A Search Model of Marriage with Differential Fertility*

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Abstract

Women go through menopause after which time fertility is zero. This is driven largely by biology. Even though some literature on marriage markets introduces the limited fertility horizon issue, my work differs from these papers in important ways. First, the work below is an equilibrium search model with two-sided matching and where utility is non transferable. Second, the dynamic in my model are driven solely from the more limited fertility time horizon for women and not from assumptions on how wages change with age. This is important for two reasons. First to investigate how many of the stylized facts about marriage markets (for example than women generally marry older men) can be explained without resorting to the wage process. Second, while the wage process varies considerably across societies, biology varies very little. Therefore, predictions from this model should be more applicable in a variety of contexts.

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1 Introduction

Since the publication of Gary Becker's first paper about marriage (Becker 1973) there has been growing interest in investigating decisions about marriage as if they occurred in a market. Becker argues that marriage has many aspects that are similar to trade of any other good in a market. Marriage is a voluntary contract between two people, or two families, who believe that they will be better off married than remaining single. Further, like buyers and sellers, many men and women compete to find mates. These aspects make marriage amenable to investigation as voluntary trade in a competitive market.

The compelling logic of this argument has spawned a large volume of research in both economics and sociology of both a theoretical and empirical nature. Much of this early literature followed up on Becker's insight and thought about one person, usually the women, "purchasing" a mate in the marriage market. In this literature, women made decisions on marriage based on "meeting" men from the available the pool and choosing whether to marry them or remain single. Men were passive agents and the bilateral nature of the marriage market was ignored. While these models were perhaps unrealistic in construct, they have had a major impact on the literature and on public policy. One well cited example is the work of Wilson and Neckerman (1986) who argue that the rise in out-of-wedlock childbearing among African-Americans is primarily a result of African-American women increasingly choosing not to marry from a shrinking pool of African-American men who are deemed to be of sufficient quality to be "marriageable" (i.e. close to the women's age and education level, not in prison and with a job).

Investigating this theory, Brien (1997) finds that while the pool of marriageable men does affect the age at first marriage, this mechanism explains very little of the difference in the time of marriage (and fertility) between African-Americans and Whites. Other empirical examples where women are seen as choosing from a pool of available men include Fitzgerald (1991), Lichter, LeClere and McLaughlin (1992), Lichter et al. (1992), Wood (1995) and Schmidt (2002). Another literature, equally one sided, focuses the analysis on the decision of men in the marriage market as they face a pool of marriageable women (i.e. Siow (1998)).

The bilateral nature of the marriage market has been addressed in some theoretical literature. For example, in their book, Roth and Sotomayor (1990), discuss two sided matching models in a marriage market context.

They show that a given match is stable if is not blocked by any individual or any pair of agents. To understand the "blocking pairs", they consider a given matching structure such that there exist a man and a woman that are not matched to one another, but that prefer each other to their assignments in that matching structure. The man and the woman will be said to block that matching structure. Moreover, Pollak and Winter (1997), show that as markets become large, the probability that a random matching is stable goes to zero, and the number of blocking pairs for a given unstable matching goes to infinity. For empirical researchers, the sensitivity of the equilibrium to aspects of the problem that can never be observed in the data is troubling.

Most of the literature view utility arising solely from the quality of the match between the husband and the wife (Siow is an exception). Clearly, one of the main reasons that marriage has occurred is for the production of children. While more recently, bearing children outside of marriage has become more common in developed countries, there are still reasons to believe that it is less costly, or of higher utility for parents to raise a child within marriage. For example, Willis and Weiss (1993) argue that children are a public good within marriage and as such both parents can derive utility from the child at the same time while sharing the cost of raising the child. This advantage is lost when a child's time need be divided between a custodial and non-custodial parent outside of marriage. In a real sense, the distinction of utility arising both from the marriage itself as well as from the children produced by it is unimportant when the marriage market is viewed as static (as the utility from the marriage can simply be redefined as the marriages intrinsic value plus the expected utility from children produced from it at the time of marriage). But as we discuss below, when men and women are forward looking, and when fertility falls with age, this distinction becomes important.

There is some literature that recognizes the bilateral nature of marriage within a dynamic search context (i.e. Mortensen (1988))[1] . The interaction between marriage, labor market and human capital accumulation has also been addressed in the literature. Recent examples are Aiyagari et al. (2000), Seitz (2002) and Greenwood, Guner and Knowles (2002). Also, in a recent paper, Brien, Lillard and Stern (2002) analyze cohabitation before marriage as a learning process about match quality.

[1] For detailed surveys about the search and matching literature, see Burdett and Coles (1999) and also Pissarides (2000)

While this work shares the dynamic search approach, it differs by recognizing and incorporating an undeniable fact - women go through menopause after which time fertility is zero. This is driven largely by biology. A typical woman is born with four million eggs. Each month, one egg ripens and passes into the uterus and about 10,000 eggs deteriorate. When she runs out of functioning eggs, she is in menopause. In the U.S., menopause occurs between the ages of 40 and 60 with age 50.5 the median age of menopause and with 1-in-10 women beginning menopause after age 54. The distribution of the onset of menopause appears not to have changed over the last 100 years. While there appears to be heritable variation of the age at menopause, the mechanism is not well understood. Fertility also falls for men with age. But men are clearly fertile for more of their adult years than women. This differential decline in fertility of men and women becomes salient when men and women care both about the intrinsic utility from marriage as well as the utility of having children as childbearing can only occur if both the husband and wife are fertile.

There is another literature that incorporates the limited time horizon for childbearing into a marriage market model[2]. Siow (1998) introduced the issue of the shorter fecundity period of women In a model with capital accumulation and where utility comes exclusively from having children, old and young men (all fertile) compete for young women as by assumption infertile women do not participate in the market. Young men would always marry young women except that Siow allows wages to rise with age as well. Because of this some old men, those who successfully obtain a higher wage, are able to marry. This displaces some of the young men in the competition over scarce fertile women. Moreover, Siow argues that there is a relationship between the scarcity of fertile women and the fact that men are more likely to remarry after divorce[3]. While it is hard to argue that, at any point of time, the stock of single fertile women is smaller than the stock of single

[2]Schmidt (2002) recognizes this fact using a search framework but concentrates only on women's decision.

[3]As Siow(1998) states in the introduction (page 335) "First, in monogamous societies with divorce and remarriage, fecund women are relatively scarce. For example, in North America, at least 30 percent of first marriages fail. Twenty percent of divorced women and 60 percent of divorced men will remarry. This differential in remarriage rates suggest that 12 percent of women who marry for the first time will marry divorced men. There are at least 12 percent fewer never-married to match with never-married men. Women will behave differently than men in response to this relative scarcity."

fertile men, it is not clear whether this will be true in a dynamic framework. Women, on one hand, recognizing their more limited fertility, may marry earlier reducing the size of the pool of fertile marriageable women. Men the other hand, may face over their longer fertile cycle more than one cohort of fertile women and that is what make them able to wait. Therefore, in a dynamic world it is not at all clear that fertile women will be the scarce resource in a steady state.

While my work also emphasizes the limited fertility horizon, it differs from these papers in important ways. First, the work below is an equilibrium search model with two-sided matching. Second, the dynamics in my model are driven solely from the more limited fertility time horizon for women and not from assumptions on how wages change with age. In other words, the objective of this paper is to address the question: What can be explained solely by biology? This is important for two reasons. First it is helpful to clarify how many of the stylized facts about marriage markets can be explained without resorting to the wage process. For example, I show below that limited time horizons alone are sufficient to generate a stylized fact that holds across many societies - on average younger women marry older men. In addition, the model below shows that biology alone would generate what is often reported in the popular press (but to date has little empirical evidence) - that women find in the 30s that there are "no good men" to marry. A second reason to start by isolating the effects of biology independent of the wage processes is that while the wage process varies considerably across societies, biology varies very little. Therefore, any prediction gleaned from a model that does not rely on upward sloping wage profiles is likely more applicable in a variety of social contexts.

The model developed below is a two sided general equilibrium search model where (as in most of the labor related search literature) men and women are ex ante homogenous and utility is non transferable[4].Only after a random meeting does the man and the women receive an independent signal about the match quality (match specific heterogeneity). Using a non transferable utility is helpful in order to provide a framework that is able to explain stilyzed facts marriage independently of potential gains of specialization, as it is common in the literature. In this model utility depend both

[4]There is an increasing theoretical literature about ex ante heterogeneous agents (for example Burdett and Coles (1997) and Smith (2002). However, the standard framework is sufficient for the goals of this paper.

on the love share by the couple and on joy derived from of having children. Unlike most of the previous marriage market literature, neither employment decisions nor capital accumulation is analyzed here. The only asymmetry between men and women is the fertility horizon. The total number of single men and women, and therefore the sex ratio, is determined endogenously in the model.

The structure of the paper is as follows. In Section 2, I outline a simple 2 period model where women are only fertile in the first period and men in both periods. This model is solved analytically for an uniform distribution of match qualities. This model is intended to set the basic conceptual framework in a simplified way. In Section 3, I extend the work by modeling the marriage choice as a finite horizon dynamic programming problem and solve the model numerically, extension that generates several stylized facts about marriage. Section 4 is devoted to compare the implications of the model with US Census data and with data of selected countries. Section 5 concludes.

2 A Simple 2 Period Model

In this section we develop a simple model where people live two periods, women are fertile only in the first period and men are fertile in both periods. This simplification will allow us to obtain closed form solutions of the strategies and to proof existence and uniqueness. In next section we will generalize this model allowing people live a finite number of periods, and where fertility horizon for women is less or equal the one for men. The numerical solution for the generalized model can be compared with Census data in order to analyze the results more accurately.

2.1 Assumptions

There is a continuum of single women of measure $W(t)$, and of men , $M(t)$. We will focus in the steady state, so $W(t) = W$ and $M(t) = M$.

