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HOW TRADE SPLITS UP INFORMATION SETS

AND DEALERS CARRY OUT

THEIR BROKERAGE OF ASYMMETRIC INFORMATION

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Abstract

In this paper we set forth a new perspective from which to understand and measure the brokerage of asymmetric information that intermediaries usually carry out. Firstly, we deal with partitions of a given set so as to lay grounds to our line of research. Secondly, we argue that trade splits up imperfect information sets, over which traders try to negotiate and profit, but also hide their opportunistic behavior from their counterparts. Next, the brokerage of asymmetric information is framed so as to stress the fact that any exchange is dual, entailing not only bargaining property rights but also information value. Lastly, we bring to light the linkage between differential rates, residual information sets and trading environments, which seems to be a functional toolkit for assessing how much asymmetric information is brokered eventually.

JEL: *G14, D82, D80, C78*

Key words: *asymmetric information, brokerage, differential rates, residual information sets, financial intermediaries*

INTRODUCTION

Whenever any intermediary fulfills his professional tasks, he also carries out the role of a **broker of asymmetric information**[2]. To qualify this statement it seems helpful to proceed with a set of environments that brings to light how the intermediary performs his function. In other words, we will attempt to lay bare what could be named the geometric topology of information sets.

The paper will go on developing the following topics:

Sections 1 and 2 lay the groundwork, the former by establishing a general outcome that holds for partitions of a given set; the latter by introducing imperfect information sets and the dual nature of any trade.

In section 3, it will be shown how trade between economic actors and intermediaries brings about a natural partition of their underlying information sets . It is for section 4 to set forth the framework where the brokerage of asymmetric information takes place. Last of all, section 5 discusses trading environments with a focus on opportunistic behavior, by means of differential rates and residual information sets.

This paper follows up a line of research started with a former work devoted to the transaction cost approach to financial assets, developed later with an extensive discussion about the brokerage of asymmetric information; and finally carried out by an inquiry about differential rates and residual information sets (namely, Apreda 2000 a, b; 2001; 2004).

1. PARTITION OF A GIVEN SET

As the idea of partition plays a key role in this chapter, it's worth recalling its mathematical meaning.

Definition 1 Set partition

Given any non-empty set X, and a collection of subsets of X,

$$\Phi = \{ A_\lambda \subseteq X : \lambda \in J, \; J \text{ an index set} \}$$

we say that Φ *is* ***a partition of X*** *(or that* Φ *partitions* ***X****) if it holds that*

[2] An introduction to this issue, and the place where we coined the expression "the brokerage of asymmetric information" can be found in Apreda (2001).

a) $\boldsymbol{X = \cup A_\lambda}$,

b) $\boldsymbol{A_\lambda \cap A_\mu} \neq \varnothing$, *for every pair* $\boldsymbol{\lambda, \mu}$ *of different elements in the index set*[3].

Before moving on to what is the main concern of this paper, the brokerage of information sets, we need to bring into play the following lemmas, of which we are going to produce their proofs.

Lemma 1 ***Given two non empty sets A and B, their union can be partitioned in the following way:***

$$\boldsymbol{A \cup B = (A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)}$$

Proof: firstly, we have to prove the double inclusion; secondly, that the right side in the equation above really partitions the left side.

Let start supposing that $x \in \mathbf{A \cup B}$, then it must belong to **A** or to **B**. If it is not the case that $x \in \mathbf{A \cap B}$, it follows that either it belongs to **A** without belonging to **B**, or the other way round. That is to say, $x \in \mathbf{(A \cap B^c)}$ or $x \in \mathbf{(A^c \cap B)}$. Hence, this shows that the set on the left side is included in the set on the right side of the equation.

Now, let us assume that

$$x \in \mathbf{(A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)}$$

then it must belong to one of these sets. Without loss of generality, if it were the case that $x \in \mathbf{(A \cap B^c)}$, then x belongs to A without belonging to B, so in the end we have that $x \in \mathbf{A \cup B}$, and the same procedure holds in the other two cases. In conclusion it is true that

$$\mathbf{(A \cap B^c) \cup (A^c \cap B) \cup (A \cap B) \subseteq A \cup B}$$

and the equality of both sets it comes after.

To prove that we have partitioned the union of both sets, the only thing it remains to bring to light is that the three subsets on the right side are disjoint, that is, they do not share not even a single element. But this comes from their definition, since the two first sets have each of them

[3] The index set can be finite or non-finite. In the remaining of this book our focus will be on finite structures of sets, however.

points that belong only to one of them at a turn, while the third have point that only belong to both sets. < END >

It is worth remarking here that the conclusion of the lemma

$$\mathbf{A \cup B = (A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)}$$

can be rewritten in a more operational fashion that will come in handy whenever we handle an arbitrary collection of sets.

$$\begin{cases} \mathbf{A_1 \cup A_2 = \cup \{ (A_1^{k(1)} \cap A_2^{k(2)}) \}} \\ \text{such that} \\ k(1), k(2) \in \{ 1, -1 \} \text{ and the case } k(1) = k(2) = -1 \text{ is forbidden} \end{cases}$$

When the exponent of a set is 1, it means we work with the set itself; when it marked by −1, it means its complement[4].

Plain as it is, the former lemma allows us to give a general statement that it will follow from complete induction.

Lemma 2 ***Given a finite family of non empty sets***

$$\{ A_1 , A_2 , A_3 , \ldots\ldots\ldots , A_N \}$$

their union can be partitioned in the following way:

$$\cup_{1 \leq j \leq N} A_j = \cup \{ (A_1^{k(1)} \cap A_2^{k(2)} \cap A_3^{k(3)} \cap \ldots\ldots\ldots \cap A_N^{k(N)}) \}$$

such that k(1), k(2), k(3), ….. , k(N) ∈ { 1, -1 }

and the case k(1) = k(2) = k(3) = ….. = k(N) = -1 is forbidden

Proof: the inductive foundation is provided by lemma 1. So the statement is true when n = 2.

Let us assume that the statement is true when n = p, and let us see if it follows that is also true for n = p + 1. We start with the left side of the target statement.

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[4] So, A_1 reads as $A_1{}^1$, while $A_2{}^c$ reads as $A_2{}^{-1}$.

