & -\bar{C} & 0 & 0 & 0 & 0 \\ 0 & 1+\theta & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ -\varphi_{11}^1(t-1) & 0 & -\varphi_{12}^1(t-1) & 0 & 0 & 1 & 0 & 0 \\ -\varphi_{21}^1(t-1) & 0 & -\varphi_{22}^1(t-1) & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The dynamic version of the model (solving the model)

$$B_t = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(1-\delta)\bar{K} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma \end{bmatrix},$$

and

$$C = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1]'$$

Finding an impulse response function

- Constant gain OLS can be very slow to converge
- the economy is run for 40,000 periods with a forgetting factor of .99999
- then run again using the average values for Φ over the last 20,000 periods as a new starting point.
- The economy is run twice for 40,199 periods beginning with the coefficients found above (with the forgetting factor = 1).
- the same normally distributed shocks are applied to the economy
- except that in period 40,001 of the second running, an additional impulse of .1 is applied to the technology shock
- The impulse response function to a technology shock for this economy is found by subtracting the last 199 observations of the first run without forgetting from the second run.

How measurement error affects the parameters

- For an example economy with a variance of measurement error of .001,
- The range of the signal to noise ratio is from 10 to .0001

τ	10	1	.1	.01	.001	.0001
φ_{11}	0.4989	0.4856	0.4295	0.3258	0.2260	0.1978
φ_{12}	0.2892	0.2763	0.2698	0.2053	0.1163	0.0914
φ_{21}	-1.0079	-0.9936	-0.8935	-0.6341	-0.4987	-0.5149
φ_{22}	0.9702	0.9583	0.8572	0.5134	0.2101	0.1088

Impulse response functions

- τ is the ratio of $\sigma_{\varepsilon\lambda}^2$ to σ_{μ}^2 , shock is to technology

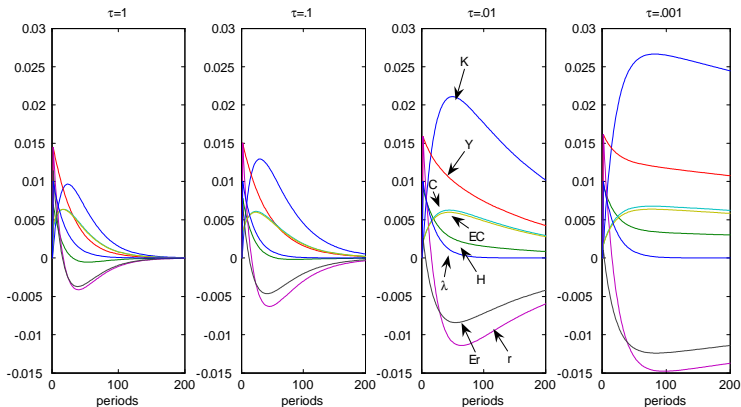


Figure: Impulse-response functions

- The government does a survey of a fraction of the population
 - collects the household's observations of the data
 - publishes the average of sample data
- Private collection of data unlikely
 - it isn't very expensive
 - but not clear how much it can be sold for
 - at first it doesn't change function estimates much
 - even with forgetting
- Data will need to be collected for a long period before it has effects on the equilibrium

Equilibrium with public information

- Public information gradually removes the bias in forecasting equation estimates
 - speed of adjustment depends on forgetting factor
 - without forgetting, could be extremely long
- Expectations adjust upward
- With some small amount of forgetting,
 - for stationary state, expectations equal current stationary state values of variables
 - for dynamics, impulse response functions go to the rational expectations ones

Conclusions

- This simple economy with learning and idiosyncratic measurement error
- With greater measurement error
 - higher stationary state values for output, consumption, and utility
 - greater persistence of shocks
- Public provision of correct sample averages
 - moves economy to RE equilibrium
- Comments on INDEC
 - If public information has increasingly large errors (larger than private)
 - households shift to using private information with idiosyncratic errors
 - economy gradually moves to higher SS
 - persistence of shocks increases

Comments on paper of Morris and Shin

- Dynamic game with individual measurement error
- Economy with multiple Nash equilibria (stable and crisis, for example)
- The measurement error smooths out equilibrium formation
 - produces a continuum of responses
 - one of these is an equilibrium
- If public information is better than private
 - can improve utility for decision makers
 - but may remove the smoothing behavior of the noise