In the spirit of Pissarides (1990), the number of contacts between single women and men is determined by a constant return to scale meeting function. Women will meet at most one man per period and vice versa. The probability of meeting someone of the opposite sex each period will depend the relative scarcity of each sex. Therefore, for women, the probability of meeting a single man will be

$$\lambda = \eta \frac{M^{\theta} W^{1-\theta}}{W} = \eta \left(\frac{M}{W} \right)^{\theta} \tag{1}$$

where $0 < \theta < 1$ and η a constant lower than 1.[5]

Similarly, men will meet a single women

$$\alpha = \eta \frac{M^{\theta} W^{1-\theta}}{M} = \eta \left(\frac{M}{W} \right)^{\theta - 1} \tag{2}$$

All singles are ex-ante identical with respect to their type. However, they differ in age and potential fertility. While all men are fertile no matter their are young or old, women are only fertile at age 1.

Both men and women live two periods, ages 1 (young) and 2 (old) . In any moment, there will be a number of women from both generations, $w_1$of age1and w_2 of age 2 looking for a husband. Similarly these women will face a market of $m_1$young and m_2 mature bachelors.

Since men and women get married in pairs we need the number of young and old women that get married each period (v_1 and v_2) to be equal to the total of men (h_1 young plus h_2 old) who enter into marriage. That is:

$$v_1 + v_2 = h_1 + h_2$$

Each period, an exogenous flow of single young people of age 1, w_1 women and m_1 men assumed to be equal, enter the market and the ones who have not matched in the previous period will remain in the market. In the steady state, this flow of young people entering the market will be equal to the number of people who exit the market through marriage at any age plus the ones that die single after period 2 ($\widehat{w^s}$ old maids and $\widehat{m^s}$old bachelors).[6]

People who divorce or whose spouse dies don't re enter the market. The meaning of this assumption is that, when single, people plan to marry only once in life. In addition, the motivations of an eventual divorce and remarriage could be very different that the ones for first marriage. Therefore, this

[5] This constant is merely a time scaling parameter introduced to ensure that the probability of meeting is lower than 1 and allow to replicate the same model in an arbitrary number of periods.

[6] Here we implicitly assume that the actual number of children that people have is the quantity needed to ensure the steady state with no population growth. Since the goal of this paper is to explain only the decision of marriage we assume the decision about the number of children as exogenous.

assumption is made to restrict the goal of this paper to the explanation of first marriage.[7]

The above assumptions are summarized in the following identities and in the figure below

$$\begin{aligned} w_1 &= v_1 + v_2 + \widehat{w^s} \\ m_1 &= h_1 + h_2 + \widehat{m^s} \end{aligned}$$

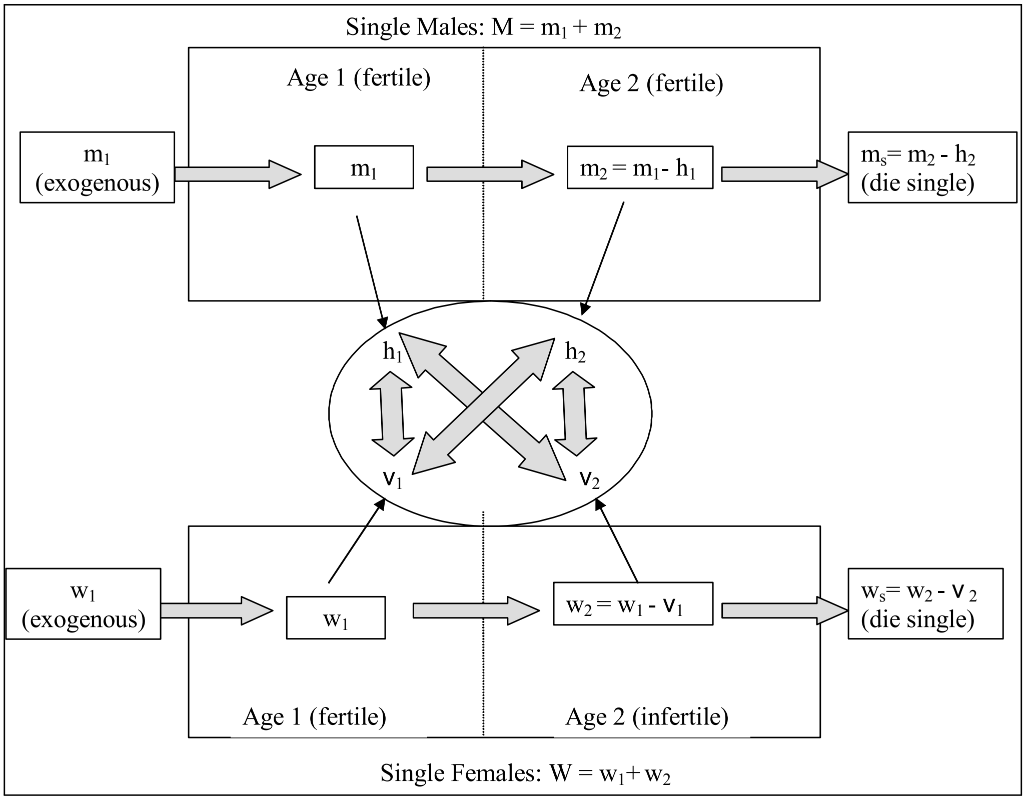
The stock of single female of each age at any moment will be

$$\begin{aligned} w_2 &= w_1 - v_1 \\ \widehat{w^s} &= w_2 - v_2 \\ W &= w_1 + w_2 \end{aligned}$$

Similarly, the stock for single men

$$\begin{aligned} m_2 &= m_1 - h_1 \\ \widehat{m^s} &= m_2 - h_2 \\ M &= m_1 + m_2 \end{aligned}$$

[7] For a model of marriage with "on the job" search and therefore endogenous separations, see Cornelius (2003)



The discount factor is set to be equal to $\beta < 1$

The age composition of the marriage market is endogenously determined in the model. Therefore the fraction of young women and men $p = \frac{w_1}{W}$ and $q = \frac{m_1}{M}$ are simultaneously determined as a function of the reservation strategies of men and women.

2.1.1 Payoffs

Given a man and a woman meet, their potential payoffs come from love, and the utility of having children within marriage. We assume that both men and women will receive zero utility if they don't marry either in period one or two.

The specific utility that a woman receives from a man and vice versa are considered as independent random draws from the distribution $G(y)$ and $F(x)$, respectively. Assume that $G(y)$ have support $[0, \overline{y}]$ and mean μ_y, and $F(x)$ have support $[0, \overline{x}]$ and mean μ_x.[8] Both distributions are strictly

[8]In theory, there would be no inconvenient on allowing y or x to take negative values

increasing on x and y respectively.

In addition, if a fertile man and a fertile woman meet, the utility is increased by a multiplicative parameter $k > 1$ because of the possibility of having children together. For example, if a fertile man marries a fertile woman he will receive kx per period and she will receive ky per period. If either the man or the woman involved is infertile, both of them will only receive x or y respectively, that is, only the love of the other person.

The rationale for the parameter k is that the value of a "having a family" will be a function of the feelings to their significant other. That is, people enjoy more to have a family with the person they are in love with. If we assume that people always receive utility for having children, we can separate it in two components, one come specifically from parenthood, and the other depend who are you having children with. Since it is not necessary either love nor a stable relationship in order to have children, the specific joy of having children (and therefore the utility out of wedlock parenthood) is normalized to 0 in this model. We assume further that the multiplicative parameter k has a maximum such that the utility of marrying and have children with an average person can not be higher than the joy of finding a soulmate. That is,

$$k\mu_x \leq \overline{x} \quad \text{and} \quad k\mu_y \leq \overline{y} \tag{3}$$

Thus, the payoffs of marriage for men and women are the following:

Women	Husband Age 1	Husband Age 2
Marry at age 1	$ky(1+\beta)$	ky
Marry at age 2	y	

Men	Wife Age 1	Wife Age 2
Marry at age 1	$kx(1+\beta)$	x
Marry at age 2	kx	x

2.2 The Male's Optimization Problem

if the mean of both distributions are strictly positive. It sounds perfectly plausible that any man or woman could find that marrying certain canditates to be worse than staying single, and having children with these potential mates as a discount over having them out of wedlock. However, since the utility of being single is equal to 0, the reservation values set by men and women will be always nonnegative and that assumption will become irrelevant.

Let us analyze first the Male Problem. In each period a male will meet a woman of type x with probability α. The man will meet a single young woman (age 1) with probability p (that will equal $\frac{w_1}{W}$, the fraction of young single women, but is taken as given by each individual) , or an old single woman (age 2), with probability $(1-p)$. However, the fact that he meets a young woman does not mean that he has a concrete opportunity to marry her. Even though all men are fertile, a given young woman will not be indifferent between a man of age 1 and of age 2, because if she marries a senior bachelor she will enjoy his company for only one period. Therefore she will set two different reservation values, R^w for young men and R^{wold} for men of age 2. That is the same to say that a given senior bachelor will have a probability of an offer (that is, to meet and also being accepted by a young woman) of

$$\alpha_2^{young} = \alpha \left[p \left(1 - G \left(R^{wold} \right) \right) \right]$$

and a young man will receive an offer from a young lady with probability

$$\alpha_1^{young} = \alpha \left[p \left(1 - G \left(R^w \right) \right) \right]$$

Since old women will have reservation utility equal to 0 , they will accept any proposal. Therefore the probability that a given male receives and offer from an old woman will be

$$\alpha^{old} = \alpha \left(1 - p \right)$$

Given that a concrete opportunity for a match is available, the man receives a signal drawn from the distribution $F(x)$ and decide to match or not.

