Job market signaling under two-dimensional asymmetric information*

Jorge Miguel Streb†

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Abstract

This paper analyzes what happens to the Spence signaling model when there is heterogeneity in two dimensions, competency and character. Competency is productivity at work. Character is the taste for study. If heterogeneity in character is low, the equilibrium is separating. If heterogeneity in character is high, the equilibrium is partially pooling. In the partially pooling equilibrium, only extreme types can be distinguished. Expected competency is monotonically increasing in the level of education. Supplementary information can reveal personal characteristics, acting as a sign of work productivity.

Key words: asymmetric information, adverse selection, signals, indices, signs

JEL: J31, D10

1 Introduction

In the Spence model, education acts as a signal of productivity because more productive individuals have lower costs of education. This paper asks what happens when the costs of education also depend on other personal traits. The paper will specifically consider differences in the taste for study.

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†Jorge M. Streb, Universidad del CEMA, Av. Córdoba 374, C1054AAP Buenos Aires, Argentina; e-mail jms@cema.edu.ar; tel. 54-11-4314-2269.
The influence of a person’s character on the costs of education leads to asymmetric information in an additional dimension. The reason for asymmetric information is quite simple. Differences in the taste for study are part of personal preferences, so this is private information. These preferences have to be inferred from actions, just like competence at work.

That individuals can differ both in competency and in character is at the heart of the Akerlof (1970) lemons model. The problem with lemons arises not only because there are different quality cars, but also because there are dishonest sellers who are willing to misstate the quality of a used car. Heterogeneity along the dimensions of competency and character is quite widespread. For instance, Covey et al. (1995, pp. 240-1) focus on the importance of both competency and character for business organizations. They also give more homely examples, such as wanting physicians to be competent, to give us the right treatment, and honest, to not prescribe a costly treatment we do not need.¹

With two-dimensional asymmetric information, a separating equilibrium exists if heterogeneity in character is low. If heterogeneity in character is high, no separating equilibrium exists. Instead, there is a partially pooling equilibrium. In the partially pooling equilibrium, the probability the worker is competent is monotonically increasing in the signal. Signaling is still informative, but only extreme types can be told apart.

An implication of the result that education may not lead to a separating signal is that other types of information will be used by employers to sort out the productivity of workers. Education may be specially important as a signal on entry to the job market. Previous job experience, and the continuity of employment relationships, may work as a sign for workers already in the job market.

2 Preferences

The present signaling model is a variant of Spence (1973). The principals are firms. The agents are workers. The signal is given by education. Let a workers’ utility depend positively on wages and negatively on the cost of

¹Levine (1998) considers how heterogeneity in character (specifically, whether an individual is nice to others or not) can help explain anomalous results in experimental economics. Heterogeneity in competency and in honesty has been analyzed, for example, in papers on corruption (Weinschelbaum, 1998) and governance (Dixit, 2001).
education,

\[ U(w, e | \theta, \nu) \equiv w - c(e | \theta, \nu), \]

where \( w \) stands for wage and \( e \) for education level. The utility cost of education \( c \), as well as utility \( U \), are assumed to be conditional on a worker’s productivity type \( \theta \), and on taste for education \( \nu \).

In keeping with the original Spence model, the influence of the parameters \( \theta \) and \( \nu \) on the costs of education are given an extremely simple formulation,

\[ c(e | \theta, \nu) \equiv \frac{c(e)}{\theta \nu}, \]

where high productivity \( \theta \) and high taste for education \( \nu \) both lower the costs of education, and \( c(e) > 0 \). In the figures below, we will assume \( c(e) = e^2 \) for concreteness. The formulation in (2) assures that the single-crossing property is satisfied.

Firms are risk-neutral and maximize profits. Profits equal wages minus a worker’s productivity:

\[ \pi \equiv w - \theta \]

In a setting with perfectly competitive markets, expected profits will be zero, so in expected value wages will equal productivity. The behavior of competitive firms can be represented by a single player that minimizes a loss function given by the quadratic difference between wages and productivity (Fudenberg and Tirole 1991, chap. 11).

