Strategic Effect of Option Contracts

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Abstract

Option contracts are introduced to a supply function equilibrium (SFE) model. The uniqueness of the equilibrium in the spot market is established. Comparative statics results on the effect of option contracts are presented. A multi-stage game where option contracts are traded before the spot market stage is considered. When contracts are optimally procured by a central authority, the selected profile of option contracts is such that the spot market price equals marginal cost for any load level resulting in a significant reduction in cost. If load serving entities (LSEs) are price takers, in equilibrium, there is no trade of option contracts. Even when LSEs have market power, the central authority's solution cannot be implemented in equilibrium.

1 Introduction

Wholesale electricity markets exhibit characteristics that can result in severe strategic supply reduction. Highly inelastic demand, transmission constraints and non-storability are among these characteristics. Supply reduction not only affects equilibrium prices but also network security and the cost of providing ancillary services. Option contracts and other electricity derivatives have been considered to mitigate market power while keeping an adequate level of investment (see [9] and [10]). In this work, we study the potential benefits of option contracts in reducing market power, i.e. supply reduction, and how different market environments affect the realization of these potential gains.

One of the contributions of this work is to introduce option contracts to a Supply Function Equilibrium (SFE) model. Following common practice in the literature, we use the expression Supply Function Equilibrium (SFE) to refer to the equilibrium of a uniform price reverse auction, in which there is no asymmetric information. By issuing a "call" option, a supplier commits to provide a certain amount of units at a specific price in case these units are demanded. We analyze in detail how these contracts affect the incentives of generators to exercise market power.

We also consider multistage games in which option contracts are traded in rounds of negotiations that precede the spot market. We model these interactions explicitly. In this, we depart from common practice in the literature that simply imposes a no-arbitrage condition. Explicit modeling of bargaining is needed because in imperfectly competitive industries there are nontrivial interactions between spot and derivatives markets.

Two different settings will be considered. In a first scenario a central authority (or social planner) procures option contracts with the objective of minimizing the total cost of energy. Total cost of energy includes the cost of procuring option contracts and the net revenue in the spot market. The outcome from this setting will be used as a benchmark result reflecting the potential gains from option contracts in terms of market power mitigation.

Next, we analyze the effect of option contracts in a decentralized environment. LSEs are modeled alternatively as price taking agents and agents with market power. LSEs are able to perform bilateral bargaining of option contracts with generators. We will focus on how much of the gains presented in the centralized setting can be attained in a setting in which LSEs act independently.

Our results can also be applied to other industries. Multiunit auctions are increasingly used in the private and public sector. For example, uniform price auctions are used by governments to sell bonds and lumber.

In the following section we comment on the related literature. Section 3 presents the spot market model; this is followed by the analysis of multistage games in which financial positions are endogenously determined. The last section considers extensions and concludes.

2 Related Literature

The study of SFE was started by Klemperer and Meyer [4]. These models constitute a generalization of more traditional models used in the study of oligopolies. Instead of assuming that firms compete in price or quantities, these authors allowed firms to select a supply function, that is, a mapping that assigns to each price a quantity offered.

Most of the literature focuses on the case in which the position of the demand function is uncertain. Tractability and multiplicity of equilibrium are common problems when selecting this approach.

SFE has received renewed attention as an application to wholesale electricity markets. This is because the model of supply function equilibrium is consistent with actual wholesale electricity markets. See for example Newbery [5], Green [6] and Rudkevich et alt. [7].

Our model is close to the approach taken by Holmberg [8] in allowing for equilibrium excess of demand. That is, demand might be higher than firms installed capacity with strictly positive probability. That will result in a boundary condition that guarantees uniqueness of the spot market equilibrium.

To our knowledge, there is no piece of literature that considers option contracts in a SFE. There are some works considering contracts for differences (e.g. Green

[6]).
Another related piece of literature is the one started by Allaz and Vilas [1]. They analyze the effect of futures contracts on market power in a model of Cournot competition. They conclude that future contracts result in significantly more competitive market outcomes. We extend their work in many dimensions. First we work with a model of supply function equilibrium, second, we are allowing for option contracts. Observe that futures contracts might be viewed as option contracts with a strike price of 0. We also model the financial contract trading stage explicitly, while in their work a simple no-arbitrage condition is imposed. Ferreira [3] and Hughes et alt. [2] extend this literature by analyzing the effect of observability in the model.
Finally, as mentioned in the introduction, there are some electricity markets policy papers that stress the role of financial derivatives in allowing for more competitive outcomes and an adequate level of investment(see Wilson et alt. [9] and Oren [10]).

