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ECONOMIES OF SCALE AND DEGREE OF CAPACITY UTILIZATION. EVIDENCE FROM RETAIL BANKS IN ARGENTINA

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Economies of scale and degree of capacity utilization.

Evidence from retail banks in Argentina

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Abstract The permanent income/transitory income distinction from consumption functions can be applied to cost functions. Transitory deviations of actual output from potential output, i.e. variations in capacity utilization, are relevant for the pattern of U-shaped average costs found in econometric studies. Data from retail banks in Argentina are used to illustrate this issue, with the number of branches as a proxy for potential output, and product per branch as a proxy for the utilization level. Economies of scale at the plant level can be reinterpreted as an indication of excess capacity in the banking industry.

JEL classification codes: D24, G2.

Key words: short-run costs, potential output, capacity utilization.

Economies of scale and degree of capacity utilization. Evidence from retail banks in Argentina^{*}

1. Introduction

Cross-section data has been widely used to estimate industry cost functions. In the specific case of the banking industry, studies have usually led to reject log cost functions in favor of more flexible functional forms such as translog functions and nonparametric estimates (cf. Humphrey, 1990, and McAllister and McManus, 1993).

This paper shows that even if the data exhibit U-shaped average costs, the log cost function might be the correct specification: a Cobb-Douglas constant returns to scale technology with fixed factors exhibits exactly that behavior.

The econometric issue is similar to the problem Friedman faced when estimating the consumption function. In the consumption function, high income levels can signal either high permanent income or high transitory income. Likewise, in the cost function high output can indicate either high permanent output or high transitory output. This leads to a bias against the acceptance of a constant returns to scale technology, since costs can first increase less than proportionally with output, due to excess

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capacity, and then they can increase more than proportionately, due to fixed factors that cannot be adjusted optimally in the short run. One way out of this problem is to introduce a measure of permanent or potential output.

The structure of the paper is as follows. Section Two reviews time-series data on average operating costs in private banks in Argentina, to suggest that demand fluctuations have been a predominant factor in recent variations in productivity. In Section Three, the deviations of actual output from potential output are cast as changes in the degree of capacity utilization. fluctuations in the degree of capacity utilization are then linked to the difference between short- and long-run costs. In Section Four, we illustrate the distinction between potential output and degree of capacity utilization with data from retail banks, which typically operate with many branches. We treat them as multi-plant firms whose level of potential output is given by the number of plants, while output per plant is a proxy for the degree of capacity utilization. The cross-section of retail banks shows that U-shaped cost curves can be explained by capacity utilization effects. Furthermore, the fact that the banking industry operates on average with excess capacity can explain why econometric studies apparently show economies of scale at the plant level, but not at the firm level. Section Five concludes.

2. Fluctuations of productivity in banks in Argentina

A stylized fact of business cycles is that productivity is

procyclical (e.g. Zarnowitz 1985). This is also true of seasonal cycles, determined mainly by demand fluctuations, so this evidence in particular points to the presence of labor hoarding and excess capacity more than to an interpretation based on technological shocks (Barsky and Miron 1989).

We focus specifically on the issue of productivity fluctuations in the banking sector, using data from private banks in Argentina. In high-inflation countries, banks are particularly exposed to macroeconomic instability, since booms are often closely related to price stabilizations that encourage the remonetization of the economy, while recessions are associated to devaluations that provoke a flight from domestic assets, as López, Streb et al. (1993) document for Argentina. This pattern fits into the exchange rate-based stabilizations described by Kiguel and Liviatan (1992).

If productivity in financial intermediation is measured as the ratio between operating costs and deposits, over the last few years productivity in private banks in Argentina has been very influenced by cyclical fluctuations. Figure 1 shows that there is an inverse relationship between productivity and the degree of monetization of the economy, defined as the ratio of M3* to GDP (both sight and time deposits, in pesos and in dollars, are included in M3*).

The ratio of operating costs to deposits in private banks stood on average at 5.1% per quarter in 1988, reaching a peak of 14.6% in 1990. This ratio has recovered since early 1991, when the Convertibility Plan pegged the peso to the dollar. It was down to 4.5% per quarter in 1993, three times the U.S. level, and it

Productivity and degree of monetization

Figure 1

continued to fall in 1994.1

The ups and downs of productivity do not suggest that technological change was the dominant force in the recent period, but rather that firms could not readjust their potential scale of production quickly in reaction to the sharp changes in the degree of monetization, which thus entailed large jumps in the degree of capacity utilization. The recovery of deposits since 1990 took place with a network of branches of private banks that did not vary much, with employment levels that reacted with a certain lag, allowing a recovery in the degree of capacity utilization.²

Though we do not analyze the time-series data econometrically, in Section Four we use cross-section data to attempt to prove that differences in productivity across banks have a component that is precisely due to differences in the level of capacity utilization.

¹In the U.S., the ratio of operating costs to deposits hovered around 1.5% per quarter over the 1985-1993 period (noninterest expenses as a percentage of deposits of all banks, Federal Reserve Board, Federal Reserve Bulletin, June 1994).

These comparisons are in monetary terms. According to Mckinsey Global Institute (1994), in 1992 the productivity gap was even larger in physical terms: retail banking in Argentina had 1/5 of U.S. labor productivity (a larger gap than in other sectors, were productivity was around 1/3 of the U.S.). However, this overstates the technological gap in banking, because it reflects in part a low level of capacity utilization, as Section Four attempts to show.

²We are not ruling out technological progress altogether (though it seems to have been more significant in manufacturing sectors affected by foreign competition). Similarly, Vicens and Rivas (1994) explain the rapid decrease of average operating costs between 1991 and 1994 mainly through the recovery in the amount of deposits managed by the banks, stressing the increase in average deposits per account.

3. Short-run and long-run costs

Here we develop the implications of fluctuations in the degree of capacity utilization. We interpret the issue of capacity utilization as a distinction between actual and potential output, that causes a divergence between short-run and long-run costs. If this distinction is ignored, there can be an errors-in-variables problem in econometric estimates of the cost function.

3.1. Permanent and transitory components of output and costs

It is helpful to think of the issue of capacity utilization in terms of permanent and transitory components of output. Following Friedman, current output is the sum of permanent and transitory components, $Y=Y_p+Y_t$. Actual output Y can depart from the permanent or potential level of output Y_p in the short-run due to unexpectedly high or low demand. These deviations will imply that capacity utilization is above or below the "normal" level.

Similarly, current costs C can be broken down into permanent and transitory components. Potential output Y_p will determine permanent or long-run costs C_p . The transitory cost component C_t captures the deviations of short-run costs from long-run costs, due to differences between current output and potential output.

$$C(Y_{p}, Y_{t}) = C_{p}(Y_{p}) + C_{t}(Y_{p}, Y_{t})$$
(1)

This distinction between transitory and permanent components of output is key to our interpretation, because a technology that possesses constant returns to scale can exhibit U-shaped average costs when estimated econometrically, if current output is used

instead of potential output as the explanatory variable.

