The Threat of Insurance.
On the Robustness of Principal-Agent Models

Mariano Tommasi
Universidad de San Andrés &
Centro de Estudios para el Desarrollo Institucional

Federico Weinschelbaum¹
Universidad de San Andrés

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Abstract

The traditional principal-agent model assumes that the principal offers an exclusive contract to the agent. This paper shows that the standard results are not robust to the introduction of additional contracting opportunities for the agent. We analyze equilibria of an extended game with the presence of additional players who might trade risk away from the agent. There are settings (and parameter values) in which the principal is worse off, total welfare is lower, and suboptimal effort is implemented in equilibrium. There are other settings in which the principal can manage to be as well off as in the standard case, but in these cases he has to offer a steeper contract for performance. These findings may call for a revision of some previous theoretical and applied conclusions.

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¹<fweinsch@udesa.edu.ar> Vito Dumas 284 (1644) Victoria -Provincia de Buenos Aires- Argentina. Phone (54-11)4725-7041, Fax (54-11)4725-7010. We received valuable comments from Federico Echenique, Bryan Ellickson, Hugo Hopenhayn, David Levine, Alejandro Manelli, David Perez-Castrillo, Jorge Streb and Bill Zame, and from participants at various seminars. We received very valuable suggestions and excellent research assistance from Ignacio Esponda.
1 Introduction.

Contractual relationships have traditionally been modeled as if they were exclusive, implying that no outside party can interfere with the incentives provided by the original contract. Most of agency theory proceeds under the implicit assumption that the principal can sign an exclusive contract with the agent. This might seem an innocuous assumption in many situations. For instance, a CEO spends her entire day working for the firm, so it seems unlikely that conflicts could arise from competition of other firms for her effort. Enforcing exclusivity seems easy: the board simply controls that the CEO does not work for other firms. As a result, competition over efforts is not a problem, and studying the problem in isolation seems to be the proper assumption.

However, it seems demanding to assume that the principal is able to control the agent’s consumption vector. Recall that in the traditional model the principal can give incentives to a risk-averse agent, only at the cost of exposing her to some risk. But this opens the door for an outside contract in which the agent might reduce her risk exposure. Since this contract does not need to be tied to any “productive” activity, the enforcement of exclusivity seems more demanding.

For example, the CEO can be given incentives and exposed to some risk by giving her shares in the company, but the CEO can undo this position by going short on the company’s shares. As a result, the company might end up paying a high cost for a manager that, once insured, might provide suboptimal levels of effort. If the company anticipates this, it might not offer incentives in the first place, or at least it might not offer the type of incentives predicted by the literature. A related case is that of mutual risk insurance among co-workers or informal insurance among family members.

\[\text{\textsuperscript{2}}\text{See for instance the textbooks by Salanié (1997), by Macho-Stadler and Pérez-Castrillo (1997) and by Laffont and Martimort (2002), and the surveys by Gibbons (1996) and by Prendergast (1999).}\]

\[\text{\textsuperscript{3}}\text{There might be explicit restrictions to such trading, but it is not hard to imagine ways to circumvent that through relatives, friends, investment companies, etc. Ofek and Yermack (2000) provide empirical evidence showing that such diversification does indeed occur.}\]

\[\text{\textsuperscript{4}}\text{When shares are used to provide incentives to managers, it will be desirable to control their trading. Bebchuk, Fried and Walker (2002) claim that boards rarely restrict and control the sale of managers’ shares. (They use this fact to criticize the optimal contracting approach to managerial compensation).}\]
Such side contracting can bring some benefits if it permits better monitoring, but it might also introduce inefficiencies by increasing moral hazard problems (Arnott and Stiglitz, 1991; Itoh, 1993). More generally, the spirit of the non-robustness results that we will present in this paper, carry over to any situation in which the agent’s marginal risk aversion is affected by additional income.\(^5\)

In this paper we investigate to what extent the results from the standard principal-agent model are robust to the possible presence of these outside insurance opportunities. We work out the case with two levels of effort and a continuum of outcomes. We first show that in a typical principal-agent setting the second-best contract is not robust to the introduction of these additional contracting opportunities for the agent. That is due to the fact that the part of the risk born by the agent in the second-best contract creates a surplus for an agent-insurer relationship.

We then characterize equilibria of an extended principal-agent-insurer(s) game under different settings. Our settings differ in two main dimensions: one is the number of insurers, the other is the sequence of play. We vary the number of (potential) insurers all the way from one to free entry. We study two broad categories of sequential settings: sequential offering and sequential contracting. In the first case, the agent receives all offers and can then decide which of them, if any, to accept. In the latter case the agent signs the original contract first, and only then do new contractual opportunities appear. Our different settings allow us to study robustness under different degrees of outside competition.

It turns out that when the principal faces weak outside competition he can still manage to induce the (second best) optimal level of effort at the same cost, so that the threat of insurance does not hurt him. For example, consider the simplest case of an insurer making an offer after the principal, and then the agent choosing which contract(s) to accept (sequential offering). In that case, the principal can offer a contract riskier than the second best, but with the same expected cost,\(^6\) and have the insurer reduce this extra risk and take the agent to the second best. The expected payoff in case of low effort in the principal’s contract will be so low, that the insurer would have to put money from his own pocket in order to take the agent to low effort.

\(^5\)There is a recent literature challenging the assumption of exclusive contracts. The common theme is that every transaction affects incentives, so that outside opportunities should be taken into account when designing contracts. We discuss that literature below.

\(^6\)The principal’s contract, by itself, leaves the agent below her reservation utility.
This equilibrium collapses if we extend the setting to free entry of insurers. The threat of further insurance will prevent any player from offering a contract to the agent that (if last contract) will take her to the second best. The result of the previous paragraph can collapse also even without free entry, in the case of sequential contracting (when the agent has to first sign the principal’s contract) if the insurer has the bargaining power in this second relationship. In that case, the agent will refuse to sign an interim contract that leaves her below her reservation utility. In the case of free entry, as well as in this latter case, the principal must pay a higher cost in order to implement high effort, and we are in a “third-best” situation. One of the problems now is that, for some economies (i.e., parameter values), the principal will not find this extra cost worthwhile and optimal high effort will not be implemented. In that case, the threat of insurance eliminates incentives.

But incentives are not always eliminated in our model; furthermore, in some cases they are stressed. Older analyses of non-exclusive contracting have predicted that they eliminate effort incentives. We concur with Kahn and Mookherjee (1998; KM) and Bisin and Guaitoli (1999; BG) in challenging those results. In our setting, the third-best contract gives more intense incentives than the second-best contract.

Our paper shares other features of the work of KM and BG, who also study non-exclusive contracts in moral hazard economies with hidden action. KM study the effects of nonexclusive credit or insurance contracts from multiple risk neutral firms with sequential free entry. They find that competition between firms can induce a reduction in customer’s effort, and that the lack of coordination among insurers may affect the cost of implementation even without affecting effort levels. One of the settings of our model can be seen as an extension of their work in which their first insurer

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7 Third best in the sense of adding a third constraint to a planning problem. This third condition arises from non-exclusivity and avoids later contracts inducing low effort. A similar terminology has been used in related papers by Kahn and Mookherjee (1998) and Bisin and Guaitoli (1999).

8 See Pauly, 1974; Helpman and LaFont, 1975; Kletzer, 1984; and Bizer and DeMarzo, 1992.

9 The issue of non-exclusivity has been studied originally in a sequence of papers by Arnott-Stiglitz [Arnott and Stiglitz (1988), (1990), (1991b), and Arnott, Greenwald and Stiglitz (1993)] and by Hellwig (1983). They study an insurance market where offers are made simultaneously and there is no principal (player that has a “productive” relationship with the agent). As we comment in section 7.3, one of our proposed extensions could be thought of as an extension of these papers.
is replaced by a productive principal.\textsuperscript{10}

BG analyze a case where “intermediaries” design and offer contracts simultaneously. They show, as we do, that the optimal action is not implemented in equilibrium for some economies. They also show that whenever the equilibrium contracts implement the optimal action, intermediaries make positive profits and equilibrium allocations are inefficient.

Contractual externalities in a sequential banking setting are studied by Bizer and de Marzo (1992). A bank cannot prevent a borrower from getting a loan at another bank. The terms of this new contract will not reflect the devaluation of existing debt. As a result, interest rates are higher and welfare is lower, due to the fact that banks have to operate under a “no further borrowing” constraint, which plays a similar role than a non-insurability constraint in our paper.