3 The spot market

There are n suppliers selecting supply functions in a uniform price reverse auction with a finite grid of prices $\{p_k\}_{k=0}^{M}$, with $p_0 = 0$. An strategy for firm i consist of a maximum quantity offered q_k^i for each price in the grid p_k, $\left\{q_k^i\right\}_{k=0}^{M}$. This sequence must be increasing in k.
The demand function is inelastic, the quantity demanded equals x and is distributed according to $F(x)$ with support $[0, \infty)$. Suppliers face constant marginal cost normalized to zero up to *cap*, the total capacity of the firm.
There are also option contracts: o_k^i is the quantity committed by firm i at a price of k, $O_k^i = \sum_{j=0}^{k} o_j^i$. Firms always comply with options contracts, this means that the maximum quantity that is offered by a firm in the spot market is $cap - O_M$, otherwise with positive probability a firm might not be able to supply quantities committed under option contracts. We are assumming that the sanctions to generators are such they do not engage in this behavior. When the superindex i is omitted we refer to the aggregate variable.
The equilibrium price is the minimum p_k such that $q_k + O_k \geq x$. We assummed proportional rationing over incremental quantities. There are two situations in which rationing might occur in equilibrium. If $q_{k-1} + O_{k-1} < x < q_{k-1} + O_k$, not spot market quantities are called at the clearing price. The quantity supplied by agent i equals the quantity offered below the clearing price (including exercised options) plus a fraction of the excess demand at p_{k-1} proportional to o_k^i / o_k. In the case $q_{k-1} + O_k < x < q_k + O_k$ similar proportional rationing is applied on the additional quantities offered at p_k.

For a given demand level x the profit function equals:

$$\Pi(q_k^i, o_k^i, x) = p_k q_{k-1}^i + \sum_{j=0}^{k-1} p_j o_j^i + p_k o_k^i min\left[1, (x - q_{k-1} - O_{k-1})/o_k\right]$$
$$+p_k max\left[0, (x - q_{k-1} - O_k)\frac{(q_k^i - q_{k-1}^i)}{(q_k - q_{k-1})}\right] \quad (1)$$

The first two terms refer two the units offered at a price below the clearing price. For the last two terms, the proportional rationing rule is applied, the quantities offered at the clearing price are used in the calculation. As x is not observed, the expected value must be calculated in order to find an expression for the objective function:

$$E_x\left(\Pi(q_k^i, o_k^i, x)\right) = \sum_{k=0}^{M} \int_{O_{k-1}+q_{k-1}}^{O_k+q_{k-1}} \left[p_k q_{k-1}^i + \sum_{j=0}^{k-1} p_j o_j^i + p_k(x - q_{k-1} - O_{k-1})o_k^i/o_k\right] dF(x)$$
$$+\sum_{k=0}^{M} \int_{O_k+q_{k-1}}^{O_k+q_k} \left[p_k q_{k-1}^i + \sum_{j=0}^{k} p_j o_j^i + p_k(x - q_{k-1} - O_k))\frac{(q_k^i - q_{k-1}^i)}{(q_k - q_{k-1})}\right] dF(x) \quad (2)$$

The first group of integrals corresponds to the profit levels attained when only option contracts are demanded at the clearing price. The second group of integrals corresponds to the cases in which quantities offered at the clearing price in the spot market are also demanded. The constraints of the problem are:

$$q_k^i \geq q_{k-1}^i \forall k$$
$$q_M^i \leq cap - O_M \quad (3)$$

The first condition implies that the submitted supply function must be increasing. The second implies that the maximum quantity offered cannot be higher than the installed capacity minus the total amount committed according to the option contracts. This way, with a discrete grid, each supplier simply has to solve a programming problem in R^n with a set of linear constraints.
For future reference we provide here an expression for the partial derivative with respect to q_{k-1}^i:

$$\frac{\partial \Pi(.,.)}{\partial q_{k-1}^i} = -f(O_{k-1} + q_{k-1})q_{k-1}^i(p_k - p_{k-1})$$
$$+p_{k-1}\int_{O_{k-1}+q_{k-2}}^{O_{k-1}+q_{k-1}} \frac{(x - q_{k-2} - O_{k-1})(q_{k-1}^{-i} - q_{k-2}^{-i})}{(q_{k-1} - q_{k-2})^2} dF(x)$$
$$+p_k \int_{O_{k-1}+q_{k-1}}^{O_k+q_{k-1}} \frac{o_k^{-i}}{o_k} dF(x) + p_k \int_{O_k+q_{k-1}}^{O_k+q_k} \frac{(q_k^{-i} - q_{k-1}^{-i})(q_k + O_k - x)}{(q_k - q_{k-1})^2} dF(x) \quad (4)$$

The usual quantity-price trade off can be observed. The first term is negative and reflects the fall in the clearing priced. The remaining three terms reflect the increase in quantities call as a consequence of the higher q_k^i.