3.2. Short- and long-run Cobb-Douglas cost functions

We study short- and long-run costs in the specific setting of a generalized Cobb-Douglas technology, as in e.g. Varian (1992), and relate it to our distinction between permanent and transitory cost components.

Let the production function be $Y=K^{\alpha}L^{\beta}$, where Y stands for output, K for capital and L for labor. Operating costs C are given by the sum of capital and labor costs, $C=W_1K+W_2L$.

In the long-run, both outputs can be adjusted optimally. Solving the problem of cost minimization, the cost function Cp that corresponds to potential output is given by

Long-run:
$$C_p(Y_p, W_1, W_2) = \frac{\alpha + \beta}{\alpha} \left(\frac{\alpha}{\beta} \frac{W_2}{W_1}\right)^{\frac{\beta}{\alpha + \beta}} W_1 Y_p^{\frac{1}{\alpha + \beta}}$$
 (2)

Taking logs, the returns to scale are constant when $\alpha+\beta=1$, increasing when $\alpha+\beta>1$ and decreasing when $\alpha+\beta<1$.

$$\frac{\partial \ln C_p}{\partial \ln Y_p} = \frac{1}{\alpha + \beta} \tag{3}$$

Let the short-run cost function C be restricted by a fixed capital stock.

Short-run:
$$C(Y, W_1, W_2, K) = W_1 K + W_2 (\frac{Y}{K^{\alpha}})^{\frac{1}{\beta}}$$
 (4)

We interpret the ratio U=Y/K as a measure of capacity utilization. There is excess capacity when U is below $U^*=(Y/K)^*$

 $=(eta/lpha~W_1/W_2)^{eta/(lpha+eta)}~Y_p^{(lpha+eta-1)/(lpha+eta)},$ and overused capacity when it is above this point. This is analogous to the idea in macro of variations in the degree of capacity utilization around full-employment, due to the fluctuation of actual output around potential output.

Taking logs, the short run elasticity is increasing in output, or equivalently in the degree of capacity utilization, which reflects the fact that marginal costs are increasing.

$$\frac{\partial \ln C}{\partial \ln Y} = \frac{1}{\beta + \alpha \left(\frac{U^*}{U}\right)^{\frac{1}{\beta}}}, \qquad \frac{\partial^2 \ln C}{\partial (\ln Y)^2} = \left(\frac{\partial \ln C}{\partial \ln Y}\right)^2 \frac{\alpha}{\beta} \left(\frac{U^*}{U}\right)^{\frac{2}{\beta}} > 0$$
 (5)

The log of the short-run cost function can be approximated by a second-order Taylor expansion in $\ln U/U^*$, yielding a translog function similar to that often found in the applied literature.

$$Ln C = \ln C_p + \ln (C/C_p) = \ln C_p + \ln (1 + C_t/C_p)$$

$$= \ln \left(\frac{\alpha + \beta}{\alpha} \left(\frac{\alpha}{\beta} \frac{W_2}{W_1}\right)^{\frac{\beta}{\alpha + \beta}} W_1 Y_p^{\frac{1}{\alpha + \beta}}\right) + \ln \left(\frac{\alpha}{\alpha + \beta} + \frac{\beta}{\alpha + \beta} \left(\frac{U}{U^*}\right)^{1/\beta}\right) \qquad (6)$$

$$\approx \ln \left(\frac{\alpha + \beta}{\alpha} \left(\frac{\alpha}{\beta} \frac{W_2}{W_1}\right)^{\frac{\beta}{\alpha + \beta}} W_1\right) + \frac{1}{\alpha + \beta} \ln Y_p + \frac{1}{\alpha + \beta} \ln \frac{U}{U^*} + \frac{\alpha}{\beta (\alpha + \beta)^2} \frac{(\ln \frac{U}{U^*})^2}{2}$$

The cost function can thus be decomposed into the permanent components, terms one and two, and the transitory components, terms three and four. The transitory components depend on the deviation of actual output from potential output, using the fact that $U/U^*\equiv (Y/K^*)\,/\,(Y_p/K^*)=Y/Y_p=1+Y_t/Y_p.$

Figure 2 graphs the log of average costs, ln C/Y, when $\alpha+\beta=1$ and $\alpha=0.4$. This function exhibits flat average costs in the long run and U-shaped average costs in the short run. Average costs are

