Área: Finanzas

SHAPING UP THE COMPANY’S INTERNAL INVESTMENT FUND THROUGH SEPARATION PORTFOLIOS

Rodolfo Apreda

Febrero 2010
Nro. 416
SHAPING UP THE COMPANY’S INTERNAL INVESTMENT FUND THROUGH SEPARATION PORTFOLIOS

Rodolfo APREDA
ra@cema.edu.ar

Professor Apreda holds a Ph. D in Economics (University of Buenos Aires) and a Master in Political Sciences (University of Cema). He is Director of the Ph D Program in Finance, and Director of the Center for the Study of Private and Public Governance, at the University of Cema.
Mail address: Av Cordoba 374, Room 612, C1054AAP, Buenos Aires, Argentina.
E-Mail address: ra@cema.edu.ar. Personal Web Page: www.cema.edu.ar/u/ra
ABSTRACT

This research paper sets forth that an alternative for managing the internal investment fund of any company, lies on separation portfolios. Firstly, the company’s internal investment portfolio is built up within the context of the incremental cash-flow model. Next, separation portfolios are introduced and consequential features for this paper are predicated upon them: firstly, they provide an easier framework for risk-management; secondly, their risk-return profile bring about a down-to-earth performance benchmark. Afterwards, the internal investment portfolio is mapped out like a distinctive separation portfolio. Lastly, pragmatic consequences and some corporate governance advantages of this financial engineering will follow.

**JEL codes:** G11, G34, G32

**Key Words:** separation portfolios, portfolio management, incremental cash-flow model, corporate governance, internal investment fund, risk metrics.

**Institutional Disclaimer**
Statements or opinions conveyed in this paper are attributable to the author only, and the University of Cema disclaims any responsibility for them.
INTRODUCTION

Idle cash balances often raise the issue of whether to manage or not a portfolio of financial assets, from which we could meet transactional, investment or hoarding targets, on the one hand, and solvency margins\(^1\) on the other.

Several financial assets qualify for membership in such internal investment fund, which can be broadly classified like current or non-current financials.

*Current financials:* they are securities for which their contractual term-to-maturity date is less than a year\(^2\). For instance,

- Time-deposits
- Repurchase agreements (Repo’s)
- Treasury Bills
- Commercial Papers
- Mutual Funds’ Equity
- Derivative products
- Short-term zero-coupon bonds

*Non-current financials:* they are securities for which their contractual term-to-maturity date will take place beyond a year. For instance,

- Government Treasury Notes and Bonds
- Ordinary stock

---

\(^1\) By solvency margin we understand either cash or short-term financial assets, that a company sets aside in addition to the amount needed for paying current bills, mainly to provide hedging finance to short-term liabilities and contingencies. For banks and institutional dealers, solvency margins are usually strongly regulated.

\(^2\) It is worthy of being remarked that a five-year bond, for instance, when its contractual term-to-maturity date becomes less than a year, it goes assimilated into a current or short-term financial on accountancy grounds.
• Simple bonds and convertible bonds
• Preferred stock
• Convertible preferred stock
• Bonds with Warrants
• Medium- and long-term zero-coupon bonds

Although the list may be larger, the company’s Treasurer ought to follow a golden rule when choosing among them: *buy only those financial assets that grant liquidity and investment grade*.

This paper will unfold the following way:
In section 1, we go into the company’s investment portfolio from the viewpoint not only of stock variables but also of incremental cash flow variables. It is for section 2 to delve into separation portfolios, bringing to light their key risk-management features. Afterwards, in section 3, we show how the company’s internal investment portfolio can be shaped up like a separation portfolio. We close with pragmatic consequences of using separation portfolios.

1. THE COMPANY’S INTERNAL INVESTMENT PORTFOLIO

Firstly, we are going to denote as

\[
\text{INVP}(t)
\]

\[\text{In this context, liquidity will mean that financials could be sold}
\]
\[\text{a) whenever we need,}
\]
\[\text{b) getting lower transaction costs,}
\]
\[\text{c) in the shortest time available.}
\]

\[\text{Hence, the Treasurer should buy only those assets that are publicly placed and for which there is a dynamic secondary market.}
\]

**Investment grade** endows financials the best risk rating, that is to say, they are issued by reliable and solvent companies whose debt might be considered an advisable investment for fiduciaries and institutional investors.
the company’s cash flows committed to the purchasing or selling of current and non-current financial assets, at date \( t \). In point of fact, \( \text{INVP}(t) \) stands for the monetary value of the company’s investment portfolio at such date, and also performs as a stock variable that can be split down into two components:

\[
\text{INVP}(t) = \text{Current Financials} (t) + \text{Non-Current Financials} (t)
\]

Secondly, let us assume that we are planning the company’s cash-flow structure along a horizon \( H = [t; T] \), that is to say, a time span that starts at date \( t \), and ends at date \( T \).

