

**UNIVERSIDAD DEL CEMA**  
**Buenos Aires**  
**Argentina**

Serie  
**DOCUMENTOS DE TRABAJO**

**Área: Economía**

**USING THE BOX-COX TRANSFORMATION TO  
APPROXIMATE THE SHAPE OF THE RELATIONSHIP  
BETWEEN CO<sub>2</sub> EMISSIONS AND GDP: A NOTE**

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**Julio 2013**  
**Nro. 513**



USING THE BOX-COX TRANSFORMATION TO APPROXIMATE THE SHAPE OF THE RELATIONSHIP  
BETWEEN CO<sub>2</sub> EMISSIONS AND GDP: A NOTE

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*Resumen:* En base a datos para los países que anunciaron reducciones de sus emisiones de CO<sub>2</sub> bajo el Acuerdo de Copenhague, esta nota apoya la existencia de una relación de largo entre las emisiones de CO<sub>2</sub> y el PIB en 11 de los 26 países de la muestra durante el período 1980-2008. Sin embargo, la especificación funcional de esa relación no es homogénea entre las naciones, siendo lineal para 2 países, log-log para otros 2 casos, y siguiendo la forma funcional Box-Cox para 7 naciones. Las elasticidades emisiones-ingreso también difieren entre países. Pero en la mayoría de los casos (8 de 11), la magnitud de la elasticidad media es menor a 1 (las emisiones aumentan menos que el PIB).

**Palabras claves:** emisiones de CO<sub>2</sub>, desarrollo sostenible, Box-Cox

**Códigos JEL:** Q01, Q54, 044

*Abstract:* With CAIT WRI data for those countries which submitted quantifiable CO<sub>2</sub> emission caps under the Copenhagen Agreement, this note supports the existence of a long run relationship between CO<sub>2</sub> emissions and GDP in 11 of the 26 countries in our sample over the period 1980-2008. However, the functional specification of that relationship is not homogenous among nations, being linear for 2 countries, log-log for 2 other cases, while the relationship follows a Box-Cox functional form for 7 nations. Elasticities of the emissions-income relationship also differ among countries. But in most cases (8 out of 11), the magnitude of the average elasticity is less than 1 (emissions increase less than GDP).

**Key words:** CO<sub>2</sub> emissions, sustainable development, Box-Cox

**JEL Literature:** Q01, Q54, 044

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\* Las opiniones de este trabajo son personales y no representan necesariamente las de la Universidad del CEMA.

## I. Introduction

The international community agrees that, in order to avoid massive damages due to climate, the average increase of global temperature should be kept below 2 degree Celsius with respect to pre industrial levels (Copenhagen Accord, Point 1). To attain the carbon concentration in the atmosphere of 450 parts per million required for the 2 degree goal, developed countries' emissions should decrease by 2020 between 25 and 40% with respect to 1990 levels while developing countries' emissions need to be reduced substantially (Gupta et al, 2007).

Countries seem to understand more clearly the threat of climate change now than in the past. But, the division between developing and developed countries remains. Developing countries historically argue that they have the right to increase emissions in order to meet their development needs since they are not responsible for the GHG (greenhouse gases) concentration levels that resulted from developed countries' economic growth.<sup>1</sup> The analysis of the relationship between emissions and GDP plays an important role in that discussion.

Indeed, a large body of literature emerges around the beginning of the 90s under the name of "Environmental Kuznets curve" (EKC). It has its origin in the works of Grossman y Krueger (1991, 1995), Shafik and Bandyopadhyay (1992) and Panayotou (1993). The idea behind the EKC is that starting from low per capita income levels, emissions per capita tend to increase at a lower rate up to a "turning point", where those emissions begin to decrease as income per capita continues to evolve because of changes in people's waste as well as technological shifts. Those seminal articles are based on panel data or cross-section data for multiple countries and diverse pollutants, and are derived from reduced pollution-income regressions forms.

After "twenty-year fascination with the EKC" Carson (2010, p.3), several authors began to review the theoretical foundations of EKC as well as the empirical weaknesses behind the EKC (Dasgupta et al 2002; Stern 1998, 2004; Dinda 2005; Kijima et al 2010). Two of the main problems pointed out by the critics of the EKC are: 1) time-series problem associated to the data (ordinary least squares regression assumes that all the variables are stationary, so if they are not, further scrutiny is needed) and 2) the functional form used to estimate the relationship between emissions and GDP. Conventional estimations did not consider the time-series properties of the variables chosen and the standard EKC estimation was assumed linear, quadratic or cubic patterns.