3 \hspace{1cm} \textbf{Education as a signal}

The Spence model implicitly assumes that heterogeneity in character is null. Hence, there is only asymmetric information in one dimension, the degree of productivity. Once there is heterogeneity in character, there will be asymmetric information in two dimensions.

\[ ^2 \text{Alternatively, the desire to achieve social recognition, or other factors, could explain differences in the psychic costs of education.} \]
We analyze a discrete setting. Productivity may be either low or high, $\theta \in \{\theta, \bar{\theta}\}$, and taste for education may also be low or high, $\nu \in \{\nu, \bar{\nu}\}$. Heterogeneity among individuals implies that there are four types of agents:

Let heterogeneity in character be denoted by

\begin{equation}
    h \equiv \tau - \nu
\end{equation}

The knife-edge case of heterogeneity $\tilde{h}$ that separates low and high heterogeneity is given by

\begin{equation}
    \theta (\nu + \tilde{h}) = \theta \tau
\end{equation}

The interval of low heterogeneity corresponds to $h \in [0, \tilde{h})$, while the interval of high heterogeneity corresponds to $h \in [\tilde{h}, H]$, for some positive $H > \tilde{h}$. In the knife-edge case $h = \tilde{h}$, the indifference curves of types $(\theta, \tau)$ and $(\nu, \bar{\nu})$ are exactly superimposed on each other.

### 3.1 Low heterogeneity in character

In terms of the present notation, the original Spence model corresponds to $h = 0$. This case boils down to two types of workers, high and low productivity. Spence (1973) shows there are a continuum of separating equilibria. One can select the least cost separating signal, where the unproductive worker is just indifferent between studying or not, to define a unique separating equilibrium.

There is also a pooling equilibrium in Spence (1973), which can be discarded applying the Cho-Kreps intuitive criterion. A competent worker has lower signaling costs, so it will be willing to deviate to levels of education higher than what an incompetent worker would ever pick.

These results generalize to the case of $h \in (0, \tilde{h})$. In a separating equilibrium, beliefs will be given by

\begin{align*}
    e = 0 & \quad \Rightarrow \quad \hat{\theta} = \theta \\
    e = e^s & \quad \Rightarrow \quad \hat{\theta} = \bar{\theta}
\end{align*}
For out-of-equilibrium values of education $e$, we assume a firm will assign
$\widehat{\theta} = \underline{\theta}$ if $e < e^*$, and $\widehat{\theta} = \overline{\theta}$ if $e > e^*$. One can define the signal $e^*$ uniquely
by picking as signal the least-cost level of education that will differentiate
productive and unproductive workers, as Figure 1 shows.

The least cost separating signal is determined by the unproductive worker
with a high taste for study, at point $A$ in Figure 1. At point $A$, worker type $(\underline{\theta}, \overline{\tau})$ is indifferent between getting a high wage
$w = \overline{\theta}$ with education $e = e^*$, and a low wage $w = \underline{\theta}$ with education $e = 0$. One can assume it will not
signal when it is just indifferent (to break indifference, it would suffice to
consider a signal $e^* + \epsilon$, with $\epsilon > 0$ that is arbitrarily small). Hence, behavior
will conform to (6), so this is indeed a separating equilibrium.

One can discard a pooling equilibrium applying the Cho-Kreps criterion,
as Figure 2 shows.

The farthest that an incompetent worker is willing to deviate is point $B$, with education $e^d$. Productive workers have lower signaling costs, so they can
be better off to the right of that point. Since those deviations are dominated
in equilibrium for unproductive types, but not for productive types, firms can infer that a worker is productive if levels of education larger than (or equal
to) $e^d$ are observed. That restriction on out-of-equilibrium beliefs destroys
any pooling equilibria.

Likewise, one can also discard partially pooling equilibria. The indifference
curves of productive workers are flatter than the indifference curves of
unproductive types, so productive workers will always be willing to deviate
farther to the right than unproductive workers.

These results can be summarized as follows.

**Proposition 1** With low heterogeneity in character, there is a unique sepa-
rating equilibrium. Unproductive workers pick low education and productive
workers pick high education.

Hence, with low heterogeneity in character the signaling results of the
basic Spence model are robust to two-dimensional asymmetric information.
In this interval, only a separating equilibrium survives refinements of the
Perfect Bayesian equilibrium that rule out dominated equilibria and apply
the intuitive criterion.
3.2 High heterogeneity in character

The interval of high heterogeneity corresponds to \( h \in [\tilde{h}, H] \). We will show that no separating equilibrium exists. Instead, there will be a partially pooling equilibrium with the following features. Type \((\theta, \nu)\) will pick zero education. Worker types \((\bar{\theta}, \bar{\nu})\) and \((\tilde{\theta}, \tilde{\nu})\) will send the same, intermediate, signal. Finally, type \((\bar{\theta}, \bar{\nu})\) will pick the highest level of education.

Why no separating equilibrium exists is easy to see from Figure 3.

When heterogeneity in character is high, an unproductive worker with high taste for studying is willing to invest in more education than a productive worker with low taste for studying. That is, a productive worker of type \((\bar{\theta}, \bar{\nu})\) is not willing to go farther than point \( C \) in Figure 3, while an unproductive worker of type \((\bar{\theta}, \bar{\nu})\) is.