Thirdly, and as from now, we are interested in the so-called incremental cash-flow model⁴. A cash flow is meant to be incremental if it comes into existence along \( H \), neither before nor after, a feature which may be figured out as

\[
\Delta \text{CF INVP} (t; T) = \text{INVP}(T) - \text{INVP}(t)
\]

or, equivalently, and profiting from (1),

\[
\Delta \text{CF INVP} (t; T) = \Delta \text{CF Current Financials} (t; T) + \Delta \text{CF Non-Current Financials} (t; T)
\]

Following Markowitz’s approach to portfolio management⁵, the company’s investment portfolio will come defined as the vector of wealth proportions allotted to

---

⁴ More background about this model in Ross et al. (1995).

⁵ A straightforward introduction to Markowitz’s contributions is put forth in Elton-Gruber (2006).
financial assets available in the market. Let us denote such portfolio \textbf{INVP}, and move on to develop such a construct in more detail.

a) If we regard \textbf{INVP} as a stock variable like in (1), the proportions will be determined as

\begin{equation}
\begin{align*}
x_{CF} &= \frac{\text{Current Financials} (t)}{\text{INVP} (t)} \\
x_{NCF} &= \frac{\text{Non-Current Financials} (t)}{\text{INVP} (t)}
\end{align*}
\end{equation}

which lead to the portfolio:

\[
\text{INVP} = < x_{CF}; x_{NCF} >
\]

b) Alternatively, if we look upon \textbf{INVP} as a flow variable like in (2),

\[
\Delta \text{CF INVP} (t; T)
\]

the proportions will be

\begin{equation}
\begin{align*}
x_{\Delta CF} &= \frac{\Delta \text{CF Current Financials} (t; T)}{\Delta \text{CF INVP} (t; T)} \\
x_{\Delta NCF} &= \frac{\Delta \text{CF Non-Current Financials} (t)}{\Delta \text{CF INVP} (t; T)}
\end{align*}
\end{equation}

to give rise to the incremental cash-flow portfolio structure

\[
\Delta \text{CF INVP} = < x_{\Delta CF}; x_{\Delta NCF} >
\]
2. SEPARATION PORTFOLIOS

It is our contention that separation portfolios\(^6\) may become a powerful vehicle to foster accountability and transparency to the internal investment portfolio of any company\(^7\). Let us define them and expand upon their main properties along this section. A further development including the axiomatic treatment of such portfolios, can be found in Apreda (2009).

**Definition 1**

*By a Separation Portfolio is meant a portfolio*

\[
S = < x_F; x_M >
\]

*such that*

\[
x_F + x_M = 1
\]

*where F stands for a risk-free asset\(^8\) and M for a market-indexed portfolio.*

---

\(^6\) Worthy precedents for this matter can be found in Tobin (1958), Sharpe (1964), Elton-Gruber (1997) and Brennan (1999).

As it has already been argued in An Axiomatic Treatment of Enlarged Separation Portfolios and Treasurer’s Portfolios (Apreda, 2009), the method followed in this paper deals with down-to-earth proxies of the theoretical Capital Market Line (CML). In point of fact, these proxies spring from a concrete risk-free rate and a distinctively available market-index.

\(^7\) Transparency and accountability are deeply engrained with Corporate Governance. On this latter subject, we refer the reader to Apreda (2006, 2005b).

\(^8\) Bear in mind that a financial asset is risk-free when it holds that

\[
E( R(F) ) = R(F)
\]

And this is true if and only if

\[
E( R(F) ) - R(F) = 0
\]

if and only if

\[
s^2 ( F ) = E [ E( R(F) ) - R(F) ]^2 = 0
\]
Among the upsides that separation portfolios bear over other risky portfolios, we must highlight three of them:

a) They are easily affordable since we only need to choose up a risk-free asset and a market asset under the guise of a market index or a matching proxy.

b) They are cheaper; in fact, transactions costs are lower because we have to buy only two distinctive assets to get hold of a separation portfolio.

c) But a far-reaching implication of separation portfolios links to their risk-metric, as next lemma will make it clear.

