Inflation Tax and Inflation Subsidies: Working Capital in a Cash-in-advance model

George T. McCandless

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Abstract

This paper studies the nature of monetary policy with financial intermediaries that provide loans for working capital in a cash-in-advance model with indivisible labor. Monetary policy occurs through money injections either directly to families or to the financial intermediary. Stationary state injection to the families produce an inflation tax while injection directly to the financial intermediary provide an inflation subsidy that improves output, consumption, and welfare. The dynamic properties, as responses to a monetary inpulse, of both models are compared.

1 Introduction

Cash in advance models of money traditionally result in equilibrium where money injections work as a tax, reducing welfare and creating misallocation of resources. The model of Cooley and Hansen [3] is the classic example. However, these models do not generally include any form of a financial system. In this paper we add a simple financial system that lends to firms to finance working capital. We assume that the wage bill has to be paid before the firm receives payment for its production so the firms must borrow from a financial intermediary to cover these expenses. Households acquire physical capital as well as deposits in the financial system and rent out the physical capital to the firms. We assume that rents on physical capital are paid after the firms are paid for their output.

Monetary policy works through lump sum transfers of money into the economy. We consider two ways that money can enter the economy. One way, as in Cooley and Hansen, money can be transfered directly to the households who can use it to make in period consumption purchases or can lend it to the financial intermediary. The alternative way of injecting money that we consider is through lump sum transfers directly into the financial system. As we will see, the form of monetary injection is very important in determining the effect monetary policy has on the economy. Of course, one could also consider the case of lump sum money transfers going directly to the firms. The equilibrium for this economy turns out to be identical to that with transfers directly to the families.

Models of real business cycles have been incorporating working capital as a way of generating a positive hump shaped response to a monetary impulse. Christiano [1], Dotsey and Ireland [4], and Christiano and Eichenbaum [2] are examples of real business cycle models where variants of working capital are included in real business cycle models. The model studied here is more stripped down than theirs, there are fewer other elements added, and we get a clearer picture of the implications of how money enters the economy.

The point of this paper is the importance that modeling the monetary system and how monetary policy is carried out. Many papers on monetary policy invoke helicopter money drops directly to the citizens as their way of injecting money into the economy. Central banks work through the financial system as they implement their monetary policy and without a well modeled financial system, we may well be missing extremely important characteristics of monetary policy.

2 A model of working capital

We construct a model with financial intermediaries, mutual funds or a banking system, that borrows money from the households and lends money to the firms to finance the wage bill. Our basic assumption is that the firms must finance the entire wage bill by borrowing money from the financial intermediaries. Compared to a simple cash in advance model, households have an additional asset in which to save, the lending to the financial intermediary. The behavior of the financial intermediaries needs to be defined. This is important because the firms may be constrained in their production decisions by their access to financing for working capital.

In addition, one needs to determine exactly how new money issues or money withdrawals will occur. In the Cooley-Hansen model, new money enters the economy through direct lump sum transfers to the households. This method is also possible in a model with working capital. However, it is not the only method available. The monetary authority might choose to inject money into the economy through lump sum transfers to the financial intermediaries. As we will see, this choice is far from innocuous.

We describe separately the behavior of the households, the firms, and the financial intermediaries. In the presentation of the model, we present both versions at the same time, showing how the budget constraints are different in the model where transfers go directly to households compared to when they go to the financial intermediary. Then we compare the stationary states for the two methods of injecting money into the economy. Finally, we compare the impulse reponse functions of the two models responding to monetary shocks.

2.1 Households

We begin with households. Every household i from a unit mass of households maximizes the same utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t^i + B h_t^i \right],$$

where $B = A \ln(1 - h_0)/h_0$, h_0 is the number of indivisible hours of work that a household provides to the market if it is one of the families chosen to work, and c_t^i is the consumption of family i in period t. The common discount factor is $0 < \beta < 1$. The fraction of families that work is h_t^t/h_0 and the unemployment rate is $1 - h_t^t/h_0$. Families face a cash-in-advance constraint. The cash-in-advance constraint for the family depends on the method used for introducing money into or removing money from the economy. Constraint 1 holds when the transfer goes directly to the families and constraint 2 applies when the transfer goes directly to the financial intermediary. The cash in advance constraints are

$$P_t c_t^i = m_{t-1}^i + (g_t - 1) M_{t-1} - N_t^i, \tag{1}$$

when the transfer of money goes directly to the families and

$$P_t c_t^i = m_{t-1}^i - N_t^i, \tag{2}$$

when it does not, when it goes to the financial intermediary. In these constraints, m_t^i is the money that family i carried over from period $t-1$, $(g_t - 1) M_{t-1}$ is the lump sum transfer or tax of money, and N_t^i is the amount of money that family i lends to the financial intermediary at the beginning of period t. The families also face an in-period real flow constraint (after the cash-in-advance constraint has been removed from both sides of the equation) of

$$\frac{m_t^i}{P_t} + k_{t+1}^i = w_t h_t^i + r_t k_t^i + (1 - \delta) k_t^i + r_t^n \frac{N_t^i}{P_t},$$

where k_s^i is family i's holdings of capital, which is a credit good and money is not required to purchase it, w_t and r_t and real wages and rentals, respectively, r_t^n is the gross interest rate paid on lending to the financial intermediary.