A pooling equilibrium can be discarded, as in the case of low heterogeneity in character, by application of the intuitive criterion. A productive worker of type \((\bar{\theta}, \bar{\nu})\) will always be willing to deviate. A partially pooling equilibrium where type \((\bar{\theta}, \bar{\nu})\) worker picks zero education and all other workers pick a common positive level of education, can be ruled out by similar arguments.

The other logical possibility is a partially pooling equilibrium where beliefs are given by:

\[
\begin{align*}
 e = 0 & \quad \Rightarrow \quad \hat{\theta} = \bar{\theta} \\
 e = e^i & \quad \Rightarrow \quad \hat{\theta} = \frac{p_{12}\bar{\theta} + p_{21}\bar{\theta}}{p_{12} + p_{21}} \\
 e = e^s & \quad \Rightarrow \quad \hat{\theta} = \bar{\theta}
\end{align*}
\]

(7)

This equilibrium is represented graphically in Figure 4.

The (least-cost) intermediate signal is determined at point \( D \), with education \( e = e^i \) and average wage \( w = w^i \), that is on the indifference curve of unproductive worker type \((\bar{\theta}, \bar{\nu})\) that goes through point with coordinates \( e = 0 \) and \( w = \bar{\theta} \). As to the (least-cost) separating signal, it is determined at point \( E \), where education \( e = e^s \) with wage \( w = \bar{\theta} \), which for unproductive worker of type \((\bar{\theta}, \bar{\nu})\) is just indifferent to point \( D \). Note that this partially pooling equilibrium exists for any set of positive probabilities \( \{p_{11}, p_{12}, p_{21}, p_{22}\} \).

Hence,
**Proposition 2**  With high heterogeneity in character, there is a unique partially pooling equilibrium. Unproductive workers with no taste for study pick low education. Unproductive workers with a taste for study and productive workers with no taste for study pick an intermediate education. Productive workers with a taste for study pick high education.

Proposition 2 implies that extreme signals are still effective to convey a workers type. It is in the middle ground that there may be imperfect revelation of a worker’s type.

### 3.3 Relation to other papers

Riley (2001) considers an extension of the original Spence model to four types of agents. This resembles our two-dimensional asymmetric information framework in that there also are four types of agents: productive and unproductive, which can have a taste for study or not. However, Riley’s focus is completely different. He is not concerned about asymmetric information on other dimensions of a workers characteristics. He is concerned about what happens when there is “noise” in the basic Spence model, i.e. when some unproductive workers have relatively low signaling costs in terms of education. He shows there is a partially pooling equilibrium with either low education (unproductive workers with high signaling costs in terms of years of formal education), or high education (productive workers, or unproductive workers with low signaling costs). He then analyzes how robust the equilibrium, and the equilibrium refinements, are to different out-of-equilibrium events.

Streb (2002) considers the consequences of two-dimensional asymmetric information in the context of the Rogoff (1990) equilibrium budget cycle model. However, the analysis in the Rogoff signaling model is much more involved because the single-crossing property fails once there is heterogeneity in character (given there by differences in the degree of opportunism).

### 4 Work experience as a sign

If the issue of heterogeneity in character is empirically relevant, signaling via education will lead to a partially pooling equilibrium. One would then expect firms to use other type of information to sort productive and unproductive workers. Work experience seems particularly relevant information. Firms get
to know a worker better through day to day contact at work. This generates private information that allows to assess a worker’s type better. The process of revelation of productivity at work takes time, so to incorporate this feature requires a minimum of dynamics.

One can extend the setup in the previous section as follows. After the first period has elapsed, firms observe the worker’s true productivity. Let the decision rule be that workers whose wage is equal to or larger than their productivity are offered a renewal of their contracts. On the other hand, workers whose contract stipulates a wage larger than their productivity are fired (alternatively, one could have wage reductions, but this is not the way the job market usually works for reasons of what is seen as fair, cf. Akerlof and Yellen 1988). To not enter into the issue of the duration of unemployment spells, we will simply assume that workers that are fired have a discontinuous job experience, while workers that are not fired have a continuous job experience.

In our simple setup, the renewal decision rule will provoke a total revelation of types. Types with low or high education will get their contracts renewed, since in the first-period equilibrium their wages equaled productivity. As to types with intermediate education, those with low productivity will be fired, while those with high productivity will get a renewal offer.

If these decisions become public information, i.e. workers that are fired become visibly unemployed, this reveals information to other firms. In a competitive environment, the wages of the workers that get a renewal offer will be bid up. On the contrary, the wages of the workers that are discontinued will be bid down. In this environment, one can hence expect that high productivity workers will be paid in equilibrium a high wage, regardless of their level of education, while low productivity workers will be paid a low wage, regardless of their level of education.