To address the first concern, articles began to deal with time-series analysis. Until the late 90s, the EKC literature ignored that environmental degradation and income could be non stationary (their statistics are not constant through time), therefore, the EKC regressions could be spurious, unless the two series were cointegrated. Perman and Stern (2003) add to that analysis the consideration of panel data time series properties. In particular, with sulfur emissions and GDP data for 74 countries along 31 years (from 1960 to 1990), they found that for many of the countries emissions per capita or GDP are non stationary series and a long-run cointegrating relationship between those variables only exists in 35 of the countries, which as a consequence implies that the EKC cannot be estimated for the remaining countries or for the panel as a whole. Similarly, Wang (2013) tests the EKC for SO<sub>2</sub> and CO<sub>2</sub> emissions from 1850 to 1990 for several countries individually and within a panel and find that none of the EKC he estimates for single countries are cointegrated equations.

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<sup>1</sup> GHG gases are of six types, being the carbon dioxide (CO<sub>2</sub>) the most important in term of GHG emissions.

With respect to the second issue, a few articles have provided alternative formulations to the usual functional form specifications. Wang (2013) uses different exponentials for income, from 0 to 2 (0 is the specific case of a linear EKC function and 2 is the case of a quadratic EKC function), Schmalensee et al (1998) and Vollebergh et al (2009) use semi parametric specifications of the B-spline method,<sup>2</sup> while Galeotti et al (2006) estimate a non-linear three parameter Weibull function in order to capture the EKC CO<sub>2</sub> emissions-GDP relationship for groups of OECD and non-OECD countries.<sup>3</sup>

Another change in the EKC literature in the last few years has been the shift from cross-country and panel data analysis to individual countries' assessments (for example, Lindmark 2002 for Sweden; later on Ang 2007 for France, among many others). This is due to the fact that despite that the former provide a general understanding on how pollution variables are related to economic activity, they offer no guidance for predictions in each country. Individual countries do not possess neither the same pollution or income paths nor do the form of the relationship between income and CO<sub>2</sub> emissions have the same shape. Hence, the emissions-income per capita relationships can be very different across countries.

The EKC literature has become a very fruitful and independent research area. However, there are other narrower fields in the environmental economics literature that also analyze the CO<sub>2</sub> emissions-income relationship. That is the case of the studies on the relationship between carbon emissions and GDP which were introduced to discuss the advantages of GDP linked CO<sub>2</sub> quantified reduction targets.<sup>4</sup> For example, Höhne and Harnisch (2002) with IEA data from 1971 to 1999 looked at GDP and emissions over time for four countries (India, the former Soviet Union, USA and the UK). They find significant GHG emissions to GDP elasticities (except for the UK) between 0.45 and 1.48 depending on the period and the country. Kim and Baumert (2002) calculate an elasticity of 0.95 for Korea based on EIA data for 1981 to 1998, Barros and Conte Grand (2002) estimate a 0.5 elasticity for Argentina with local data for 1990 to 2005. In all those cases, the relationship between emissions and GDP is assumed to have the form of a specific log-log functional form. That shape results from taking logarithms to both sides of the following equation:  $E_t = b \cdot GDP_t^a$ , where  $E_t$  denotes emissions and  $GDP_t$  is the gross domestic product and  $a$  is the elasticity of emissions with respect to GDP.

The main innovation of this note is to use the Box-Cox specification to capture the CO<sub>2</sub> emissions-GDP link, taking into consideration the time-series properties of both variables depending on the transformation that is considered more appropriate for each individual country. The advantage of this formulation is employing a nonlinear transformation of variables that subsumes several other functional forms as nested cases (for example, both the linear model and log-log model). We use CAIT-WRI international database to analyze the behavior of CO<sub>2</sub> emissions with respect to GDP for those countries who submitted quantified reductions under the Copenhagen Accord. We select that group of countries in order to capture if the metric they used for the Accord has to do with the shape of their emissions-GDP long-run relationship.

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<sup>2</sup> B-splines take the form of piecewise polynomials. B is related to the smoothness of the function. If B = 0, there is no smoothness, if B=1, the function used is piecewise linear, if B=2, it is piecewise quadratic, and so on (de Boor, 2001).