2.2.1 Men of Age 2 (Old)

Suppose first the man is of age 2, so if he does not marry this time he will die single, earning zero utility. Therefore, his reservation value would be equal to 0, that means that he would be willing to marry any woman who accept him as a husband. If he happen to meet a woman also age 2 (and also with Reservation utility equal to 0), they will marry with certainty. If he meets a young woman (age 1) and he marries her, he will also enjoy the extra utility of having children (k times the type of the woman).

Therefore, the value of search for a man of age 2 will be

$$V_2^m = \left(\alpha_2^{young} k + \alpha^{old} \right) \mu_x \tag{4}$$

2.2.2 Men of Age 1 (Young)

Since young men are able to wait until they are old in order to find the right mate, in period 1 men set a reservation value for accepting a woman taking into account next period prospects. As before, they can meet young or old ladies. Of course, if a young man marry a young woman, he will enjoy having children with his wife, but that will not happen if he marries an old woman. Consequently, the reservation values a young man will set for marrying a young or an old bachelorette of the same type will not be the same. Let's call them $R^m$and R^{mold}, respectively. Moreover, in order to marry a young woman, he must be accepted by this girl , that will happen with probability $(1 - G(R^w))$ (all old ladies will accept him). So, the problem that a young man faces is the following

$$V_1^m = \underset{R^m, R^{mold}}{Max} \alpha_1^{young} k (1+\beta) \int_{R^m}^{\overline{x}} x f(x) dx + \alpha^{old} \int_{R^{mold}}^{\overline{x}} x f(x) dx + (1-\phi_1) \beta V_2^m \tag{5}$$

where

$$\phi_1 = \alpha_1^{young} (1 - F(R^m)) + \alpha^{old} \left(1 - F\left(R^{mold}\right)\right) \tag{6}$$

is the probability that a man matches with a young or a mature woman at age 1.

2.3 The Female's Optimization Problem

Now we can analyze the female problem. In each period , given the meeting rate λ, a female will meet a young man (age 1) with probability q, or an old (age 2), with probability $(1 - q)$. She will marry him if the utility of marrying the guy she meets, drawn from the distribution $G(y)$ is greater than the value of search for a better mate for one more period.

2.3.1 Women of Age 2 (Old)

Suppose the woman is of age 2. Before the last period of her life, she knows two things: First, that she will die at the end of the period, what means her reservation value will be $= 0$. Second, in the last period she is not fertile , which means that neither will she receive the extra utility of having children, nor will she be able to provide that extra utility to any man she marries.

We can define the offer rates that a senior woman faces in the following way. A woman will meet a man each period with probability λ. If she happen to meet an old man (with probability $(1-q)$) he will propose with probability 1 (the reservation value for an old man is 0), and so she will have a concrete offer from an old bachelor with probability

$$\lambda^{old} = \lambda(1-q)$$

However, if she meets a young man (with probability q) she will only marry him if her type x is as least as large as his reservation utility for a senior bachelorette, R^{mold}. Therefore, the probability that a young man proposes to a senior woman will be $\left(1 - F\left(R^{mold}\right)\right)$,what means that a senior bachelorette will receive a proposal from a young man with probability

$$\lambda_2^{young} = \lambda q \left(1 - F\left(R^{mold}\right)\right)$$

Therefore the value of continue searching for a woman is

$$V_2^w = \left(\lambda_2^{young} + \lambda^{old}\right)\mu_y$$

2.3.2 Women of Age 1 (Young)

A young woman sets a reservation value taking into account that she may have future opportunities to find a better spouse. However, if she doesn't marry at age 1 she will not be able to have children. Since men of all ages are fertile, a young woman will be indifferent to marry a young or an old man of the same match quality. Consequently she will choose only one reservation value independently of the man age. Of course, while any old men will accept her, he will only be able to marry a young man if her match quality is higher than the reservation value set by the young potential husband , R^m.Since a young man proposes to a young woman with probability $(1 - F(R^m))$, so a young lady will receive a proposal from a young man with probability

$$\lambda_1^{young} = \lambda q \left(1 - F\left(R^m\right)\right)$$

Therefore, the problem of a young female can be defined as follows,

$$V_1^w = \underset{R^w, R^{wold}}{Max} \lambda_1^{young} k (1+\beta) \int_{R^w}^{\overline{y}} y g(y) dy + \lambda^{old} k \int_{R^{wold}}^{\overline{y}} y g(y) dy + (1-\gamma_1)\beta V_2^w \quad (7)$$

where

$$\gamma_1 = \lambda_1^{young} \left(1 - G\left(R^w\right)\right) + \lambda^{old} \left(1 - G\left(R^{wold}\right)\right) \tag{8}$$

are the probabilities that a woman find her spouse at age 1.

2.4 Steady State Equilibrium

Solving the problems stated in equations(5) and (7), the reaction functions for men and women, respectively, are the following:

$$\begin{aligned} R^m &= \frac{\alpha\beta\left[(1-p) + kp\left(1 - G\left(R^{wold}\right)\right)\right]}{k(1+\beta)}\mu_x \qquad (9) \\ R^{mold} &= \alpha\beta\left[(1-p) + kp\left(1 - G\left(R^{wold}\right)\right)\right]\mu_x \\ R^w &= \frac{\beta\lambda\left(1 - qF\left(R^{mold}\right)\right)}{k(1+\beta)}\mu_y \qquad (10) \\ R^{wold} &= \frac{\beta\lambda\left(1 - qF\left(R^{mold}\right)\right)}{k}\mu_y \end{aligned}$$

As shown, the relationship in the reservation value that men set for older women is $k(1+\beta)$ times the one for young women. The reservation set for old women is exactly equal to the value to remain in the market at age 2. Clearly, the higher the reservation value for a man when young, the greater the probability of him still being in the market when old. Therefore, the higher the probability of being accepted by a women when he is older, the higher the minimum type of the bride he may require when young. For women, the intuition is as follows. The reservation value of the woman depends positively in the average"quality" of the available men, the degree of patience and the meeting rate. Conversely, women will decrease their reservation value the higher the value of having children and the higher the reservation value that men set for older women, times the fraction of young men in the market. The explanation for this last factor is the following: the more choosy young men are in order to marry a mature woman (and the greater the fraction of young men in the market), the more the incentives of young women to be worried about their future and marry at age 1.

To facilitate the following proof, we define

$$\begin{aligned} T_1(R^{wold}) &= \alpha\beta\left[(1-p) + kp\left(1 - G\left(R^{wold}\right)\right)\right]\mu_x \\ T_2(R^{mold}) &= \frac{\beta\lambda\left(1 - qF\left(R^{mold}\right)\right)\mu_y}{k} \end{aligned}$$

In this notation, an equilibrium is characterized simply by the following equations:

$$R^{mold*} = T_1(R^{wold*}) \quad (11)$$
$$R^{wold*} = T_2(R^{mold*}) \quad (12)$$

In the following theorem we will show that there exists a unique solution system formed by Equations (9) and(10) and hence, there exists a unique steady state equilibrium.

Theorem 1 *Assume that $F(x)$ and $G(y)$ have the same support $[0, \overline{x}]$ and that their respective means, μ_x and μ_y, are both strictly less than 1. Assume further that there exists a constant $C < \frac{1}{\mu_x \mu_y}$ such that the distributions' densities f and g satisfy*

$$f(x)\, g\left(R^{wold}(x)\right) \leq C$$

for all $x \in [0, \overline{x}]$. Then there exists a unique equilibrium for the system formed by equations (9) and (10). This equilibrium will be an interior solution, means that both men and women will marry either at age 1 or 2 with positive probability.

Outline of Proof. Define

$$H(x) = T_1(T_2(x)) \quad (13)$$

In Equations (11) and (12) we show that every steady state equilibrium of the model corresponds to a fixed point of $H(.)$. A long calculation, relegated to the Appendix, shows that under the hypothesis of the theorem

$$| H'(x) | \leq \mu_x \mu_y < 1$$

Consequently, $H(x)$ is a contraction mapping.

By the contraction mapping theorem, there exists a unique fixed point of $H(.)$. Call it R^{mold*}. A short argument in the Appendix shows that $R^{mold*} \in [0, \overline{x})$, and the associated $R^{wold*} \in [0, \overline{x})$, and that the uniqueness is ensured for R^m and R^w. We conclude that the unique fixed point of $H(.)$ corresponds to a steady state equilibrium of the model. ■

Remark 2 *The hypothesis of Theorem 1 is trivially satisfied if $F(x)$ and $G(y)$ are uniformly distributed with support $[0, 1]$*

Given the existence of an equilibrium, we can characterize the steady state number of single men and women using equations (1), (2), (6),(8) , (9) and (10). Therefore the number of people that marry at the young age is

$$\begin{aligned} h_1 &= m_1\phi_1 \\ \upsilon_1 &= w_1\gamma_1 \end{aligned}$$

so the number of remaining (old) singles in the market will be

$$m_2 = m_1 - h_1 = m_1(1-\phi_1) \quad (14)$$
$$w_2 = w_1 - \upsilon_1 = w_1(1-\gamma_1) \quad (15)$$

In the same way, given the probabilities of marrying for old people are

$$\begin{aligned} \phi_2 &= \left(\alpha_2^{young} + \alpha^{old}\right) \text{ for men and} \\ \gamma_2 &= \left(\lambda_2^{young} + \lambda^{old}\right) \text{ for women} \end{aligned}$$

the number of people who marry when old are

$$\begin{aligned} h_2 &= m_2\phi_2 = m_1(1-\phi_1)\phi_2 \\ \upsilon_2 &= w_2\gamma_2 = w_1(1-\gamma_1)\gamma_2 \end{aligned}$$

2.5 Example: Uniform Distribution

In this section we will solve the model assuming that the distribution of men and women of ages 1 or 2 is uniform with support $[0,1]$. That is:

$F(x) = G(y) \sim U[0,1]$

With this specification, the unique equilibrium is:

$$R^m = \frac{1}{k(1+\beta)\left(4-pq\alpha\beta^2\lambda\right)}\alpha\beta\left[2+p\left(2(k-1)-\beta\lambda\right)\right] \quad (11')$$
$$R^{mold} = \frac{1}{\left(4-pq\alpha\beta^2\lambda\right)}\alpha\beta\left[2+p\left(2(k-1)-\beta\lambda\right)\right]$$
$$R^w = \frac{1}{k(1+\beta)\left(4-pq\alpha\beta^2\lambda\right)}\lambda\beta\left[2-q\alpha\beta\left(1+p(k-1)\right)\right] \quad (12')$$
$$R^{wold} = \frac{1}{k\left(4-pq\alpha\beta^2\lambda\right)}\lambda\beta\left[2-q\alpha\beta\left(1+p(k-1)\right)\right]$$

Equations (14) and (15) can only be solved numerically in order to find the steady state equilibrium of the model. Figure 1 show the equilibrium values of selected variables as a function of $k \in [1, 2]$, considering the following values for the parameters:

$\theta = 0.5 \quad \beta = 0.9 \quad \eta = 0.9 \quad m_1 = w_1 = 100$

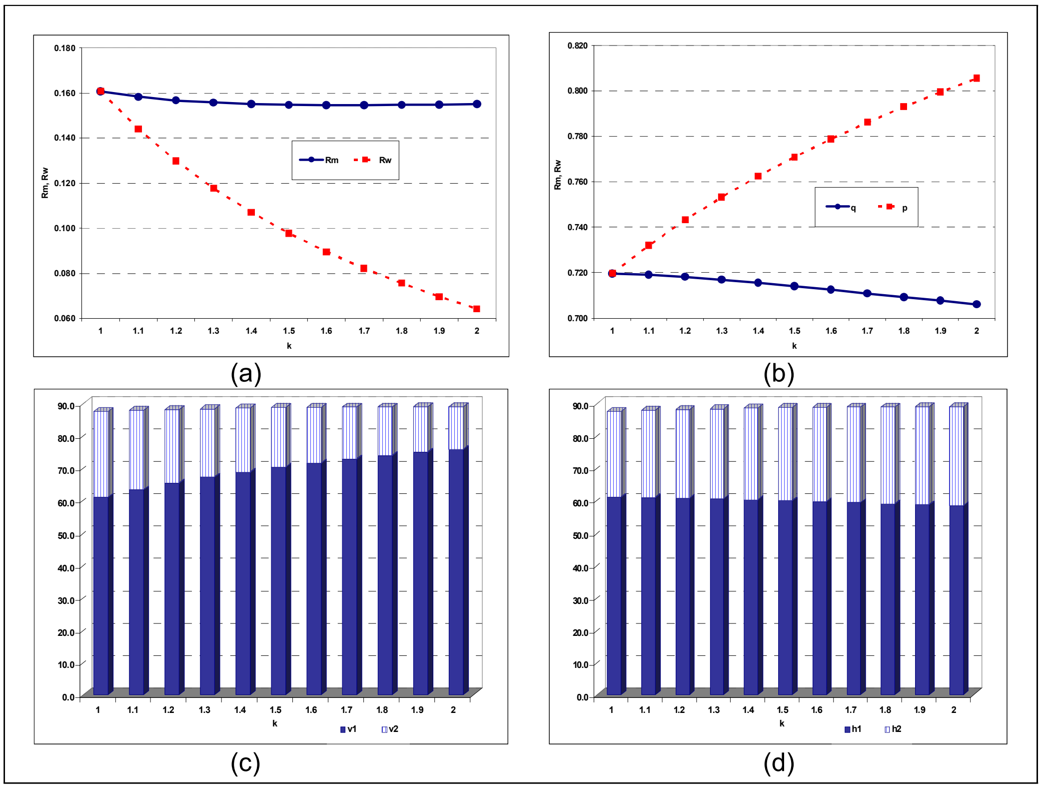


Figure 1: Equilibrium of the 2 Period Model for Different Values of k. Top: (a) Reservation Values, (b) Fraction of Young Men and Women, Bottom: Number of Women and Men Married at Each Age (c) Women, (d) Men

In the top of Figure 1, Panel (a) show the reservation values of men and women and Panel (b) the fraction of young people over the total of single men and women (q and p). Panels (c) and (d) in the bottom show the age distribution of marrying women (v_1 and v_2) and men (h_1 and h_2), respectively. As can be observed, and starting from the point where the problem for men and women is symmetric ($k = 1$), the higher the increase in the utility of marriage for having children, the lower the reservation values for women, and the higher the number of women who marry young. Since women tend to marry younger, the fraction of young women over the total of single ladies is increasing with k, On the other hand, men tend to behave contrarily as women but the patterns seem to be relatively more stable. Therefore, the predictions of this simple 2 period model are the following:

- As long of having children matters ($k > 1$), women tend to marry younger and men older. Therefore, the age difference in marriage tend to increase with higher values of k.

- The average single woman in the marriage market tend to be younger than the average single man. That is, for a given man is more likely to meet a young woman than the contrary for a given woman.

- As their reservation values decrease with age, the marriage quality for women also tend to decrease.

In other words, a the greater importance of children in marriage tend to increase age difference, make women marry younger and with relatively lower quality mates than in the case for men.

3 A Generalized Model

In this section we will extend the simple two period model to a more general finite horizon model, that we will solve numerically. The following are the modified assumptions:

Both men and women live T periods, and differ in the fertility horizon. Women are fertile for L periods, men are fertile for N periods, with $L \leq N \leq T$.

As in the two period model above, women will meet at most one man per period and vice versa. The probability of meeting is determined by equations (1) and (2).

As before, an exogenous flow of single young people of age 1, w_1 women and m_1 men enter the market each period and the ones who have not matched in the previous period will remain in the market. Therefore, the total number of single women and men will be the sum of the stock of bachelorettes and bachelors ages i and j respectively, $i, j \in [1, T]$

$$M = \sum_{j=1}^{T} m_j$$
$$W = \sum_{i=1}^{T} w_i$$

Therefore, the fraction of women and men of ages i, j will be

$$p_i = \frac{w_i}{W} \tag{16}$$
$$q_j = \frac{mj}{M} \tag{17}$$

where which will be endogenously determined.

We redefine the extra utility for having children for a man of age j as k_j^m where

$$\begin{aligned} k_j^m &= k \text{ if } j \leq N \\ &= 1 \text{ if } N < j \leq T \end{aligned} \tag{18}$$

Similarly, for a woman of age i,

$$\begin{aligned} k_i^w &= k \text{ if } i \leq L \\ &= 1 \text{ if } L < i \leq T \end{aligned} \tag{19}$$

where $k > 1$ and subject to the condition established in (3).

For simplicity reasons, we will make the assumption that people expect to enjoy the gains of marriage for the rest of their lives no matter the potential spouse's age. Although strong, this assumption will not have significant effects in the results and will help to keep things manageable[9].

The characteristics of the utility functions for men and women remain as the ones modeled in the previous section. Even though we assume ex post heterogeneity, ex ante everybody is equally attractive except for their potential fertility. Therefore, a fertile woman will be ex ante indifferent between any fertile bachelor, who will be preferred over infertile man. Infertile women will be ex ante indifferent over the age of their potential mate. The same considerations hold in the case of fertile and infertile men in the market.

[9]Relaxing this assumption would imply that people at each age would set different reservation values for each age of the other sex. Concretely, in a 15 period model, this would increase the number of equations from 60 to 480.

3.1 The Male's Optimization Problem

Each period a given man of age j will meet a woman of age i with probability αp_i (by equations (2) and (16)). Since men are fertile for N periods and women for L periods, the probability of being accepted for that woman will depend on the age of both of the man and the woman he meet. Suppose the woman is "young" , that is, she is still fertile (so her age, i, is less or equal than L). Since she is still capable of having children, she will not be indifferent between an old or a young (fertile) man, so she will set a higher reservation value for infertile men, R_i^{wold}. So, if the man meet a young woman $(i \leq L)$,the probability that a man receives an offer from a woman of age i will depend in the age of the man, j , as follows.

3.1.1 Offers from Young Women ($i \leq L$)

$$\begin{aligned} \alpha_j^i &= \alpha p_i(1 - G(R_i^w) \quad \text{if } \ j \leq N \\ &= \alpha p_i(1 - G(R_i^{wold}) \quad \text{if } \ j > N \end{aligned}$$

Since the man will set the same reservation value for all fertile women, it will be useful to define the probability of an offer from any young woman to a man of age j, α_j^{young}

$$\alpha_j^{young} = \sum_{i=1}^{L} \alpha_j^i \tag{20}$$

3.1.2 Offers from Infertile Women($i > L$)

$$\alpha^i = \alpha p_i(1 - G(R_i^w) \qquad \forall j$$

In the same way, we define the probability of an offer from an infertile woman to a man of any age j, α^{old}

$$\alpha^{old} = \sum_{i=L+1}^{T} \alpha^i, \forall j \tag{21}$$

Infertile men will be indifferent about woman's fertility, then they will set the same reservation value for all women. So,

$$R_j^{mold} = R_j^m \ \text{ if } \ j > N \tag{22}$$

3.1.3 Expected Utility of Marrying at age j

Suppose a man of age j who is considering the possibility of getting married at the current age. Given the probabilities of offers from young and old women at the reservation values that the man in question set for each group of ladies, the expected utility of marrying will be the discounted sum of flows of expected payoffs of marriage through the remaining of his life (following the assumption above). Therefore,

$$U_j^m = \sum_{s=j}^{T} \beta^{s-j} \left(\alpha_j^{young} k_j^m \int_{R_j^m}^{\overline{x}} x f(x) dx + \alpha^{old} \int_{R_j^{mold}}^{\overline{x}} x f(x) dx \right) \tag{23}$$