The same expression for the case in which all suppliers are selecting the same strategy and have the same financial position(symmetry):

$$
\begin{aligned}
\frac{\partial \Pi(.,.)}{\partial q^i_{k-1}} &= -f(O_{k-1}+q_{k-1})\frac{q_{k-1}}{n}(p_k - p_{k-1}) \\
+p_{k-1}\frac{(n-1)}{n}&\int_{O_{k-1}+q_{k-2}}^{O_{k-1}+q_{k-1}} \frac{(x-q_{k-2}-O_{k-1}))}{(q_{k-1}-q_{k-2})}dF(x) \\
+p_k\frac{(n-1)}{n}\int_{O_{k-1}+q_{k-1}}^{O_k+q_{k-1}} dF(x) + p_k\frac{(n-1)}{n}&\int_{O_k+q_{k-1}}^{O_k+q_k} \frac{(q_k+O_k-x)}{(q_k-q_{k-1})}dF(x)
\end{aligned}
\tag{5}
$$

3.1 Equilibrium characterization

These propositions partially characterize the equilibrium:

Claim 3.1 *In a symmetric equilibrium a positive quantity is offered at* p_0.

Proof
Suppose not. Then it must be the case that the partial derivative of the objective function with respect to q^i_0 is non positive. But observe that in that case the partial derivative with respect to q^i_0 is positive as long as $q_1 > 0$ or $o_1 > 0$. If none of this condition is satisfied consider $q_{i+1} = 0$ where q_i is the variable just analyzed. We arrive to a contradiction. □

Claim 3.2 *In a symmetric equilibrium all available capacity is offered at the cap price, that is* $q^i_M + O^i_{M-1} = cap$.

Proof
Suppose the equation above is not satisfied. Then, observe that increasing the quantity offered at p_M results in a weakly larger number of quantities sold without any negative effect on price. This implies that the supply function is not a best response. □

According to our previous observation, in an equilibrium of the game, suppliers solve a programming problem in R^n. In particular we have that if $q_{k-1} < q_k < q_{k+1}$ then the partial derivative of the profit function with respect to q_i must equal zero. Similarly if $q_{k-1} = q_k < q_{k+1}$, then the partial derivative with respect to q_k must be non positive. Finally if $q_{k-1} < q_k = q_{k+1}$, then, the optimality conditions imply that the partial derivative with respect to q_k must be nonnegative.
Observe that when all q's are different, the partial derivatives define a second order nonlinear difference equation.

3.2 Uniform Distribution

The simplest case to analyze is the case in which the quantity demanded (or load) is distributed uniformly. We will keep this assumption for the rest of the analysis. This approximation is reasonable since, as we will show below, for any continuous distribution, the partial derivative of the objective function converges to the uniform distribution case as the price grid is large enough.
With a uniform distribution the objective function is given by:

$$\Pi(o,s) = \sum_{k=1}^{M} [o_k [p_k [q_{k-1}^i - \frac{o_k^i}{2}] + \sum_{j=1}^{k-1} p_j o_j]$$
$$+(q_k - q_{k-1})[p_k[q_{k-1}^i + \frac{q_k^i - q_{k-1}^i}{2}] + \sum_{i=1}^{k} p_j o_j^i] \tag{6}$$

Note that the expression is, for each type of rationing (option contracts or spot market quantities), the sum of the probability of a certain price being the equilibrium price times the expected quantity supplied at a given price. There are two terms since in some cases there is rationing in option contracts and in other cases the rationing is on the spot market quantities. With a uniform distribution we have a simpler expression for the partial derivative:

$$\frac{\partial \Pi(.,.)}{\partial q_k^i} = -q_k^i \Delta + p_k \frac{(q_k^{-i} - q_{k-1}^{-i})}{2} + p_{k+1} \frac{(q_{k+1}^{-i} - q_k^{-i})}{2} + o_{k+1}^{-i} p_{k+1} \tag{7}$$