(1)

$$\cup_{1 \leq j \leq p+1} A_j = (\cup_{1 \leq j \leq p} A_j) \cup A_{p+1}$$

Now, we can see that the expression between square brackets in (1) fulfills the inductive statement. Hence,

(2)

$$(\cup_{1 \leq j \leq p} A_j) \cup A_{p+1} =$$

$$= \cup \{ (A_1^{k(1)} \cap A_2^{k(2)} \cap A_3^{k(3)} \cap \ldots\ldots\ldots \cap A_p^{k(p)}) \} \cup A_{p+1}$$

We can regard the big set on the left side of (2) as consisting of the union of two distinctive sets:

$$A = \cup_{1 \leq j \leq p} A_j$$

$$B = A_{p+1}$$

Therefore, by using Lemma 1 we are able to write:

$$A \cup B = (A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)$$

but this amounts to, namely,

a)

$$(A \cap B^c) = (\cup_{1 \leq j \leq p} A_j) \cap A_{p+1}{}^c$$

b)

$$(A^c \cap B) = (\cup_{1 \leq j \leq p} A_j)^c \cap A_{p+1}$$

c)

$$(A \cap B) = (\cup_{1 \leq j \leq p} A_j) \cap A_{p+1}$$

We take advantage of (2) and plug the result in the three relationships just to reach the following outcomes:

a')

$$(A \cap B^c) =$$

$$= (\cup \{ (A_1^{k(1)} \cap A_2^{k(2)} \cap A_3^{k(3)} \cap \ldots\ldots\ldots \cap A_p^{k(p)}) \}) \cap A_{p+1}{}^c$$

and by the distributive property,

(3)

$$(A \cap B^c) =$$

$$= \cup \{ (A_1^{k(1)} \cap A_2^{k(2)} \cap A_3^{k(3)} \cap \ldots\ldots\ldots \cap A_p^{k(p)} \cap A_{p+1}{}^c) \}$$

b')

$$(A^c \cap B) =$$

$$= (\cup_{1 \leq j \leq p} A_j)^c \cap A_{p+1}$$

and profiting from one of the well-known De Morgan's generalized properties of complementation, it holds that[5]

(4)

$$(A^c \cap B) =$$

$$= A_1^c \cap A_2^c \cap A_3^c \cap \ldots\ldots \cap A_p^c \cap A_{p+1}$$

c')

$$(A \cap B) =$$

$$= (\cup \{ (A_1^{k(1)} \cap A_2^{k(2)} \cap A_3^{k(3)} \cap \ldots\ldots \cap A_p^{k(p)}) \}) \cap A_{p+1}$$

this is easier, since we can straightforwardly apply the distributive property that leads to

(5)

$$(A \cap B) =$$

$$= \cup \{ (A_1^{k(1)} \cap A_2^{k(2)} \cap A_3^{k(3)} \cap \ldots\ldots \cap A_p^{k(p)} \cap A_{p+1}) \}$$

It is time to come back to (1) profiting from (3), (4), (5).

$$\cup_{1 \leq j \leq p+1} A_j =$$

$$\cup \{ (A_1^{k(1)} \cap A_2^{k(2)} \cap A_3^{k(3)} \cap \ldots\ldots \cap A_{p+1}^{k(p+1)}) \}$$

Applying the principle of complete induction, the statement is true for any value of n. < END >

2. INFORMATION SETS

Either by trading straight with another party, or by means of a third agent who acts on his behalf as an intermediary, any economic actor[6] makes up his mind on the basis of available information. Such information can stem from his own stock or, still further, from the one supplied by his

[5] A harder way would be to develop the right side of (2) with the complement, as the reader can verify when N = 2, getting the same outcome in the end.

[6] It goes without saying that "economic actor"(or agent) stands for both individuals and organizations.

counterpart or intermediary. In this context, trading refers to a wide range of distinctive events or processes which involve buying or selling, intermediating, contracting, keeping long or short future positions in the commodities, financial or labor markets. Narrowing down the examples to the subject of this paper, namely the financial markets, trading commit economic agents to manifold choices: from buying or selling financial assets to lending or borrowing in the credit market; from merging two companies to signing a partnership contract in a venture capital concern; from enacting a plain vanilla swap to the underwriting of an Initial Public Offer for either equity or debt. It seems sensible, therefore, to make precise what is meant by information sets.

Definition 2 Information Sets

*By the **Information Set** of certain economic actor " **e** " at date " **t** " is meant all the available information he gets access up to that date.*

We denote such a set as

$$\Omega(t \,;\, e)$$

and the fact that past information up to that date is also stored in the current information set can be translated by the nesting condition:

$$\Omega(t - j \,;\, e) \subseteq \Omega(t \,;\, e) \quad ; \quad j : 1, 2, 3, \ldots\ldots$$

Therefore, decision-making turns out to be contingent upon information sets. Although this is the customary format of what we understand as an information set, some qualifications should be borne in mind for the sake not only of semantics but rigor, as well.

a) We said that the agent stores only attainable information to him. This means he reaches his decision, in most cases, with only a fraction of the whole information to which he could get access.

b) Although information sets can be regarded as plausible databanks for any agent, they also convey the idea of toolkits for decision-making. In fact, they include mathematical models and heuristic procedures, points of view and beliefs, market trends, biases, error analysis and learning processes. Also past experiences, technological resources, and professional qualifications, just to give some particular examples of such pervasive notion.

c) A problem arises from this customary format and it hinges upon its vagueness. For the agent to reach his decisions, the information set should be freed from any content that has nothing to do with her purposes. This could prove an uneasy task to accomplish, but for an intermediary dealing with Treasury Bonds it doesn't seem sensible to keep in his information set an unrelated passion for gardening or painting; even less his own home affairs linked with neither his investments nor his trading behavior[7]. Even worse, we could not draw a distinction between useful information and garbage[8].

d) It is usually assumed that information sets fulfill a **nesting property**: prior information sets are contained in the current one. In other words: past information is not lost. **Ω(t ; k)** includes, for instance, the information set that existed two periods before, **Ω(t-2 ; k).** It goes without saying that most of the old information sets might have been as incomplete as the current one, mastering the economic actor only a fraction of the available information at each date.

Turning now to the most representative actors who trade upon their underlying information sets, we have to bear in mind that in most commodities exchanges and capital markets since the XIX century[9] at least, trading has required at least three parties:

the buyer,

the seller,

and the intermediary.

Most of the time, counterparts agree to bind themselves to commitments which can be as simple as an implicit contract (for instance, any consumer shopping for groceries), or an explicit and complex one like the one binding a corporation with its newly appointed Chief Executive Officer[10].