**Lema 1**

*In separation portfolios, risk management is reduced to the handling of the following relationship:*

\[ \sigma(S) = x_M \cdot \sigma(M) \]  \hspace{1cm} (5)

**Proof:**

For any portfolio out of \( N \) available financial assets, total risk can be translated by the following expression:

\[ \sigma^2(P) = \sum x_j \cdot x_k \cdot \sigma(j; k) \]

where indexes \( j \) and \( k \) takes any and every value between 1 and \( N \).

Firstly, separation portfolios comprise two assets only. Hence, it holds that

\[ \sigma^2(S) = \sum x_j \cdot x_k \cdot \sigma(j; k) \]

\[ \sigma^2(S) = x_F \cdot x_F \cdot \sigma(F; F) + x_F \cdot x_M \cdot \sigma(F; M) + \]


By definition, risk-free assets have a null total risk:

\[ \sigma^2(F) = \sigma(F; F) = \text{cov}(R(F); R(F)) \]

\[ \sigma(F; F) = E[<R(F) - E[R(F)]> <R(F) - E[R(F)]>] = 0 \]

On the other hand, they do not covariate with the market asset:

\[ \sigma(F; M) = \text{cov}(R(F); R(M)) \]

\[ \sigma(F; F) = E[<R(F) - E[R(F)]> <R(M) - E[R(M)]>] = 0 \]

so that

\[ \sigma^2(S) = x_M . x_M . \sigma(M; M) = x_M^2 . \sigma^2(M) \]

Or, equivalently,

\[ \sigma(S) = x_M . \sigma(M) \]

but this is (5).

Remarks on Lemma 1
We wish to point out that Lemma 1 brings forth a very consequential outcome. Let us assume that risk management policies prevent the company’s Treasurer from going beyond certain maximum level of risk

\[ \sigma_{\text{max}} \]
Thus, in order to set up a separation portfolio the following upper-limit to risk must hold:

\[ \sigma(S') = x'_M \cdot \sigma(M) < \sigma_{\text{max}} \]

and the proportion \( x'_M \) to buy in the market portfolio must fulfill

\[ x'_M < \frac{\sigma_{\text{max}}}{\sigma(M)} \]

Moreover, the structure of the separation portfolio will be

\[ S' = < x'_F; x'_M > \]

such that

\[ x'_F + x'_M = 1 \]

In short,

\[ S = < 1 - x'_M; x'_M > \]

It’s worthy of being noticed that separation portfolios entails a rather plain, albeit powerful, “risk-return profile”.

**Definition 2**

*Given a separation portfolio \( S \), by its risk-return profile it is meant the vector*

\[ < \sigma(S); E[R(S)] > \]

Let us delve into the nature of such a profile\(^9\).

---

\(^9\) A deeper insight about the financial engineering of separation portfolios is to be found in Apreda (2006b, 2005b, 2003, 2001a, 2001b). For practitioners’ needs, related sections in Bodie-Kane and Markus (2006) are still very useful.
Lema 2
If S is a separation portfolio, its risk-profile can be translated by the following relationship:

\[
\frac{R(S) - E[R(S)]}{R(M) - E[R(M)]} = \frac{\sigma(S)}{\sigma(M)}
\] (6)

Proof:
a) On date \( t \) (ex-ante basis), we have:

\[
\sigma(S) = x_M \cdot \sigma(M)
\]

\[
E[R(S)] = x_F \cdot R(F) + x_M \cdot E[R(M)]
\]

b) On date \( T \) (ex-post basis), we have

\[
\sigma(S) = x_M \cdot \sigma(M)
\]

\[
R(S) = x_F \cdot R(F) + x_M \cdot R(M)
\]

c) We can single out the premium or surprise in returns along the horizon \( H \), by doing:

\[
R(S) - E[R(S)] = x_M \cdot (R(M) - E[R(M)])
\]

d) Furthermore, and taking advantage of (5), we get:

\[
R(S) - E[R(S)] = \frac{\sigma(S)}{\sigma(M)} \cdot (R(M) - E[R(M)])
\]

e) Hence,
Remarks on Lemma 2

To what extent does relationship (6) come in handy for practitioners or market analysts? This question boils down to risk-metrics and risk-caps.