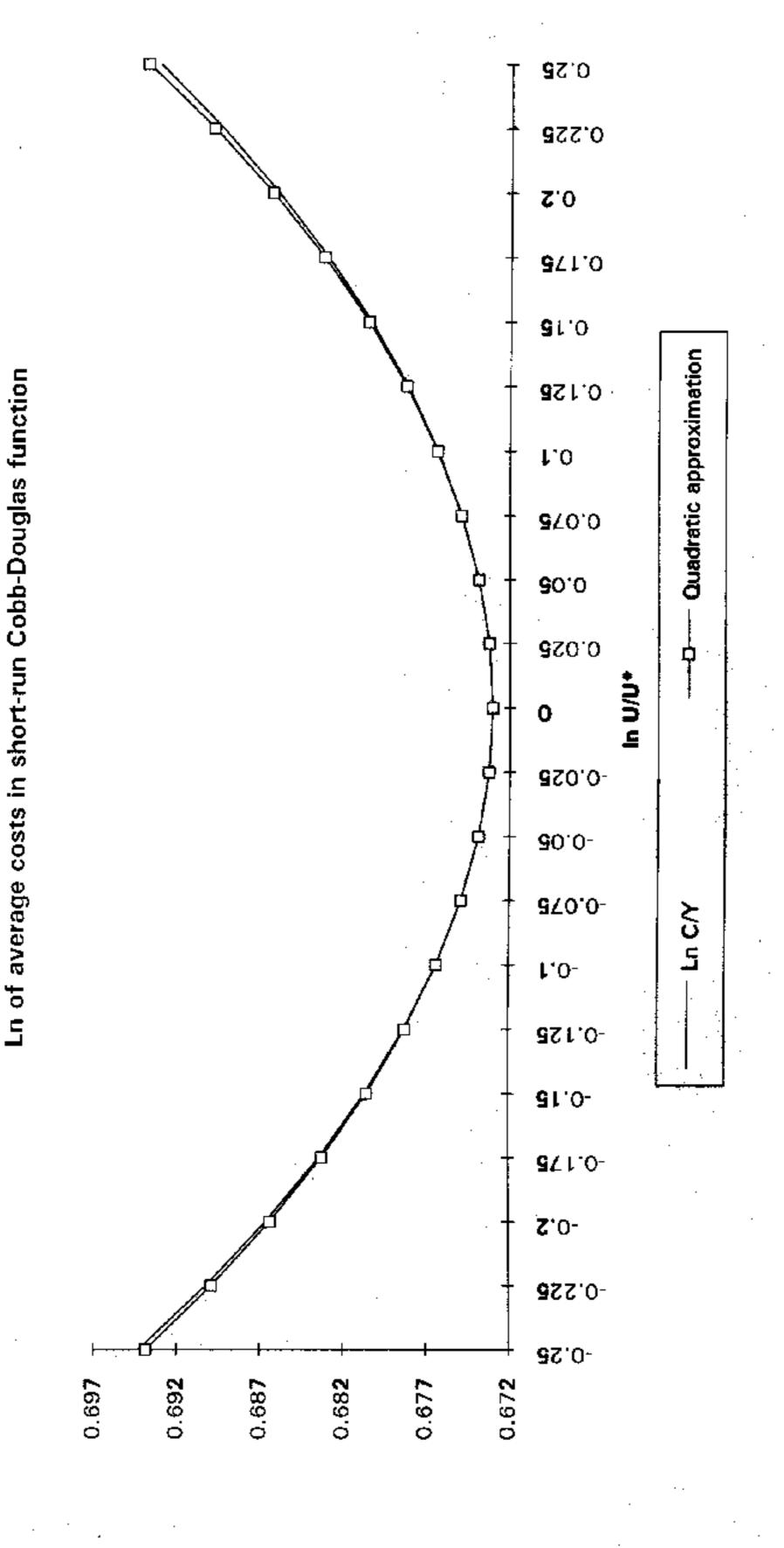


Figure 2

more symmetric in logs, so a quadratic approximation around the full-employment level U* works better after taking logs.

3.3. Bias in econometric estimates of cost functions

We now address the errors-in-variables problem that may be present in econometric estimates of the cost function.

The econometric issue involved in what we have set forth above is similar to the horse-race problem faced by Friedman (1957): the winner has more than her share of good luck. Firms with high output are not only firms with high permanent output, but also firms with high transitory output. And firms with low output can have either low permanent output or low transitory output.

To pose this question explicitly in a stochastic context, we can express the log of costs as the sum of permanent and transitory components, plus a white noise term, using suitably adapted notation.

$$C = C_p + C_t + \epsilon, \text{ with } \epsilon \sim N(0, \sigma_{\epsilon}^2),$$

$$\text{where } c \equiv \ln C, \ C_p \equiv \ln C_p, \ C_i \equiv \ln (1 + C_T/C_p).$$

$$(7)$$

This same notation can be used to express the log of output as the sum of permanent and transitory components. We assume that both components are normal, independent variables, which are not correlated to the error term ϵ .

If
$$y=\ln Y$$
, $y_p=\ln Y_p$, $y_t=\ln (Y/Y_p)$, then $y=y_p+y_t$.

$$(Y_p) \sim N[(\mu_{y_p}), (\sigma_{y_p}^2 = 0)].$$
 (8)

Going back to the second-order approximation of the log of the short-run Cobb Douglas cost function in equation (6), the permanent and transitory components fit neatly in this framework.

$$c_{p} = \alpha_{0} + \alpha_{1} y_{p}, \text{ where } \alpha_{0} \equiv \ln\left(\frac{\alpha + \beta}{\alpha} \left(\frac{\alpha}{\beta} \frac{W_{2}}{W_{1}}\right)^{\frac{\beta}{\alpha + \beta}} W_{1}\right), \quad \alpha_{1} \equiv \frac{1}{\alpha + \beta}$$

$$c_{t} = \alpha_{1} y_{t} + \frac{\alpha_{2}}{2} y_{t}^{2}, \text{ where } \alpha_{1} \equiv \frac{1}{\alpha + \beta}, \quad \alpha_{2} \equiv \frac{\alpha}{\beta (\alpha + \beta)^{2}}$$

$$(9)$$

Actual costs depend on both the permanent and the transitory components of output. If current output is used instead as the explanatory variable, there will be a classic errors-in-variables problem, and the error term will be correlated to the explanatory variable.

$$c = \alpha_0 + \alpha_1 (y_p + y_t) + \frac{\alpha_2}{2} y^2 + (\epsilon + \frac{\alpha_2}{2} (y_t^2 - y^2)) = \alpha_0 + \alpha_1 y + \frac{\alpha_2}{2} y^2 + u$$
 (10)

For the consumption function, transitory income was assumed not to be correlated to transitory consumption. For the cost function, the issue is instead that positive demand shocks are strongly correlated to transitory costs, while negative shocks are weakly correlated to them: the covariances between current costs and transitory output are larger for positive demand shocks.

$$E(C | Y_{t} | Y_{t} < 0) = \alpha_{1} E((y_{t})^{2} | Y_{t} < 0) + \frac{\beta_{2}}{2} E((y_{t})^{3} | Y_{t} < 0)$$

$$= \frac{\alpha_{1}}{2} \sigma_{y_{t}}^{2} - \frac{\beta_{2}}{2} E((y_{t})^{3} | Y_{t} > 0)$$

$$< \frac{\alpha_{1}}{2} \sigma_{y_{t}}^{2} + \frac{\beta_{2}}{2} E((y_{t})^{3} | Y^{t} > 0)$$

$$= E(C | Y_{t} | Y_{t} > 0)$$

$$= E(C | Y_{t} | Y_{t} > 0)$$
(11)

The errors-in-variables problem is treated in Appendix One. If current income is used as the explanatory variable, there is a bias towards the rejection of a generalized Cobb-Douglas technology. The bias increases as the ratio of the variance of the transitory component of output to the total variance of output increases. In the limit, the estimation will be picking up β_2 , the coefficient of the transitory component $(y_i)^2/2$.

Plim
$$b_2 = 0$$
 if $\frac{\sigma_{y_i}^2}{\sigma_y^2} = 0$, but plim $b_2 \rightarrow \beta_2$ as $\frac{\sigma_{y_i}^2}{\sigma_y^2} \rightarrow 1$ (12)

4. Estimates of retail bank cost functions

We illustrate our ideas on the distinction between actual and potential output taking retail banks as an example. We apply the distinction between transitory and permanent components of output to untangle the effects of economies of scale from changes in the level of capacity utilization.

To analyze the costs of banking firms, we follow the "production approach", concentrating on operating costs (Clark 1988). Deposits are classified as an output of the firm, not an

input, so the interest paid on deposits is not included in costs.³
4.1. Proxies of permanent and transitory output

A replication argument can justify constant returns to scale: if all inputs increase by a factor λ , production will increase by λ . Firms can of course gear potential output to expected demand. But when actual demand does not meet expectations, inputs cannot be adjusted instantaneously (Hunter and Timme 1995 explore the issue of quasi-fixed inputs).

Though there is no direct measure of potential output, we use the number of plants S (i.e. the total branches of banking firms, including the head office) as a proxy for potential output Y_p in our cross-section analysis. The reason for this is that retail banks depend to a great extent on the deposits they receive to carry on business, and hence on their geographical coverage through a network of branches.

While the number of plants is a signal of potential output, actual output can cause capacity utilization U deviate from U*. To measure the degree of capacity utilization, we use output per plant as a proxy, U=Y/S (this is strictly correct only when there are constant returns to scale, in which case fixed inputs, represented

Humphrey (1990) points out that the use of operating costs may lead to a bias in the analysis of economies of scale, due to systematic differences in banks' funding mix: in the U.S., average operating costs (operating expenses divided by total assets) fall more rapidly than average total cost (operating plus interest expenses divided by total assets), because larger banks use more purchased funds, which have low operating expenses.