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7 More background in Apreda (2001)

8 By all means, it is bounded rationality that lies at the root of this constraint.

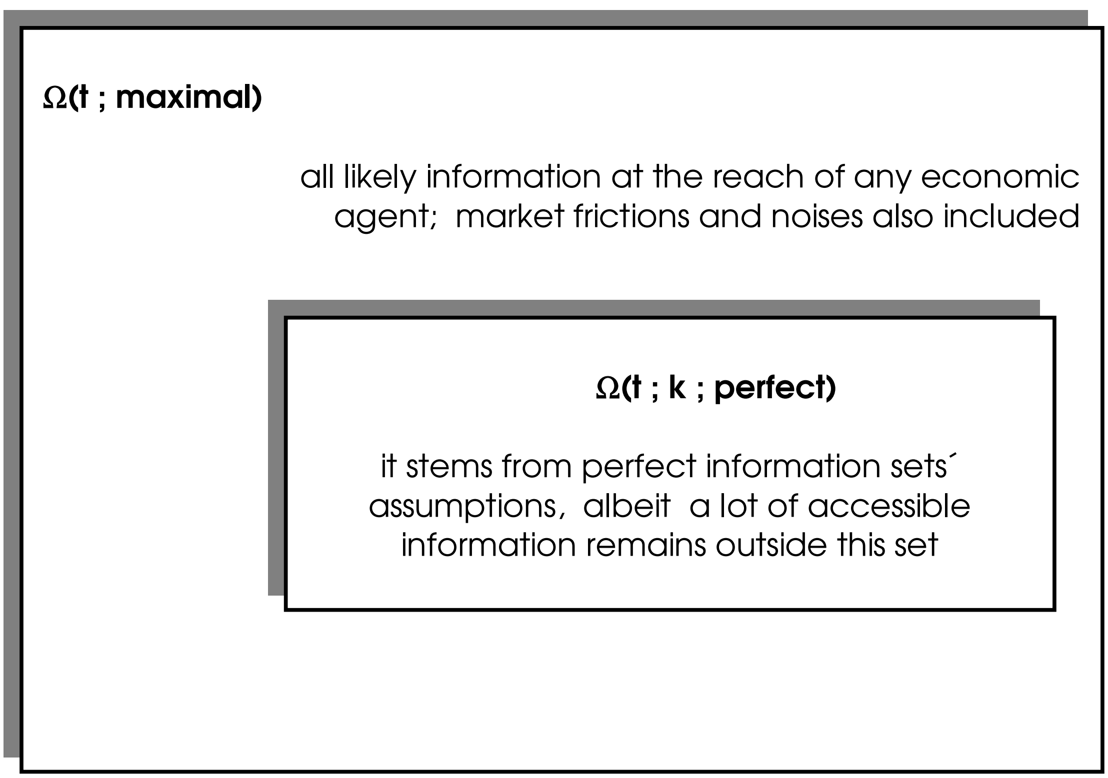
9 Capital markets, at least with their nowadays features, were already established by 1850, in spite of wide gaps with our current information technology, financial engineering and regulatory frameworks. However, they are much older institutions and Meier Kohn (1999) gives a surprising account of capital markets prior to 1600.

10 An appointment of this sort may uncover a complex spadework, starting with a professional search process with screening and tight competition, being followed by a detailed and costly contract design, ending with the engineering of compensation schemes.

Therefore, as we approach real world transactions, either transient or permanent ones, information sets not only store the records about the things to be traded, but also how trade is to be performed, inclusive of the agents' needs and those expectations closely connected with the whole process.

Although it is usually said that in a frictionless market any agent gets access to all available information, such a statement seems misplaced. In fact, transaction costs, (even taxes), credit risk, bankruptcy chances, agency problems and information asymmetries are utterly ruled out. Even worse, such an ideal world does not take intermediaries into account. In fact, neoclassical analysis kept the whole issue of information sets as a given notion in the dark, oblivious of any costly and non-shared information[11].

However, in real markets a lot of relevant information lies outside this ideal perfect world. It is for picture 1 to translate this fact.



Picture 1 : The Perfect and the Maximal Information Sets

[11] The first two chapters of our latest book "Capital Markets, Portfolio Management and Corporate Governance" may be useful to the interested reader (Apreda, 2005a).

The universal set, as mathematical Set Theory requires[12], would be

$$\Omega(\mathbf{t}\ ;\ \mathbf{maximal})$$

that reads "the maximal set of information available to all economic agents, at date **t**". It certainly includes the ideal perfect information set, beyond which we can find for instance transaction costs, taxes, market microstructure issues, and private information that might be held back because of opportunistic behavior. It contains all information available to all economic agents, either useful or useless.

2.1 IMPERFECT INFORMATION SETS

Whereas we would be the last to deny the importance of the perfect market paradigm as a theoretical benchmark, when we come down to single and observable markets, however, imperfect information and inefficient markets are not only the rule but also a fact of life[13].

Indeed, the economic agent **k** gets access to an information set which is rather a subset of the maximal information set at date **t** (see picture 2),

$$\Omega(\mathbf{t}\ ;\ \mathbf{perfect}) \subseteq \Omega(\mathbf{t}\ ;\ \mathbf{k}\ ;\ \mathbf{imperfect}) \subseteq \Omega(\mathbf{t}\ ;\ \mathbf{maximal})$$

Not to be surprised, the perfect set is the minimal one, contained in any imperfect set, because it is available to any economic agent free of charge (costlessly and immediately).