Observe that the value of the partial derivative does not depend on the own financial position. That is to say, contracts affect incentives through the financial positions of the other suppliers. The effect is positive, that is, when the rivals increase the number of contracts at p_k there are more incentive to expand the supply at that price. Also observe that q_k^i is the only element of the supply function of generator i entering the partial derivative. Finally note that the partial derivative increases in q_{k+1}^{-i} and decreases in q_{k-1}^{-i}.
Note that this effect is the opposite of the one found in Allaz and Vilas.
With symmetry:

$$\frac{\partial \Pi(.,.)}{\partial q_k^i} = -q_k \Delta \frac{n+1}{2n} + \frac{n-1}{2n} [p_{k+1} q_{k+1} - p_k q_{k-1} + 2 p_{k+1} o_{k+1}] \tag{8}$$

Theorem 3.1 *In the symmetric spot market model there exists a unique symmetric equilibrium.*

Proof
First, for each possible value of q_{M-1} find there is a unique value for q_M such that the first order condition is satisfied. This is because the partial derivative is always decreasing in q_k for any k. Also observe that the value for q_M that satisfies the first order condition is decreasing in q_{M-1}.
Now we are going to use an induction argument. Consider k such that for all

j with $k < j \leq M$ there exist a unique decreasing function $q_j(q_{j-1})$ that gives the quantity for which the first order conditions are satisfied for all $z > j-1$. Then we can prove that there exist a function $q_{j-1}(q_{j-2})$ with the same properties. Replacing $q_j(q_{j-1})$ on the partial derivative with respect to q_{j-1} and the applying the implicit function theorem, we can check that in fact the inductive hipothesis is satisfied. Finally we have that for $k = 0$, the value below is equal to 0 and this results in the unique equilibrium that can be recovered by using the functions mentioned above.□

As we mentioned above, we consider that the assumption of a uniform distribution is a reasonable one. We have that, for any continuous distribution, as the price grid becomes big, the conditions characterizing the first order conditions converge to the conditions of the continuous grid case which is independent of the distribution of the quantities demanded (see Holmberg [8]).
We already observed that option contracts have a procompetitive effect on the incentives of generators. The partial derivative for a quantity at any price level, increases with the number of contracts. Below we extend that result by prensenting comparative statics of the best response function.

Claim 1: For given supply functions and financial position of other generators, an increase in the level of financial contracts for generator g results in lower equilibrium prices for all the quantities in the support. Also, the spot market supply function for firm g remains the same except the maximum quantity falls one to one with the change in the sum of option contracts.

Proof
Simply observe that the partial derivative is not affected, so the optimum quantities offered in the spot market of each price do not change but given that more contracts have been signed the equilibrium price must decrease for all quantities.□

The following lemma is a comparative statics results of the equilibrium. It will be useful in the following section, when we characterize the optimal procurement policy of a central planner whose objective is to minimize the total cost of electricity.

Lemma 3.1 *Consider the spot market model with option contracts such that p_k is the maximum price in equilibrium then, an increment in o_k^i for all i results in a new symmetric equilibrium with $q_k^{i\prime} = q_k^i - \Delta o_k^i$ and higher quantities offered in the spot market for all prices below p_k.*

Proof
First we note that at the new $q_k^{i\prime}$ keeping k_{k-1}^i constant, the partial derivative with respect to quantity is still nonnegative. For q_{k-1}^i by inspecting the partial derivative we check that the positive effect of a higher o_k is higher than the negative effect of the lower q_k. This implies that for any q_{k-2} the new q_{k-1} that satisfies the equilibrium is higher. Now applying induction we can check that this is true for any q_j^i with $j \leq k-1$. This concludes de proof.□

Also we can consider the partial derivative of the objective function with respect to the financial position, this will be useful below when using the envelope theorem.

$$\frac{\partial \Pi(.,.)}{\partial o_k^i} = p_k(cap - q_k - O_k) + p_k q_k^i + \sum^{k-1} p_j o_j^i + \frac{p_k o_k^i}{2} + \frac{p_k o_k}{2}$$
$$- \left[p_M q_{M-1}^i + \sum^{M-1} o_j^i p_j + p_M \frac{q_M^i - q_{M-1}^i}{2} \right] - p_M \frac{q_M - q_{M-1}}{2} - o_M p_M \tag{9}$$