Our paper is also related to the literature on common agency under moral hazard (pioneered by Bernheim and Whinston 1986), in which many principals simultaneously attempt to influence the actions of one agent. Our model can be read as a common agency situation in which there is only one dimension of effort, and only one of the principals cares about that action. It turns out that our main point, the negative externality of insurance, is a feature present also in traditional common-agency models, on top of the externality coming from competition for effort.\textsuperscript{11}

The principal-agent model has also been extended in other directions that could be related to our work. Fudenberg and Tirole (1990) study renegotiation in agency contracts. They consider the situation in which the original contract can be renegotiated after the agent chooses the level of effort, but before output levels are realized. The traditional contract is not robust, since once effort is made, the principal would rather offer full insurance to the agent. Foreseeing this, the agent would choose low effort in the first place. The equilibrium with renegotiation involves mixed strategies. The optimal contract may give the agent a positive rent. The key difference between that paper and ours is their timing assumption in which some contracts (the “renegotiated” one) are signed after effort, transforming the original moral

\textsuperscript{10}More precisely, one of our settings coincide with the one of KM in all matches after the first, but the first match is different not only in the fact that one player (the principal) is endowed with a productive technology, but also in the bargaining context.

\textsuperscript{11}To our knowledge, previous common-agency papers (with the exception of Dixit, 2000, p. 19) have not explicitly noted the presence of that insurance effect.
hazard problem into an adverse selection one.\textsuperscript{12} Other aspects of the set up are equivalent: the second incarnation of the principal, “the renegotiator”, is analogous to our insurer; and their renegotiation proofness requirement plays a similar role than our non-insurability constraint.

Notice that in our setting, the introduction of new actors has (weakly) negative implications for welfare in the original relationship. This contrasts with other scenarios in which the introduction of new players can be beneficial for the principal. Those scenarios require that the new players have better information than the principal. A notable paper in this line, Itoh (1993), considers a framework with one principal and two agents. When the agents can monitor each other efforts’ perfectly, the principal is better off. In a small introductory section, Itoh establishes that if each agent does not have an informational advantage over the principal with respect to the other agent’s actions, then side contracting cannot make the principal better off.\textsuperscript{13} In this paper we work out the exact welfare implications under different settings for side contracting, finding under what conditions the principal is strictly worse off.

Arnott and Stiglitz (1991) is another paper that highlights the role of informational assumptions in multi-contract situations. They study how a client’s access to informal markets, such as family-provided insurance, affects formal insurance markets and welfare. If the family observes effort, peer monitoring increases welfare. But if they only observe output, the role of family interactions is that of providing mutual risk-sharing, and the economy is worse off as the formal sector must deal with these additional contractual constraints.

These considerations on the role of information bring to mind the “intermediate” case of imperfect observability of effort via signals other than output. According to the informativeness principle, any signal that gives information about the agent’s effort is valuable (Holmstrom, 1979; see also Shavell, 1979, and Grossman and Hart, 1983). Our model suggests that the usefulness of including different indicators in an incentive contract might depend not only on their observability by the principal, but also on their observability by other actors who might engage in side contracting with the agent.

\textsuperscript{12}It might be an interesting extension to combine the timing assumption of Fudenberg and Tirole, with the presence of additional players from our set up.
\textsuperscript{13}See also Holmstrom and Milgrom (1990), Varian (1990) and Dana (1993).
In order to specify empirical implications from our analysis, we might consider institutional, legal or market scenarios leading to the assumption of exclusive contracts to be a valid one or not. We predict that in cases of non-exclusivity, (1) performance contracts will be less common and effort (and hence) output will be lower, (2) when performance contracts are indeed used, they will be more intense than in the case of exclusivity, and (3) they might not make so much use of some indicators which are also observable by outside agents.\footnote{Whi
chi ni t s e l fi sar e q u i r e m e n tf o rv e r i fi a bility. Yet, the non-verifi
able indicators can be part of an implicit contract enforced through reputation.}

The rest of the paper is organized as follows. In Section 2 we show non-robustness of the traditional second-best equilibrium. Using some preliminary results from Section 3, we characterize equilibria in a game of sequential offering with finite number of insurers in Section 4. In section 5 we extend to free entry, and in Section 6 we extend to a game of sequential contracting. We conclude with a summary of results, welfare and empirical implications, and possible extensions.

## 2 Non-Robustness of the Standard Results

### 2.1 The Standard Case

We consider the traditional principal-agent problem where a principal wishes to hire an agent. Their interests are opposed since the agent bears a cost from effort while the principal is benefited from the agent’s effort. There is asymmetric information (effort is not contractible). Assume the possible outcomes, given the agent’s effort, are distributed in the set $\Pi \subset \mathbb{R}$ with density $f(\pi/e) > 0$ for all $\pi \in \Pi$ and for all $e$. To keep the framework simple, we assume that there are only two levels of effort, $e \in \{e_l, e_h\}$, low and high effort respectively. The principal’s gross benefits under high effort are greater than under low effort, $\int \pi f(\pi/e_h)d\pi > \int \pi f(\pi/e_l)d\pi$.

The agent maximizes expected utility\footnote{Thus, his utility function is $u_a(w, e) = v(w) - g(e)$. We assume that $v(0) - g(e_l) < \pi$; namely, that the agent will not work for free.}

$$U_a(w(\pi), e) = \int v(w(\pi))f(\pi/e)d\pi - g(e),$$

14 Which in itself is a requirement for verifiability. Yet, the non-verifiable indicators can be part of an implicit contract enforced through reputation.

15 Thus, his utility function is $u_a(w, e) = v(w) - g(e)$. We assume that $v(0) - g(e_l) < \pi$; namely, that the agent will not work for free.
where the cost of high effort is strictly greater than the cost of low effort, i.e. \( g(e_h) > g(e_l) \). Additionally, we assume \( \nu' > 0 \) and \( \nu'' < 0 \), so that the agent is strictly risk averse.

The sequence of the game is the following. The principal offers a contract \( w_p \) to the agent. The agent accepts or rejects the contract. Rejection gives the agent reservation utility \( \overline{u} \). Upon acceptance, the agent must decide on the level of effort. Given the level of effort, payoffs are realized according to the density \( f(\pi/e) \). The solution concept is subgame perfect equilibrium (SPE).

Following Grossman and Hart (1983), the problem of the principal can be decomposed in two steps. First, he finds the optimal incentive scheme for each level of effort; then he selects the optimal level of effort.

Once step 1 is solved, step 2 is trivial: the principal compares profits from implementing low and high effort and decides which level, if any, to implement. We therefore concentrate on part 1 of the problem.

The optimal incentive scheme for implementing \( e \) must solve

\[
\text{Min}_{w(\pi)} \int w(\pi) f(\pi/e) d\pi
\]

s.t.

\[
\int v(w(\pi)) f(\pi/e) d\pi - g(e) \geq \overline{u} \quad \text{(PC)}
\]

\[
e \text{ solves } \text{Max}_{\overline{e}} \int v(w(\pi)) f(\pi/\overline{e}) d\pi - g(\overline{e}) \quad \text{(IC)}
\]

where PC is the participation constraint and IC is the incentive (compatibility) constraint.

Let \( \phi \equiv \nu^{-1}(\cdot) \). The minimum cost of implementing \( e_l \) is achieved by offering \( w_p = \phi[g(e_l) + \overline{u}] > 0 \), a flat wage that makes the agent just indifferent between accepting and rejecting the contract.