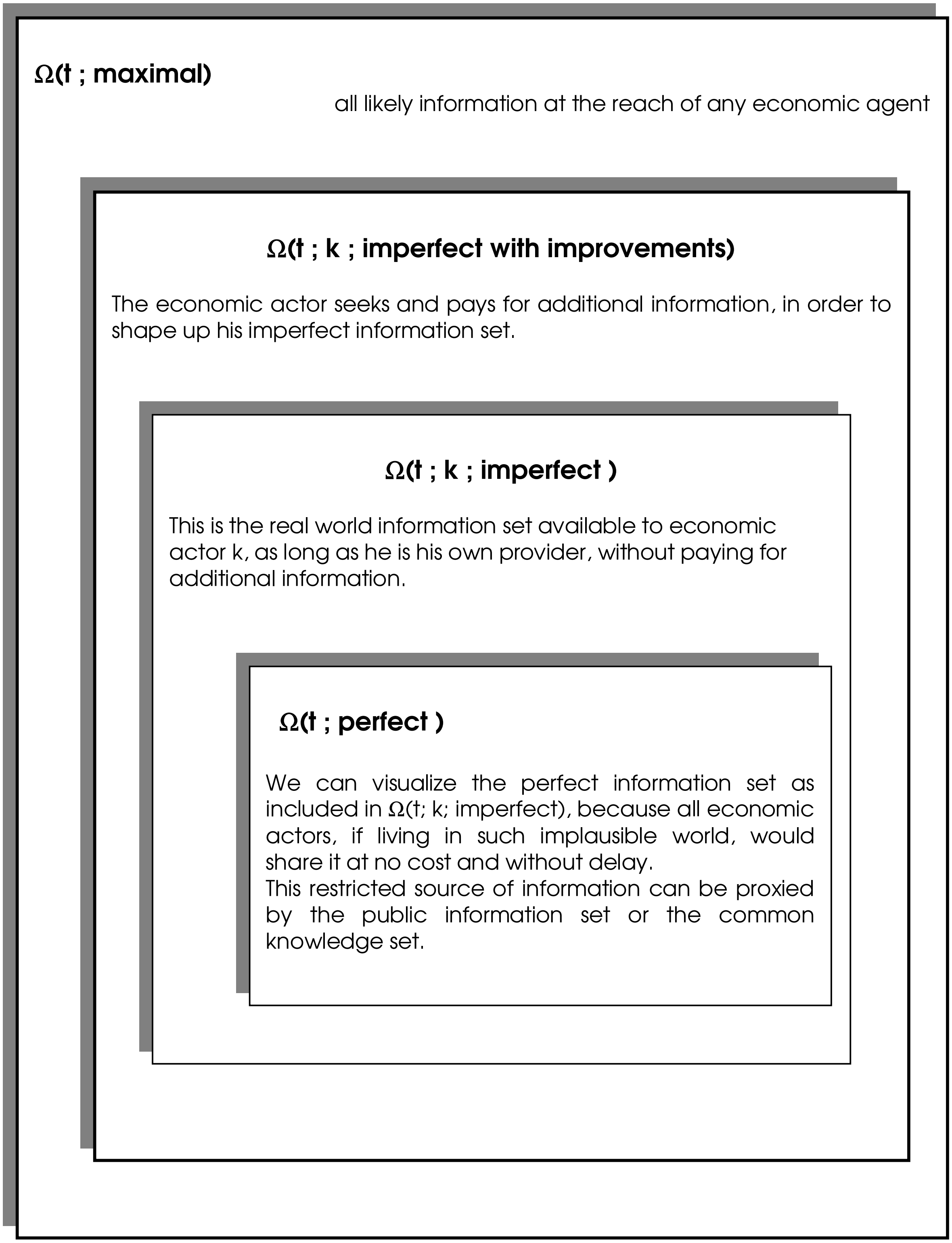
As from now and for ease of notation, any imperfect information set will be denoted

$$\Omega(\mathbf{t}\ ;\ \mathbf{k}\ )$$

whenever this will be clear from the context. Otherwise, we will resort to add qualifications by means of vectorial notation, as when we wrote above Ω**(t ; k ; imperfect)**.

[12] In a mathematical setting, this would usually be denoted by U or X (the universal set), which comes as the precondition upon which all other sets are well-defined.

[13] If markets were efficient in Fama's sense, information sets would be identical for all economic agents. In this case, prices could convey all available information instantly and costlessly (Fama, 1970, 1990), to the extent of making intermediaries redundant. But this is not granted in the real world and, therefore, inefficient markets have become a worthy field of study (Shleifer, 2000). By contrasting an ideal economy where there is perfect information with another one endowed with imperfect information only, Goldman and Sosin (1979) established a simple and interesting measure of market inefficiency.



Picture 2 : Imperfect Information Sets and improvements

Imperfect information sets exhibit some remarkable properties:

(a) Different agents have different information sets.

(b) **Ω(t ; perfect)** is the common ground knowledge they would attain in the realm of a frictionless world with perfect markets and symmetric information.

(c) Each agent can shape up his own information set by adding better or new information at the moment **t**. A foremost source of information comes out of market regulations on disclosure of information. Besides, there has grown an information industry in financial markets, trading on data assembly and securities analysis, fostered by the financial press, government agencies, business schools, executives training programs, academic or institutional research centers.

For instance, by buying such information from an expert or intermediary, the agent **k** could enhance her efficiency at valuation and forecasting. But this is contingent upon the agent's willingness to budget for new and better information, or to preclude her from doing it.

(d) Any time the information set is improved, the new one not only contains the older, but it is also contained in the limit set. That is to say:

$$\Omega(t;\, k;\, \text{imperfect}) \subseteq \Omega(t;\, k;\, \text{imperfect with improvements}) \subseteq$$

$$\subseteq \Omega(t;\, \text{maximal})$$

(e) The imperfect information set features the property of **conditional accountability**: the agent could give account of its contents, although he may be willing to disclose most of them hardly ever.

(f) Changes in information definitely trigger off arbitrage opportunities and market adjustments.

Hence, improvements in the agent´s information at moment **t** could come from the outside, mainly by acquisition of private or professional information at his own cost, although sometimes he may pursue this process by taking advantage of inside information supplied by a third party at the risk of flouting the rules, or even breaking the law.

Markets that run at variance with the assumptions of the standard paradigm are labeled ***IMPERFECT MARKETS***[14]***.***

If we wondered what is really at stake when trading or brokering, then we should stress the duality of this process[15].

Definition 3 The Duality of any Trade

Whenever two parties carry out any trade and bring it to completion, two processes are at play simultaneously:

a) there is a swap of property rights;

b) but there is also an exchange and bargaining of asymmetric information.

3. HOW TRADE SPLITS UP INFORMATION SETS

If we take a look at picture 3, we see the three information sets partitioned on their own. For instance, $\Omega(t\,;k)$ can be explained by four components:

$$\Omega(t\,;k) = (\Omega(t;k) \cap \Omega^C(t;s) \cap \Omega^C(t;l)) \cup (\Omega(t;k) \cap \Omega(t;s) \cap \Omega^C(t;l)) \cup$$

$$\cup\, (\Omega(t;k) \cap \Omega^C(t;s) \cap \Omega(t;l)) \cup (\Omega(t;k) \cap \Omega(t;s) \cap \Omega(t;l))$$