However, we control for differences in the funding mix, since it is one of the variables used to stratify banks as either retail or wholesale (cf. Appendix 3).

by factor K in Section Three, vary proportionally with potential output). Though this measure of capacity utilization can also differ between banks because of technological differences between more and less efficient firms, we work under the assumption that there is a technology common to all firms.

We proceed in a two-step fashion to measure output per plant, starting with an aggregate measure of current output similar to the gross value of production in the national income accounts that allows us to illustrate our main point. We then disaggregate this flow measure according to the different services offered by the banking firms, to allow both for economies of scale and of scope.

The information comes from a cross-section of private banks in Argentina described in Appendix Two. The banks are stratified into retail and wholesale banks, since the number of branches makes sense as a measure of potential output only for retail banks. The clustering methods are described in Appendix Three.

4.2. Costs and aggregate output

Our starting point is a standard translog cost function. We do not have reliable data on the difference of input costs in different regions of the country, so these prices are not included. All variables are in logs: c stands for operating costs, y for output, s for plants, u for utilization level, where u=c-y.

Table 1 shows the estimates of cost functions for retail and

⁴In earlier estimates, we used average wages paid in each bank as a measure of wage rates, but the conditions for concavity of the cost function were violated. The wage differences possibly reflected differences in input quality, rather than differences in wage rates for a homogeneous class of labor.

Table 1. Cost functions for retail and wholesale banks

Standard error of estimate	0.178	0.148	0.361	0.322
Rc^2	0.981	0.987	0.764	0.812
s *>		-0.163 0.047**		0.319
s^2/2		0.161		-0.011 0.361
w		0.919		-2.151 1.332
y^2/2	0.058	0.184	-0.342	-0.745 0.254**
>	0.504	-0.081 0. 1 68	3.193 1.459*	5.702 1.670**
Constant	Retail banks 1) 1.794 0.272**	2.859 0.350**	Wholesale banks 1) -7.242 4.858	-14.949 5.426*
	Retail 1)	7	Whole 1)	7

cients; statistics significant at 1% level are marked with two asterisks, Note: standard errors are below coeffi at 5% level with one asterisk.

Table 2. Cost functions for retail banks with aggregate measure of output

F-test for	reductions				F(2,80) = 1.403	•	F(3,80) = 1.049	
Standard	error of	estimate	0.148		0.148		0.148	
Rc^2			0.987		0.987		0.987	
u hat*s			0.021	0.016				
s^2/2			0.020	0.020				
w			0.936	0.053 * *	0.991	0.015**	1.000	
u hat^2/2			0.184	0.039**	0.214	0.034 * *	0.221	0.032**
u hat			0.776	0.040**	0.823	0.021 * *	0.823	0.021 * *
Constant			4.478	0.073 * *	4.403	0.047 * *	4.378	0.018**
			3)		3.)		3)	

Note: standard errors are below coefficientes; stastistics significant at 1% level are marked with two asterisks, at 5% level with one asterisk. wholesale banks. Output is the scale variable, in the manner customarily posited in the literature. When the number of branches is added, it is a significant explanatory variable.

In what follows, we estimate cost functions for the subset of retail banks. A common cost function for both types of banks can be rejected at the 1% probability level: taking specification (1), F(3,113)=6.93, while with specification (2) F(6,107)=3.78.

Table 1 shows U-shaped cost curves for retail banks. To interpret the source of that behavior, our reference point is the short-run Cobb-Douglas function discussed in Section Three. We work with an approximation around the average level of capacity utilization \bar{U} , which influences the regression curve fitted to the data.

$$Ln C \approx \ln\left(\frac{\alpha+\beta}{\alpha}\left(\frac{\alpha}{\beta}\frac{W_{2}}{W_{1}}\right)^{\frac{\beta}{\alpha+\beta}}W_{1}\right) + \ln\left(\frac{\alpha}{\alpha+\beta} + \frac{\beta}{\alpha+\beta}\left(\frac{\overline{U}}{U^{*}}\right)^{\frac{1}{\beta}}\right)$$

$$+ \frac{1}{\alpha+\beta}\ln Y_{p} + \frac{\left(\frac{\overline{U}}{U^{*}}\right)^{\frac{1}{\beta}}}{\alpha+\beta\left(\frac{\overline{U}}{U^{*}}\right)^{\frac{1}{\beta}}} \ln U/\overline{U} + \frac{\frac{\alpha}{\beta}\left(\frac{\overline{U}}{U^{*}}\right)^{\frac{1}{\beta}}}{(\alpha+\beta\left(\frac{\overline{U}}{U^{*}}\right)^{\frac{1}{\beta}})^{2}} \frac{(\ln U/\overline{U})^{2}}{2}$$

$$(13)$$

Let $\hat{u}=\ln (U/\bar{U})$, denoting the deviations of the log of utilization from its average level. The coefficient of \hat{u} reflects the short-run elasticity of costs with respect to utilization. It can convey information on whether the system is on average below full-capacity utilization U^* or not, under the null hypothesis that a constant returns to scale technology is the correct specification. With constant returns to scale, it equals 1 when $\bar{U}=U^*$, it is smaller than 1 when the system is below the full-

employment level U*, and it is above 1 when the opposite holds, as the simulations in Table 3 show.

The output variable in Table 1 can be rescaled by the number of plants to yield a proxy of the utilization level, u=y-s, and then expressed in terms of the deviations û=u-ū around the average utilization level ū. Taking s and û as explanatory variables avoids the large degree of multicollinearity between s and y. The main point, however, is that according to our interpretation scale effects are represented by s, rather than by y which is affected by transitory components. Scale effects can be directly distinguished from the capacity utilization effects given by û.

In Table 2, estimation (3) is equivalent to estimation (2). The data do not reject a reduction from (3) to (3'), with a constant elasticity of scale, or to (3"), with constant returns to scale. On the contrary, the usual interpretation of specification in Table 1 leads to reject a Cobb-Douglas form.

According to this representation, depicted in Figure 3, there are U-shaped average costs due to fixed factors in the short run, but in the long run there are constant returns to scale.⁵

The coefficient of the linear utilization term in equation

⁵In the scatter diagram, there is an outlier. However, it is not an influential point because its removal does not alter the results significantly. Taking average costs $c-y\equiv ln$ C/Y as the dependent variable in specification (3"), the adjusted R² falls from 0.63 to 0.41, but the standard error of estimate is practically identical (0.147, vs. 0.148 before). The linear and quadratic terms in û remain highly significant: the coefficients are -.166 (t=-7.21) for the linear term, vs. -.177 (t=-8.26) before, and .176 (t=3.74) for the quadratic term, vs .221 (t=6.93) before.