The cost-minimizing contract that implements \( e_h \) is the solution to the two binding constraints PC and IC, and to the first order condition

\[
\frac{1}{\nu'(w(\pi))} = \lambda + \gamma \left[ 1 - \frac{f(\pi/e_l)}{f(\pi/e_h)} \right],
\]

where \( \lambda > 0 \) and \( \gamma > 0 \) are the Lagrange multipliers of PC and IC, respectively. This is known as the second-best solution, \( w^{2nd} \), and it is unique given our assumptions (Mas-Colell et al, 1995).
Given \( g(e_h) > g(e_l) \) and the fact that the agent is risk averse, it is straightforward to show that

\[
\int w^{2\text{nd}}(\pi) f(\pi/e_h)d\pi > \phi [g(e_l) + \bar{\pi}] > 0;
\]

that is, the cost of implementing \( e_h \) is strictly greater than the cost of implementing \( e_l \). Denote these minimum costs by \( c^{2\text{nd}}(e_h) \) and \( c^{2\text{nd}}(e_l) \), respectively.\(^{16}\)

**2.2 Non-robustness**

The traditional principal-agent model implicitly assumes that either the agent has an “exclusivity” contract or that there is no other player who can provide her with insurance and has the same informational structure that the principal has. The next proposition motivates the discussion that follows, by showing that standard results are not robust to the introduction of this third player.\(^{17}\)

**Lemma 1** The principal’s contract, \( w_p \), intended to implement high effort is robust to a third risk neutral player if and only if the following General Non-Insurability Constraint holds:\(^ {18}\)

\[
\psi \geq v \left( \int w_p(\pi) f(\pi/e_l) d\pi \right) - g(e_l), \tag{GNIC}
\]

where \( \psi = \max \{ \bar{\pi}, \int v(w_p(\pi)) f(\pi/e_h) d\pi - g(e_h) \} \).

The intuition is simple: when an insurer is present, the agent’s utility under low effort is no longer given by the traditional equation, but now takes the form of the RHS of GNIC. Then the above expression simply requires that choosing low effort is not the (only) best alternative. Notice that we could also think of GNIC as a New Incentive Compatibility constraint, where the presence of an insurer changes (increases) the agent’s benefits from low effort. A formal proof of the lemma is provided below.

\(^{16}\)Given the optimal incentive scheme for each level of effort (step 1), the principal now chooses the level of effort that maximizes his benefits, \( \int \pi f(\pi/e)d\pi - c^{2\text{nd}}(e) \) (step 2).

\(^{17}\)We assume risk neutrality for simplicity; the spirit of our main results will hold if this third player is risk averse.

\(^{18}\)GNIC is a condition such that a third risk-neutral player has no incentive to take the agent to low effort by offering him full insurance.
Notice that by introducing $\psi$ we have left open the possibility that the principal might leave the agent above her reservation utility. This is something that might happen in the presence of insurers. Notice however that when the principal’s contract for high effort does not offer the agent more than her reservation utility, then $\psi = \pi$, and the constraint is replaced by

$$\pi \geq v \left( \int w_p(\pi)f(\pi/e_l)d\pi - g(e_l) \right).$$

(NIC)

The above condition, which we refer to as the Non-Insurability Constraint, will play a crucial role in the settings where the principal can manage to overcome the presence of insurers at no additional cost. Notice that NIC is indeed a sufficient condition for robustness, as NIC implies GNIC. When $\psi = \pi$, NIC is equivalent to GNIC, hence it is also necessary (Lemma 1).

**Proof.** Notice first that $\psi$ is what the agent gets from doing something other than low effort, so that the insurer must offer her at least $\psi$ to induce low effort.

To prove sufficiency notice that the insurer’s optimal contract ($w_I$) giving incentives for low effort is

$$w_I(\pi) = \phi[\psi + g(e_l)] - w_p(\pi),$$

so that the aggregate contract is a flat wage. Integrating we see that the insurer’s expected profits are

$$-\int w_I(\pi)f(\pi/e_l)d\pi = \int w_p(\pi)f(\pi/e_l)d\pi - \phi[\psi + g(e_l)].$$

GNIC implies that profits from taking the agent to low effort are less than or equal to zero. This can be seen by rewriting GNIC as

$$\int w_p(\pi)f(\pi/e_l)d\pi \leq \phi[\psi + g(e_l)]$$

(2)

To establish necessity, we will prove that if GNIC does not hold then there are incentives to insure the agent.

Notice that if GNIC does not hold, then

$$v \left( \int w_p(\pi)f(\pi/e_l)d\pi \right) - g(e_l) > \psi$$
Then, there exists a $\rho$ such that

$$v \left( \int w_p(\pi) f(\pi/e_i) d\pi - \rho \right) - g(e_i) = \psi$$

If the insurer pays $\int w_p(\pi) f(\pi/e_i) d\pi - \rho - w_p(\pi^0)$ when the profits are $\pi^0$ the insurer will get an expected profit of $\rho$ (the risk premium) and the agent will accept the contract. ■

The intuition is also clear from equation (2), which is simply a restatement of GNIC. The principal must pay less than what the agent requires to make low effort, so that anyone wishing to take the agent to low effort is forced to put money from his own pocket. An outside player uninterested in the agent’s effort would surely not do so.

We now show our non-robustness result.

**Theorem 1** The second-best contract is not robust to the presence of a third, risk-neutral, player.

**Proof.** The second best contract satisfies PC (and IC) with equality. As a result, we have $\psi = \pi$. Therefore, NIC is a necessary (and sufficient) condition for robustness. We now show that the second best contract does not satisfy NIC. From PC and IC one obtains

$$\int v(w^{2nd}(\pi)) f(\pi/e_i) d\pi = \pi + g(e_i).$$

By Jensen’s inequality we get

$$v \left( \int w^{2nd}(\pi) f(\pi/e_i) d\pi \right) > \pi + g(e_i).$$

This violates NIC. ■

We have seen that, when the principal wants to implement high effort, results are not robust to the threat of insurance. This is not the case when the principal wants to implement low effort. This is because the traditional optimal contract is a flat wage, so that new players have nothing further to offer to the agent.\(^{19}\)

\(^{19}\)When the principal wants to implement low effort in the presence of insurers, there are also other equilibria. However, they share most of the properties with the flat-wage equilibrium, as we will briefly comment in the next section.
There are settings where implementation of high effort will cost the principal more than the second-best cost. When this happens, two things might occur. If the principal’s gross benefits from high effort are large enough he will still prefer to pay the higher cost. But when the opposite occurs, the principal will implement low effort when it would have otherwise implemented high effort.

In the rest of the paper we analyze under what conditions the implementation of high effort is indeed costlier for the principal when we allow for the existence of these risk-neutral players which we call, from now on, the insurers.

3 Preliminary Results

Before characterizing equilibria for some specific settings, we provide some useful results that generalize to all of our settings. In all the settings: the players move sequentially rather than simultaneously, the principal moves first, and the insurers and the principal have the same information (in particular, they can all observe and contract on outcomes, but not on efforts).

Throughout the paper, we denote by \( w^j_\pi \) the contract offered by party \( j \) (either the principal, an insurer, or, as in Section 6.1, the agent). This is taken to be the amount that the agent receives (pays, if negative) when the output is \( \pi \).

Let \( I = \{1, 2, ..., N\} \) denote the set of insurers (where \( N \) is infinity in the free entry case). Then our set of players is given by \( \{p\} \cup I \cup \{a\} \), where \( p \) and \( a \) stand for the principal and the agent, respectively.

We say a player (other than the agent) is active if he signs a contract \( w \neq 0 \) with the agent. Denote the set of active players by \( A \).

Let \( \Pi_i(e), i \in I \), stand for insurer \( i \)'s benefits from implementing \( e \), and let

\[
    w_A(\pi) = \sum_{i \in A} w_i(\pi)
\]

represent the aggregate contract.

\(^{20}\)We say a principal or an insurer “implements \( e \)” if he makes an offer (maximizing profits taking into account both past offers and the best responses of players moving next) such that the agent accepts his contract (and possibly others) and chooses effort \( e \).
Lemma 2 A subgame perfect equilibrium of our game is characterized by the following properties:

1. The aggregate cost of implementing effort $e$ is at least as great as the cost in the traditional principal-agent problem.

2. The principal is never better off than in the traditional problem.

3. In any case where the agent chooses $e_l$ or $e_h$, the principal is always an active player.

Proof.

1. Part 1 says that it is not possible for our aggregate player (the principal and the insurers) to do better. Suppose the contrary. But then the principal could mimic such an aggregate contract in the traditional principal-agent problem and pay less to implement $e$ than the minimum found in Section 2. We know this is not true.

2. From Part 1, we know that $\int w_A(\pi)f(\pi/e)d\pi = \int w_p(\pi)f(\pi/e)d\pi - \sum_{i \in I} \Pi_i(e) \geq e^{2nd}(e)$. If the principal were better off, at least one insurer would be making negative profits. This is not possible in equilibrium.