<sup>3</sup> A few papers have used non-parametric approaches through the use of kernel models (for example, Azomahou et al 2006).

<sup>4</sup> Intensity caps, contrary to fixed caps, do not set country's allowable emission level, but define it as a linear function of GDP. Those "pure" intensity targets define emission intensity while fixed caps imply fixing emissions. At the moment, two countries submitted linearly indexed pledges to the Copenhagen Accord: China and India.

This note is organized as follows. Section II describes the empirical strategy and describes the data sources. Section III shows our results and Section IV concludes.

## II. Empirical strategy

It has been acknowledged in econometric studies that the determination of the functional relationship that may exist between some variables of interest could be derived applying the Box-Cox transformation technique (Box-Cox, 1964). Applied to our research, the functional form would be:

$$E_t^\theta = \delta_0 + \delta_1 \cdot GDP_t^\lambda + u_t \quad (1)$$

where  $E_t^\theta = \frac{E_t^\theta - 1}{\theta}$  and  $GDP_t^\lambda = \frac{GDP_t^\lambda - 1}{\lambda}$  and  $\theta, \lambda \neq 0$ . Then,  $\theta$  and  $\lambda$  can be estimated in order to "choose" a functional form sufficiently flexible to "fit" the data.

Then, it is possible to test if more usual (restricted) relationship between emissions and GDP are preferred to the Box-Cox flexible model:  $\theta=\lambda=1$  (linear model) and  $\theta=0, \lambda=1$  (log – lin model), while if  $\theta=\lambda=0$ , the function is defined as log – log). Those are usual functional forms that can make emissions and GDP positively related and preclude negative emissions as could be the case for some alternative models (e.g., the lin-log functional form). Box-Cox regressions are run here using STATA 11 and the tests performed are Likelihood Ratio tests based on the log-likelihood estimates for the unrestricted (Box-Cox) versus the restricted (lin-lin, log-log, or log-lin) models (Greene, 2003). A high calculated  $\chi^2$  statistics implies rejection of the null hypothesis that the restrictions to the functional form are correct. More specifically, the test we use is:

$$LRT = -2 \cdot (\ln \Omega^* - \ln \Omega) \sim \chi_J^2 \quad (2)$$

Where  $\ln \Omega^*$  is the log likelihood evaluated at the restricted estimates,  $\ln \Omega$  is the log-likelihood evaluated at the unrestricted Box-Cox estimates, and  $J$  are the number of restricted parameters. The Box-Cox method begins by computing the maximum likelihood estimation (MLE) score when the parameters  $\theta=\lambda=1$ . Then, other values for the parameters are tried and, at the end, the method reports the values of  $\theta$  and  $\lambda$  that maximizes the MLE score.

Once selected the functional form that best fits emissions and GDP data of each country, it is crucial to test for time-series properties of the variables used in the regression because a distinction has to be made between the true relation between GDP and pollution and the change in emission levels occurring merely due to the passage of time. More specifically, if dependent and independent variables are stationary, they can be used in a regression. If they are not stationary, but are integrated of the same order and cointegrated, they can also be included in a regression and the estimation of the model by OLS and the classical analysis of series is valid (Greene, 2003). If neither of the two conditions holds, the regression would be spurious (Granger and Newbold, 1974). This means that results would indicate that the two variables are related when in fact they are not, it is only their dependence to time that relates them. Separating the relationship between pollution and income from the correlation between both variables and the passage of time is difficult. It can be the case that they are related by time and not by themselves.

A time series is stationary if it has a constant mean and its covariance between t and t-s does depend on s but not on time (t) (or, which is the same, the variance of the series is not growing with time). One of the most popular tests of stationarity is the Dickey-Fuller (DF) test. The starting point is the following general model:

$$Y_t = \alpha_0 + \emptyset \cdot Y_{t-1} + \alpha_1 \cdot t + u_t, \quad (3)$$

Where  $\alpha_0$  is a constant (also called "drift"),  $Y_t$  is the serie whose stationarity we test,  $t$  is a trend and  $u_t$  are taken to be independently normally distributed. Based on the significance of the constant and the trend test we then, on the remaining model, contrast the null hypothesis  $H_0: \emptyset = 1$  (unit root or non-stationarity). But we cannot estimate a model regressing the series on its lagged value to see if the estimated  $\emptyset$  is equal to 1 because in the presence of a unit root the t-statistics for that coefficient is severely biased. Therefore, Eq. (3) is expressed in terms of differences subtracting the lagged value from both sides. Then,

$$Y_t - Y_{t-1} = \alpha_0 + (\emptyset - 1) \cdot Y_{t-1} + \alpha_1 \cdot t + u_t, \quad (4)$$