3.1.4 Matching Probabilities for Men

Using equations (6) and (22), we define the hazard rate for a man of age j to match in period t, as follows

$$\phi_t = \begin{cases} \left(\alpha_j^{young} \left(1 - F\left(R_j^m\right)\right) + \alpha_j^{old} \left(1 - F\left(R_j^{mold}\right)\right)\right) & \text{if } j \leq N \\ \left(\alpha_j^{young} + \alpha_j^{old}\right) \left(1 - F\left(R_j^m\right)\right) & \text{otherwise} \end{cases} \tag{24}$$

and then we can define the unconditional probability that a man matches at age j as

$$\Phi_j = \phi_j \prod_{t=1}^{j-1} (1 - \phi_t) \tag{25}$$

3.1.5 Objective Function for Men

Given Equations (23) and (24), the problem faced for any man at a given period t is the following

$$\underset{R_j^m,\, R_j^{mold}}{Max} \sum_{j=t}^{T} \beta^{j-t} U_j^m \prod_{s=t+1}^{j} \left(1 - \phi_{s-1}\right)$$

Making $j = t$, the respective Bellmann Equation is

$$V_t^m = \underset{R_t^m,\, R_t^{mold}}{Max} U_t^m + (1 - \phi_t) \beta V_{t+1}^m \tag{26}$$

3.1.6 Stock of Single Males in the Market

Similarly than in the 2 period Model, we can define the total number of single men as follows

$$m_j = m_{j-1}\left(1 - \phi_{j-1}\right) \tag{27}$$

3.2 The Female's Optimization Problem

Each period a given woman of age i will meet a man of age j with probability λq_j (by equations(1) and (17)). Since women are fertile for L periods and men for N periods, the probability of being accepted for that bachelor will depend on the age of both of the man and the woman she meet. Suppose the man is "young" ($j \leq N$). Similarly as the case stated above, so the man will set a higher reservation value for infertile women, R_j^{mold}. So, if the woman meet a young gentleman ($j \leq N$) ,the probability that the lady receives an offer from a man of age j will depend in the age of the woman, i , as follows.

3.2.1 Offers from Young Men ($j \leq N$)

$$\begin{aligned}\lambda_i^j &= \lambda q_j(1 - F(R_j^m) \quad \text{if } \; i \leq L \\ &= \lambda q_j(1 - F(R_j^{mold}) \;\; \text{if } \; i > L\end{aligned}$$

Since the woman will set the same reservation value for all fertile men, we define the probability that a woman of age i receives an offer from a young bachelor , λ_i^{young}.

$$\lambda_i^{young} = \sum_{j=1}^{N} \lambda_i^j \tag{28}$$

3.2.2 Offers from Infertile Men ($j > N$)

Since old men are indifferent about woman's age, they will set a unique reservation value for all ladies. Then,

$$\lambda^j = \lambda q_j(1 - F(R_j^m) \;\; \forall i$$

Therefore, the probability of an offer from an infertile man to a woman of any age i, λ^{old} will be

$$\lambda^{old} = \sum_{j=N+1}^{T} \lambda^{j} \tag{29}$$

Infertile women will be indifferent about man's fertility,so

$$R_i^{wold} = R_i^w \text{ if } i > L \tag{30}$$

3.2.3 Expected Utility of Marrying at age i

As is the case of a man, the expected utility of marrying at the current period for a woman of age i will be

$$U_i^w = \sum_{s=i}^{T} \beta^{s-i} \left(\lambda_i^{young} k_i^w \int_{R_i^w}^{\overline{y}} yg(y)dy + \lambda^{old} \int_{R_i^{wold}}^{\overline{y}} yg(y)dy \right) \tag{31}$$

3.2.4 Matching Probabilities for Women

The hazard rate that a bachelorette of age i matches in period t, as follows

$$\begin{aligned} \gamma_t &= \left(\lambda_i^{young} \left(1 - G\left(R_i^w\right)\right) + \lambda_i^{old} \left(1 - G\left(R_i^{wold}\right)\right)\right) \text{ if } i \leq L \quad (32) \\ & \left(\lambda_i^{young} + \lambda_i^{old}\right) \left(1 - G\left(R_i^w\right)\right) \qquad \text{otherwise} \end{aligned}$$

and the unconditional probability that a woman matches at age i

$$\Gamma_i = \gamma_i \prod_{t=1}^{i-1} (1 - \gamma_t) \tag{33}$$

3.2.5 Objective Function for Women

Given Equations (31) and (32) , the problem faced for a single lady at a given period t is the following

$$\underset{R_i^w,\, R_i^{wold}}{Max} \sum_{i=t}^{T} \beta^{j-t} U_i^w \prod_{s=t+1}^{i} \left(1 - \gamma_{s-1}\right)$$

Making $i = t$, the Bellmann Equation for the female problem is

$$V_t^w = \underset{R_t^w,\, R_t^{wold}}{Max} U_t^w + (1 - \gamma_t) \beta V_{t+1}^w \tag{34}$$

3.2.6 Number of Single Females of age i

Similarly than in the previous case, we can define the number of bachelorettes of age i as follows

$$w_i = w_{i-1}\left(1 - \gamma_{i-1}\right) \tag{35}$$

3.3 Solution

Similarly to the 2 Period model above,by the linearity of the problem we have that

$$\begin{aligned} R_j^{mold} &= kR_j^m \text{ if } \; j \leq N \\ R_i^{wold} &= kR_i^w \text{ if } \; i \leq L \end{aligned}$$

Now we can solve numerically the system formed by Equations (26), (27), (34) and (35). The distribution functions $F(x)$ and $G(y)$ are both uniform with support $[0, 1]$ and the values given to the parameters will be the following:

$T = 15$ $\quad N = 12$ $\quad L = 7$ $\quad \eta = 0.9$
$k = 1.25$ $\quad \beta = 0.9$ $\quad \theta = 0.5$ $\quad m_1 = w_1 = 100$

Figure 2 shows the reservation values for men and women with respect to young prospective spouses at different ages. Interpreting the figure will help us to summarize several of the predictions of the model about marriage behavior

- Since women are fertile for a shorter period of time, they lower their reservation values faster than men, reaching a minimum at the end of fertility (period 7 in this example). After they became infertile, they raise their minimum acceptable level for a mate, and then start decreasing their reservation value until death (point where the minimum level of zero is reached). This behavior can be interpreted as follows: As the end of her fertile period is getting closer, assuming the woman is single and without children, she is willing to accept a relatively lower quality match in order to have children. Once they become infertile, the only point of getting married is the quality of the match (they want to marry somebody they really like) and this is the meaning of the increase in the reservation value after fertility. Since celibacy has zero utility in this model, the decrease in the minimum value through the time of death make sense as an "anything is better than loneliness" kind of behavior.

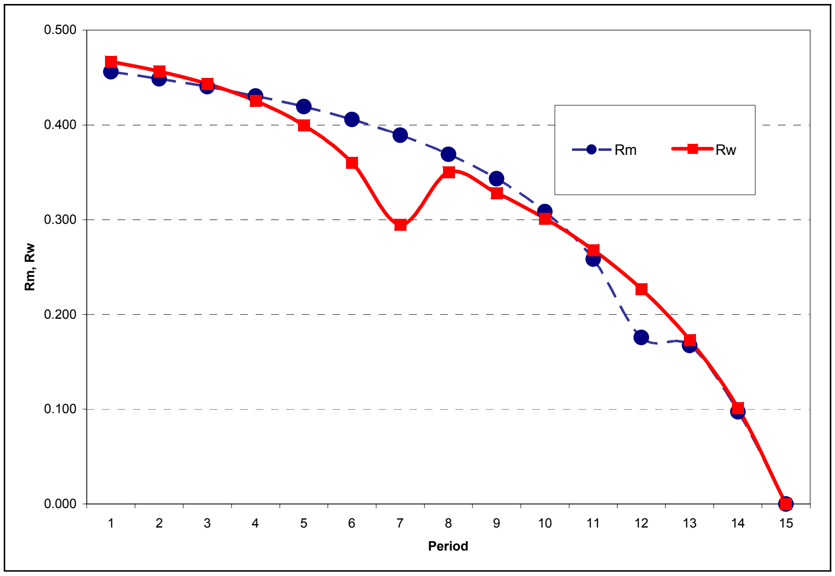


Figure 2: Reservation Values for Men and Women

- The behavior of men is similar than the one of women, but the timing is different. Since the fertility horizon is longer for men, so the time to searching. Men decrease their reservation values slower than women do, until reaching a minimum at period 12 (they became infertile). Then, similarly as what happen with women, they raise their reservation value to finally decrease until death. Not surprisingly, the reservation value for infertile men is the same as the one for infertile women.

- Very young women set a higher reservation value than men of their same age. The reason for this is, on the one hand, because the relatively shorter fertility horizon should not be an issue for teenage girls. On the other hand, since the stock of fertile men outnumber the one fertile women, a young woman faces relatively better market conditions than a man of her age. Concretely in this example, the probability of an offer from a young bachelor to a young woman (see equation (28)) is 0.52, while the same probability for young bachelors (by (20)) is only 0.45.

- When men and women are very old (both are infertile), reservation values are the same, simply because fertility is just not an issue at all.

- Another interesting point is what happen at an age where men are still fertile and women not (in this example between periods 7 and 12). As stated above, after menopause women increase their reservation value for a husband since the only utility they will receive from a spouse will be the quality of the match. However, men of the same age are still fertile and somewhat desperate to find a spouse and decrease their reservation value rapidly. The example shows the interesting case that infertile women (at period 11) become choosier than (still fertile) men.

Here we can explore in more detail the behavior of men and women and compare this result with the ones established in the simple model of the previous section. Remember that one of the principal results of the two period model was that women set (in period 1) a reservation value lower than the one for men. Here apparently the result is different because in the early periods women set a higher reservation value. However, if we compare the first period of the model of the previous section with the reservation values set by women during their whole fertile period (1 through 7), the results are broadly consistent.