3. Notice that, ex-ante, insurers are not interested in the agent’s effort. This implies that, by themselves, they can never offer a profitable contract that gives an agent incentives to choose $e$. The principal must be active in every such case. Formally, we know from equation (1) that the aggregate cost of implementing $e$ is greater than zero, so that $\int w_A(\pi)f(\pi/e)d\pi = \int w_p(\pi)f(\pi/e)d\pi - \sum_{i \in I} \Pi_i(e) > 0$. If the principal is not active, then $\int w_p(\pi)f(\pi/e) = 0$, and at least one insurer makes negative profits, which is not possible in equilibrium.

Lemma 2 simply says that the introduction of these new players can add nothing to our previous economy. If our insurers had an informational advantage over the principal, such that they could observe the agent’s level of effort, then the principal could be better off, as in Itoh (1993) and Arnott and Stiglitz (1991). We will comment in the conclusions how this alternative assumption might alter our results.
4 Sequential Offering

4.1 The one-insurer case

In this subsection we consider the implementation of high effort in a setting in which 1) the principal offers a non-exclusive contract \( w_p \); 2) an insurer observes \( w_p \) and offers a contract \( w_1 \); 3) the agent considers which contracts (if any) to accept, and chooses the level of effort; 4) payoffs are realized. We denote this setting by PIA, which gives the order in which each player moves.

From Lemma 2 we know that

\[
\int w_p(\pi)f(\pi/e_h)d\pi \geq \int w^{2nd}(\pi)f(\pi/e_h)d\pi. \tag{3}
\]

We also know, from Lemma 1, that the principal’s contract for implementing high effort must satisfy GNIC. If that is the case, then the insurer can either stay out or implement high effort. We will show that the latter is the equilibrium action, induced by the first move of the principal.

If the insurer were to stay out, the combination of the non-insurability constraint with the participation constraint will lead to a cost to induce high effort which is higher than the second best, leaving the principal worse off.

The other possibility (the one that will be chosen in equilibrium) is for the principal to offer a contract such that the insurer is willing to participate and take the agent to high effort. In order to implement \( e_h \) the insurer will offer the contract such that the aggregate contract is the second-best contract:

\[
w_1(\pi) = w^{2nd}(\pi) - w_p(\pi),
\]

The expected benefits to the insurer from implementing \( e_h \) are therefore

\[
\Pi_1(e_h) = -\int w_1(\pi)f(\pi/e_h)d\pi = \int w_p(\pi)f(\pi/e_h)d\pi - \int w^{2nd}(\pi)f(\pi/e_h)d\pi.
\]

Equation (3) implies that \( \Pi_1(e_h) \geq 0 \), so an insurer will have incentives to take the agent to high effort (and he will not have incentives to take her to low effort given that the contract satisfies GNIC). The principal’s least costly way of implementing \( e_h \) is by setting \( w_p \) such that \( \Pi_1(e_h) = 0 \). We
show next that this is possible, so that it is a property shared by any SPE of this game.

From the zero profit condition the contract must satisfy

\[ \int w_p(\pi)f(\pi/e_h)d\pi = \int w_{2nd}(\pi)f(\pi/e_h)d\pi ; \]  

(SBEC)

that is, the principal is paying the same expected cost as in the traditional problem. (SBEC stands for Second-Best Expected Cost).

**Lemma 3** If the principal’s contract satisfies SBEC, then \( \psi = \pi \) (and therefore GNIC becomes NIC).

**Proof.** Assume that \( \psi \neq \pi \), so that \( \int v(w_p(\pi))f(\pi/e_h)d\pi - g(e_h) > \pi \). We show that in that case the principal’s contract cannot satisfy SBEC. If the principal’s contract satisfies IC, then costs must be higher than second best costs, so that SBEC cannot be true. Suppose the principal’s contract does not satisfy IC. But then an insurer must take the agent to high effort, offering a contract such that \( w_p(\pi) + w_1(\pi) \) satisfies IC. The cost of this aggregate contract must be higher than SBEC, since the insurer cannot leave the agent below \( \psi \). If the principal’s contract satisfied SBEC, then the insurer would be making negative profits.  

We are therefore looking for a contract that satisfies both SBEC and NIC:

\[ \int w_p(\pi)f(\pi/e_h)d\pi = \int w_{2nd}(\pi)f(\pi/e_h)d\pi \]

\[ > \phi [g(e_1) + \pi] \geq \int w_p(\pi)f(\pi/e_1)d\pi, \]

where the equality is SBEC, the first inequality follows from equation (1) and the second inequality follows from a version of NIC where \( \phi \) is applied to both sides.\(^{21}\)

Equation (4) has a nice interpretation. Compared to the second-best contract, the principal offers higher payments in the “good” states and lower payments in the “bad” ones in such a way that average compensation is the same when the agent makes high effort, but lower when she makes low effort. Therefore, the principal is offering a riskier contract to the agent but still paying the cost of the second-best contract.\(^{22}\) Since this alternative is

\(^{21}\)It is easy to see that existence is guaranteed.

\(^{22}\)The contract is riskier since the agent is being paid the same expected wage, but receiving lower utility than in the second best. Indeed, the interim payoff is below her reservation utility.
less costly than preempting the insurer’s entry, this is the option chosen in equilibrium, as mentioned before.

We therefore have the following result:

**Proposition 1** In the setting PIA, the principal’s cost of implementing high effort is identical to the traditional second-best cost. Furthermore, equilibria where high effort is implemented are characterized by the following properties:

(i) The principal offers a contract $w_p(\pi)$ that satisfies both SBEC and NIC.

(ii) The insurer takes the agent to the second-best contract, $w_A(\pi) = w_{2nd}(\pi)$, and makes zero profits.

(iii) The agent accepts both contracts, chooses $e_h$ and makes her reservation level of utility $\pi$.

Three points should be noticed. First, greater risk to the agent implies that the principal’s contract, by itself, leaves the agent under her reservation utility $\pi$. Call this the agent’s interim utility.\(^{23}\)

Second, there are multiple optimal contracts because many contracts satisfy (i) in Proposition 1. This multiplicity appears because the principal can expose the agent to different amounts of risk. That is, while there is an upper bound to the agent’s interim utility (which is SBEC and NIC with equality), there is no lower bound. Of course, we can assume that lower bound if we introduce limited liability. In that case, we get a bounded range of possible contracts.\(^{24}\)

Third, the insurer is given the incentives necessary to insure the agent and take her to the second-best aggregate contract and, consequently, to her reservation utility $\pi$.

The intuition is simple. Knowing that an insurer has incentives to reduce any risk the agent might take and eventually give her incentives to choose $e_h$, the principal offers a riskier contract such that the insurer is just willing to participate and insure the agent up to the second-best contract. As a result,

\(^{23}\)We will see later that when the principal cannot leave the agent below her reservation utility, then he is worse off.

\(^{24}\)Of course, the introduction of limited liability can jeopardize existence. Notice also that uniqueness of the contract can also be achieved by assuming that the insurer has a slight degree of risk aversion.
the principal manages to pay the traditional second-best cost, even in the presence of an insurer.

Given that results also remain the same for the case of \( e_l \), the resulting equilibria of this extended game shares most of the properties of the traditional equilibria. The principal still pays the second-best cost to implement high effort, the agent makes her reservation level of utility, and the insurer makes zero profits. What differs is the principal’s optimal contract, which is now steeper. However, the aggregate contract is still the second-best contract.

So far there have been no fundamental changes to the standard results. From Lemma 2 we know that, given our informational assumption, the principal cannot be better off. We have just seen a particular setting under which the principal is not worse off either. We now proceed to generalize this setting, and it turns out that in some cases the threat of insurance actually hurts the principal (implementation of high effort costs more).

We extend the model in two dimensions: 1) increasing the number of insurers both to a finite number and to its limiting case, free entry; and 2) considering alternative time sequences.

### 4.2 \( N \) insurers

In the last subsection we found that with one insurer the principal can implement the high level of effort at the same cost as before, by offering a contract that is “steeper” than the second-best contract. We now show that this result generalizes to a finite number of insurers.

We extend the setting of the previous section to the case where \( N \) insurers offer contracts to the agent, and the agent finally decides which of the \( N+1 \) contracts to accept, and which level of effort to exert. We denote this setting by \( \Pi^{\ldots}IA \).

The straightforward extension to the finite case is stated in the next proposition in order to emphasize the role played by the last insurer. The proof, together with some details specific to our multi-insurers setting, are left for the Appendix.