In this model we test the null hypothesis  $H_0: (\emptyset - 1) = 0$  (i.e. there is a unit root and the series is nonstationary) against  $H_1: H_0: (\emptyset - 1) < 0$  (i.e. there is no unit root and the series is stationary). Under the null hypothesis, the statistic of the regression has a distribution which was first estimated by Dickey and Fuller (1979) and then obtained analytically by Phillips (1987). If the series is stationary in levels, it is integrated of order 0:  $I(0)$ . If the series became stationary after differentiating it  $d$  times, the series is integrated of order  $d$ :  $I(d)$ .

However, the Dickey-Fuller test assumes that the error terms are uncorrelated. When the residuals are serially correlated, the Augmented Dickey-Fuller (ADF) test proposes to include in the regression several lags of the dependent variable  $\Delta Y_t$  to eliminate the serial correlation. In this paper we perform the modified Dickey-Fuller  $t$  test (known as the DF-GLS test) proposed by Elliott, Rothenberg, and Stock (1996) to test for a unit root. Essentially, the test is an augmented ADF test except that the time series is transformed via a generalized least squares regression. Elliott et al (1996) and later studies have shown that this test has significantly greater power than the previous versions of the ADF test (Elliott et al 1996, p. 813).

The testing procedure for the DF-GLS test is the same as for the ADF test and is applied to the model with constant term and trend. This would imply (when applied to (4)):

$$\Delta Y_t = \alpha_0 + \delta \cdot Y_{t-1} + \alpha_1 \cdot t + \sum_{i=1}^m \beta_i \Delta Y_{t-i} + u_t \quad (5)$$

The lag lengths were chosen for each variable in each of the countries using the procedure suggested by Ng-Perron (1995). This criterion starts with a maximum lag length as selected by Schwert (1989) and test the highest lag coefficient for significance. When the p-value of that lag falls below 0.1, the lag is retained and is chosen as the optimal lag.<sup>5</sup>

After the DF-GLS tests, we learn the order of integration of the series. But, regressions between series that are not stationary in levels can only be run if they are integrated of the same order and cointegrated. Cointegration indicates a long run relationship between non-stationary

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<sup>5</sup> We also run the Kwiatkowski, Phillips, Schmidt and Shin (1992) test to verify our results. The evidence is in all cases supportive of the results.

time series. If the series are cointegrated it means that although they move together over time they do it in a harmonized way so that the error between the variables does not change. The long run relationship is represented by a linear combination of the variables that is stationary:

$$c_1 \cdot Y_t + c_2 \cdot X_t \text{ is } I(0), \text{ where } Y_t, X_t \text{ are } I(d), d \neq 0 \quad (6)$$

Where  $c = (c_1, c_2)$  is the cointegrating vector.

Cointegration is tested by the Engle-Granger test. Engle-Granger (1987) "residual approach" implies running a regression of  $Y_t$  against  $X_t$  (where both variables have the same order of integration) and extract the residuals.<sup>6</sup> Then, if the estimated residuals have a unit root (or, which is the same, are non-stationary), then cointegration is rejected. If residuals are  $I(0)$ , cointegration cannot be rejected. To do so, we run Eq. (5) and obtain the residuals, then the first difference of the residuals is regressed on the lagged level of the residuals without a constant. As the cointegration test is based on the estimated residuals of the long run relationship, the usual Dickey-Fuller table is not longer valid, so critical values employed for the Engle-Granger test are those calculated by MacKinnon (1990, 2010). If the residuals are stationary the two series do not drift too much apart from each other and hence, there is a cointegration between the two variables.

Our data are 1980-2008 CAIT 2012 International data for emissions (measured in thousands of metric tons of CO<sub>2</sub> equivalent) and GDP (in million constant 2005 international dollars converted by Purchasing Power Parity) from the World Bank Development Indicators. We limit our analysis to those countries having submitted pledges under the Copenhagen Accord because we try to associate if the metric of their pledges to the Accord has to with the shape of the relationship between emissions and GDP for each of the countries.