As women decrease their reservation values, they are increasing the probability of matching[10]. After they became infertile, this probability decrease dramatically because not only because they raise their reservation values but also become less attractive for men. In the case for men, the process is similar, but slower as the fertility horizon is longer. This behavior determines the steady state stock of single men and women, as shown by Figure 3 along with the sex ratio at each age.

By assumption, the stock of single men and women in the first period is equal and therefore the sex ratio is 1 at the early age. In subsequent periods, men start increasingly outnumbering women (the sex ratio at period 8 reaches a maximum of 1.37). After women became infertile, the sex ratio decreases to find a minimum at age 13 (0.93), just after men also became infertile. A comparison between Figures 2 and 3 will give us the whole picture. Women the leave the market (marry) relatively younger than men, and try to do so before becoming infertile (period 8). Specifically, the average time of marriage for women is in 3.33 periods, compared with 3.52 periods in the

[10] In this case the increase of the hazard rate will come directly from the decrease in reservation values since we made the simplyfing assumption that men are indifferent about women's age, as long they are fertile (and vice versa).

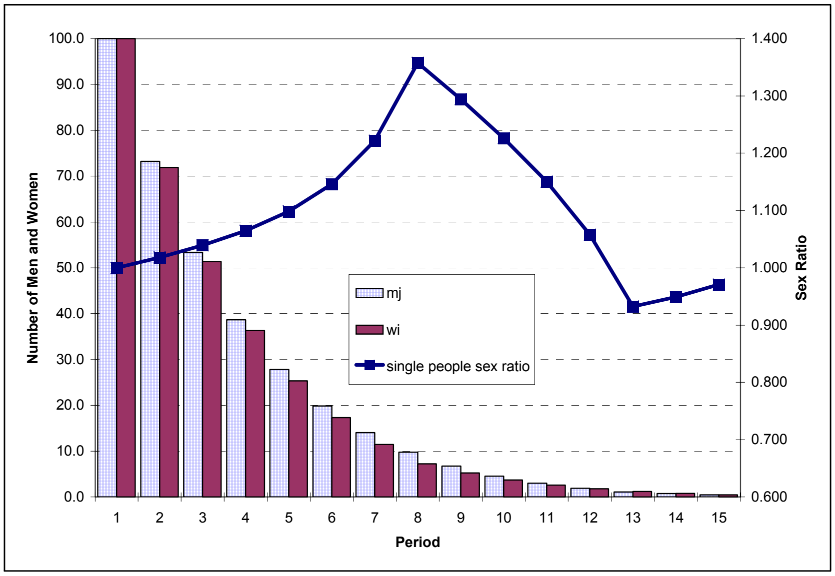


Figure 3: Stocks and Sex Ratio of Single Men and Women at each Period

case of men[11].

It would be worthy to point out that, even though the overall sex ratio is greater than 1 (1.08 with the above parameters), and fertile men outnumber fertile women even in a larger proportion, that does not mean that bachelors are more likely to die single than women. In the steady state results, and inflow of young women and men each of size 100 is compensated by 99.7 new couples and only 0.27 who die single for each of the sexes. Moreover, 7 of the women who marry do that after their fertile period is over. The older bachelors who remain in the market outnumbering bachelorettes of their age eventually marry younger women from the ones who enter the market each period. In other words, even though in any period there are more men than

[11] This result is also found in Siow (1998) and in Bergstrom and Bagnoli (1993). The latter present a model with incomplete information where men who expect to succeeed, delay marriage until they are able to give a signal that allows them to attract a more desirable females. The equilibrium of this model is that, while all women marry early in life, the most desiderable females marry successful older males and the less desiderable females marry young males who do not expect to prosper.

Even though Siow (1998) also assumes differential fertility, that assumption is not sufficient to show the age difference, unlike in this model.

women looking for a mate, the market clears dynamically because men have more opportunities during their lifetime to meet a spouse and start a family.[12]

4 Comparison with Census Data

In this section we try to calibrate the numerical results of the model with census data. For that reason it will be convenient to find a way to "transform" the periods of the model in age periods. We will assume that each period has a length of 4 years starting at age 16. Therefore we can compare with data for people from 16 to 75 years old. As we assume before, women become infertile after period 7 (40-43 years old) and men after period 12 (60-63 years old), and everybody will die at 75.

First we take a look to the 2000 US Census. The sample is constructed from the IPUMS 1% release, exclude people in institutions, and it is limited to people born in the US. Even though the several problem could arise with the data, such as mixing different cohorts, and young mortality, ignored in the model, the comparison is still useful. Figure 4 shows the comparative evolution of the stocks of single women and men at each age group predicted by the model(Top) and in the 2000 US Census (Bottom). For that purpose, the number of men and women of age 16-19 in the Census is normalized to 100. The shaded bars correspond to the model results and the solid ones to the Census data. In both cases, stocks of young singles appear to diminish more rapidly in the data than in the model until age 36-39, then the contrary occurs. One explanation of what happen latter in comes simply that in our model we assume that everybody gets utility from marriage and children. But the important point is that, even the pattern of people who do marry captured by the model is similar to the one shown by the data (women tend to marry faster), the pace appear to be slower that the one in the 2000 Census.

In Figure 5 the comparison is about the age composition of the single population. The top panel shows the results of the model and the one below what comes from the data. In both cases The fraction of very young women (the first 2 periods or from age 16 to 23) is more important in the distribution of single females than in the case for men. The explanation is simple: since women tend to marry faster, the inflow of young single women to the market has more weight in the total number of single females than the inflow of

[12] In Siow (1998), the higher number of fertile men cause the imposibility of finding a match for some of them.

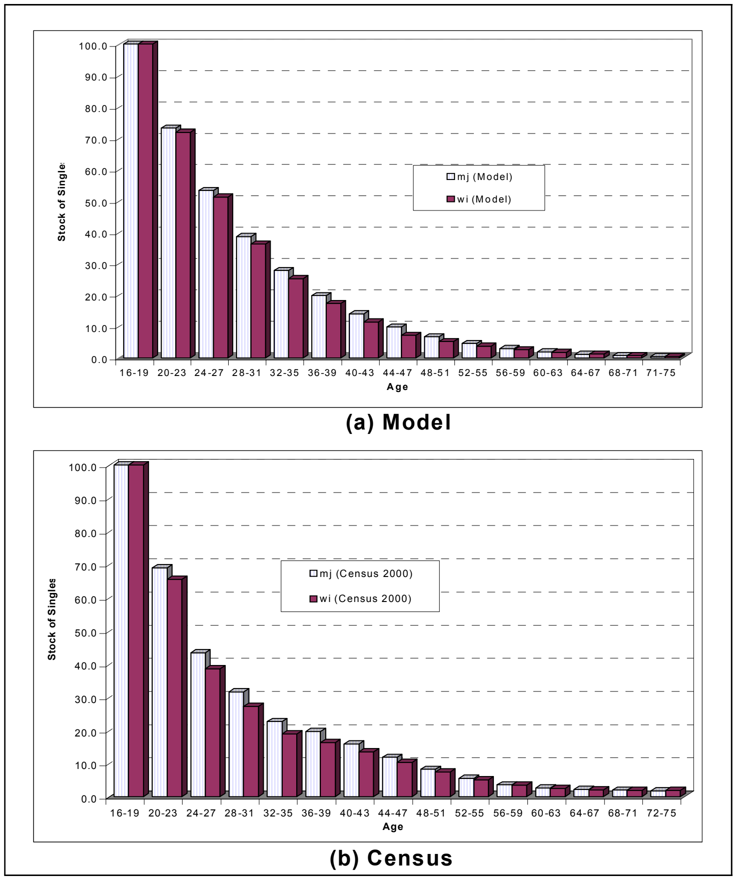


Figure 4: Comparison of Stocks of Single Men and Women by Age. Panel (a) Model. Panel (b) US Census 2000.

young bachelors that need to "share" the market with a larger number of older males. Therefore, the average age of single women should be lower than the average of single men. This result is similar to the one found analytical in the to period model: if having children matters $(k > 1)$, single women tend to be younger than men in average.

Now we can compare what happen with the evolution of the sex ratio for single people by age. In this case, as shown in Figure 6, we compare the model (solid line) results not only with the 2000 census but also with the 1960 and 1980 census. As can be observed in the figure, even though the pattern has the same shape than the one in the model, appear to have changed over time.

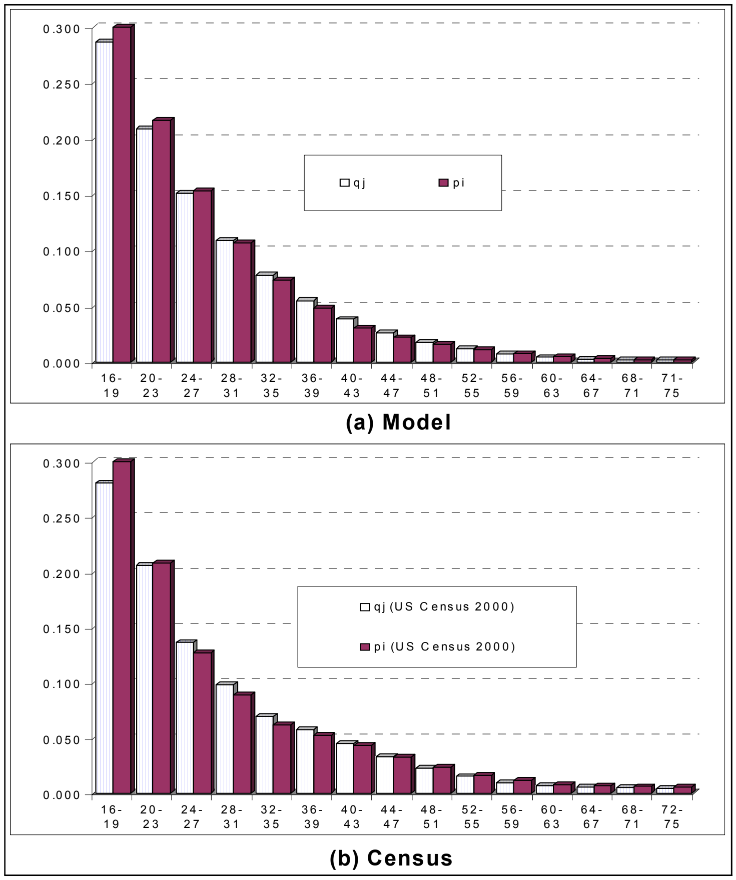


Figure 5: Proportions of Single Men and Women at Each Age Group: (a) Model, (b) US Census 2000

First, the evolution of the sex ratio looks the more "smooth" in 1980 with respect to 1960 and even more in 2000. That is a signal that the difference in age of first marriage is decreasing over time. Second, the "peak" in the graph, that in 1960 and 1980 was 24-27 years old, moved to 36-39 years old in 2000 Census. In other words, the number of women who marry at each age group outnumber the number of marrying men in lower proportion but for a longer period of time. Remember that the model implies that women are getting married at a faster rate than men until they became infertile (47 years old with the parameters chosen).