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\(^{25}\)To be precise, the principal’s cost of implementing \( e_l \) remains the same, but there are now multiple optimal contracts that implement \( e_l \). There are some contracts that give the agent somewhat greater risk, but cost \( c^{2nd}(e_l) \), and the insurer will afterwards insure the agent and take her to the flat wage \( \phi[g(e_l) + \overline{\pi}] \).
Proposition 2 In the setting PII...IA, the principal’s cost of implementing high effort is identical to the traditional second-best cost. Furthermore, equilibria where high effort is implemented are characterized by the following properties.\textsuperscript{26}

(i) The principal offers a contract $w_p(\pi)$ that satisfies SBEC and NIC.

(ii) $k \in \{0, 1, ..., N - 1\}$ insurers are active, make zero profits and offer contracts such that each subset of insurers satisfy NIC. (See the Appendix for details)

(iii) The last insurer (insurer $N$) is always active, makes zero profits, and takes the agent to the second-best contract, $w_A(\pi) = w^{2nd}(\pi)$.

(iv) The agent accepts the aggregate contract, chooses $e_h$ and makes her reservation level of utility $\pi$.

Proof: see the Appendix.

It is apparent from Proposition 2 that a key player in the result is the last insurer. He is responsible for taking the agent to the second-best aggregate contract. Of course, he is able to do so because he knows there is no next insurer willing to take the agent to an aggregate flat wage. In the traditional principal agent problem the principal possesses the technology that makes the agent’s effort valuable, as well as the power to control her consumption vector. In the case here the agent’s consumption vector is under the control of the last insurer, but the timing of the game gives the principal the ability to “appropriate” the rents generated by this power. With free-entry of insurers, there is no last insurer, and consequently no one willing to take the agent to the second best, since nobody has the power to control the agent’s consumption vector. As we will see next, that is one case where the threat of insurance hurts the principal.

5 Free entry

In this section we extend the previous setting to the case where there is free entry of insurers. Insurers continue to make their offers sequentially, but now

\textsuperscript{26}Again, we have multiple equilibria. But now there is a second source of multiplicity: equilibrium is consistent with any number of active insurers (as long as the last insurer is active). There is a simple way to get rid of the second source. If the agent bears a small cost $\varepsilon > 0$ from contracting with each insurer, then we have uniqueness in the sense that the agent will accept contracts only from the last insurer. Of course, there is still multiplicity from the first source.
there is no last insurer. To avoid the problem of infinity of offers, we assume that a) the agent can stop the game at any time;\textsuperscript{27} b) the agent gets an utility of $-\infty$ if she negotiates an infinite number of contracts.

Results from the finite case can be interpreted as follows. The principal and the $N$ insurers can be seen as an aggregate player facing a single agent. From Lemma 2, part 1, the aggregate cost of implementing a given level of effort is greater than or equal to the traditional cost, without the presence of insurers. It turns out that, given his first-mover advantage, the principal can still pay that lower bound cost in order to implement high effort.

With free-entry, it will still be true that the principal pays the lower bound aggregate cost of implementing $e_h$. However, this cost now differs from the second-best cost.

Recall that the aggregate contract is

$$w_A(\pi) = \sum_{i \in A} w_i(\pi),$$

where $A$ is the set of active players. What is the equilibrium aggregate cost of implementing high effort? In the finite case, it was simply the second-best cost; that is, the solution to the problem of minimizing cost subject to PC and IC. But now, we know that such a contract cannot implement high effort: a new insurer will take the agent to low effort. As a result, the optimal aggregate contract (i.e. the one that yields the minimum aggregate cost of implementing high effort) is the solution to the following third-best problem:

$$\max - \int w_A(\pi)f(\pi/e_h)d\pi$$

subject to

$$\int v(w_A(\pi))f(\pi/e_h)d\pi - g(e_h) \geq \psi$$ \hspace{1cm} (PC)

$$\int v(w_A(\pi))f(\pi/e_h)d\pi - g(e_h) \geq \int v(w_A(\pi))f(\pi/e_l)d\pi - g(e_l)$$ \hspace{1cm} (IC)

$$\psi \geq v(\int w_A(\pi)f(\pi/e_l)d\pi) - g(e_l),$$ \hspace{1cm} (GNIC)

where the new constraint, GNIC, follows from Lemma 1. Recall that

$$\psi = \max \left\{ \bar{\pi}, \int v(w_A(\pi))f(\pi/e_h)d\pi - g(e_h) \right\}.$$

Call the cost minimizing contract the third best, and denote it by $w^{3rd}(\pi)$. It is an immediate result that the third best contract costs more than the second best contract.

\textsuperscript{27}The agent, however, cannot commit to stop the game at a certain time.
Proposition 3 The minimum aggregate cost of implementing $e_h$ with sequential free entry of insurers is strictly greater than the cost of implementing $e_h$ without insurers or with a finite number of insurers; that is
\[ \int w^{3\text{rd}}(\pi)f(\pi/e_h)d\pi > \int w^{2\text{nd}}(\pi)f(\pi/e_h)d\pi. \]

**Proof.** Relative to the second best problem, there is an additional constraint, implying that \( \int w^{3\text{rd}}(\pi)f(\pi/e_h)d\pi \geq \int w^{2\text{nd}}(\pi)f(\pi/e_h)d\pi \). The strict inequality comes from the fact that the solution to the second best problem does not satisfy the new restriction, GNIC.

Given the sequential nature of the game and following the analysis from the finite case, it is easy to show that the principal can implement high effort by paying the minimum possible cost; in this case, the third-best cost. One possibility is for the principal to offer the third-best contract, so that no insurer participates. Another possibility is for the principal to offer a contract satisfying third-best expected cost, \( \int w_p(\pi)f(\pi/e_h)d\pi = \int w^{3\text{rd}}(\pi)f(\pi/e_h)d\pi \), (TBEC) and GNIC, but with \( w_p(\pi) \neq w^{3\text{rd}}(\pi) \). In this case the principal’s contract gives the agent higher risk than the third-best contract, and leaves her under her reservation utility. An insurer (or many insurers) will take the agent to the third-best aggregate contract. Thus, we continue to have multiple equilibria.

The following proposition characterizes the equilibrium in the free entry case.

**Proposition 4** In a setting with Free Entry, the principal’s cost of implementing $e_h$ is equal to the third-best cost, which is greater than the traditional second best cost. Furthermore, equilibria where high effort is implemented are characterized by the following properties:

(i) The principal offers a contract $w_p(\pi)$ that satisfies TBEC and GNIC.
(ii) If $w_p(\pi) = w^{3\text{rd}}(\pi)$, no insurers are active. If $w_p(\pi) \neq w^{3\text{rd}}(\pi)$, any number of finite insurers are active, make zero profits, and offer contracts such that each subset of insurers satisfy GNIC.
(iii) The equilibrium aggregate contract satisfies $w_A(\pi) = w^{3\text{rd}}(\pi)$.

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28 Again, we can get rid of this multiplicity with a small cost of contracting.
29 The requirement that a subset of insurers satisfy GNIC is identical to the requirement formally stated in the proof of proposition 2, except that NIC is now replaced by GNIC.
(iv) The agent accepts that aggregate contract, stops the game, chooses \( e_h \) and makes, at least, her reservation utility level, \( \overline{u} \).

Notice that we have used the more general constraint, GNIC; simply because we have not yet specified whether the optimal contract for implementing high effort might leave the agent above her reservation utility.

### 5.1 Characterization of the Third-Best Contract

Next we provide a characterization of the third best contract under a particular assumption over the utility function. In particular, we show a sufficient condition on the utility function such that the principal leaves the agent at her reservation utility (so that NIC is the relevant constraint).\(^{30}\)

We first show that IC is a redundant constraint.