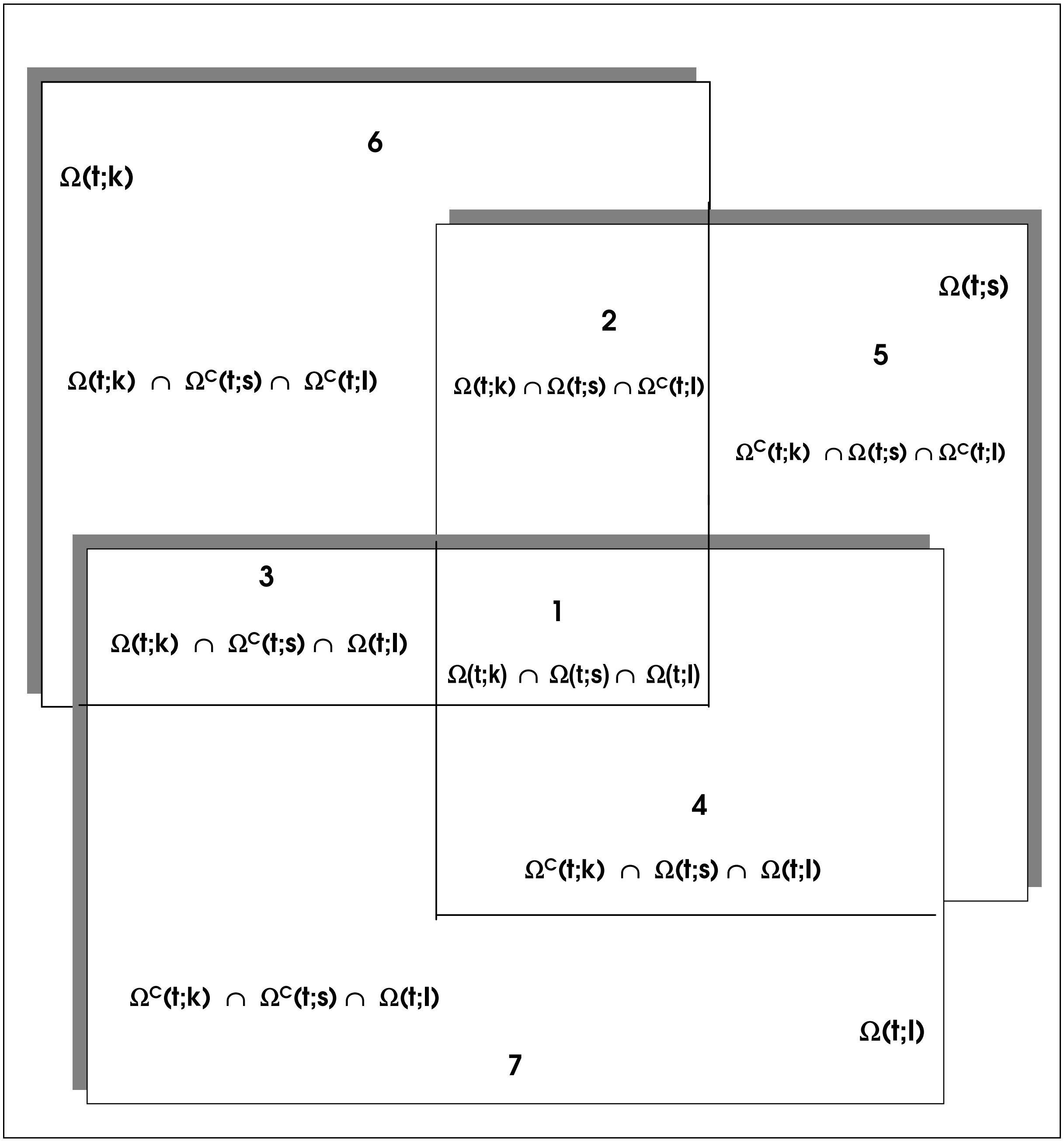
On the other hand, the union of the three main information sets gives rise to seven mutually exclusive and exhaustive subsets. We had laid down the foundations of this statement by means of lemmas 1 and 2, in section 1.

Hence, the economic actors **k** and **s** trade with the intermediary **i**, and asymmetric information pervades their relationships to the extent of bringing about a full partition of their information sets. We are going to set up mutually exclusive environments, in order to describe this partition[16].

[14] It is because of imperfect information, division of labor and specialization that these markets call for intermediaries and institutional environments (Spulber, 1999).

[15] This could hold in explicit or implicit way. What agent X sells is not only a good (either merchandise, service, or a financial) but, in the first place, underlying property rights attached to the good and, in the second place, the extent to which information pertaining the good or the actual transaction is actually exchanged, hidden, even misrepresented.

[16] This method provides what it could be denoted the geometric topology of information sets.



Picture 3 : Partitioning information sets: dealers as brokers of asymmetric Information

Environment 1
Pooling Information Sets

When agents **k** and **s** trade with the dealer, both of them may wish to buy (or sell) him a financial asset. Perhaps they are willing to take different positions, one of them going long while the other short.

Anyway, for the trade to be effective, all participants carry on their purposes starting from the common ground information set that it will be named subset 1 and defined by

$$\Omega(t;k) \cap \Omega(t;s) \cap \Omega(t;i)$$

By the same token, when a single economic actor **k** trades with an intermediary both pool their information sets, to the extent of sharing information so as to round off the exchange. That is to say:

$$\Omega(t;k) \cap \Omega(t;l)$$

which is the outcome of adding sets 3 and 1.

From picture 3, we can get a grasp that pooling doesn't mean losing all the private information endowment each economic actor is entitled to.

Environment 2
Private knowledge on the side of both agents

Next, let us now suppose that both agents share some sort of information not easily accessible to the intermediary. This could be the case when investors are conversant about restrictions to their portfolios set up by regulators, or profit from inside information (perhaps they have knowledge of an impending change in the company whose securities they are interested in).

Another example can be drawn from two traders who believe they have a compelling rationale to buy or sell certain financial asset, disregarding fundamental values[17]. Furthermore, let imagine that both traders (perhaps chartists) are willing to overcome the dealer and bring him into a loss.

Where is such information to be located? It is embedded in subset 2:

$$\Omega(t;k) \cap \Omega(t;s) \cap \Omega^{C}(t;i)$$

More precisely, we should write

$$\Omega(t;k) \cap \Omega(t;s) \cap \Omega^{C}(t;i) =$$

[17] In some current models, they would be labeled non-informed traders, which is rather a misleading expression. Both are informed actually, although their knowledge is at variance with the information claimed by those that reach their decision-making on the grounds of fundamental values.

$$= \{ x \in \Omega(t\,;\,\text{maximal}):\ x \in \Omega(t\,;\,s) \text{ and } x \in \Omega(t\,;\,k) \text{ and } x \notin \Omega(t\,;\,i) \}$$