First order conditions for the family's optimization problem are

$$\begin{aligned} \frac{1}{w_t} &= E_t \frac{\beta}{w_{t+1}} \left(r_{t+1} + (1 - \delta) \right), \\ \frac{B}{w_t} &= -\frac{\beta P_t}{E_t P_{t+1} c_{t+1}^i}, \\ \frac{1}{r_t^n} &= \frac{\beta P_t c_t^i}{E_t P_{t+1} c_{t+1}^i}. \end{aligned}$$

Notice that the household's first order conditions are the same for both methods of introducing money into the economy. The differences in the two methods show up in the budget constraints.

2.2 Firms

Firms are perfectly competitive and produce the same good. They rent capital from the households and hire labor. Labor must be paid its wages before the good is sold, so firms borrow working capital from the financial intermediaries to pay wages. All firms are alike so we just consider a generic firm. Given that firms are perfectly competitive, they make no profits and the real budget constraint for each firm is

$$Y_t = r_t^f w_t H_t + r_t K_t,$$

where r_t^f is the gross interest rate that each firm pays the financial intermediaries for borrowing working capital to finance their wage bill of $w_t H_t$. The production function is Cobb-Douglas,

$$Y_t = \lambda_t K_t^{\theta} H_t^{1-\theta},$$

where λ_t is a technology shock with

$$\ln \lambda_t = \gamma \ln \lambda_{t-1} + \varepsilon_t^{\lambda},$$

$0 < \gamma < 1$, and $\varepsilon_t^{\lambda} \sim N(0, \sigma^{\lambda})$. Under conditions of perfect competition, equilibrium conditions for the factor markets are

$$r_t^f w_t = (1-\theta)\, \lambda_t K_t^{\theta} H_t^{-\theta},$$

and

$$r_t = \theta \lambda_t K_t^{\theta-1} H_t^{1-\theta}.$$

2.3 Financial intermediaries

The financial intermediaries are perfectly competitive and take deposits (loans) from the households and lend them to the firms. The lending to the firms is paid back at the end of the period. The conditions for the financial intermediary depends on the way the monetary authority introduces money into the economy. When money is introduced by direct lump sum transfers to the families, lending to the firms is

$$N_t = \int_0^1 N_t^i di = P_t w_t H_t,$$

and since the financial intermediaries make no profits, their budget constraint is simply

$$r_t^n N_t = r_t^f N_t. \tag{3}$$

When money is injected into or withdrawn from the economy by transfers to the financial intermediary, then lending to the firms is

$$N_t + (g_t - 1) M_{t-1} = P_t w_t H_t, \tag{4}$$

and the budget constraint for financial intermediaries is

$$r_t^n N_t = r_t^f \left[N_t + (g_t - 1) M_{t-1} \right]. \tag{5}$$

Notice in this second method of introducing new money to the economy, injection of money will cause the lending rate to the firms to be different from borrowing rate from the households.

The monetary policy rule of the monetary authority follows the process

$$M_t = \overline{g} g_t M_{t-1},$$

where $\overline{g}$ is the stationary state growth rate of money and the stochastic monetary shock g_t follows the process,

$$\ln g_t = \pi \ln g_{t-1} + \varepsilon_t^g,$$

with $0 < \pi < 1$, and $\varepsilon_t^g \sim N(0, \sigma^g)$.

2.4 Aggregation conditions

Since all firms, financial intermediaries, and households are the same, and we have a unit mass of each, the results for the representative agent will be the aggregate for the economy. This means that, in equilibrium,

$$\begin{aligned} m_t^i &= M_t, \\ N_t^i &= N_t, \\ c_t^i &= C_t, \end{aligned}$$

and

$$h_t^i = H_t.$$

3 Stationary states

We define a stationary state with constant gross money supply growth rate of $\overline{g}$ as an equilibrium where all real variables are constant and the nominal variables are equal to the constants

$$\frac{M_t}{P_t} = \overline{M/P},$$

$$\frac{N_t}{P_t} = \overline{N/P},$$

and

$$\frac{P_{t+1}}{P_t} = \overline{g}.$$

From the first and third foc's of the households we get the stationary state conditions,

$$\overline{r} = \frac{1}{\beta} - (1 - \delta),$$

and

$$r_t^n = \frac{\overline{g}}{\beta}.$$

Households arbitrage between the real return on the two assets in which they can invest, capital and lending to the financial intermediary so the stationary state real return from capital, $1 + r - \delta = 1/\beta$, is equal to the stationary state real return on lending to the financial intermediary, $P_t r_t^n / P_{t+1} = 1/\beta$. Notice that the stationary state values of these two interest rates are independent of the method of injecting money into the economy.