Developed countries have submitted emission reduction pledges to be attained in 2020 and whose base years differ among the proposals but in all cases refer to a year in the past (see Table 1). On the other side, some developing countries have submitted proposals, which include economy-wide caps. Developing countries' proposals are of four types (Levin and Finnegan, 2011). Some of those targets are set as percentage reductions of emissions with respect to a given base year (the same metric of developed countries' caps). In many cases, commitments are absolute emissions reductions from the (future) business as usual (BAU) levels in 2020. Other countries set reductions in emissions intensity in comparison to a base year. For example, China made a pledge to cut in CO<sub>2</sub> emissions per unit of GDP by 40 to 45% below 2005 level by 2020 and India proposed a 20 to 25% emissions intensity reduction over the same period.<sup>7</sup> And, finally, there are nations (e.g., Costa Rica) whose aim is to achieve carbon neutrality by 2020 (i.e., zero net emissions: emissions do not exceed sequestration). Table 1 shows the different types of caps. We observe that only two countries have submitted voluntary reductions to Copenhagen that depend of GDP (i.e., China and India), and, that those specific quantified limits on emissions depend linearly of GDP.

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<sup>6</sup> Note that the constant and coefficient estimated by the regression are the estimates of the cointegrating coefficients.

<sup>7</sup> China's target has received a high level of attention in the empirical literature in part because it is based on an intensity indicator, but also because China represents a very large share of total global emissions. It has grown at an exceptionally high rate over the last years, and so has grown its emissions, but its emissions intensity has declined. China's target has raised the issue of whether or not it just represents the business as usual trend. There are no unanimous conclusions in that regard.

**Table 1. Quantified reductions proposed under the Copenhagen Accord**

States	Country	Base	Target	Metric	Stringency ( $\theta$ )
<b>Developed</b>	Australia	2000	2020	GHG Emissions	-5/-25%
	Belarus	1990	2020	GHG Emissions	-5/-10%
	Canada	2005	2020	GHG Emissions	-17%
	Croatia	1990	2020	GHG Emissions	-5%
	Iceland	1990	2020	GHG Emissions	-15/-30%
	Japan	1990	2020	GHG Emissions	-25%
	Kazakhstan	1992	2020	GHG Emissions	-15%
	Liechtenstein	1990	2020	GHG Emissions	-20/-30%
	Monaco	1990	2020	GHG Emissions	-30%
	New Zealand	1990	2020	GHG Emissions	-10/-20%
	Norway	1990	2020	GHG Emissions	-30/-40%
	Russian Federation	1990	2020	GHG Emissions	-15/-25%
	Switzerland	1990	2020	GHG Emissions	-20/-30%
	Ukraine	1990	2020	GHG Emissions	-20%
	USA	2005	2020	GHG Emissions	-17%
<b>Developing</b> ***	Antigua and Barbuda	1990	2020	GHG Emissions	-25%
	Marshall Islands	2009	2020	CO2 emissions	-40%
	Republic of Moldova	1990	2020	GHG Emissions	No less than 25%
	Brazil	BAU	2020	Emissions	-36.1/-38.9%
	Chile	BAU	2020	Emissions	-20%
	Indonesia	BAU	2020	Emissions	-26%/-41%
	Israel	BAU	2020	GHG Emissions	-20%
	Mexico	BAU	2020	GHG Emissions	Up to -30%
	Republic of Korea	BAU	2020	GHG Emissions	-30%
	Singapore	BAU	2020	GHG Emissions	-16%
	South Africa	BAU	2020	Emissions	-34%
			2025	Emissions	-42%
	China	2005	2020	Carbon dioxide emissions per unit of GDP Share of non-fossil fuels in primary energy consumption Forest coverage and forest stock volume	-40/-45% Reach 15% + 40 mill.has./+ 1.3 bill.m <sup>3</sup>
	India	2005	2020	Emissions intensity of its GDP (grams of CO2eq excluding agriculture, per Rs. of GDP)	-20/-25%
	Bhutan	2020		Emissions do not exceed its sequestration capacity	
	Costa Rica	2021		Carbon neutrality	
	Maldives	2020		Carbon neutrality	
	Papua New Guinea	2030		GHG emissions	-50%
		2050		Carbon neutrality	

Source: [http://unfccc.int/kyoto\\_protocol/items/3145.php](http://unfccc.int/kyoto_protocol/items/3145.php). Kyoto Protocol to the UNFCCC, United Nations 1998 (Article 3, Annex A,

Annex B). Appendix I Quantified economy-wide emissions targets for 2020

[http://unfccc.int/meetings/copenhagen\\_dec\\_2009/items/5264.php](http://unfccc.int/meetings/copenhagen_dec_2009/items/5264.php). Appendix II Nationally appropriate mitigation actions of developing country Parties [http://unfccc.int/meetings/cop\\_15/copenhagen\\_accord/items/5265.php](http://unfccc.int/meetings/cop_15/copenhagen_accord/items/5265.php).