3.2.1 Special Case

For sufficiently low levels of option contracts the unique equilibrium supply function is strictly increasing and is characterized by the linear difference equation below:

$$q_{k-1} = \frac{2(n-1)p_k(o_k)}{(n+1)(p_k - p_{k-1})} + \frac{(n-1)(p_k q_k - p_{k-1} q_{k-2})}{(n+1)(p_k - p_{k-1})} \tag{10}$$

There are two boundary conditions, coefficients are variable and the equations is not homogeneous.
Note that this difference equation converges to a differential equation:

$$q_{k-1} = \frac{2(n-1)p_k(o_k)}{(n+1)\Delta} + \frac{(n-1)(p_k q_k - p_{k-1} q_{k-2})}{(n+1)\Delta} \tag{11}$$
$$\to \lim_{\Delta \to 0} \frac{2(n-1)p_k(O_k - O_{k-\Delta})}{(n+1)\Delta} +$$
$$\lim_{\Delta \to 0} \frac{(n-1)(p_k(q_k - q_{k-1}) + p_{k-1}(q_{k-1} - q_{k-2})}{(n+1)\Delta} + \frac{(n-1)q_{k-1}}{(n+1)} \tag{12}$$
$$\tag{13}$$

$$q(p) = \frac{(n-1)p}{n} \frac{\partial q(p)}{\partial p} + \frac{(n-1)po(p)}{n} \tag{14}$$

This equation can be compared to similar expression obtained in continuous price models.
The system of linear difference (or recurrence equations) can also be solved explicitly. We can rewrite the equation as:

$$q_k = a_k q_{k-1} + b_k q_{k+1} + c_k \tag{15}$$

Where:

$$a_k = -\frac{(n-1)p_k}{(n+1)(p_{k+1} - p_k)} \tag{16}$$
$$b_k = \frac{(n-1)p_{k+1}}{(n+1)(p_{k+1} - p_k)}$$
$$c_k = \frac{2(n-1)p_{k+1}(o_{k+1})}{(n+1)(p_{k+1} - p_k)}$$

Since the lowest and the highest value for q_i are known, this second order difference equation defines a system of linear equations. The solution is unique if the matrix of coefficient has a determinant different from zero. We find the unique solution by solving the system through substitution and applying induction. The computed solution equals:

$$q_k = \sum_{t=k}^{M-1} \left[\prod_{i=t}^{M-1} b_i \frac{\sum_{i=1}^{t} c_i d_i \prod_{j=i+1}^{t} a_j}{\prod_{i=t}^{M} d_i} + \prod_{i=t}^{M-1} b_i \frac{d_t}{d_M} [cap - O_M] \right] \tag{17}$$

Where:

$$d_1 = d_2 = 1 \tag{18}$$
$$d_k = d_{k-1} - d_{k-2} b_{k-2} a_{k-1}$$

For sufficiently high level of option contracts, there might be some k's such that $q_k = q_{k+1}$.

4 Option markets

Two different settings will be studied, in both we introduce a previous stage in which option contracts are traded. In the first one we establish a benchmark result. A central planner procures option contracts to minimize the total cost of electricity provision. In the second setting we allow LSEs and generators bargain bilateral contracts. First assumming that there are LSE that are price taking in the spot market, that is, each LSE's financial position does not affect the equilibrium spot market price. Later we allow for LSEs that are large and affect the equilibrium spot market price through their financial positions.

4.1 Centralized problem

Now suppose that there is a central authority that procures option contracts with the objective of minimizing the total cost of electricity. The central planner makes take it or leave it offers for a profile of financial contracts, $o(p)$. This function indicates at each price the quantity of call options being provided. A strategy for a generator consists of accepting or rejecting the offer for the contracts (a_i) plus a mapping $(s_i(p))$ the supply function in the spot market. Then if a generator accepts the offer, the objective function is given by:

$$\Pi^T(a, s) = T + \Pi(o, s) \tag{19}$$

Where T is the payment to the generator for signing the contracts. If a generator rejects the offer the objective function is given by:

$$\Pi(a, s) = \Pi(0, s) \tag{20}$$

To implement a profile $o(.)$ as an equilibrium it must be the case that accepting the offer is a best response when all the other generators also accept the offer, that is:

$$T + \Pi((o_i, o_{-i}), (s_i*, s_{-i}*)) \geq \Pi((0, o_{-i}), (s_0, s_{-i}*)) \; \forall s_0 \tag{21}$$

where $s*(.)$ is the unique symmetric equilibrium of the spot market when every generator accepts the offer for o_i. Note that the condition can be simplified by considering s_0 the best response to s_i* for the corresponding financial position.
The central planner's objective is to minimize the cost of electricity, this implies that T will be selected such that the inequality above is satisfied as an equality. So the objective function of the planner depends only on the profile of contracts o_i:

$$\begin{aligned} min_{o_i} \Pi(o_i, s*) + \Pi((0, o_{-i}), (s_0, s_{-i}*)) - \Pi((o_i, o_{-i}), (s_i*, s_{-i}*) \\ = \Pi((0, o_{-i}), (s_0, s_{-i}*)) \end{aligned} \tag{22}$$