3.1 Transfers directly to families

We designate values for the stationary state with transfers directly to the families by the subscript "D". Using equation 3, we have

$$\overline{r}^{fD} = \overline{r}^n = \frac{\overline{g}}{\beta},$$

so we can use the factor market conditions for the firm to get

$$\overline{w}^D = \frac{\beta}{\overline{g}} (1-\theta) \left[\frac{\theta}{\overline{r}} \right]^{\frac{\theta}{1-\theta}}.$$

The second foc for the households gives

$$\overline{C}^D = -\frac{\beta \overline{w}^D}{B \overline{g}} = -\frac{\beta^2}{B \overline{g}^2} (1-\theta) \left[\frac{\theta}{\overline{r}} \right]^{\frac{\theta}{1-\theta}},$$

where $\overline{w}^D$ was substituted into the second expression. The cash-in-advance condition in a stationary state is

$$\overline{C}^D = \overline{M/P}^D - \overline{N/P}^D,$$

and putting this into the flow budget constraint for the households gives

$$\overline{K}^D = \frac{1}{\overline{r} - \delta} \left[\overline{C}^D - \overline{r}^n \overline{w}^D \overline{H}^D \right],$$

and from the production side of the economy, we have

$$\overline{H}^D = \frac{\overline{r}}{\overline{r}^n \overline{w}^D} \frac{1-\theta}{\theta} \overline{K}^D.$$

These two equations can be solved for $\overline{K}^D$ and $\overline{H}^D$, where

$$\overline{K}^D = \frac{\overline{C}^D}{[(\overline{r} - \delta) + \overline{r}(1-\theta)/\theta]},$$

Using the budget constraint for the financial intermediaries, one gets

$$\overline{N/P}^D = \overline{w}^D \overline{H}^D.$$

The stationary state values for the rest of the variables follow immediately. For a quarterly standard cash-in-advance economy with indivisible labor (from Cooley and Hansen), the values used for the parameters are $\beta = .99$, $\delta = .025$, $\theta = .36$, $h_0 = .583$ and $A = 1.72$. The table below shows the stationary state values for the variables of the model when money is injected into the economy through direct transfers to the households.

Annual inflation	-4%	0	10%	100%	400%
$\overline{g}$	.99	1	1.024	1.19	1.41
$\overline{r}$	.035101	.035101	.035101	.035101	.035101
$\overline{r}^n$	1.0000	1.0101	1.0343	1.2020	1.4242
$\overline{r}^f$	1.0000	1.0101	1.0343	1.2020	1.4242
$\frac{M_t}{P_t} = \overline{M/P}$	1.7093	1.6675	1.5732	1.0911	0.7266
$\frac{N_t}{P_t} = \overline{N/P}$	0.7907	0.7672	0.7145	0.4553	0.2737
$\overline{C}$	0.9187	0.9004	0.8587	0.6358	0.4529
$\overline{Y}$	1.2354	1.2108	1.1547	0.8551	0.6090
$\overline{w}$	2.3706	2.3469	2.2919	1.9722	1.6645
$\overline{H}$	0.3335	0.3269	0.3118	0.2308	0.1644
$\overline{K}$	12.6707	12.4185	11.8432	8.7695	6.2464
utility	-0.9455	-0.9485	-0.9568	-1.0485	-1.2164

Inflation functions as a tax on the economy. Higher stationary state inflation rates imply lower consumption, production, employment, and utility. These results are very similar to those of Cooley and Hansen [3].

3.2 Transfers to financial intermediaries

When the transfers of new money are made directly to the financial intermediaries, there are two important changes in the structure of the model. The first is in the equation of the household cash-in-advance constraint. Since there are not transfers of money directly to the households, equation 2 is the one that holds. In a stationary state, this equation is

$$\overline{C}^F = \frac{\overline{M/P}^F}{\overline{g}} - \overline{N/P}^F, \tag{6}$$

where the stationary state values for the economy with transfers to the financial intermediaries are designated by a superscript of "F". Given that the transfers go to the financial intermediary directly, the zero profit condition for the financial intermediaries is given by equation 5. In a stationary state, this condition is

$$\overline{r}^n \overline{N/P}^F = \overline{r}^{fF} \left[\overline{N/P}^F + (1 - \frac{1}{\overline{g}}) \overline{M/P}^F \right]. \tag{7}$$