Notes: Rs. = Rupees. \* Targets of Kyoto Protocol in carbon dioxide equivalent. \*\* EU-15=15 States who were EU members in 1997 when the Kyoto Protocol was adopted. \*\*\* Includes non-Annex I countries who submitted a pledge with their NAMAS. Guyana and Thailand are not included: they submitted pledges, but not NAMAs.

### III. Results

Regarding the functional form of the relationship between CO<sub>2</sub> emissions and GDP, the results of the Log-likelihood Ratio test reported in Table 2 indicate that the model specifications differs among countries. More precisely, the log-log shape is the one that predominates among developed countries (it provides the best fit in 4 of the 11 countries), while the Box-Cox functional form provides the best adjustment in many of the developing countries (i.e., in 8 of 15 nations).<sup>8</sup> Overall, the linear model and (to a lesser extent) the log-linear functional form are the ones that less fit the individual countries' data. More precisely, the linear shape is only preferred in the case of 3 countries in the overall sample (Canada, Mexico and Republic of Moldova).

<sup>8</sup> For those cases where the LR tests were not significant in more than one model (e.g., Croatia and Japan) we compare the log-likelihood values to select the model that fits best to the data.

**Table 2. Functional form tests with respect to the Box-Cox transformation**

Country	Calculated $\chi^2$ of Log-likelihood Ratio Test		
	Linear	Log-Log	Log-lin
<b>Developed</b>			
Australia	13.52 ***	0.96	31.12 ***
Canada	2.08	4.64 **	6.17 **
Croatia	1.66	0.93	6.84 ***
Iceland	13.32 ***	9.17 ***	8.63 ***
Japan	2.53	9.11 ***	0.42
Kazakhstan	5.08 **	1.69	0.88
New Zealand	23.18 ***	24.19 ***	30.48 ***
Norway	3.60 *	1.03	3.32 *
Switzerland	6.67 ***	6.91 ***	8.35 ***
Ukraine	10.84 ***	11.93 ***	1.22
USA	6.86 ***	2.62	8.37 ***
<b>Developing</b>			
Antigua and Barbuda	34.96 ***	59.21 ***	65.57 ***
Bhutan	27.35 ***	13.41 ***	21.72 ***
Brazil	4.32 **	0.63	6.96 ***
Chile	5.38 **	0.86	25.74 ***
China	5.52 **	0.03	23.37 ***
Costa Rica	21.97 ***	20.53 ***	42.38 ***
India	4.39 **	0.06	89.57 ***
Indonesia	13.75 ***	4.12 ***	8.43 ***
Israel	50.86 ***	44.88 ***	43.00 ***
Maldives	4.89 **	4.84 **	10.37 ***
Mexico	0.72	4.90 **	14.49 ***
Papua New Guinea	29.91 ***	16.75 ***	15.87 ***
Republic of Korea	16.14 ***	1.16	39.65 ***
Republic of Moldova	1.56	12.22 ***	8.03 ***
Singapore	40.11 ***	22.69 ***	19.66 ***
South Africa	27.88 ***	16.69 ***	10.67 ***

Notes: \*, \*\*, \*\*\* denote rejection of the null-hypothesis (the model in each column against the corresponding Box-Cox functional form) with 10%, 5% and 1% significance. There is no CO<sub>2</sub> data available from CAIT-WRI International data for Liechtenstein, Monaco and Marshall Islands. Belarus results are not reported because coefficients of the relationship between emissions and GDP are not significant for any of the alternative functional forms. The Russian Federation is not included because when trying to perform the Box Cox transformation to the dependent variable only, the data presents discontinuities that do not allow maximum likelihood to be reached and the comparison between the Box Cox and the log-lin transformations cannot be performed,