The following result characterizes the solution to the central planner's problem:

Theorem 4.1 *The solution to the planner's problem is a profile of option contracts such that price equals marginal cost for any load level.*

Proof
Suppose that o_k is such that $p_k > 0$ is the equilibrium highest price. We are going to prove that increasing the quantity of contracts at that price, o_k, results in a lower total cost of electricity. There is a direct gain in the spot market that is given by the lower price for a given supply functions. This gain more than offsets the increase in the cost of procuring contracts. We can establish that by noting, as shown above, that the planner's objective function can be expressed simply as: $\Pi((0, o_{-i}), (s_0, s_{-i}*))$, the maximum profit of a generator that rejects the offer for the option contracts when all other generators accept. Note that this is always decreasing since, first, an increase in the number of option contracts signed by the other generators o_{-i} has a direct negative effect on $\Pi_i(.)$. Additionally there is a negative effect due to the associated increase in the equilibrium supply function as proved in the lemma presented in the previous section.$\square$

The central authority makes offers to the generators in a way that they face an additional stage in which they compete. This double round of competition allows the central authority to reduce the cost of electricity.
In a model with asymmetric producers (e.g. peakers and baseload generators) options could also allow for a more fair compensation for peakers. Note that without option contracts, baseload producers would be unnecessarily compensated during peak hours.
Observe that we assumed risk neutrality for generators and the central authority, nevertheless the optimal allocation is such that the price is flat.
Under the central authority offer, there might exist other equilibria in which generators do not accept the offer for option contracts. Requiring unique implementation of the allocation would make the procurement of contracts more costly and might change the results.
To quantify the relevance of the result above the consider a numerical example en the next subsection.

4.1.1 Numerical example

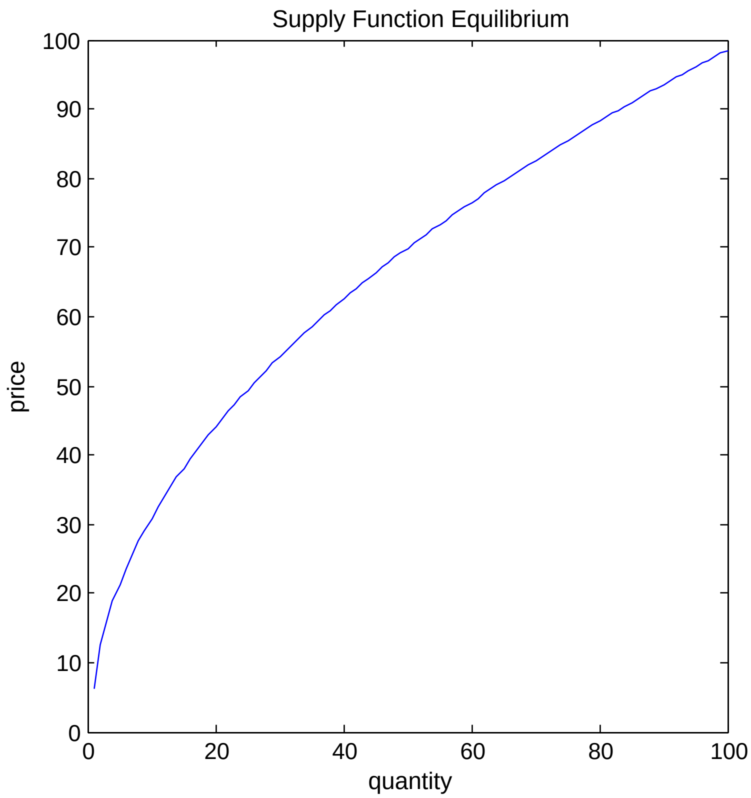
Consider a market with 3 generators, demand is distributed uniformly between 0 and 100. As before marginal cost equals zero up to a per generator capacity of 100/3. We assume that the grid price has 101 points between 0 and 100. In that case the equilibrium supply function of an equilibrium with no option contracts is represented in figure 1.
We can compute the cost of electricity associated with that equilibrium by using the expression of the generators profit function that in this case coincides with the cost of electricity. The cost is equals to 254,170.
To compute the cost under optimal procurement, note that if the remaining generators are submitting a flat supply curve at marginal cost, the best response is to offer all the capacity at a price equal to the cap price. To note that this is the best response, observe that the only increment in quantities can be attained by offering at a price equal to the marginal cost, but this results in a mark-up of zero. This way we can easily compute the maximum profit level of a firm that rejects the offer for contracts. Remember that this is the cost associated with procuring the contracts. In this case, the sum of the profits of the generators (and total cost of electricity) equals 166,830. This implies that the reduction in the cost of electricity is larger than 33 percent of the original level. We conclude this section emphasizing that option contracts can result in a significant reduction in the cost of electricity. Next we analyze different environments to study if and to what extent decentralized markets attain those potential gains.