The second foc for households can be rearranged to give

$$\overline{w}^F = -\frac{\overline{C}^F \overline{g} B}{\beta},$$

which we will use to remove wages from the model. Substituting this into equation 4, and evaluating in a stationary state gives the equation

$$-\frac{\overline{C}^F \overline{g} B}{\beta} \overline{H}^F = \overline{N/P}^F + (1 - \frac{1}{\overline{g}}) \overline{M/P}^F. \tag{8}$$

From the factor market equations, we get, after substituting out wages,

$$\overline{r}^{fF} = -\frac{\beta (1-\theta) \left(\frac{\theta}{\overline{r}}\right)^{\frac{\theta}{1-\theta}}}{\overline{C}^F \overline{g} B}. \tag{9}$$

Finally we use the household flow budget constraint to get

$$\overline{M/P}^F = \frac{\overline{g}}{\beta} \overline{N/P}^F + \left[(\overline{r} - \delta) \left[\frac{\theta}{\overline{r}}\right]^{\frac{1}{1-\theta}} - \frac{\overline{C}^F \overline{g} B}{\beta} \right] \overline{H}^F. \tag{10}$$

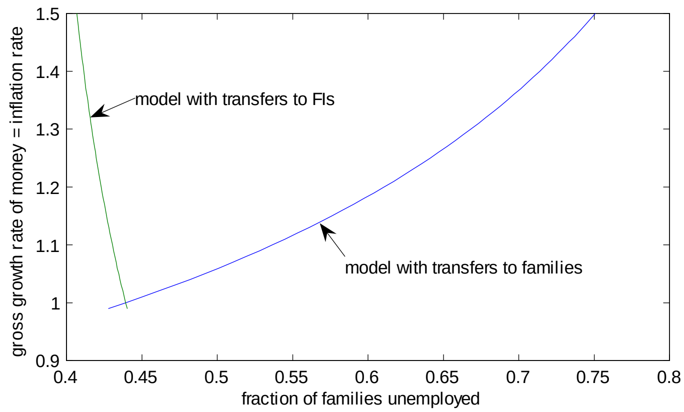
The five equations 6 to 10 are a system in the variables $\overline{M/P}^F$, $\overline{N/P}^F$, $\overline{C}^F$, $\overline{H}^F$, and $\overline{r}^{fF}$. Using the same parameter values as above, the solutions to the system were calculated using MATLAB and the rest of the values for the stationary state followed directly. The the values calculated are shown in the following table.

Annual inflation	−4%	0	10%	100%	400%
$\overline{g}$	.99	1	1.024	1.19	1.41
$\overline{r}$	.035101	.035101	.035101	.035101	.035101
$\overline{r}^n$	1.0000	1.0101	1.0343	1.2020	1.4242
$\overline{r}^f$	1.0221	1.0101	0.9824	0.8259	0.6820
$\frac{M_t}{P_t} = \overline{M/P}$	1.6557	1.6675	1.6960	1.8896	2.1395
$\frac{N_t}{P_t} = \overline{N/P}$	.7736	.76715	0.7523	0.6626	0.5716
$\overline{C}$	.8988	.90038	0.9040	0.9253	0.9458
$\overline{Y}$	1.2087	1.2108	1.2158	1.2444	1.2720
$\overline{w}$	2.3193	2.3469	2.4130	2.8702	3.4762
$\overline{H}$	.3263	.32688	0.3282	0.3360	0.3434
$\overline{K}$	12.3967	12.418	12.4690	12.7627	13.0454
utility	−0.9488	−0.9485	−0.9479	−0.9445	−0.9418

In this model, stationary states with higher rates of money growth have higher output, consumption, real wages, hours worked, capital, and utility. Calculating the unemployment rate as $1 - H/h_0$, as a function of the stationary state rate of money growth (which equals the stationary state inflation rate) for the two models, one gets the Phillips curves shown in Figure 1.

In the model with working capital and money injections via the financial intermediaries, money injections operate as a subsidy to hiring labor, reducing the real cost of labor. The reductions in the real cost of capital increases demand paper phillips

curve



$_1.pdf$

Figure 1: Phillips curves for the two models

and more labor ends up being hired. With more labor hired, the marginal product of capital increases and stationary state capital is higher. For the model with working capital where the transfers of money are made directly to the families, inflation has the same effect as in the basic Cooley-Hansen model, inflation works as a tax in the economy and reduces output and employment so the Phillips curve implies higher unemployment with higher inflation. Recall that in all cases in this graph of Phillips curves, we are comparing stationary states.

4 Transfers directly to the firms

A version of this model with lump sum money transfers going directly to the firms ends up being identical to that with transfers to the families.

The first order conditions for the families are the same as in the other models, the cash-in-advance constraint is

$$P_t c_t^i = m_{t-1}^i - N_t^i,$$

and the flow budget constraint is

$$\frac{m_t^i}{P_t} + k_{t+1}^i = w_t h_t^i + r_t k_t^i + (1-\delta)k_t^i + d_t^i + r_t^n \frac{N_t^i}{P_t},$$

where d_t^i is the lump sum dividend payment that the family recieves as its share of the profits of the firms.