Regarding the stationarity and cointegration of our series, the order of integration of transformed emissions and GDP series (as well as the cointegration among them) differs for the different countries (see Table 3). In all nations, emissions and GDP transformed are integrated of the same order. Moreover, in less than 20% of developed and of developing countries the hypothesis of non-stationarity is rejected for series in levels and, in the remaining countries, the series are stationary after taking the first difference. Hence, the series are I(0) only for 4 cases (2 developed and 2 developing countries) and are I(1) for the remaining ones validating the long-run cointegration possibility between emissions and GDP. As can be seen in Table 3, for those economies whose series are not integrated in levels but are stationary in first differences, the null hypothesis of no cointegration within the Engle-Granger test is rejected for 3 developed countries and for 4 developing ones. Hence, a valid regression analysis searching for the functional form

between CO<sub>2</sub> emissions and GDP levels can only be performed for less than half of the countries who submitted quantitative caps under the Copenhagen Agreement (more precisely, for 4+7=11, of the 26 countries in our sample).

**Table 3. Unit root and cointegration tests**

Countries	Functional form	Model in levels			Model in first difference			Integration order both series	Residuals EG Statistic	Cointegration
		Dep. Var.	DF-GLS Statistic	Indep. Var.	DF-GLS Statistic	Dep. Var.	DF-GLS Statistic	Indep. Var.	DF-GLS Statistic	
<b>Developed</b>										
Australia	Log-Log	log(E)	-2.179	log(GDP)	-1.875	Δlog(E)	-4.578 ***	Δlog(GDP)	-4.248 ***	I(1)
Canada	Linear	E	-2.751	GDP	-2.303	ΔE	-4.930 ***	ΔGDP	-2.293 *	I(1)
Croatia	Log-Log	log(E)	-1.658	log(GDP)	-1.721	Δlog(E)	-4.147 ***	Δlog(GDP)	-2.522 *	I(1)
Iceland	Box-Cox	θ̂ E	-4.541 ***	χ̂̄ P	-3.508 **					Stationary in levels
Japan	Log-lin	log(E)	-1.198	GDP	0.365	Δlog(E)	-4.732 ***	ΔGDP	-2.605 *	I(1)
Kazakhstan	Log-lin	log(E)	-2.602	GDP	-3.222 **	Δlog(E)	-3.232 *	ΔGDP	-3.125 ***	I(1)
New Zealand	Box-Cox	θ̂ E	-2.405	χ̂̄ P	-0.203	θ̂ E	-7.439 ***	ΔĜ̄ P	-3.437 **	I(1)
Norway	Log-Log	log(E)	-3.175 *	log(GDP)	-0.648	Δlog(E)	-5.384 ***	Δlog(GDP)	-2.519 **	I(1)
Switzerland	Box-Cox	θ̂ E	-3.660 **	χ̂̄ P	3.328 ***					Stationary in levels
Ukraine	Log-lin	log(E)	-3.080 *	GDP	-2.002	Δlog(E)	-2.747 ***	ΔGDP	-3.143 **	I(1)
USA	Log-Log	log(E)	-2.080	log(GDP)	-2.088	Δlog(E)	-3.807 ***	Δlog(GDP)	-3.772 ***	I(1)
<b>Developing</b>										
Antigua and Barbuda	Box-Cox	θ̂ E	-2.162	χ̂̄ P	-0.736	θ̂ E	-8.039 ***	ΔĜ̄ P	-2.945 *	I(1)
Bhutan	Box-Cox	θ̂ E	-3.042	χ̂̄ P	-3.285 **	θ̂ E	-6.530 ***	ΔĜ̄ P	-3.674 ***	I(1)
Brazil	Log-Log	log(E)	-1.822	log(GDP)	-2.434	Δlog(E)	-3.679 ***	Δlog(GDP)	-3.886 ***	I(1)
Chile	Log-Log	log(E)	-2.122	log(GDP)	-3.181 **	Δlog(E)	-3.743 ***	Δlog(GDP)	-2.540 *	I(1)
China	Log-Log	log(E)	-2.538 *	log(GDP)	-4.152 ***					Stationary in levels
Costa Rica	Box-Cox	θ̂ E	-2.609	χ̂̄ P	-2.923 *	θ̂ E	-4.924 ***	ΔĜ̄ P	-3.068 ***	I(1)
India	Log-Log	log(E)	-1.717	log(GDP)	-1.170	Δlog(E)	-3.007 ***	Δlog(GDP)	-3.561 **	I(1)
Indonesia	Box-Cox	θ̂ E	-0.982	χ̂̄ P	-1.586	θ̂ E	-4.154 ***	ΔĜ̄ P	-3.802 ***	I(1)
Israel	Box-Cox	θ̂ E	-2.646 *	χ̂̄ P	-2.588 *					Stationary in levels
Maldives	Box-Cox	θ̂ E	-1.305	χ̂̄ P	-3.520 *	θ̂ E	-3.658 **	ΔĜ̄ P	-4.667 ***	I(1)
Mexico	Linear	E	-1.999	GDP	-1.466	ΔE	-7.839 ***	ΔGDP	-4.743 ***	I(1)
Republic of Korea	Log-Log	log(E)	-0.760	log(GDP)	-0.760	Δlog(E)	-4.970 ***	Δlog(GDP)	-5.371 ***	I(1)
Republic of Moldova	Linear	E	-1.435	GDP	-3.256 **	ΔE	-3.910 ***	ΔGDP	-3.520 ***	I(1)
Singapore	Box-Cox	θ̂ E	-1.611	χ̂̄ P	-1.685	θ̂ E	-4.731 ***	ΔĜ̄ P	-4.000 ***	I(1)
South Africa	Box-Cox	θ̂ E	-2.957	χ̂̄ P	-1.943	θ̂ E	-3.614 **	ΔĜ̄ P	-3.871 ***	I(1)