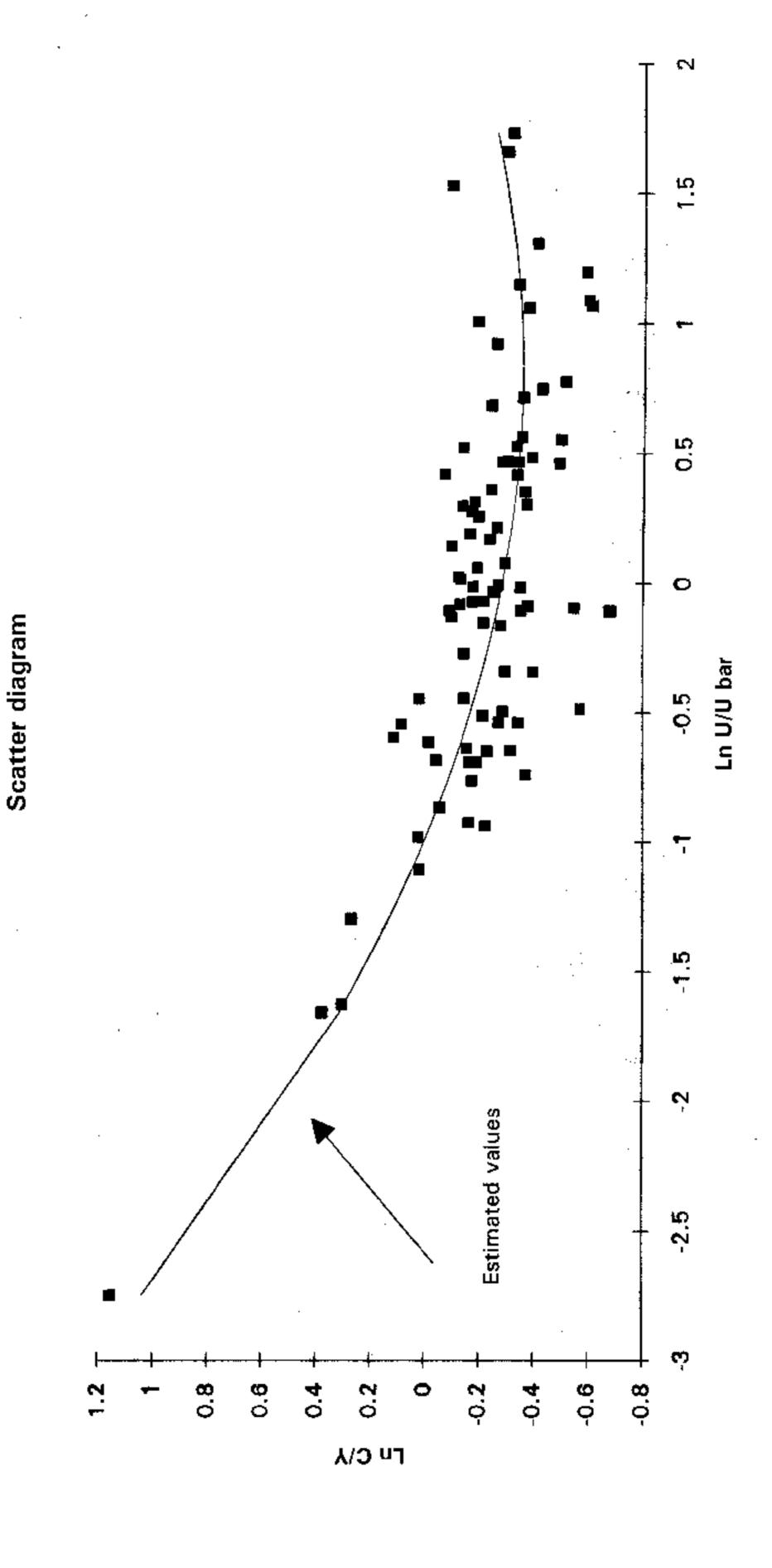
Table 3. Capacity utilization terms with constant returns to scale

1. Coefficient of In (U/U bar)

					Alpha				
U bar/u*	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.6	0.93	0.85	0.76	0.65	0.53	0.39	0.24	0.10	0.01
0.65	0.94	0.88	0.80	0.70	0.59	0.46	0.31	0.14	0.01
0.7	0.95	0.90	0.83	0.75	0.66	0.54	0.38	0.20	0.03
0.75	0.96	0.92	0.87	0.80	0.72	0.61	0.47	0.28	0.06
8.0	0.97	0.94	0.90	0.85	0.78	0.69	0.56	0.38	0.12
0.85	0.98	0.96	0.93	0.89	0.84	0.77	0.67	0.50	0.21
0.9	0.99	0.97	0.95	0.93	0.90	0.85	0.77	0.64	0.37
0.95	0.99	0.99	0.98	0.97	0.95	0.92	0.88	0.81	0.62
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.05	1.01	1.01	1.02	1.03	1.05	1.07	1.12	1.21	1.53
1.1	1.01	1.02	1.04	1.06	1.10	1.15	1.24	1.44	2.24

2. Coefficient of (ln (U/U bar))^2/2

				4	Aipna				
U bar/u*	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.6	0.17	0.34	0.51	0.66	0.78	0.83	0.75	0.47	0.07
0.65	0.16	0.33	0.50	0.68	0.84	0.94	0.93	0.68	0.15
0.7	0.15	0.32	0.50	0.69	0.88	1.05	1.13	0.97	0.31
0.75	0.14	0.30	0.49	0.69	0.92	1.16	1.35	1.32	0.62
0.8	0.13	0.29	0.48	0.69	0.95	1.25	1.56	1.75	1.17
0.85	0.13	0.28	0.46	0.69	0.97	1.33	1.77	2.25	2.09
0.9	0.12	0.27	0.45	0.69	0.99	1.40	1.98	2.80	3.59
0.95	0.12	0.26	0.44	0.68	1.00	1.46	2.17	3.40	5.85
1	0.11	0.25	0.43	0.67	1.00	1.50	2.33	4.00	9.00
1.05	0.11	0.24	0.42	0.65	1.00	1.53	2.48	4.58	12.98
1.1	0.10	0.23	0.40	0.64	0.99	1.55	2.59	5.12	17.37



(3") is smaller than 1, which can be interpreted as indicative of excess capacity in the financial system in late 1992 and early 1993. The quadratic term, however, has a smaller coefficient than one would expect according to the simulations in Table 3.

The result that the financial system was below full capacity utilization supports the interpretation in Section Two about the importance of the cyclical expansion for the observed rise in productivity in banking. Appendix 4 shows that the residuals are normal and homoskedastic, so the estimation is well-behaved.

4.3. Excess capacity as an explanation of scale economies for the banking office

If excess capacity is the normal state of affairs in the banking industry (and not only because of the cyclical reasons discussed in Argentina), that can help explain a stylized fact of banking industry surveys (Clark 1988, and Humphrey 1990): there are scale economies for the average banking office (i.e. varying y, with s constant), while for the average banking firm (i.e. incorporating the variation of s, as y varies) these economies have either disappeared or there are slight diseconomies of scale.