The firms maximize profits, D_t, where

$$D_t = \lambda_t K_t^\theta H_t^{1-\theta} - r_t^f \left[w_t H_t - (g_t - 1)\frac{M_{t-1}}{P_t} \right] - r_t K_t,$$

and $(g_t - 1)M_{t-1}/P_t$ are the real value of the lump sum transfers of money to the firms. The first order condtiions for profit maximization are

$$\frac{\partial D_t}{\partial K_t} = \theta \lambda_t K_t^{\theta-1} H_t^{1-\theta} - r_t = 0,$$

and

$$\frac{\partial D_t}{\partial H_t} = (1-\theta)\,\lambda_t K_t^{\theta} H_t^{-\theta} - r_t^f w_t = 0.$$

These first order conditions are the same as in the other models. Here, however, the firms make profits of $r_t^f(g_t - 1)M_{t-1}/P_t$ rather than the zero profits in the other models and this amount gets transfered to the families as dividends, and in equilibrium,

$$\int_0^1 d_t^i di = D_t = r_t^f (g_t - 1)\frac{M_{t-1}}{P_t}.$$

The financial intermediaries lend to the firms the difference between the wage bill and the money they receive from the new money issue,

$$N_t = P_t w_t H_t - (g_t - 1)M_{t-1}.$$

Since they make no profits,

$$r_t^n N_t = r_t^f N_t.$$

All the first order conditions in this model are the same as in the model where the money injections go directly to the families. Using the equation for dividends and for total loans, given above, and substituting them into the families cash-in-advance and flow budget constraints give

$$P_t C_t = g_t M_{t-1} - P_t w_t H_t,$$

and the flow budget constraint is

$$\begin{aligned} \frac{M_t}{P_t} + K_{t+1} &= w_t H_t + r_t K_t + (1-\delta)K_t + r_t^f (g_t - 1)\frac{M_{t-1}}{P_t} + r_t^n \frac{N_t}{P_t} \\ &= w_t H_t + r_t K_t + (1-\delta)K_t + P_t w_t H_t. \end{aligned}$$

The equations for this model, at the aggregate level, are the same as for the model with injections going directly to the families. Therefore, the stationary state equilibria (and the dynamic properties given below) are also the same.

5 Dynamic properties of the models

The short run reactions of the models to a money growth shock can be observed by first log-linearizing the models, solving the linear versions for matrix policy functions, and then comparing the impulse response functions that result from the same money shock. Log-linearization and the solution for the policy matrices of the models is done following techniques described in McCandless [5].

The following is for the log-linear version of the working capital model where money injections go directly to the family. These equations describe the approximate (first order) behavior of the model around a stationary state. The stationary state values of the variables (those indicated by a bar in the equations) are determined as above and depend on the stationary state growth rate of money.

$$
\begin{aligned}
0 &= \widetilde{w}_t + \widetilde{P}_t - E_t\widetilde{P}_{t+1} - E_t\widetilde{C}_{t+1}, && (11)\\
0 &= \widetilde{w}_t - E_t\widetilde{w}_{t+1} + \beta\overline{r}E_t\widetilde{r}_{t+1}, && (12)\\
0 &= \widetilde{r}^n_t - \widetilde{w}_t + \widetilde{C}_t, && (13)\\
0 &= \overline{C}\left[\widetilde{P}_t + \widetilde{C}_t\right] - \overline{M/P}\widetilde{M}_{t-1} - \overline{M/P}\widetilde{g}_t + \overline{N/P}\widetilde{N}_t, && (14)\\
0 &= \overline{M/P}\widetilde{M}_t + \left[\overline{r}^n\overline{N/P} - \overline{M/P}\right]\widetilde{P}_t + \overline{K}\widetilde{K}_{t+1} - \overline{w}\overline{H}(\widetilde{w}_t + \widetilde{H}_t) && (15)\\
&\quad -\overline{r}\overline{K}\widetilde{r}_t - (\overline{r}+1-\delta)\overline{K}\widetilde{K}_t - \overline{r}^n\overline{N/P}\widetilde{N}_t - \overline{r}^n\overline{N/P}\widetilde{r}^n_t, &&\\
0 &= \widetilde{w}_t + \widetilde{r}^f_t - \widetilde{\lambda}_t - \theta\widetilde{K}_t + \theta\widetilde{H}_t, && (16)\\
0 &= \widetilde{r}_t - \widetilde{\lambda}_t - (\theta-1)\widetilde{K}_t - (1-\theta)\widetilde{H}_t, && (17)\\
0 &= \widetilde{Y}_t - \widetilde{\lambda}_t - \theta\widetilde{K}_t - (1-\theta)\widetilde{H}_t, && (18)\\
0 &= \widetilde{r}^f_t - \widetilde{r}^n_t, && (19)\\
0 &= \overline{N/P}\widetilde{N}_t - \overline{N/P}\widetilde{P}_t - \overline{w}\overline{H}\widetilde{w}_t - \overline{w}\overline{H}\widetilde{H}_t && (20)\\
0 &= \widetilde{M}_t - \widetilde{g}_t - \widetilde{M}_{t-1}. && (21)
\end{aligned}
$$