4.2 Decentralized market

Now we are going to compare the benchmark result above with what can result in specific market environments. First we consider a case in which LSEs are small enough so that their respective financial position does not affect prices in the spot market. This is an extreme assumption but it will be used to illustrate that the potential gains presented in the previous section might not be realized in certain market environments. Next, we will present some results for the case in which LSE are large, that is, their individual actions and financial positions have an effect on the distribution of prices in the spot market.

4.2.1 Price taking LSEs

As indicated we first analyze a model in which LSEs are price takers in the spot market. The financial position of an individual LSE has no effect on the equilibrium distribution of the spot price. This can be considered as an approximation to the case in which LSEs are small. Another interpretation of the environment is that we are considering LSEs that are naive and do not anticipate the effect of their financial positions on the average price. They only consider the effect on net revenue for a given equilibrium distribution of the spot price. Independently of the interpretation, the analysis of this environment results in some useful insights for the analysis of the gains that can result from the existence of option contracts.


Supply Function Equilibrium
price
quantity
0
10
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80
100

Additionally, this exercise is an extention and robustness check of the literature started by Allaz and Vilas (av) which was mentioned in the Related Literature section. In all these papers, the demand side agents are modeled as price takers.
For the following proof we will make a small change to the model presented above. We will assume that when rationing a quantity at a given price, first the quantities offered in the spot market are called and then option contracts are exercised. The assumption regarding proportional rationing described above still stands. The modification should not be viewed as conceptual but as one made for expositional convenience.
The following is an illustration of the difficulties that can obstruct the realization of the potential benefits of option contracts:

Theorem 4.2 *Suppose LSEs are price taking in the spot market, then, there are no gains from trading option contracts.*

Proof
For each load level x, the gains for an LSE from procuring option contracts are simply given by the difference between the equilibrium price p and the corresponding strike price of the contract in the case the option is exercised. Next we claim that the loss to a generator is larger than the gain to a LSE. The gain to a LSE is a lower bound for the loss to a generator since by not selling the contract the generator can still mimic the prices of the equilibrium with those contracts. Under this strategy, a generator commits the same number of units at each price, but, for each price there are more units offered in the spot market and less units committed under option contracts. Now observe that this results in delivering a strictly higher number of quantities since, for a given price, quantities offered at the spot market, q_k^i, are called before the quantities committed under contracts o_k^i. We conclude that the cost to generators is higher than the benefit to a price taking LSE. □

Corollary 4.1 *Suppose LSEs are price taking in the spot market, then, in no symmetric equilibrium there is trade of option contracts.*

Proof
This is a trivial consequence of the previous theorem.□

Observe that the result above does not depend on the bargaining position of each agent. That is, even if we allow LSEs to make take it or leave it offers to generators there will be no tradeoption contracts in a symmetric equilibrium. This is a consequence of LSEs not internalizing the externalities of option contracts through the spot market price. Small LSEs do not consider that effect, while generators with market power do consider that effect.
Note that this result is in sharp contrast with the result by Allaz and Vilas ([1]). In their work the agents on the demand side do not have market power, nevertheless there exist trade of futures contracts and this result in a more competitive outcome. In their work, there are no capacity contraints, and competitors reduce the quantity supplied when they observe that a rival has increased the number of contracts signed. This implies that signing contracts is convenient for a generator/supplier. Our result does not depend on observability, since of other generators observe a higher number

of contracts signed, then they have less incentives to reduce the supply level.
It would be desirable to consider intermediate cases in which LSEs do not act as a single agent but are large enough so that a fraction of the effect on the equilibrium spot price is taken into account. This is what we consider in the following subsection.