A more detailed comparison with US Census data is shown by Table 1,

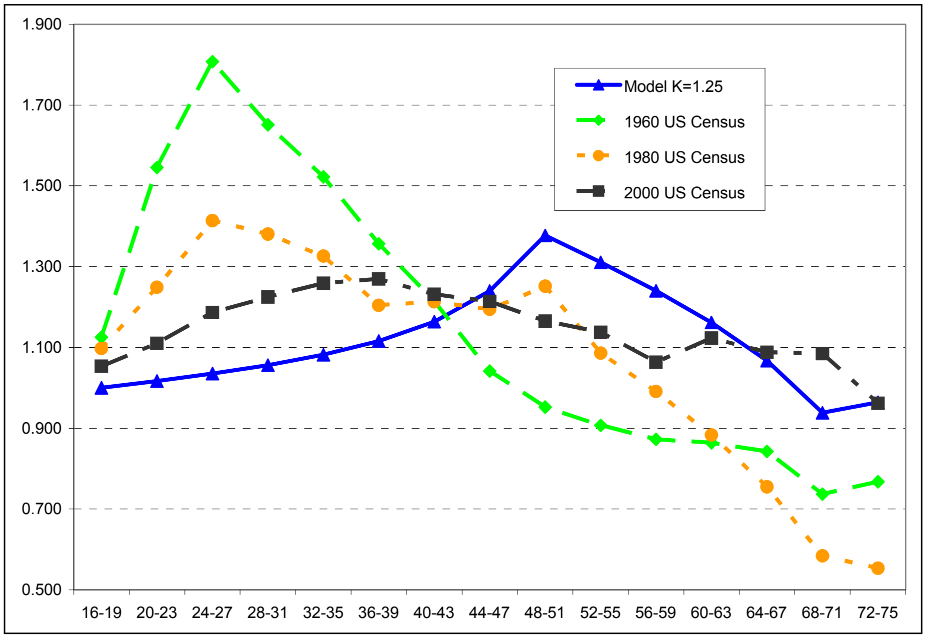


Figure 6: Sex Ratio for Single People in Model and Different US Census

where the first column show the predictions of the model and the 3 following columns show data for the 2000, 1980 and 1960 US Census, respectively[13]. The table shows clearly the tendency of delaying marriage and the decrease of age difference at first marriage in the last 40 years[14]. As can be observed, the 2000 US Census data is which match the model predictions the best. However, the model predicts that people will marry even later (men at age 28 and women at 27.3 years old against 26.8 and 25.1, respectively), and the age difference to be less.

In Figures 7 and 8 the results of the model are compared with the 1990 Census of selected countries. Figure 7 shows the evolution of the sex ratio by age in selected developed countries (France, Sweden and USA) and Figure 8 shows the pattern for Colombia, Kenya Mexico and Vietnam . It easy to tell that the data for developed countries match better with the model

[13]The Age at First marriage in census data in Table 1 corresponds to Median Age at First Marriage.

[14]For an empirical study about the change in marriage patterns in the US in last decades, see Rose (2001).

	Model		2000 Census		1980 Census		1960 Census	
	Men	Women	Men	Women	Men	Women	Men	Women
Age of First Marriage	28.08	27.34	26.80	25.10	24.70	22.00	22.80	20.30
Average Age of Singles	27.20	26.69	27.69	27.31	24.70	25.30	27.45	29.03
Marriage Rate	0.066		0.087		0.106		0.085	
Ever Married (%)	0.697	0.709	0.709	0.765	0.732	0.792	0.808	0.855
Never Married (%)	0.303	0.291	0.291	0.236	0.268	0.208	0.192	0.145
Sex Ratio of Singles	1.08		1.17		1.19		1.22	
Total Sex Ratio	1.00		0.96		0.92		0.92	

Table 1:
Comparison Between Model and US Census Data

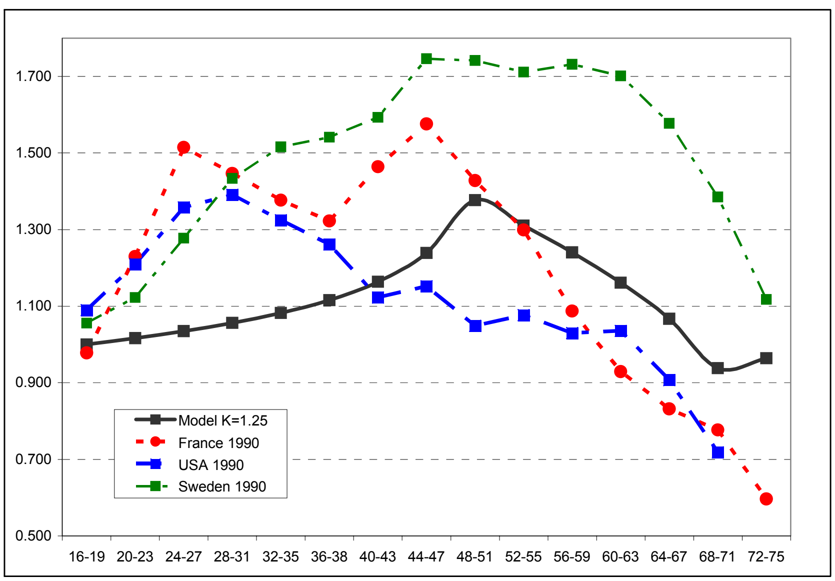


Figure 7: Comparison of Sex Ratio by Age Between Model and France, USA and Sweden 1990 Census. Source: IPUMS International

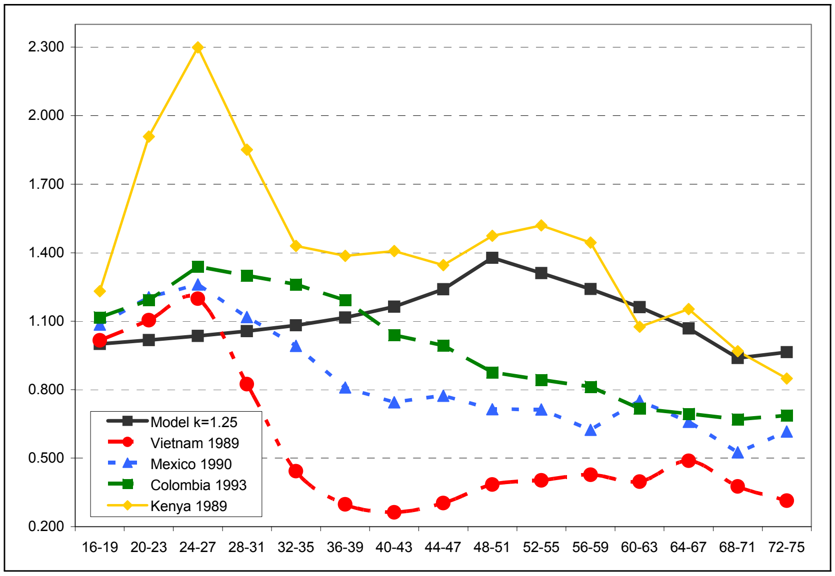
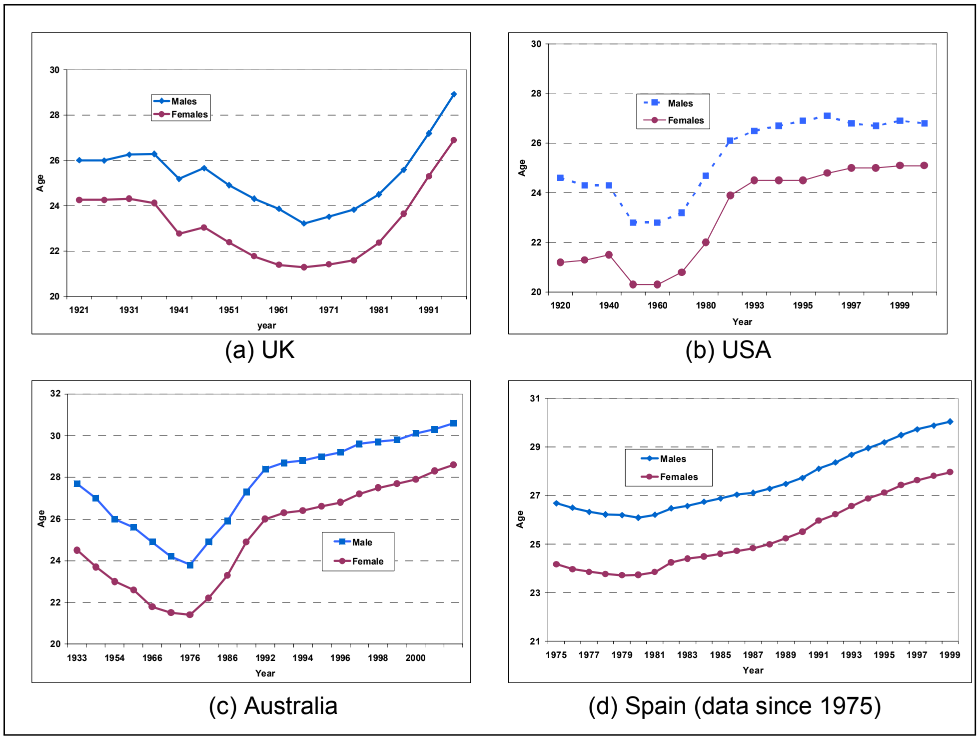


Figure 8: Comparison of Sex Ratio by Age Between Model and Colombia, Kenya, Mexico and Vietnam Census. Source: IPUMS International

that the one for developing countries. Moreover, the sex ratio in France and Sweden "peak" at 43-47 years old, closer to the model than the data for the US. However, for developing countries the pattern is different. In the three countries of the figure the sex ratio reaches a maximum at ages 24-27 and then decreases sharply, following the pattern that is somewhat similar to the one in the US in 1960 in Figure 6 above.