This setup implies there will be a positive correlation between education and wages, and between job experience (or job continuity) and wages. In the second period, more highly educated workers on average get higher wages. Workers with no unemployment spell will also tend to get higher wages.

As to what happens in the first period equilibrium, since in the second period wages are independent of education. This implies that the equilibrium in the first period are just as described in section Three above, where the costs of education have to be compared to the benefits in that period.

In summary, if there is two-dimensional asymmetric information, one would expect firms to use other pieces of information to sort workers. Previ-
ous job experience is an obvious candidate, since empirically it is a key factor in determining which job applicants an employer will interview (Behrenz 2001). Formal education is important as a signal, but this information may be specially relevant to determine entry requirements (we are abstracting from the role of education in the buildup of human capital, that enhances productivity by itself).

As to the informational status of previous work experience (or job continuity), Spence (1973) distinguishes between “indices” and “signals”. Indices are fixed attributes of job applicants, (mostly) unalterable observable attributes such as race and sex. Since age does not change at the discretion of the individual, Spence also considers it an index. Signals are observable characteristics that are subject to the manipulation by the individual, of which education was singled out by Spence. The continuity of discontinuity of work experience, because an individual may have been fired, is neither a fixed characteristic, nor is it usually subject to the direct manipulation of the worker. Since it can reveal characteristics of the worker that are very similar to signals, it might be termed as a “sign”.

Work continuity in the job market is comparable as an informational sign to credit availability in the credit market. The creditworthiness of small firms or individuals may only be privately known to the bank or lender that has carried out transactions with them and developed a relationship. This relationship generates private information. However, the very existence of a relationship, if it is observable, can act as a public sign to third parties of who is a good credit or not. Being a bank client or getting a credit card can act as a good sign, and other financial intermediaries will try to get these clients. The same happens with people that have continuing employment.

5 Conclusions

This signaling model implies a variant of Spence (1973). Differences in character imply that the taste for study (or the desire to achieve social recognition) vary. In our discrete example, this parameter can be either high or low. This trait is independent from the fact that highly competent individuals have lower costs of completing formal education.

In our discrete framework, signaling is quite resilient to the introduction of asymmetric information in two dimensions. With low heterogeneity in character, the equilibrium is separating. High heterogeneity in character
leads to a partially pooling equilibrium. Very competent individuals who are highly motivated will stand out from the rest. At the other extreme will be types with low competency and low motivation. In the middle, the mix of competent individuals who do not have a taste for education, and of incompetent individuals that do, will be difficult to tell apart. Extreme types still send unequivocal signals. Furthermore, average competency will be increasing in the degree of education.

If heterogeneity in character is indeed important, education might have a differential impact on entry level wages, in contrast to wages later on in the work cycle. More specifically, if education is an imperfect proxy for a worker’s productivity, other information would be important for firms. Job experience immediately comes to mind. In a dynamic setting, interaction in the work place gives firms direct information on worker productivity. If firms learn over time about the differences in productivity that workers have, and higher productivity workers are more likely to keep their job, this would be consistent with the finding that wages are increasing with on-the-job experience in empirical wage equations. Though on-the-job experience generates private information to the employer, the continuity of working relationships can also act as a sign of high productivity to other firms.

This analysis can be extended to consider more types of competency and of character. Though technically more complicated, the fact that a partially pooling equilibrium may arise should not change. This framework ignores that work productivity in part is a matter of matching the right person to the right job, which of course may complicates the process of getting an efficient outcome.

References


Table 1

<table>
<thead>
<tr>
<th>Types of individuals</th>
<th>Competency $\theta$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td><strong>Low Taste</strong></td>
<td>$p_{11}$</td>
</tr>
<tr>
<td><strong>High</strong></td>
<td>$p_{21}$</td>
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Table 2

<table>
<thead>
<tr>
<th>Wage in second period</th>
<th>- conditional on signal -</th>
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<table>
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<tr>
<th>Education:</th>
<th>$\theta$ low</th>
</tr>
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<tbody>
<tr>
<td>None</td>
<td>$p_{21} \theta_{\text{low}} + (p_{12} + p_{22}) \theta_{\text{high}}$</td>
</tr>
<tr>
<td>Positive</td>
<td>$p_{11} + p_{12} + p_{22}$</td>
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<table>
<thead>
<tr>
<th>Job experience:</th>
<th>$\theta$ low</th>
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<tbody>
<tr>
<td>Discontinuous</td>
<td>$p_{11} \theta_{\text{low}} + (p_{12} + p_{22}) \theta_{\text{high}}$</td>
</tr>
<tr>
<td>Continuous</td>
<td>$p_{11} + p_{12} + p_{22}$</td>
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Figure 3

Separating equilibrium: high heterogeneity

Figure 4

Partially pooling equilibrium