4.2.2 LSEs with market power

Now we turn to large LSEs that do not act as a single agent. In line with the result in the previous setting, we show that, even when we allow for large LSEs with market power, a significant fraction of the potential benefits of option contracts might not be attained in a decentralized market environment.

Consider the following market protocol:

- LSEs submit offers for bilateral option contracts simultaneously.
- Generators accept or reject those contracts simultaneously.
- Generators observe their respective financial position.
- The spot market is run as a uniform price auction.

The above procedure should be considered as an example. The result presented below holds for wide-ranging market protocols. For example we could have generators making the offers or a centralized market for option contracts. Since the proof is in terms of gains from trade, the bargaining position of agent does not modify the result.

Theorem 4.3 *The central authority solution cannot be implemented in a symmetric decentralized market equilibrium.*

Proof
The proof consists in showing that there are no gains from trade. Suppose that the planner's solution is implemented in an equilibrium. Then, the cost to a generator of accepting the offers made for quantities o_0^i by LSEs equals the product of p_M and the expected quantity called in the spot market when the price at which capacity is offered is the highest price. This is because, if contracts are not traded, a generator would select the maximum price for those units.
Observe that since that value is also an upper bound for the benefits to an LSE, then that must be the equilibrium price for the contracts.
The benefit of a contract of o_0^i units is smaller since alternatively a LSE can offer contracts for ko_0^i units to two generators at the price of k times the price for the original contract. For k close to 1, this contracts will be accepted and the difference in net revenue in the spot market will be smaller since both generators are competing and will submit a supply function which is strictly below p_M. Since the cost to generators is higher that the benefit to a LSE, we conclude there are no gains from trade and, thus, such a contract is not traded in equilibrium. □

This result is another example of the limitations of decentralized markets in capturing the potential benefits of option contracts. The intuition behind the result is that LSEs are not able to internalize the effect of their respective financial positions

of the net revenue of other LSEs in the subsequent spot market. As a consequence demand is to low to implement the solution of the central planner.
It would be interesting to simulate numerically an equilibrium of the market presented above. We would like to asses which is the fraction of the gains that are captured when LSEs are big, that is, individual LSEs financial positions affect the spot market equilibrium price distribution. Considering a market with risk adverse agents would also help to asses the impact of option contracts on market power mitigation.

5 Concluding remarks

We considered a supply function equilibrium (SFE) model with option contracts. Uniqueness of the symmetric equilibrium was verified. We also checked that option contracts result in less supply reduction and lower prices in the spot market.
In the benchmark results, the centralized setting, we show that option contracts can result in significant reduction in the total cost of electricity. Additionally the volatility of the cost is greatly diminished since in the solution to the planner's problem the spot market price always equals marginal cost.
The realization of these gains depends on market characteristics. More specifically, the social planner's solution cannot be implemented when LSEs act in a decentralized market. LSEs do not internalize the benefits of procuring their respective option contracts on the spot price paid by other LSEs. Additionally, we find that with price taking LSEs there are no gains from trading option contracts. Thus, in equilibrium, there is no trade of option contracts and there is no benefit resulting from option contracts. In this case, the result does not depend on the bargaining power of the different agents. This results are a consequence of LSE not internalizing the social benefits of procuring option contacts.
Market structure and regulatory measures shape the effect of option contracts on imperfectly competitive markets. Centralized procurement of call options and requiring generators to sell call options are among the regulatory measures that could improve the performance of wholesale electricity markets. Ignoring these possibilities might result in not implementing an important fraction of the potential gains of option contracts.
The model can be extended to include more realistic features of wholesale electricity markets. For example, allowing for asymmetric generators, LSEs with market power, and introducing risk aversion seem to be desirable features. It is not clear if numerical methods will be required to conduct those exercises. In particular with asymmetric producers, we could study how option contracts might compensate peakers that are called only during extremely high load period, that is, with low probability.
A more general cost structure is also going to be considered in future research. We speculate that our observations will stand. That is, an important fraction of the market power mitigation gains from option contracts might not be attained unless the adequate regulation is put in place.
Transmission constraints is another salient characteristic of wholesale electricity markets that has important consequences when considering market power related issues.

Network congestion generates local market power stochastically. Naturally, the modeling of such a market might result in considerable technical difficulties.
Market power reduction and investment in capacity have generally been conflicting objectives in electricity markets. It would be interesting to establish how much of this conflict goes away once when option contracts are considered. Not only the volume of investment but also the resulting technology mix should be considered.

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