**Lemma 4** Given strict risk aversion, constraint GNIC and PC imply that IC is satisfied with strict inequality.

**Proof.** Using the fact that \( v \) is a strictly concave function, Jensen’s inequality implies that

\[
\int v(w_A(\pi)) f(\pi/e_i) d\pi < v \left( \int w_A(\pi) f(\pi/e_i) d\pi \right). \tag{5}
\]

General NIC implies that

\[
\int v(w_A(\pi)) f(\pi/e_i) d\pi - g(e_i) < \psi.
\]

The result follows from PC and the definition of \( \psi \). \( \blacksquare \)

Lemma 4 allows us to rewrite the third-best problem in the following way:

\[
\max_{w_A(\pi)} - \int w_A(\pi) f(\pi/e_h) d\pi
\]

subject to

\[
\int v(w_A(\pi)) f(\pi/e_h) d\pi - g(e_h) \geq \overline{u} \tag{PC}
\]

\(^{30}\)We also conjecture, in the next footnote, a condition under which the principal might decide to leave the agent above her reservation utility (so that GNIC is the relevant constraint).
\[
\int v(w_A(\pi)) f(\pi/e_h) d\pi - g(e_h) \geq v \left( \int w_A(\pi) f(\pi/e_l) d\pi \right) - g(e_l), (\text{GNIC}')
\]

Constraint GNIC' can be seen as a modified incentive constraint, given that the presence of insurance has increased utility from low effort.

As in the traditional problem, it is easy to see that the incentive constraint GNIC' is binding. If it were not, then the optimal contract would be a flat wage satisfying PC. But this does not satisfy GNIC'.

Could the cost minimizing contract leave the agent above her reservation utility? The following proposition shows that this cannot be the case if we assume non-decreasing absolute risk aversion.

**Proposition 5** Assume the agent has non-decreasing absolute risk aversion. Then the third-best aggregate contract leaves her exactly at her reservation utility level, \( \pi \). Furthermore, the optimal contract \( w^{3rd}(\pi) \) is fully characterized by solution to constraints PC and GNIC' binding, and to the following first-order condition

\[
\frac{1}{v'(w^{3rd}(\pi))} = \frac{\lambda}{1 + \delta \frac{f(\pi/e_l)}{f(\pi/e_h)}}, \quad (\text{FOC})
\]

where \( \lambda > 0 \) and \( \delta > 0 \) are the usual Lagrange multipliers.

**Proof.** See the appendix.

The only incentive the principal could have to pay the agent more than her reservation utility is to reduce the risk premium. But when the agent has non-decreasing absolute risk aversion, increasing the utility of the agent will not reduce the risk premium.\(^{31}\) From now on we assume the agent has non-decreasing absolute risk aversion, so that she is always left at her reservation utility.

Proposition 5 characterizes the third-best contract under the assumption of non-decreasing absolute risk aversion. It is easy to verify that a property valid in the traditional problem holds for the third-best contract as well: the optimal third-best rewarding scheme is increasing in \( \pi \) under the assumption that the monotone likelihood ratio property holds.

We close this section with a comparison between the finite and the free entry case. The aggregate contract in the finite case, similar to the standard

\(^{31}\)It seems possible that when the agent absolute risk aversion is strongly decreasing, the principal could reduce the cost by giving the agent more than her reservation utility, but we do not have an example where this happens.
results, is the second-best contract. The aggregate contract in the free entry case is a third-best contract; this has different implications for welfare and (for some parameter values) for equilibrium actions.

However, both the finite number of insurers and the free entry case have some similarities with regard to the principal’s contract. In both cases, in comparison with the standard case, the original contract is steeper. Under free entry the contract’s variance is amplified by increasing the reward to high effort. This, as we have noticed, involves a greater cost. On the other hand, the principal’s optimal contract when the number of insurers is finite pays the same to high effort but decreases the expected payoff of undertaking low effort. The variance is also increased. Therefore, incentives are enhanced in both cases. The difference is that in one case the reward to high effort increases, while in the other the reward to low effort decreases.

6 Sequential Contracting

Up to now we have modeled situations in which the agent faces a set of offers before making a final decision. We found that when there is no last player, the threat of insurance hurts the principal. A different case is that in which, after signing the principal’s contract (and before exerting effort), the agent can engage in additional contractual relations.

We first consider a setting with one principal and one insurer, and continue to assume that the principal moves first. When the agent has all the bargaining power in the agent-insurer relationship, insurance does not hurt the principal. However, when we give all the bargaining power to the insurer, the principal’s optimal contract is third best.

The extensions to N and to free entry that we have worked out in the previous section are straightforward in these new settings. Once again, free entry leads to a third-best equilibrium, with only some minor modifications that we will comment along the way.

6.1 Agent makes offer

We consider a sequential setting where: 1) The principal offers a non-exclusive contract $w_p(\pi)$; 2) the agent decides whether to accept or reject the contract; 3) the agent offers a contract $w_a(\pi)$ to the insurer; 4) the insurer either accepts or rejects the contract; 5) the agent chooses the level of effort; 6) payoffs
are realized. We denote this setting by PAAI. It turns out that the results are identical to the ones from the sequential offering case, (when the agent was the last to play.)

The principal offers a contract satisfying SBEC and NIC, which by itself gives the agent a utility under her reservation level. Now the agent makes an offer $w_a(\pi)$ to the insurer. This contract could be offered with the intention of making high or low effort.

Notice that the most the agent can get is $\pi$. This is because if the agent gets more than $\pi$, it must be at the expense of an insurer making negative profits. In the case of high effort, the agent gets $\pi$ by offering a contract such that $w_a(\pi) = w^{2nd}(\pi) - w_p(\pi)$. Integrating and using SBEC we get $\int w_a(\pi)f(\pi/e_h)d\pi = 0$; that is, the insurer makes zero profits and is indifferent between accepting this contract or not.

It is easy to see that the agent will not choose low effort. In order to do so she should ask the insurer for a contract $w_p(\pi) + w_a(\pi) = \phi[g(e_l) + \pi]$. But the insurer would never accept this contract, because the principal’s contract satisfies NIC and so the insurer would make negative profits. Therefore we have an equilibrium where the principal offers a contract satisfying SBEC and NIC, and the insurer takes the agent to the second best.

Consider the extension of the above setting to the case with $N$ insurers, where the agent makes an offer to a second insurer, who can either accept or reject it, and so on, until the last insurer is reached. Once again, it is easy to show that no insurer except the last would accept to take the agent to the second-best aggregate contract. Results are almost identical to the timing PI...IA, as summarized in Proposition 2. There is a slight difference in Part (ii) of the Proposition as a result that now the contracts are not only offered sequentially but are also being signed sequentially. As a result, we do not need all subset of offered contracts (except the last insurer’s contract) to satisfy NIC, as we did in the proof of Proposition 2. That condition must now hold only for each subset of contracts that have been signed, for all stages of the game (except the last).

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32 Once again we have multiple optimal contracts; comments from Section 4 are also valid here.

33 What is important is that the agent has all the bargaining power in the last agent-insurer relationship.
In the free entry version of this timing we still have the third best. This is simply because no insurer will accept the agent’s insurance proposal since she cannot commit to not making any further proposals to other insurers.\footnote{To be precise, Proposition 4 needs a slight modification. In part (ii), even if the principal offers the third best contract, \( w^p(\pi) = w^{3rd}(\pi) \), any number of insurers may be active. These insurers might continually give and take risk away from the agent. These contracts are, of course, completely uninteresting.}

6.2 Insurer makes offer

We now consider a timing where the insurers are given all the bargaining power vis a vis the agent. As in the previous subsection, we describe the case of \( N = 1 \), in which: 1) The principal offers a non-exclusive contract \( w_p(\pi) \); 2) the agent decides whether to accept the contract; 3) the insurer offers the agent the contract \( w_I(\pi) \); 4) the agent either accepts or rejects the contract; 5) the agent chooses the level of effort; 6) payoffs are realized. Denote this setting by PAIA. It turns out that the insurer’s bargaining power will now hurt the principal.

**Proposition 6** In order to implement high effort in the timing PAIA, the principal pays the third-best cost and the insurer makes positive profits.

**Proof.** The principal would like, once again, to offer SBEC and NIC, but this gives the agent less than \( \pi \). Then the insurer, who has all the bargaining power, would leave the agent at this lower utility. The agent will never accept the principal’s contract in the first place.

Therefore, the principal now faces two constraints: 1) GNIC; 2) give the agent the reservation level of utility. This is precisely our 3rd best problem.