The continous tendency of delaying marriage in developed countries other than the US can be observed in Figure 9, that shows the evolution

In summary, this model does a better job explaining what happen in developed countries than in developing countries, and in recent times than in previous decades. In this model the evolution of the sex ratio with age is entirely determined by the different fertility horizon of men and women. Therefore, all other differences between sexes intentionally excluded in this model, obviously also play a role in the marriage behavior. What this model, driven solely by biology, is not able to explain, is explained by social norms. When we find a peak in the sex ratio at 20-23 years old, as in Figure 8, or the sharply increase in the sex ratio in the early twenties in US in 1960, perhaps we are talking about some "social limit" to the age when women



(a) UK (b) USA

(c) Australia (d) Spain (data since 1975)

Figure 9: Evolution of the Median Age at First Marriage in Selected Countries

should marry. This social norm appear to be more important in societies where traditional marital roles are still well defined.

However, the last few decades observed more similar roles for men and women. For example, in the US, the level of education have become increasingly similar and women's labor participation have increased dramatically in the last 20 years. Moreover, marriage specialization have consequently decreased[15], and traditional roles in marriage are not so common as they were in the past. Even though social norms have changed making that roles of men and women became more and more similar, the fertility horizon differences will persist and that can be an explanation of why women still tend to marry older men.

[15]For a study on the decline in marriage specialization, see Lundberg and Rose (1998)

5 Conclusion

As shown above, asymmetric fertility horizons between men and women alone are sufficient to generate a stylized fact that holds across many societies - on average younger women marry older men. Moreover, the proximity of menopause appear to be also an issue in women behavior. Even though many other factors are important in the decision and timing of marriage, they were not considered in the model above. Even though a richer model that introduce labor participation and education could explain many other stylized facts (and reinforce the results of the model outlined in the previous sections), but that questions are well addressed by the existing literature.

One contribution of this paper is to provide a framework where age of marriage is determined by biological concerns, ignoring potential gains of specialization. In the last few decades, men and women are becoming more alike in their social roles, women labor participation have increased dramatically in many societies and differences in education level tend to disappear in more developed countries. In other words, this is a model of the modern world. For the United States, the 2000 Census more closely resembles the theoretical predictions than the 1980 Census, which fits better than the 1960 Census.

Internationally, while countries with advanced post-industrial demographics (e.g. France, Sweden) have patterns closely related with the one predicted in the model, in developing countries where traditional gender roles are still important diverge more with model predictions.

Even though cultural pattern have change considerably during recent decades, either in Sweden, Kenya or the US, still women tend to marry older men. This paper shows that the only clear difference between men and women, that females have a shorter fertility horizon than men, is able to explain the still important differences in behavior.

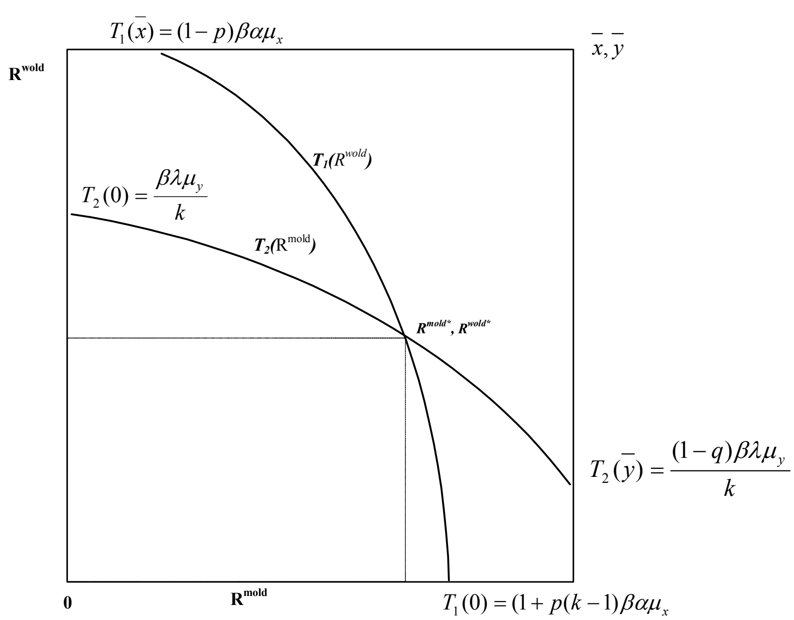
Appendix

Complete Proof of Theorem 1

By Equations (9) and (10), we know that

$$R^m = \frac{R^{mold}}{k(1+\beta)}$$
$$R^w = \frac{R^{wold}}{k(1+\beta)}$$

Therefore, it will be sufficient to show existence and uniqueness for R^{mold} and R^{wold} to show them for R^m and R^w.



Lets start defining the reaction functions

$$R^{mold} = T_1(R^{wold}) = \alpha\beta\left[(1-p)+kp\left(1-G\left(R^{wold}\right)\right)\right]\mu_x \quad \text{(A.1)}$$

$$R^{wold} = T_2(R^{mold}) = \frac{\beta\lambda\left(1-qF\left(R^{mold}\right)\right)\mu_y}{k} \quad \text{(A.2)}$$

By definition, $p \in [0.5, 1]$ and $q \in [0.5, 1]$. Differentiating (36) and (36) we see that T_1 and T_2 are strictly decreasing in R^{wold} and R^{mold} respectively.

$$T_1'(R^{wold}) = -kp\alpha\beta g\left(R^{wold}\right)\mu_x < 0 \quad (36)$$

$$T_2'(R^{mold}) = -\frac{q\beta\lambda f\left(R^{mold}\right)\mu_y}{k} < 0 \quad (37)$$

Now, using the condition in (3), it is easy to show that the intercepts are

$$\begin{aligned}
T_1(0) &= (1+p(k-1))\,\alpha\beta\mu_x < \overline{x} \\
T_2(0) &= \frac{\beta\lambda\mu_y}{k} < \overline{y} \\
T_1(\overline{y}) &= (1-p)\,\alpha\beta\mu_x \geq 0 \\
T_2(\overline{x}) &= \frac{\beta\lambda(1-q)\,\mu_y}{k} \geq 0
\end{aligned}$$

The above conditions, summarized in the figure, rule out any corner solution and at the same time guarantee the existence of at least one interior solution. The next step will be to show that there is a unique equilibrium in the model. We know that

$$\begin{aligned}
R^{mold} &= T_1(R^{wold}) = T_1(T_2\left(R^{mold}\right)) \\
&\equiv H\left(R^{mold}\right)
\end{aligned}$$

where

$$H\left(R^{mold}\right) = \alpha\beta\left[(1-p)+kp\left(1-G\left(T_2\left(R^{mold}\right)\right)\right)\right]\mu_x$$

To ensure uniqueness, we need a fixed point of $H(.)$. Therefore, we need to show that $H\left(R^{mold}\right)$ is a contraction mapping. that is

$$\begin{aligned}
\|H\left(R_1^{mold}\right) - H\left(R_2^{mold}\right)\| &\leq \delta\|R_2^{mold} - R_1^{mold}\| \\
\text{where } 0 &< \delta < 1
\end{aligned}$$

To show that H is a contraction mapping, it is enough to prove that

$$H'\left(R^{mold}\right) \leq \delta < 1 \quad \forall R^{mold} \in [0, \overline{x}]$$

But

$$H'\left(R^{mold}\right) = T_1'(T_2\left(R^{mold}\right)).T_2'\left(R^{mold}\right) \tag{38}$$

Using (1), (2) (**??**), (36), $(37)$and the assumption that $m_1 = w_1$, substituting in (38) and manipulating, we get

$$H'\left(R^{mold}\right) = \left(\eta\left(\frac{M}{W}\right)^{\theta}\right)^{2} \beta^2 \left(\frac{m_1}{M}\right)^2 f\left(R^{mold}\right) g\left(T_2\left(R^{mold}\right)\right) \mu_y \mu_x$$

By assumption, we know that $\left(\eta\left(\frac{M}{W}\right)^{\theta}\right) \leq 1$, that $\frac{m_1}{M} \leq 1$ and that $\beta < 1$. Hence, it will be sufficient for $H'\left(R^{mold}\right)$ to be a contraction mapping if we have

$$f\left(R^{mold}\right) g\left(T_2\left(R^{mold}\right)\right) \leq C$$

where

$$C = \frac{1}{\mu_y \mu_x} > 1$$

Therefore, we have that

$$H'\left(R^{mold}\right) \leq \mu_y \mu_x < 1$$

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