- Firstly, it links return surprises with the underlying $\sigma$-metrics.

- Secondly, it provides with risk-caps. That is to say, whenever we set forth assessments for $\sigma_{\min}$ and $\sigma_{\max}$ so as to choose a suitable $\sigma(S)$ between both floor and ceiling, we can assess how well or badly the risk premium has performed at the end of the day.

In other words, the risk-cap evolves from

$$\frac{\sigma_{\min}}{\sigma(M)} < \frac{(R(S) - E[R(S)])}{(R(M) - E[R(M)])} < \frac{\sigma_{\max}}{\sigma(M)}$$

or, equivalently,

$$\frac{\sigma_{\min}}{\sigma(M)} < \frac{\sigma(S)}{\sigma(M)} < \frac{\sigma_{\max}}{\sigma(M)}$$

- Thirdly, and it is actually a corollary to Lemma 2, to meet the risk-cap we have to choose the proportion of the market index so that

$$\frac{\sigma_{\min}}{\sigma(M)} < x_M < \frac{\sigma_{\max}}{\sigma(M)}$$

Before closing this section, we wonder whether there would be a direct way to figure out the premium gap (to be denoted as $\text{pg}(S)$) arising out of the expected
return at date $t$, and the actual ex post return of $S$ at date $T$. This involves a yardstick for the risk metric.

**Lemma 3**

*The premium gap ensues from the relationship*

\[
pg(S) = \frac{(R(S) - E[R(S)])}{<1 + E[R(S)]}> / <1 + E[R(S)]>
\]

*Proof:*

By using a multiplicative model of returns\(^{10}\),

\[
<1 + R(S)> = <1 + E[R(S)]> . <1 + pg(S)>
\]

from which we have

\[
<1 + pg(S)> = <1 + R(S)> / <1 + E[R(S)]>
\]

and, lastly,

\[
pg(S) = \frac{(R(S) - E[R(S)])}{<1 + E[R(S)]>}/<1 + E[R(S)]>
\]

**Remarks on Lemma 3**

- For the purposes of budgetary control and report, either to the CEO’s office, the Board of Directors, or the Auditing Committee, Lemma 3 furnishes the latter with a metric to improve accountability, compliance and transparency.

---

\(^{10}\) An additive model would produce

\[
pg(S) = R(S) - E[R(S)]
\]

which is meaningful only when the second-order expression

\[
E[R(S)] . pg(S)
\]

becomes negligible. This is a topic often bypassed in most treatments. Within the context of transactional algebras, it has been enlarged upon by Apreda (2006b).
which are core variables at the interface of Portfolio Management and Corporate Governance\textsuperscript{11}.

\begin{itemize}
  \item \(pg(S) > 0\) signals that the actual return outperformed the forecast, while \(pg(S) > 0\) points to an actual return below the forecast.
\end{itemize}

3. THE COMPANY’S INVESTMENT PORTFOLIO AS A SEPARATION PORTFOLIO

As it was established in section 1, the company’s investment portfolio conveys a dual structure either as a stock (1) or as a flow (2). Starting from monetary stocks,

\begin{equation}
\text{INVP}(t) = \text{Current Financials} (t) + \text{Non-Current Financials} (t)
\end{equation}

whereas the incremental-cash-flow construct amounts to

\begin{equation}
\Delta CF \text{ INVP} (t; T) = \Delta CF \text{ Current Financials} (t; T) + \Delta CF \text{ Non-Current Financials} (t; T)
\end{equation}

At this juncture, let us suppose that the CFO’s primary concern hinges upon two kinds of securities, exclusive of any other:

a) current and non-current risk-free financial assets, like Treasury Bills, bank term deposits, or zero-coupon bonds held till their contractual maturity date;

\textsuperscript{11} See footnote 7.
b) current and non-current financial risky assets\textsuperscript{12} that strongly follow a well-known market index or, directly, purchasing forward a distinctive index.