Therefore, we have a unique equilibrium where, in order to implement high effort, the principal offers the 3rd best contract

\[
w_p(\pi) = w^{3rd}(\pi).
\]

(6)

Given this contract, the insurer offers to take the extra risk away from the agent and makes positive profits. This is done by offering \( w_I(\pi) = w^{2nd}(\pi) - w_p(\pi) \), so that integrating and using equation (6) we get

\[
\Pi_1(e_h) = - \int w_I(\pi) f(\pi/e_h) d\pi = \int w^{3rd}(\pi) f(\pi/e_h) d\pi - \int w^{2nd}(\pi) f(\pi/e_h) d\pi > 0.
\]
In this case, the insurer appropriates the rents from being able to control the agent’s consumption vector, which is the difference between the third-best and the second best cost.

Once again, with $N$ insurers only the last insurer takes her to the second-best aggregate contract and makes positive profits. Notice also that under this particular sequence, free entry leads to a unique equilibrium: the principal’s contract is always the third-best contract. The reason is that any contract satisfying TBEC and NIC, but different from the third-best contract, would leave the agent below her reservation utility.

7 Conclusion: Summary of Results, Implications and Possible Extensions

In this paper we have studied how the threat of insurance affects contracting in moral-hazard agency relationships. We first showed that the traditional second-best equilibrium is not robust to the introduction of this additional risk-sharing opportunities. We then characterized the equilibria of our new game under different settings, and found that the principal can sometimes manage to overcome this threat by offering steeper (more intense) rewarding schemes. However, when “outside competition” is strong enough, the principal is worse off.

Our results might have important practical implications for the design of compensation schemes, in making the point that attention should be paid to outside opportunities even if they seem productively unrelated (as in the common-agency reading of this paper).

In this section we summarize the results and implications, and discuss possible extensions

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35 The common agency literature has concentrated on competition for the agent’s limited effort. On the other hand, this paper has shown that there is an important insurance effect, so that there are contractual externalities in two dimensions. The degree to which this second externality bites in our setup is determined by factors such as bargaining power and the ease of entry into “the industry”.
7.1 Results and welfare comparisons

Table 1 lists results for the three different sequences (with a finite number of insurers) and for the free entry case. The principal manages to still pay the second-best cost in the case of sequential offering and in the sub-case of sequential contracting in which the agent has the bargaining power in the insurance contract. This is accomplished by offering a contract that gives the agent both more risk and more incentives than in the second best. (After that, the last insurer takes the agent to an aggregate contract equal to the second-best contract.)

It is when insurers have the bargaining power in the insurance relationship, or when there is always the threat of further insurance, that the results differ more substantially from the standard case. In the first case, the equilibrium aggregate contract is the second best, but the principal is worse off, since he has to pay a higher cost for implementing high effort. In the latter, the same occurs, except that the aggregate contract is the third best. Furthermore, the difference between the third and fourth rows of the table is that in the first case the last insurer can engage in a non-threatable contract with the agent, and can make positive profits, while in the free entry case, those rents are just diluted to zero, and no one in the economy receives what the principal has paid in excess of the second best.

Furthermore, there will be situations (parameter values) in which that increased cost will prevent the principal from choosing to implement high effort. A comparison of the situations between the first two (and the standard case) and the last two rows of the table, in terms of equilibrium efforts, is given in Figure 1. The horizontal axis measures the difference in expected profits to the principal under high versus low effort. To the left of $w^{2nd}(\pi) - \phi[g(e_1) + \overline{\pi}]$, low effort is chosen in equilibrium in all setups. To the right of $w^{3nd}(\pi) - \phi[g(e_1) + \overline{\pi}]$, high effort is chosen in equilibrium in all set ups. In the region in between those two values, the effective threat of insurance moves the economy from high to low effort.

There are two sources of (potential) inefficiency in comparison to the standard principal agent problem. A first effect (1) is that the principal
not always implements the second-best optimal level of effort. This is the case in rows 3 and 4. There is a second source of inefficiency (2) in the free entry case. Because the insurers do not appropriate any of the principal’s losses, even when the principal insists on implementing high effort, there is an efficiency loss given by the higher implementation cost. It is therefore in the free entry case where the economy suffers the greatest welfare loss.

7.2 Empirical implications

The motivation of the paper is that contract exclusivity may not be enforceable in many real world situations. Empirical implications can be obtained from the comparison between cases where the threat of insurance is totally absent and cases where it might be an issue.36 The threat of insurance may not be a problem, either because the risk in question is non insurable, or because, being insurable, further contracting can be legally forbidden. This in turn, may relate to the overall institutional and legal capabilities of a given country, or to the observability and verifiability of the additional contracts, which might vary across markets or across activities.

The main implications are:

1) in cases of non-exclusivity, performance contracts will be less common, and effort (and hence output) will be lower. This comparison might be undertaken across industries and / or across geographical markets. There might be potential comparisons across legal rules.

2) when performance contracts are indeed used, they will tend to be steeper. The steepness of contracts comes from a lowering of the payoffs to low outcomes in rows 1 and 2 of Table 1, and from increasing the expected payoff to high effort in rows 3 and 4.

Some additional predictions might obtain, although they would require further theoretical analysis, relating to:

a) the tendency to observe exclusivity clauses (where their use will depend on the ex ante incentives of the principal and of the agent to utilize such a commitment technology if it was available); and

b) to the use of different indicators in performance contracts, depending on their observability not only by the principal, but also by outside parties.

36Within the set of insurable (threatable) contracts, we have some different implications depending on the details of sequencing, bargaining power, and number of players.
7.3 Extensions

Some possible extensions have already been hinted throughout the paper. We list here a few additional exercises which might deliver interesting insights.\(^{37}\)

One possible extension would be to work with a simultaneous setting in which everybody offers contracts at the same time, or a semi-simultaneous setting in which all insurers simultaneously offer contracts after the principal. This might generate different results, since the notion of last player was crucial for part of the analysis. One might conjecture that when the insurers make the offers simultaneously and there is limited liability, the second-best solution would be implementable only if the quantity of insurers is small enough.\(^{38}\)

Alternatively, the appearance of new contractual opportunities might not be an event known with certainty by the principal or the agent. One can model a sequential setting in which the appearance of an insurer occurs with an interior probability. Notice that we already know the results when that probability is zero (the standard case), and when it is one (this paper). Preliminary computations suggest that, even in the cases where the second best is implementable with insurers present for sure, it might not be implementable when the existence of insurers is uncertain. The intuition is that in these cases the uncertainty about the presence of insurers precludes an agent from accepting a principal’s contract that leaves her below her reservation utility.

The standard principal-agent problem might also be non robust to other contracts by the agent even if not related to the output of this productive relationship: for instance other contracts that generate income for the agent and affect his risk aversion at the margin. It could be interesting to model how different specifications of such a situation affect the original contract.

In this paper we have emphasized the “negative effects of insurance opportunities.” Itoh (1993) and Arnott and Stiglitz (1991) have shown that

\(^{37}\)Of course there is the possibly straightforward, but certainly tedious, extension to multiple effort levels. In that case we would have a non-insurability condition for each effort level above the lowest one. The model might also be extended to the continuous case, where several technical details will appear; all in the spirit of the traditional model’s extension to continuous effort.

\(^{38}\)This could also be thought as an extension of the Arnott-Stiglitz, Hellwig papers. A principal will offer a contract knowing that after him the agent will face the insurance market studied by them. So the principal will provide a contract that is the lottery with which the agent arrives to the insurance market.
when these “third players” have perfect information — i.e., they observe effort — welfare improves. It would be quite interesting to interact the different settings developed in our paper, with different degrees of informational advantage by the insurers.

Also, as stated before, this paper might lead to a sort of “qualification” of the Informativeness Principle. Briefly, such a principle states that any signal that gives information about the agent’s level of effort has positive value and (if costless) should be included in the reward scheme. Our results suggest that the value of information might depend also on the observability (and contractibility) by other potential contractual partners. This intuition would require a multi-signal extension of our work.\(^{39}\)

More generally, outside opportunities should be taken into account when designing incentive schemes. We have shown that this is true even when those outside opportunities are seemingly unrelated. This might require the revision of some additional previous results in the applied literature on incentives.