Environment 3
Private knowledge on the side of the dealer

By the same token, let us rephrase the former environment so as to make the dealer the owner of some private information that allows him to overwhelm those economic agents that come to him asking to sell securities because of liquidity or portfolio rebalancing targets. Being knowledgeable, the dealer will be able to buy assets paying less than otherwise (that is to say, sellers would perform like liquidity-traders, as they are referred in some current models). The distinctive information subset 7 that grants the dealer with such an advantage is defined as

$$\Omega^{c}(t\,;\,k) \cap \Omega^{c}(t\,;\,s) \cap \Omega(t\,;\,i)$$

Environment 4
Exclusive private information

This is the usual environment where the most customary adverse selection problems arise. Let us assume that

$$\Omega(t\,;\,k)$$

is the information set of a corporation which moves towards a public offering in the capital market for either a forthcoming bond or stock of their own. A plausible dealer could be an investment bank,

$$\Omega(t\,;\,i)$$

while

$$\Omega(t\,;\,s)$$

stands for any broker acting on behalf of its customers and ready to place their purchasing orders. For the company, opportunistic behavior is fostered by subset 6:

$$\Omega(t\,;\,k) \cap \Omega^{c}(t\,;\,s) \cap \Omega^{c}(t\,;\,i)$$

The broker conveys private information and financial secrecy when dealing on his customers' account, hence resorting to information subset 5:

$$\Omega^{c}(t\,;\,k) \cap \Omega(t\,;\,s) \cap \Omega^{c}(t\,;\,i)$$

In actual practice, perhaps the broker is submitting significant buying orders from institutional investors, or he is covering up a transaction in the market for corporate control.

Therefore, any information set holds a subset that is privy to each actor. That is to say, agent **k** hoards some information that he keeps under wraps, and the same holds true for agent **s**. It goes without saying that the intermediary also profits from his private knowledge, as we saw in environment 3 that led to subset 7.

At this juncture, we must be aware that two opposing and driving actors are at play: informed and uninformed agents. Informed agents may have better information than the intermediaries about the value of the asset. For instance, they may know the asset is undervalued at the ask price, or overvalued at the bid price. As they go on trading, it is the intermediary who makes a loss. On the other hand, uninformed agents trade for liquidity. It is within such dynamic contest that intermediaries manage their bid-ask spread so as to balance the losses from trading with informed investors by means of profits made with uninformed investors.

Another development, on the side of speculators, is worthy of being noticed. Speculators can distort prices by introducing "misinformation" into a market. By so acting, speculators behave not as more rational traders do; it is said that they behave irrationally[18]. Hence, they buy or sell securities without taking into account midstream information. They become "noise traders", mistakenly following irrational information as it was fundamental or rational information. They hardly meet the passive investment strategies that the Efficient Markets Model would forecast for traders with poor or none information.

But if "irrational traders" become a greater crowd, they may overwhelm rational traders and bring about price distortions. They can be misinformed but not stupid at coping with transaction prices or price changes. What is at stake here is nothing less than the grabbing of informational rents and the arbitrage against information sets.

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[18] "Rational and irrational traders" seems a misleading phrase, to say the least. Both agents are rational most of the time (we dare say they are extremely rational). But they do not follow the same type of rationality. One keeps himself within the common knowledge set, whereas the other contests this information set, even by resorting to a paradoxical logic.

Environment 5
An agent shares information with the intermediary but not with the other agent

This environment comes to be the customary concern, as far as bilateral negotiations are at stake. The dealer can improve his relationship with agent **k** because both of them may share information not accessible to agent **s** by means of

$$\Omega(t\,;k) \cap \Omega^{C}(t\,;s) \cap \Omega(t\,;i)$$

which leads to subset 3 that could be seen as information production on behalf of agent **k**, and it comes at a cost for him. The dealer can claim this service through a higher fee than the one he would charge otherwise, by only using the common knowledge set.

By the same token, the dealer can improve his relationship with agent **s** because both of them may share information not easily accessible for agent **k**, as depicted in subset 4:

$$\Omega^{C}(t\,;k) \cap \Omega(t\,;s) \cap \Omega(t\,;i)$$

4. FRAMING THE BROKERAGE OF ASYMMETRIC INFORMATION

For the time being, let us assume that the information set claimed by a decision-maker **k** at date **t**

$$\Omega(t\,;k)$$

allows him to handle a rate of return linked to a distinctive economic variable[19]

that comes defined along the investment horizon **H = (t ; T)**, contingent upon the stated information set. If we knew that we can assess another information set $\Omega_1(t\,;k)$ such that it holds

$$\Omega_1(t\,;k) \subseteq \Omega(t\,;k)$$

which explains reasonably well the value of a rate of return

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[19] In the case of a financial asset, the variable is price (or some proxy of it); therefore we mean here the rate of return of that financial asset. But there are other natural examples, as in the foreign exchange market when dealing with the appreciation or depreciation of the foreign currency against the domestic one. Also we could point to the rate that measures the change undergone by certain index.

$$\mathbf{r_1 (t, T, \Omega_1 (t ; k))}$$

then we could attempt to measure how much of the starting rate remains unexplained by this new one. This is the logic behind the complementary notions of differential rate of return and residual information set. We are going to provide a simplified definition of both concepts, good enough for the purposes of this paper. But the reader should keep in mind that when we try to deal with more than two differential rates and residual information sets, the mathematics of the subject become more intricate and we have to deal with algebras of sets, a complete development of which we have provided elsewhere (Apreda, 2000a, 2004).

Definition 4 Differential Rates and Residual Information Sets

The rate of return ***g(.)*** *that solves the following equation*

$$(1 + r(t, T, \Omega(t ; k))) =$$

$$= (1 + g(t, T, \Omega_R (t ; k))) . (1 + r_1 (t, T, \Omega_1 (t ; k)))$$

is called the ***differential rate*** *of* ***r(.)*** *given* $\mathbf{r_1 (.)}$ *, whereas the underlying information set for this rate*

$$\Omega_R (t ; k)$$

gets the label of ***residual****.*

At this juncture, let us assume that agent **k** is a seller of security ABC, for example, and agent **s** is a prospective buyer of it. A dealer could handle the transaction outright as a round-off exchange. If there were not an agent **s** available at the moment, it would be for the dealer to provide liquidity to agent **k**, buying the security and holding it in stock. This is a usual way of trading, rather mechanical.