Equations 11, 12, and 13 are the log-linear version of the family's first order condtions. Equation 14 is the log-linear version of the cash in advance constrant with money injections going directly to the family. Equation 15 is the family flow budget constraint. Equations 16, 17, and 18 come from the production section, the first two are the factor market conditions and the third is the aggregate production function. Equation 19 comes from the zero profit condition of the financial intermediary. Equation 20 is the working capital condition, that borrowing equals the wage bill. Equation 21 is the money growth rule. In addition to this set of equations, we have the rules for the evolution of the technological and monetary shocks,

$$\widetilde{\lambda}_t = \gamma\widetilde{\lambda}_{t-1} + \varepsilon^{\lambda}_t,$$

and

$$\widetilde{g}_t = \pi\widetilde{g}_{t-1} + \varepsilon^{g}_t.$$

Defining $x_t = \left[\widetilde{K}_{t+1}, \widetilde{M}_t, \widetilde{P}_t\right]'$ as the state variables, $y_t = \left[\widetilde{r}_t, \widetilde{w}_t, \widetilde{Y}_t, \widetilde{C}_t, \widetilde{H}_t, \widetilde{N}_t, \widetilde{r}^n_t, \widetilde{r}^f_t\right]'$ as the jump variables, and $z_t = \left[\widetilde{\lambda}_t, \widetilde{g}_t\right]'$ as the stochastic variables, the system can be written as

$$
\begin{aligned}
0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t,\\
0 &= E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right],\\
z_{t+1} &= Nz_t + \varepsilon_{t+1}.
\end{aligned}
$$

This system can be solved (using MATLAB) for a set of policy matrices of the form

$$x_t = Px_{t-1} + Qz_t,$$

and

$$y_t = Rx_{t-1} + Sz_t,$$

where ($\overline{g} = 1$ or the inflation rate equals 0 for the example shown here),

$$P = \begin{bmatrix} 0.9430 & 0 & 0 \\ 0 & 1 & 0 \\ -0.3340 & 1 & 0 \end{bmatrix},$$

$$Q = \begin{bmatrix} 0.1490 & -0.0306 \\ 0 & 1 \\ -1.0337 & 1.6124 \end{bmatrix},$$

$$R = \begin{bmatrix} -0.9236 & 0 & 0 \\ 0.5315 & 0 & 0 \\ 0.0764 & 0 & 0 \\ 0.5434 & 0 & 0 \\ -0.4432 & 0 & 0 \\ -0.2457 & 1 & 0 \\ -0.0119 & 0 & 0 \\ -0.0119 & 0 & 0 \end{bmatrix},$$

$$S = \begin{bmatrix} 1.8309 & -0.5768 \\ 0.4701 & -0.0147 \\ 1.8309 & -0.5768 \\ 0.4077 & -0.3538 \\ 1.2982 & -0.9013 \\ 0.7347 & 0.6965 \\ 0.0625 & 0.3391 \\ 0.0625 & 0.3391 \end{bmatrix}.$$

The log-linear version of the model with money injection going to the financial intermediary is the same as the one given above except for equations 14, 19, and 20. These equation become, respectively,

$$0 = \overline{C}\left[\widetilde{P}_t + \widetilde{C}_t\right] - \frac{\overline{M/P}}{\overline{g}}\widetilde{M}_{t-1} + \overline{N/P}\widetilde{N}_t,$$

$$\begin{aligned} 0 \quad = \quad & \overline{r}^f \left[\overline{N/P} + \overline{M/P}\left(1 - \frac{1}{\overline{g}}\right)\right]\widetilde{r}_t^f + \left(\overline{r}^f - \overline{r}^n\right)\overline{N/P}\widetilde{N}_t \\ & - \left[\left(\overline{r}^f - \overline{r}^n\right)\overline{N/P} + \overline{r}^f\overline{M/P}\left(1 - \frac{1}{\overline{g}}\right)\right]\widetilde{P}_t \\ & + \overline{r}^f\overline{M/P}\widetilde{g}_t + \overline{r}^f\overline{M/P}\left(1 - \frac{1}{\overline{g}}\right)\widetilde{M}_{t-1} - \overline{r}^n\overline{N/P}\widetilde{r}_t^n, \end{aligned}$$

and

$$
\begin{aligned}
0 &= \overline{N/P}\widetilde{N}_t + \overline{M/P}\left(1-\frac{1}{\overline{g}}\right)\widetilde{M}_{t-1} - \left[\overline{N/P} + \overline{M/P}\left(1-\frac{1}{\overline{g}}\right)\right]\widetilde{P}_t \\
&\quad + \overline{M/P}\widetilde{g}_t - \overline{w}\overline{H}\widetilde{w}_t - \overline{w}\overline{H}\widetilde{H}_t.
\end{aligned}
$$