With a constant returns to scale Cobb-Douglas technology, the difference between the cost elasticities at the firm and the plant level equals the elasticity of costs with respect to s (as can be verified operating with (13), transforming it into an expression in y and s, taking $\alpha+\beta=1$). This elasticity is positive at the average utilization level \overline{u} when there is excess capacity:

$$\frac{\partial C}{\partial S} = \left[1 - \frac{\left(\frac{\overline{U}}{U^*}\right)^{\frac{1}{\beta}}}{\left(1 - \beta\right) + \beta\left(\frac{\overline{U}}{U^*}\right)^{\frac{1}{\beta}}}\right] - \frac{\frac{1 - \beta}{\beta}\left(\frac{\overline{U}}{U^*}\right)^{\frac{1}{\beta}}}{\left(\left(1 - \beta\right) + \beta\left(\frac{\overline{U}}{U^*}\right)^{\frac{1}{\beta}}\right)^{2}} \left(u - \overline{u}\right) \Rightarrow \frac{\partial C}{\partial S} \Big|_{u = \overline{u}, \text{ when } \overline{u} < u^{-}} > 0$$
(14)

The observation that a positive coefficient for the number of branches can indicate excess capacity was already suggested by Santibañes (1975), and is discussed in D'Amato et al. (1994).

4.4. Costs and the multiproduct character of banks

A caveat applies to the results using an aggregate output measure: they are strictly valid only if it is possible to find a consistent output estimate (Kim, 1986). An aggregate measure of output permitted us to isolate the issue of scale versus degree of capacity utilization, but this overlooks the issue of economies of scope, which are significant in the banking sector (Humphrey 1990).

Banks create deposits to extend loans, and they also engage in a variety of services that make the output of banks difficult to identify. To take the multiproduct character of banks into account, total output (measured by the flow of income of the banking firm) is disaggregated into net interest income and income from other services. Equation (4) is a multiproduct analogous to (3).

The variable s that represents the number of plants is once again the measure of scale. The level of capacity utilization is measured as the log of the ratio of each product to the number of branches, net of the average utilization level, to express each in terms of deviations from the mean: $\hat{u}_i = u_i - \bar{u}_i$ denotes the utilization

level of financial services, $\hat{u}_s = u_s - \bar{u}_s$ the utilization of other services.

In Table 4, we also try out an alternative measurement of financial product, since there may be problems with the use of net interest income: in (5), the monetary value of loans and deposits, which is a more standard definition, is employed instead (see e.g. Humphrey 1990 on usual output measures). We do not have a comparable stock variable to represent the other services from banks, so flow and stock variables are combined in this estimation.

According to representation (4), there is an interrelation between scale and product mix, while according to (5') these interrelations are not significant. The next point compares the different estimates. Appendix 4 shows that the residuals of both of these regressions are normal and homoskedastic.

4.5. Scale and utilization elasticities of different estimates

Unlike estimation (3") with an aggregate measure of output, the multiproduct estimates (4) and (5') reject the hypothesis of constant returns to scale, pointing to increasing returns to scale.

Table 5 shows that the elasticity of costs with respect to capacity utilization is pretty low for all specifications, which signals sizeable decreases in average costs due to an expansion in demand at that point in time (late 1992 - early 1993). Though the results on economies of scale are not clearcut, the Table seems to show that when average excess capacity is larger (i.e., the estimated elasticity of costs with respect to average capacity utilization is lower), the economies of scale are larger.

Table 4. Multiproduct cost functions for retail banks

T-test for reductions		(3,76) = 4.464*	(4,76) = 4.700*		F(3,76) = 1.552	(4,76)=4.494*
Standard error of estimate	0.162	0.172	0.176	0.167	0.169	0.181
Rc^2	0.984	0.982	0.981	0.983	0.983	0.980
ui hat* us hat	-0.043	-0.124 0.045**	-0.132 0.045**	-0.198	-0.323 0.085**	-0.348 0.091**
s* us hat	0.123			0.107		
s* ui hat	-0.090			-0.081		
us hat square/2	0.251	0.378	0.412	0.212	0.364	0.410
us hat	0.200	0.469	0.437	0.204	0.445	0.398
	ui hat 0.091 0.030*	0.136 0.026*	0.150	s ui hat 0.313 0.136*	0.430	0.476
ui hat	activities 0.513 0.076*	0.331	0.363	al activitie 0.496 0.135*	0.312	0.350
s^2/2	financial -0.010 0.025			r financia -0.001 0.028		
ග	ariable for 0.974 0.069*	0.957	1.000	/ariable fc 0.929 0.074*	0.934	1.000
Constant	Using a flow variable for financial activ 4)	4.424 0.063**	4.297	Using a stock variable for financial activities ui hat 5) 4.535 0.929 -0.001 0.496 0.313 0.103** 0.074* 0.028 0.135* 0.136	4.520 0.058**	4.328 0.026**
	Using 4}	4.`	4	Usinç 5)	5,)	2

Note: standard errors are below coefficients;statistics significant at 1% level are marked with two asterisks, at 5% level with one asterisk.

Table 5. Scale and utilization elasticities

Equation	Elasticity of costs to scale	Elasticity of costs to capacity utilization
3)	0.986	0.831
3')	0.991	0.823
3")	1	0.823
4)	0.949	0.802
4')	0.957	0.8
4")	1	0.8
5)	0.928	0.771
5')	0.934	0.757
5")	1	0.747

Note: the cost elasticities are evaluated at the average values of branches and capacity utilization

Equation (5') is closest to the standard output measures (the problem with net interest earnings in (3) and (4) is that they may be influenced by credit risk and other factors). Taking (5') as the preferred estimate, there are sizeable economies of scale, with a cost elasticity of 0.93.6 This estimate is not consistent with a Cobb-Douglas form, however, because the cross-product term should be positive (see Appendix 5).7

In relation to our central argument, the distinction between the transitory and permanent components of output, the results are more robust. The U-shaped form of the cost curve can be ascribed to the presence of fixed factors: in no case is the quadratic term $s^2/2$

⁷In estimate (5') there are significant economies of scope, since the condition for economies of scope is satisfied:

$$\frac{\partial^2 C}{\partial YI \partial YS} = \frac{C}{YI * YS} \left[\eta_{YI} * \eta_{YS} + \alpha_{YI,YS} \right] < 0,$$

where η_i are product i-elasticities of costs,

 $\alpha_{i,j}$ is regression coefficient of cross-product term

The sign of the second-order cross-derivative depends on the term in brackets, which has a value of -.184 (t=-2.069, using an approximate t-test due to Fuller, 1962). On the contrary, in estimate (4) it has a value of .352 (t=6.673).

Note that though our regressions are expressed in terms of utilization levels, these values are exactly the same as those that result from using current output. This indicates that the usual measures of economies of scope might be distorted by capacity utilization effects, so a rejection of economies of scope should not be taken at face value.

⁶Humphrey (1990) reports that early studies with a Cobb-Douglas form found that average scale economies existed, with an average value of 0.92. The results of these early studies would not be biased estimates of scale economies, even if current output is used as the explanatory variable, as long as economies of scope are not significant (this can be verified extending the argument in Appendix 1 on the plim of linear parameter, when there are no quadratic terms).

significant, while the quadratic terms in capacity utilization are significant.