But let us regard a more complex setting, as when big blocks of bonds or shares are at stake, also when new private or public placements are the concern of an agent **k**, and let us assume that agent **s** is a portfolio arbitrageur. In this case, instead of designing a spread on the grounds of a common knowledge set

$$\Omega(t ; \text{common knowledge}) = \Omega(t ; k) \cap \Omega(t ; s) \cap \Omega(t ; i)$$

that stems from the following relationship

$$\mathbf{P(asked, \Omega(t ; common\ knowledge)) =}$$

$$\mathbf{= P(bid, \Omega(t ; common\ knowledge)) \ . \ (1 + spread(common\ knowledge))}$$

the dealer engineers a truly distinctive information structure because he knows that both economic actors would be taking advantage not only of the common knowledge subset, but also of their more hidden information subsets:

Agent **k**:

$$(\Omega(t ; k) \cap \Omega^C (t ; s) \cap \Omega^C (t ; i)) \cup (\Omega(t ; k) \cap \Omega(t ; s) \cap \Omega(t ; i))$$

Agent **s**:

$$(\Omega^C (t ; k) \cap \Omega(t ; s) \cap \Omega^C (t ; I)) \cup (\Omega(t ; k) \cap \Omega(t ; s) \cap \Omega(t ; i))$$

so the actual spread the dealer is going to set up will be grounded on his own enhanced information structure, so as to hedge his informational risk:

$$\Omega(t ; common\ knowledge) \cup \Omega(t ; private\ knowledge ; i)$$

where

$$\Omega(t ; private\ knowledge ; i) = (\Omega(t ; k) \cap \Omega^C (t ; s) \cap \Omega (t ; i)) \cup$$

$$\cup (\Omega(t ; k)^C \cap \Omega(t ; s)^C \cap \Omega(t ; i)) \cup (\Omega^C (t ; k) \cap \Omega(t ; s) \cap \Omega (t ; i))$$

that means that agent **k** may accept a bid price less than the bid price assessed with only the common knowledge set. On the other hand, agent **s** will accept an ask price higher than the ask price that would have obtained through the common knowledge set. Therefore, the final spread is wider than otherwise, because it has impounded a differential rate of return the dealer charges because of his brokerage of asymmetric information. Mathematically, this runs as follows:

Lemma 3 ***The final spread the dealer charges on the grounds of the common knowledge and his private information can be broken down by means of the following relationship***

$$(1 + spread) = (1 + spread (t ; bid)) \ .$$

$$. \ (1 + spread (t ; asked)) \ . \ (1 + spread(common\ knowledge))$$

Proof:

The dealer charges an implicit spread to both legs of the transaction:

c) additional spread on the buying side

$$\mathbf{P(bid,\ \Omega(\ t\ ;\ common\ knowledge\ ))\ =}$$

$$\mathbf{=\ P(bid,\ \Omega(\ t\ ;\ private\ knowledge\ ;\ I\ ))\ .\ (\ 1\ +\ spread\ (t\ ;\ bid)\ )}$$

d) additional spread on the selling side

$$\mathbf{P(asked,\ \Omega(\ t\ ;\ private\ knowledge\ ;\ i\ ))\ =} \tag{6}$$

$$\mathbf{=\ P(asked,\ \Omega(\ t\ ;\ common\ knowledge))\ .\ (\ 1\ +\ spread\ (t\ ;\ asked)\ )}$$

Now, recalling that with only the common knowledge set the spread comes out of

$$\mathbf{P(asked,\ \Omega(\ t\ ;\ common\ knowledge\ ))\ =} \tag{7}$$

$$\mathbf{=\ P(bid,\ \Omega(\ t\ ;\ common\ knowledge\ ))\ .\ (\ 1 + spread(\ common\ knowledge\ )\ )}$$

if we multiply both sides of (7) by

$$\mathbf{(\ 1\ +\ spread\ (t\ ;\ asked)\ )}$$

it holds that

$$\mathbf{P(asked,\ \Omega(\ t\ ;\ common\ knowledge))\ .\ (\ 1\ +\ spread\ (t\ ;\ asked)\ )\ =}$$

$$\mathbf{=\ P(bid,\ \Omega(\ t\ ;\ common\ knowledge))\ .\ (\ 1\ +\ spread\ (t\ ;\ asked)\ )\ .}$$

$$\mathbf{.\ (\ 1\ +\ spread\ (t\ ;\ common\ knowledge)\ )}$$

and by using (6) and (7) we get

$$\mathbf{P(asked,\ \Omega(\ t\ ;\ private\ knowledge\ ;\ i))\ =}$$

$$\mathbf{=\ P(bid,\ \Omega(\ t\ ;\ private\ knowledge\ ;\ i))\ .\ (\ 1\ +\ spread\ (t\ ;\ bid)\ )\ .}$$

$$\mathbf{.\ (\ 1\ +\ spread\ (t\ ;\ asked)\ )\ .\ (\ 1 + spread(common\ knowledge)\ )}$$

Now we can work out the final spread the dealer charges from using both the common knowledge subset and the private one.

$$\mathbf{(1 + spread) = (1 + spread\ (t\ ;\ bid)) \ .}$$

$$\mathbf{.\ (1 + spread\ (t\ ;\ asked))\ .\ (1 + spread(common\ knowledge))}\ \text{< END >}$$

5. TRADING ENVIRONMENTS UNDER OPPORTUNISTIC BEHAVIOR

In this section, we are going to approach our subject from another viewpoint. Instead of stressing asymmetric information, we will focus on opportunistic behavior. In fact, it performs as a complementary line of analysis, since asymmetric information does not necessarily imply opportunistic behavior, albeit the latter almost always matches up the former.

Let us assume two traders, S and B, the former wanting to sell asset A, the latter wanting to buy it. The information sets available at date **t** are the following:

$$\Omega^{S} = \Omega\ \text{(common knowledge)} \cup \Omega^{S}\ \text{(private, technical)} \cup$$

$$\cup\ \Omega^{S}\ \text{(private, opportunistic)}$$

$$\Omega^{B} = \Omega\ \text{(common knowledge)} \cup \Omega^{B}\ \text{(private, technical)} \cup$$

$$\cup\ \Omega^{B}\ \text{(private, opportunistic)}$$

For ease of notation, we are going to rewrite these sets as:

$$\Omega^{S} = \Omega\ (c\ k) \cup \Omega^{S}\ (p, tech) \cup \Omega^{S}\ (p, opp)$$

$$\Omega^{B} = \Omega\ (c\ k) \cup \Omega^{B}\ (p, tech) \cup \Omega^{B}\ (p, opp)$$

There are some environments that come in handy at this stage to illustrate how residual information sets are naturally involved with the brokerage of asymmetric information.

Environment 1
Seller and buyer agree on the underlying value and close the deal in what we can term a fair exchange.

In the language of portfolio managers, for that asset would not be arbitrage opportunities from which to reap any extraordinary rent. In this case, both traders reach to almost the same expected return for the asset:

$$1 + E(r(A; \Omega^{S})) = 1 + E(r(A; \Omega^{B}))$$

and the analysis for each side of the exchange would show that the common knowledge set was the main one in nurturing that return assessment. In other words,

$$\Omega^{S} = \Omega (c\ k) = \Omega^{B}$$

This would be the outcome in a very competitive market, fully arbitraged, close to the paradigm of efficient markets in Fama's sense (bear in mind, however, that nothing grants that this setting might be suitable for other assets traded in the market at that moment).