The policy matrices that come from solving this linear system are

$$x_{t+1} = Px_t + Qz_t,$$

and

$$y_t = Rx_t + Sz_t,$$

where (again for the case where $\overline{g} = 0$),

$$P = \begin{bmatrix} 0.9430 & 0 & 0 \\ 0 & 1 & 0 \\ -0.3340 & 1 & 0 \end{bmatrix},$$

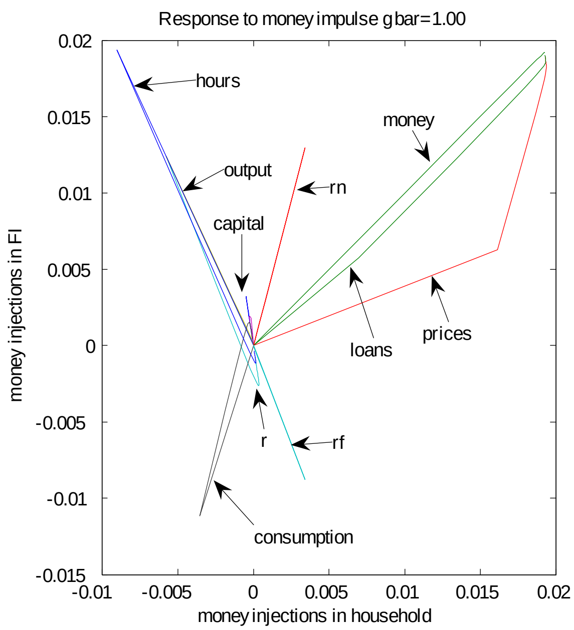
$$Q = \begin{bmatrix} 0.1490 & 0.2020 \\ 0 & 1 \\ -1.0337 & 0.6287 \end{bmatrix},$$

$$R = \begin{bmatrix} -0.9236 & 0 & 0 \\ 0.5315 & 0 & 0 \\ 0.0764 & 0 & 0 \\ 0.5434 & 0 & 0 \\ -0.4432 & 0 & 0 \\ -0.2457 & 1 & 0 \\ -0.0119 & 0 & 0 \\ -0.0119 & 0 & 0 \end{bmatrix},$$

$$S = \begin{bmatrix} 1.8309 & 1.2411 \\ 0.4701 & 0.1791 \\ 1.8309 & 1.2411 \\ 0.4077 & -1.1172 \\ 1.2982 & 1.9392 \\ 0.7347 & 0.5734 \\ 0.0625 & 1.2964 \\ 0.0625 & -0.8772 \end{bmatrix}.$$

The two models are identical in their response to technology shocks. In addition, the deterministic part of the policy functions (matrices P and R) are identical. The difference is the way the two economies respond to monetary shocks. This can be seen by comparing the second columns of matrices Q and S. The responses to a monetary shock of capital, real rentals, real wages, output, hours worked, and the interest rate paid by firms are of opposite signs in the two models. Figure 2 shows the responses to a .01 monetary shock of for the two models over thirty periods.