5. Conclusions

Observed cost functions are inherently short-run functions. This paper attempts to take this feature into consideration when economies of scale are estimated, applying the idea in macro that fluctuations in effective demand make current output diverge from potential output.

Just as Friedman finds that in the estimates of the consumption function there is a bias against the hypothesis of a marginal propensity to consume equal to one in the long run, here we show that in the econometric estimates of the cost function there is a bias against the acceptance of constant returns to scale in the long-run. The problem is similar in both instances: the use of current income or output, instead of the permanent component.

The solution we propose is to separate the transitory and permanent components of current output, which lead short-run and long-run costs to diverge. Current output varies with changes of the degree of capacity utilization around potential output.

An application to retail banking in Argentina, using the number of branches as a proxy for potential output and output per plant as a proxy for the level of capacity utilization, shows that a U-shaped average cost curve does not necessarily lead to reject constant returns to scale, since this can be due to the effect of fixed factors of production in the short-run. An extension to a

multiproduct framework, however, does lead to reject a Cobb-Douglas form, because of the presence of economies of scope (but not because economies of scale are U-shaped).

Furthermore, the stylized fact in studies of the banking industry that there are scale economies for the average banking office, while they do not exist for the average banking firm, can be reinterpreted in this framework as an indication that banks normally operate with excess capacity.

Appendix 1: the errors-in-variables problem

In matrix form, equation (10) can be expressed as follows.

$$c = X \beta + u$$
, where $\beta \equiv \begin{bmatrix} \alpha_1 \\ \alpha_2/2 \end{bmatrix}$ (15)

Doing ordinary least squares, the estimated coefficients are

$$b = (X'X)^{-1}X'c = (X'X)^{-1}X'(X\beta + u) = \beta + (\frac{X'X}{T})^{-1}\frac{X'u}{T}$$
 (16)

By the properties of probability limits (cf. Judge et al. 1988, pp. 266-7), extending to matrices Slutzky's theorem which states that if g(.) is a continuous function and Z_t is a random variable that depends on T, then plim $g(Z_t) = g(\text{plim } Z_t)$,

Plim
$$b=\beta+plim\left(\frac{X'X}{T}\right)^{-1}\frac{X'u}{T}$$
] = $\beta+plim\left(\frac{X'X}{T}\right)^{-1}plim\frac{X'u}{T}$
= $\beta+(plim\frac{X'X}{T})^{-1}plim\frac{X'u}{T}$ (17)

The first probability limit is the matrix Σ of variances and covariances of the explanatory variables, while the second probability limit is the vector of covariances between the explanatory variables and the error term u.

We assume that the transitory and permanent components are not correlated, so $\sigma_y^2 = \sigma_{yp}^2 + \sigma_{yt}^2$ and $\sigma_y^4 = \sigma_{yp}^4 + 2\sigma_{yp}^2\sigma_{yt}^2 + \sigma_{yt}^4$. Using these facts, and inverting Σ , the probability limit of the vector b is

$$\frac{3}{2} \quad 0 \quad \frac{-1}{2\sigma_{y}^{2}} \quad \frac{-\beta_{2}}{2}\sigma_{y_{p}}^{2}$$

$$Plim \ b = \beta + \begin{bmatrix} 0 & \frac{1}{\sigma_{y}^{2}} & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2\sigma_{y}^{2} & \frac{-\beta_{2}}{2}(3\sigma_{y_{p}}^{4} + 5\sigma_{y_{p}}^{2}\sigma_{y_{p}}^{2}) \end{bmatrix}$$

$$\frac{-1}{2\sigma_{y}^{2}} \quad 0 \quad \frac{1}{2\sigma_{y}^{2}} \quad \frac{-\beta_{2}}{2}(3\sigma_{y_{p}}^{4} + 5\sigma_{y_{p}}^{2}\sigma_{y_{p}}^{2})$$

The third row is the coefficient $b_2/2$ of the quadratic term.

Plim
$$b_2 = \beta_2 + \beta_2 \left(\frac{\sigma_{y_p}^2}{2\sigma_y^2} - \frac{3\sigma_{y_p}^4}{2\sigma_y^4} - \frac{5\sigma_{y_p}^2\sigma_{y_p}^2}{2\sigma_y^4} \right)$$
 (19)

Since $\sigma_{yp}^{\ 2} = \sigma_y^{\ 2} - \sigma_{yt}^{\ 2}$, and thus $0 \le \sigma_{yt}^{\ 2} \le \sigma_y^{\ 2}$, the bias against the true coefficient 0 on the permanent output component will increase as $\sigma_{yt}^{\ 2}/\sigma_y^{\ 2}$ grows.

Appendix 2: the data base

The data used in the regressions are an average of monthly figures from four months, August, October and December 1992, and February 1993. The observations cover 118 private banks in Argentina, which are classified in Table 6 as wholesale and retail banks according to the procedure described in Appendix 3.

Appendix 3: stratification of private banks

Burdisso et al. (1994) used two variables to stratify banks,
(i) the ratio of deposits to total assets, and (ii) the ratio of
commissions on deposit and loan accounts, plus the rental of safe
deposit boxes, to total income from services provided to clients.
They acted on the hypothesis that retail banks counted to a greater
extent on deposits to fund their operations, and that most of the
services they provided had to do precisely with deposit and loan

Table 6. Descriptive statistics of sample of banks

	in thousands of	hesos, exce	shr nomber o	i branciles	
1) Retail banks (86 case	s)				
	Variable	Min.	Max.	Mean	St. dev.
Operating costs	С	22	20059	2597	3806
Net income	Υ	7	25550	3489	5306
Net interest income	ΥI	1	15417	1904	2991
Other income	YS	3	13977	1712	2732
Loans + deposits	L+D	434	2640569	326370	537708
Branches	S	1	169	25	30
Capacity utilization	U = Y/S	7	598	139	113
Cap. util. fin. services 1	UI1 = YI/\$	0	466	84	88
Cap. util. fin. services 2	U12 = (L + D)/	434	62851	12737	12403
Cap. util. other serv.	US = YS/S	3	277	60	47
2)Wholesale banks (32 d	cases)				
	Variable	Min.	Max.	Mean	St. dev.
Operating costs	С	100	2944	831	570
Net income	Υ	196	3757	1151	771
Net interest income	Y 1	19	2182	711	575
Other income	YS	30	1648	501	490
Loans + deposits	L + D	7649	582514	125619	114545
Branches	s	1	18	3	3
Capacity utilization	U = Y/S	133	2740	758	669
Cap. util. fin. services 1	UI1 = YI/S	11	1750	466	485
Cap. util. fin. services 2	U12 = (L + D)/	4371	307509	71164	63100
Cap. util. other serv.	US=YS/S	17	1564	334	437
	In logs				
1) Retail banks (86 cases	s)				
	Variable	Min.	Max.	Mean	St. dev.
Operating costs	. C	3.068	9.906	7.088	1.293
Net income	y	1.915	10.148	7.309	1.405
Net interest income	γì	0.012	9.643	6.648	1.556
Other income	ys	1.099	9.545	6.437	1.586
Loans + deposits	l + d	6.073	14.787	11.722	1.474
Branches	s	0.000	5.13 1	2.646	1.161
Capacity utilization	u = y-s	1.915	6.394	4.663	0.765
Cap. util. fin. services 1	ui1 = yi-s	-1.380	6.144	4.002	1.066
Cap. util. fin. services 2	ui2 = (1+d)-s	6.073	11.048	9.076	0.885
Cap. util. other serv.	us = ys-s	1.099	5.626	3.792	0.836
2)Wholesale banks /32 o	anna!				