Bringing this line of argument into sharper view, equation (7) becomes:

\begin{equation}
\text{INVP}(t) = \text{Risk Free Financials}(t) + \text{Market-Indexed Financials}(t)
\end{equation}

whereas equation (8) turns out to be

\begin{equation}
\Delta \text{CF INVP}(t; T) = \Delta \text{CF Risk-Free Financials}(t; T) + \Delta \text{CF Market-Indexed Financials}(t; T)
\end{equation}

Taking advantage of relationships (3), (4), (9) and (10) together, we can just build up separation portfolios, either from a stock or an incremental cash-flow standpoint.

\textbf{a) Stock version of the separation portfolio:}

Defining

\begin{align*}
x_F &= \frac{\text{Risk Free Financials}(t)}{\text{INVP}(t)} \\
x_M &= \frac{\text{Market-Indexed Financials}(t)}{\text{INVP}(t)}
\end{align*}

the portfolio comes to be

\[ \text{INVP} = \langle x_F; x_M \rangle \]

\textsuperscript{12} A healthy constraint on this issue consists in purchasing investment-grade risky assets that convey investment grade and grant liquidity. Footnote 3 adds precision to this remark.
**b) Incremental cash flow version of the separation portfolio:**

Defining

\[
x_{\Delta F} = \Delta CF \text{ Risk Free Financials (t; T) } / \Delta CF \text{ INVP (t; T)}
\]

\[
x_{\Delta M} = \Delta CF \text{ Market-Indexed Financials (t; T) } / \Delta CF \text{ INVP (t; T)}
\]

the portfolio will be given by the vector

\[\Delta CF \text{ INVP } = < x_{\Delta F} ; x_{\Delta M} >\]

There is a last point to notice. In actual practice, instead of a single risk-free asset we come across a portfolio of different risk-free assets, which could raise the question whether such portfolio still remains risk-free on its own. The following lemma shows that there is a positive answer to this query.

**Lemma 4**

If we have a portfolio \( F \)

\[ F = \{ F_1; F_2; F_3; \ldots \ldots ; F_L \} \]

where \( F_k \) is a risk-free asset ( \( k: 1, 2, \ldots, L \) ), then \( F \) is a risk-free portfolio.

**Proof:**

Let us work out the variance of \( F \):

\[
\sigma^2 (F) = \sum x_j \cdot x_k \cdot \sigma(j; k) \quad (j, k: 1, 2, \ldots, L)
\]

but

\[
\sigma(j; k) = \text{cov} (R(F_j); R(F_k))
\]

\[
\sigma(j; k) = E [ < R(F_j) - E[R(F_j)] > \cdot < R(F_k) - E[R(F_k)] > ]
\]
However, all of them are risk-free assets. Therefore,

$$\sigma(j; k) = 0$$

Hence,

$$\sigma^2(F) = 0$$

which makes $F$ a risk-free portfolio. ■

4. PRAGMATIC CONSEQUENCES AND CONCLUSIONS

Whenever a company sets about framing its internal investment fund as a separation portfolio, such a decision-making triggers off manifold pragmatic consequences. Let us give account of the most distinctive among them.

a) By their own nature (see definition 1 and section 3), separate portfolios have a very simple structure, consisting only of risk-free asset and a market portfolio. On these grounds, their composition and performance can be efficiently tracked down. Moreover, they are easily affordable and cheaper to purchase, with lower transaction costs than otherwise.

b) From sections 1 and 3, the management of separation portfolios is embedded into the general framework furnished by the incremental cash flows model. This is not a minor issue, since budgetary control and valuation techniques become more transparent.

c) Lemma 1 brings forth a risk-multiplier which is grounded on the proportion of the market portfolio. By the same token, Lemma 2 provides managers and directors with risk-caps to handle their risk policies, whereas Lemma 3 gives them a benchmark that sets a standard for risk-management.
d) When Financial Statements and the Annual Report account for how the internal investment fund is ultimately run, they disclose material information to both stockholders and creditors. For the latter, a separation portfolio may witness to the fulfillment of debt covenants, including sinking-fund provisions\textsuperscript{13}.

In conclusion: separation portfolios afford the company with a superior investment fund that adds to the company’s governance by all intents and purposes.

REFERENCES


\textsuperscript{13} Sinking-fund provisions and their relationship with Corporate Governance have been dealt with in Apreda (2007). A broader analysis in debt covenants is to be found in Smith and Warner (1979).