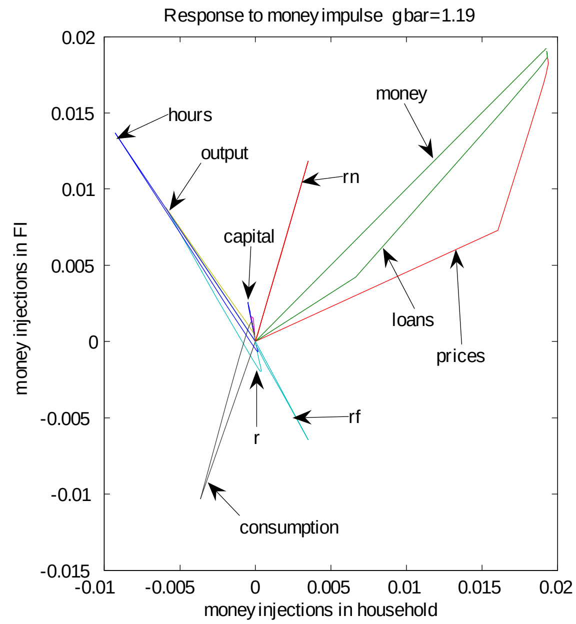
two models imrs 0 ss



$_2.pdf$

Figure 2: Models with 0 annual inflation

two models imrs



$_3.pdf$

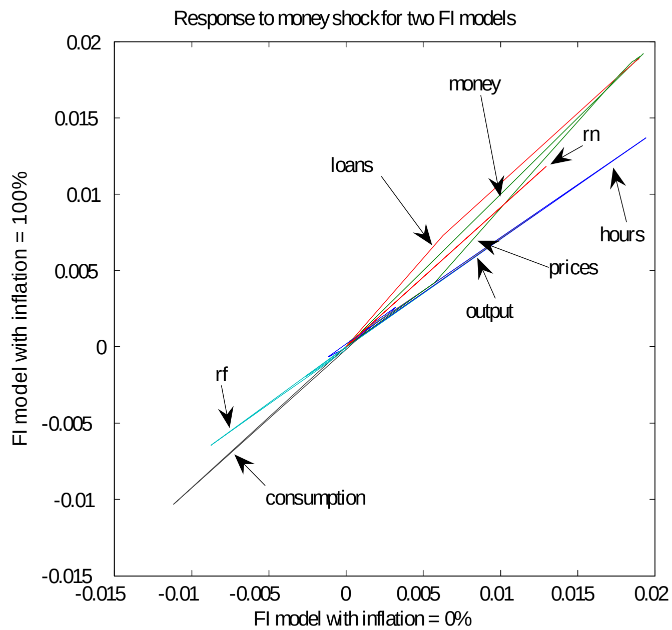
Figure 3: Models with 100% annual inflation

If a variable responds exactly the same way to a monetary shock in the the two models, the line in the graph for that variable would fall on the 45° line. This is the case for the response of the money stock to the monetary shock. Lines with positive slopes indicate that the variable is responding in the same direction, but possible with different timing and magnitures to a monetary shock. Lines with negative slopes indicate that the variable responds in the opposite direction to a monetary shock. For example, in Figure 2, capital, hours, output, and r^f have negative slopes, indicating that the response is in the opposite direction. In the model where the injections are made directly to the families, a monetary injection reduces these variables while in the model where injections are made through the financial intermediary, a monetary injection increases these variables. Note that wages responds the same and of about the same magniture as does the capital stock so it can not be observed separtely in the graph.

The policy matrices for the two economies where the stationary state annual inflation rate is 100% was also calculated. Figure 3 presents the same information as Figure 2 but for the economy with 100% inflation. The reactions are very similar. The main changes are that the maginitures of the response of some of the real variables is reduced and there is a slight change in the timing of the response of some of the nominal variables.

two FI models imrs 0 and

100



$_4.pdf$

Figure 4: Comparing effects of rates of inflation

Another way to compare the responses of the model as a functions of the stationary state inflation rate is to view graphs similar to those given above but where one is comparing the same model at the two different inflation rates. Figure 4 gives this information for the model where the money injections go to the financial intermediary. The horizontal axis gives the responses for the economy with a zero annual inflation rate and the vertical axis for the same economy but with a 100% annual inflation rate. The lines for all variables fall near the 45 degree line, so that responses are all similar. However, one can observe that output and hours respone more in the lower inflation model, since they are below the 45 degree line. The timing on loans and prices are slightly different at the two inflation rates. The results for the real varialbes are consistent with the observation that the interest rate that the firms pay, r^f, responds slightly less when inflation is higher. Since in stationary states, higher money injections to the financial intermediary imply a higher subsidy (and a decreasing marginal subsidy) to production, the impact of a positive monetary shock is likely to be smaller with higher initial inflation.

5.1 Conclusions

The way monetary policy injects new money (and removes old money) into the economy is crucial for determining its impact on real variables. The traditional stationary state result of an inflation tax in a cash-in-advance model continues to hold when financial intermediaries that lend for financing working capital are added as long as the mechanism for injecting money is the same. When stationary state monetary policy via money injections works directly on the financial iintermediaries, then money injections work as a subsidy and with higher rates of inflation come higher output, consumption, and utility. With monetary policy working through the financial intermediaries, a stationary state Phillips curve exists.

The short run responses of the economy to monetary shocks is of a similar nature to those of the stationary states to different long run inflation rates. When money injections go directly to the families, positive money shocks tend to have a negative effect on the real side of the economy. For economies where money injections go to the financial intermediaries, positive monetary shocks result in positive real responses from capital, output, and hours worked but with a negative immediate response for consumption. The humped shaped response of real variables to monetary shocks exists, but is of relatively short duration. Christiano and Eichenbaum [2] and Dotsey and Ireland [4] have looked at ways of changing the timing of the firms' decisions so as to reduce the difficulty that Christiano and Eichenbaum describe as the result that "disproportionately large share of monetary injections is absorbed by firms to finance variable inputs." However, since the models are corner solutions in that in one all money injections go to families and in the other all go to the financial intermediary, these results should not be surprising. What we lack is evidence on how monetary injections enter the economy, what fraction can be considered going directly to families (and not directly into the financial system) and what part works as a subsidy to the financial system.

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