Environment 2
The seller fixes a price that is higher than the fundamental value. The buyer, however, signs out the deal in spite of the overvaluation.

This is a situation for which there are manifold explanations: for instance, the superior technical knowledge on the side of the seller or the relative ignorance of the buyer. It is also typical of the smart-money trader against a noise trader, both of them with conflicting assessments of the asset returns. Finally, the seller may misrepresent the features of the trade with guile, by taking advantage of superior information and opportunistic behavior. Let us discuss two clear-cut examples.

- Scenario 1: There is no opportunistic behavior.

The relevant information set for the seller, seems to be

$$\Omega^{S} = \Omega (c\ k) \cup \Omega^{S} (p, tech)$$

and the final assessment is conveyed by differential rates and residual information sets this way:

(8)

$$1 + E(r(A; \Omega^{S})) =$$

$$= \langle 1 + E(r(A; \Omega (c\ k))) \rangle \ . \ \langle 1 + E(r(A; \Omega^{S} (p, tech))) \rangle$$

In this setting, the buyer is a price-taker. And the only residual information set that comes in handy for him is the common knowledge one from which his own assessment leads to

$$\mathbf{1 + E(r(A; \Omega^{B})) = < 1 + E(r(A; \Omega (c\ k))) >}$$

As they go on with the deal, the seller is charging the buyer with a higher price than the one the buyer would have wished to meet[20]. Hence, the buyer's expected return is lessened to the extent of the seller's mark-up who proceeds to grab an arbitrage profit over his own assessment of the expected return of the asset. The differential rate that measures up this grabbing will be denoted by g(S).

$$\mathbf{1 + E(r(A; \Omega^{S})) = < 1 + E(r(A; \Omega^{B})) > . < 1 + g(S) >} \qquad (9)$$

What this relationship reports is that the seller seizes upon the underlying price from a fair valuation model gross of transaction costs, and also benefits from an arbitrage gap that gives account of his superior information and power at setting up the final price. That is to say, his actual return comes out of

$$\mathbf{1 + E(r(A; \Omega^{S}))}$$

whereas the consequence for the buyer is to pay more, but only profit from

$$\mathbf{< 1 + E(r(A; \Omega^{B})) >}$$

If we now look at (8) and (9), it doesn't come as a surprise that

$$\mathbf{< 1 + g(S) > = < 1 + E(r(A; \Omega^{S} (p, tech))) >}$$

Last of all, and taking a step further in the argument, we could assume that when B and S worked out the "break-even" expected return provided by the common knowledge set, what they actually intended to do was to blend a forecast from valuation models with a transaction costs rate. Let us delve into this process with greater detail.

Firstly, by means of a valuation model they should have arrived to

[20] The buyer pays more because he agrees about the fair return provided by

$$\mathbf{E(r(A; \Omega (c\ k)))}$$

which amounts to a lesser than expected return, hence a higher price than otherwise.

$$\textbf{< 1 + E(r(A; } \Omega \textbf{ (valuation model))) >}$$

where the underlying information set, that in most equilibrium models is predicated on perfect and symmetric information, is a subset of the common knowledge set.

Secondly, they should have made use of a transaction cost rate[21], for instance the following one

$$< 1 + TC (t, T, \Omega^{TC}{}_t) > =$$

$$< 1 + INT (t, T, \Omega^{INT}{}_t) > . < 1 + MICR(t, T, \Omega^{MICR}{}_t) > .$$

$$. < 1 + TAX (t, T, \Omega^{TAX}{}_t) > . < 1 + INF (t, T, \Omega^{INF}{}_t) > . < 1 + FIN (t, T, \Omega^{FIN}{}_t) >$$

with the restriction

$$\Omega^{k}{}_t \subseteq \Omega^{TC}{}_t \quad \text{for } k: \text{ INT, MICR, INF, FIN, TAX}^{22}$$

Finally, they would have worked out the expected return gross of enlarged transaction costs

$$< 1 + E(r(A; \Omega \text{ (c k)})) > =$$

$$= < 1 + E(r(A; \Omega \text{ (valuation model)})) > . < 1 + TC (t, T, \Omega^{TC}{}_t) >$$

- Scenario 2: Opportunistic behavior is added to superior technical information.

Following the same line of reasoning, the seller's information set now would turn out to be

$$\Omega^{S} = \Omega \text{ (c k)} \cup \Omega^{S} \text{ (p, tech)} \cup \Omega^{S} \text{ (p, opp)}$$

and when both parties cut the deal the expected returns for them would be depicted as

$$1 + E(r(A; \Omega^{S})) = < 1 + E(r(A; \Omega^{B})) > . < 1 + g(S) >$$

[21] This multiplicative model is suitable for a transaction cost structure and has been fully developed in Apreda (2000a, 2000b, 2004)

[22] Int stands for intermediation, micr for microstructure, inf for information, fin for financial, and tax for taxes.

but now the seller's expected return would be greater than in the former scenario and the arbitrage gap richer than before, amounting to:

$$< 1 + g(S) > =$$

$$= < 1 + E(r(A; \Omega^S (p, tech))) > . < 1 + E(r(A; \Omega^S (p, opp))) >$$

CONCLUSIONS

We have argued that trade splits up the information sets of the economic actors who involve each other in such process, laying the foundations of this point of view, firstly, by means of partitions of the underlying sets in the first place, and secondly, by giving heed to imperfect information sets from which asymmetric information stems from.

Next, we defined the dual nature of any trade, which led to regard the role the intermediary performs as one of broker of asymmetric information. At departure from generally held views, we have shown that intermediaries pursue their trade charging differential rates of return to their counterparts, rates that come defined over residual information sets. In this virtual exchange they are able to enhance the market efficiency, but also to follow opportunistic and rent-seeking behavior[23]. We also proved that the final spread the dealer marks up eventually, can be broken down into spreads from the bide and ask side of the transaction, as well as the spread that comes from common knowledge.

In the last part of the paper, we dealt with trading environments under opportunistic behavior. Here information sets were split up into three components: common, technical and opportunistic knowledge. The groundwork was done through two environments. In the first one, seller and buyer agree on the underlying value and close the deal in what we can term a fair exchange. In the second, the seller fixes a price that is higher than the fundamental value and the buyer signs out the deal in spite of the overvaluation. In the latter environment we distinguished two scenarios, one with no opportunistic behavior, the other including such behavior and superior technical information.

[23] On this issue, a line of research leading to corporate governance has recently been presented by Apreda (2005b).

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