8 Appendix:

Proof of Proposition 2

To prove Proposition 2 notice that Lemma 2 implies that

\[
\int w_A(\pi)f(\pi/e_h)d\pi \geq c^{2nd}(e_h)
\]

or

\[
\int w_p(\pi)f(\pi/e_h)d\pi - \sum_{i=1}^{N} \Pi_i(e) \geq c^{2nd}(e_h).
\] (7)

Consider the case where the principal offers a contract with second-best expected cost, as in (SBEC). Insurers can respond by implementing \(e_h, e_l\) or staying out. Assume for the moment that the best response for insurer 1 is to implement \(e_h\), and that he chooses to play that best response. Then from

\(^{39}\)Note that here, as in the basic model of the standard case, “output” is the only signal of effort.
equation (7) profits must be zero, and profits from deviating to \( e_l \) must be less than or equal to zero. Now insurer 2 will take as given

\[
\int [w_p(\pi) + w_1(\pi)]f(\pi/e_h)d\pi = c^{2nd}(e_h) - \Pi_1(e_h) \\
= c^{2nd}(e_h) - 0 \\
= c^{2nd}(e_h),
\]

Insurer 2 simply faces the same situation as insurer 1, so his best response must again include \( e_h \). All we need to check is that insurers have no incentives to deviate to \( e_l \).

From Lemma 1 and a trivial generalization of the results from the one insurer case (i.e. lemma 3), insurer one will have no incentives to take the agent to low effort if and only if NIC holds. So again the principal can offer a contract satisfying SBEC and NIC and incur the same cost to make the agent implement high effort.

The remaining insurers, except for the last insurer, are themselves subject to NIC. This means that none of them can take the agent to the second-best aggregate contract (if they did, they would make negative profits because the next insurer would take the agent to low effort). Therefore, they must satisfy a notion of aggregate NIC that we define below.

Let \( O_h = \{p, 1, 2, ..., h - 1\}, h = 1, ..., N \), represent the set of players that have already made their offers by the time it is the turn of insurer \( h \) to move, where \( p \) is the principal. Let \( \wp(O_h) \) denote the set whose elements are all subsets of \( O_h \); that is, the power set. We then require that NIC is satisfied for all subset of players \( X_h \in \wp(O_h) \), for all \( h = 1, ..., N \). To state this formally, let \( W_{X_h} \) stand for the contract made up of players in the subset \( X_h \),

\[
W_{X_h} = \sum_{i \in X_h} w_i(\pi).
\]

Then the equilibrium contracts \((w_p, w_1, ..., w_N)\) must satisfy

\[
v(\int W_{X_h}(\pi)f(\pi/e_l)d\pi) - g(e_l) \leq \varphi \text{ for all } X_h \in \wp(O_h), \text{ for all } h = 1, ..., N.
\]

Notice also that the equilibrium aggregate contract up to insurer N-1 gives the agent a utility that is below her reservation utility. The agent will accept to put the high level of effort only in the case the last insurer takes her to the second best aggregate contract.
Proof of Proposition 5

i) Our first step is to show that non-decreasing absolute risk aversion implies that under the optimal solution, constraint PC is binding. Let \( w^* \) be an optimal solution with PC non-binding. Define \( \hat{w}(\pi) = w^*(\pi) - \varepsilon \) for all \( \pi \), for \( \varepsilon > 0 \) small enough so that PC holds. It is straightforward to see that costs are lower under \( \hat{w} \). Next, we show that constraint GNIC' remains satisfied with this new contract (and hence that \( w^* \) cannot be an optimal solution.)

To see this, use the definition of risk premium to rewrite the agent’s utility under high effort as \( v \left( \int w^*(\pi) f(\pi/e_h) d\pi - \rho(w^*(\pi)) \right) - g(e_h) \). Use this new definition to rewrite GNIC’. We must show that

\[
v \left( \int w^*(\pi) f(\pi/e_h) d\pi - \rho(w^*(\pi)) \right) - v \left( \int w^*(\pi) f(\pi/e_h) d\pi - \rho(\hat{w}(\pi)) - \varepsilon \right) \leq v \left( \int w^*(\pi) f(\pi/e_1) d\pi \right) - v \left( \int w^*(\pi) f(\pi/e_1) d\pi - \varepsilon \right).
\]

Nondecreasing absolute risk aversion implies that \( \rho(w^*(\pi)) \geq \rho(\hat{w}(\pi)) \). Therefore,

\[
v \left( \int w^*(\pi) f(\pi/e_h) d\pi - \rho(w^*(\pi)) \right) - v \left( \int w^*(\pi) f(\pi/e_h) d\pi - \rho(\hat{w}(\pi)) - \varepsilon \right) \leq v \left( \int w^*(\pi) f(\pi/e_h) d\pi - \rho(w^*(\pi)) \right) - v \left( \int w^*(\pi) f(\pi/e_h) d\pi - \rho(w^*(\pi)) - \varepsilon \right) < v \left( \int w^*(\pi) f(\pi/e_1) d\pi \right) - v \left( \int w^*(\pi) f(\pi/e_1) d\pi - \varepsilon \right).
\]

The first inequality follows directly from nondecreasing absolute risk aversion. The second inequality holds due to the strict concavity of \( v \). To see the latter, let \( A = \int w^*(\pi) f(\pi/e_h) d\pi - \rho(w^*(\pi)) \) and \( B = \int w^*(\pi) f(\pi/e_1) d\pi \).

Notice that as \( w^* \) satisfies GNIC’, then \( A > B \). Strict concavity of \( v \) implies that \( v(A) - v(A - \varepsilon) < v(B) - v(B - \varepsilon) \) for \( \varepsilon > 0 \).

ii) Our next step is to use the fact that the solution will be such that \( \psi = \pi \) to rewrite the third-best problem as

\[
\max_{w_A(\pi)} \int w_A(\pi) f(\pi/e_h) d\pi
\]

subject to

\[
\int v(w_A(\pi)) f(\pi/e_h) d\pi - g(e_h) = \pi \quad \text{(PC)}
\]
Rewriting NIC as in equation 2 in the text and applying the standard trick of defining \( x(\pi) = v(w_A(\pi)) \) so that \( w_A(\pi) = \phi(x(\pi)) \) (see Grossman and Hart (1983)), the problem becomes

\[
\max_{x(\pi)} - \int \phi(x(\pi)) f(\pi/e_i) d\pi
\]

subject to

\[
\int x(\pi) f(\pi/e_h) d\pi - g(e_h) = \overline{u}
\]

\[
\int \phi(x(\pi)) f(\pi/e_i) d\pi \leq \phi \left[ g(e_i) + \overline{u} \right].
\]

To apply the Kuhn-Tucker Theorem, notice that the objective function is strictly concave and that the constraint set is convex. To see the latter let PCt and NICt hold for \( x \) and \( y \). We need to show that both equations also hold for \( z = \alpha x + (1 - \alpha) y \). That the first holds is trivial since it is linear. That the second holds follows from the strict convexity of \( \phi \):

\[
\int \phi \left[ \alpha x(\pi) + (1 - \alpha) y(\pi) \right] f(\pi/e_i) < \int \alpha \phi(x(\pi)) + (1 - \alpha) \phi(y(\pi)) f(\pi/e_i) \leq \phi \left[ g(e_i) + \overline{u} \right].
\]

As a result, the first order conditions are both necessary and sufficient for a global optimum. The FOC are:

\[
\frac{1}{v'(w_{3rd}(\pi))} = \frac{\lambda}{1 + \delta \frac{f(\pi/e_i)}{f(\pi/e_h)}},
\]

where \( \lambda \) and \( \delta \geq 0 \) are the Lagrange multipliers associated to PCt and NICt respectively.

Notice that we must have \( \delta > 0 \), if it were \( \delta = 0 \) then the solution would be a flat wage, and this does not satisfy NICt. Given that \( \delta > 0 \) and \( v' > 0 \) (and finite!), then we must also have \( \lambda > 0 \). This proves the desired result. \[\Box\]
References


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<td>Even Lower</td>
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Table 1. Summary of results
\[ e_l \quad e_h \]

\[ \int w^{2nd} f(\pi / e_h) d\pi - \phi[\bar{u} + g(e_l)] \quad \int \pi f(\pi / e_h) d\pi - \int \pi f(\pi / e_i) d\pi \]

**Figure 1A.** Effort implemented in equilibrium: standard case, PIA, PAAI.

\[ e_l \quad e_h \]

\[ \int w^{3rd} f(\pi / e_h) d\pi - \phi[\bar{u} + g(e_l)] \quad \int \pi f(\pi / e_h) d\pi - \int \pi f(\pi / e_i) d\pi \]

**Figure 1B.** Effort implemented in equilibrium: PAIA, Free Entry.