2) Wholesale banks (32 cases)

	Variable	Min.	Max.	Mean	St. dev.
Operating costs	С	4.603	7.988	6.491	0.742
Net income	У	5.277	8.231	6.823	0.718
Net interest income	yi	2.959	7.688	6.135	1.116
Other income	γs	3.384	7.407	5.727	1.054
Loans + deposits	l + d	8.942	13.275	11.396	0.890
Branches	S	0.000	2.890	0.551	0.778
Capacity utilization	u = y-s	4.890	7.916	6.272	0.860
Cap. util. fin. services 1	ui1 = yi-s	2.399	7.467	5.584	1.178
Cap. util. fin. services 2	ui2 = (l + d)-s	8.382	12.636	10.844	0.865
Cap. util. other serv.	us = ys-s	2.825	7.355	5.175	1.098

accounts.

Using the two-stage density linkage method (SAS/STAT, 1988), Burdisso et al. (1994) found two clusters which they labelled retail and wholesale banks. We applied this same procedure, adding a third variable to stratify the sample, ln (total loans/number of clients). Wholesale banks typically operate with large loans, so their average loan size is larger.

We found two clusters that had a great deal of overlap with their classification (in the four months there were 4 borderline banks, 2 of which were ascribed to the retail group, while 2 other were ascribed to the wholesale group; a retail bank that behaved like an outlier in the cost estimates was removed from the sample).

No significant linear relation between the classifying variables remained within the clusters. Furthermore, Table 7 shows that in the cluster of retail banks the classifying variables are not related to our scale measure either (the log of the number of branches).

Appendix 4: behavior of the residuals

4.1. Normality

The normality of the residuals of the final models (equations 3", 4 and 5') was analyzed through the Kolmogorov-Smirnov test. Table 8 shows that the null hypothesis that their distribution is normal is not rejected.

4.2. Heteroskedasticity

To test the presence of heteroskedasticity, we used the Breusch-Pagan test (cf. Judge et al. 1988). Through visual

Table 7. Correlations in sample and within clusters

	Deposits/ total assets	Commissions on loans and deposits/ income from services	Ln(average loan)
1.Sample of banks			
(117 cases)			
Deposits/total assets	1	0.616**	-0.630**
Ln(branches)	0.543**	0.492**	-0.583**
2.Retail banks (86 cases)			
Deposits/total assets	1	0.119	0.08
Ln(branches)	-0.021	-0.102	-0.025
3.Wholesale banks		•	
(31 cases from 32)			
Deposits/total assets	1	0.308	-0.345
Ln(branches)	0.627**	0.547**	-0.339

Table 8. Normality of residuals: Kolmogorov-Smirnov test

Equation	KS. statistic	Probability level
3'')	0.051	1
4)	0.0749	0.9999
5'}	0.0456	1

Table 9. Breusch-Pagan heteroskedasticity test

Equation	Observed value	Values of statistic at 5% probability level
3'')	0.4011	CHI2(1) = 3.84
4)	3.9664	CHI2(1) = 5.99
5')	2.8899	CHI2(1) = 5.99

inspection, the square of the utilization level was picked as explanatory variable in the case of residuals of estimation (3"), and in the case of residuals of the multiproduct estimates (4) and (5') the squares of both utilization levels were used.

We ran a regression between the square of the residuals, normalized by the estimated variance, and these explanatory variables. Z denotes the matrix of the S explanatory variables, which are either two or three, since a constant is always included.

$$\frac{e^2}{s^2} = Z\alpha + v \tag{20}$$

Under the null hypothesis that the parameters α equal 0, and the hypothesis that residuals are normal (which was verified above), half of the explained sum of squares is asymptotically distributed as a Chi-square with S-1 degrees of freedom. The test in Table 9 permits us to reject heteroskedastic errors.

Appendix 5: Multiproduct Cobb-Douglas form

If we express the Cobb-Douglas production function, in a multiproduct context, as $Y^{\lambda}Z^{1-\lambda}=K^{\alpha}L^{\beta}$, we can find the short-run and long-run cost functions in a manner similar to what was done in the text. The difference with the case of a single output is that the optimal utilization level is defined only in terms of the product of both outputs. A second-order approximation to the log of costs, similar to equation (13) in text for the single output case, yields the following equation:

$$Ln \ C \approx \ln\left(\frac{\alpha+\beta}{\alpha}\left(\frac{\alpha}{\beta}\frac{W_{2}}{W_{1}}\right)^{\frac{\beta}{\alpha+\beta}}W_{1}\right) + \ln\left(\frac{\alpha}{\alpha+\beta} + \frac{\beta}{\alpha+\beta}\left(\frac{\overline{U}}{U^{*}}\right)^{\frac{1}{\beta}}\right)$$

$$+ \frac{1}{\alpha+\beta}\left[\lambda \ln Y_{p} + (1-\lambda) \ln Z_{p}\right] + \frac{\left(\frac{\overline{U}}{U^{*}}\right)^{\frac{1}{\beta}}}{\alpha+\beta\left(\frac{\overline{U}}{U^{*}}\right)^{\frac{1}{\beta}}}\left[\lambda \ln U_{y}/\overline{U}_{y} + (1-\lambda) \ln U_{z}/\overline{U}_{z}\right]$$

$$+ \frac{\frac{\alpha}{\beta}\left(\frac{\overline{U}}{U^{*}}\right)^{\frac{1}{\beta}}}{(\alpha+\beta\left(\frac{\overline{U}}{U^{*}}\right)^{\frac{1}{\beta}})^{2}}\left[\lambda^{2}\frac{(\ln U_{y}/\overline{U}_{y})^{2}}{2} + \lambda(1-\lambda) \ln U_{y}/\overline{U}_{y} \ln U_{z}/\overline{U}_{z} + (1-\lambda)^{2}\frac{(\ln U_{z}/\overline{U}_{z})^{2}}{2}\right],$$

$$where U \equiv U_{y}^{\lambda}U_{z}^{1-\lambda} \equiv \left(\frac{Y}{K}\right)^{\lambda}\left(\frac{Z}{K}\right)^{1-\lambda} = \frac{Y^{\lambda}Z^{1-\lambda}}{K}. \tag{21}$$

A restriction imposed by the Cobb-Douglas form is that the coefficients of the quadratic utilization terms be positive, while the estimates in Table 4 yield a significantly negative coefficient on the cross-product of the utilization levels (this is a necessary condition for cost complementarities, pointing to economies of scope that are ruled out by the Cobb-Douglas form).

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