Note: \*, \*\*, \*\*\* denote 10%, 5% and 1% of significance.

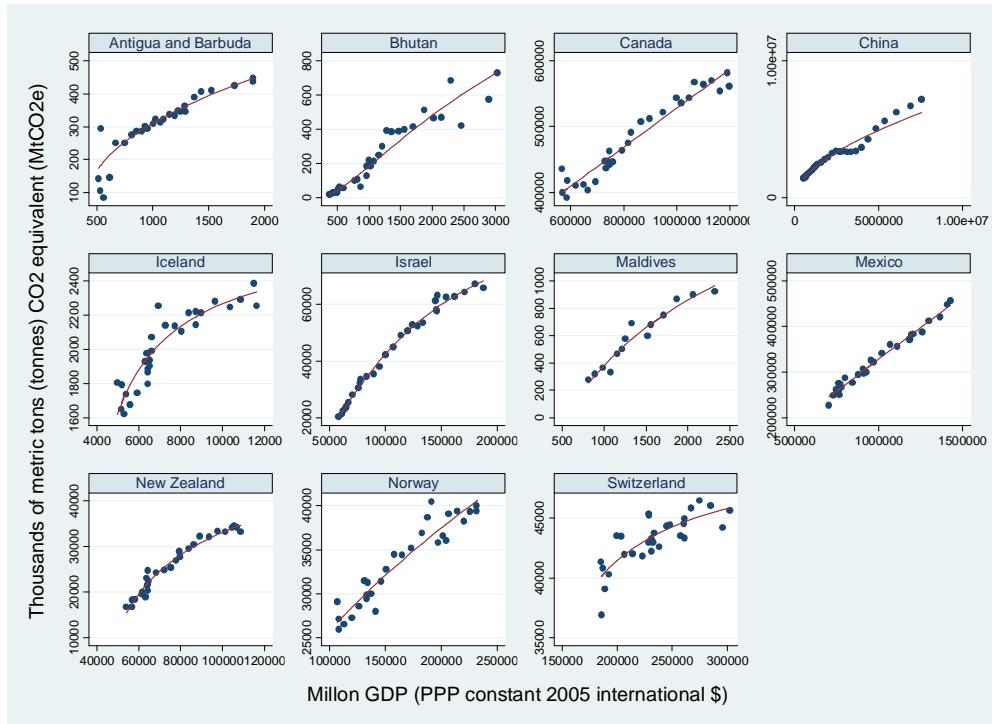
Note that for those individual countries for which the regression between emissions and GDP is not spurious, the Box-Cox functional form predominates (it constitutes 7 of the 11 cases). This result is clearly depicted in Table 4 and Figure 1.

**Table 4. Slopes and elasticities under alternative functional forms**

Country	Fn. Form	Slope	Elasticity	CO <sub>2</sub> mean	GDP mean
<b>Developed</b>					
Canada	Linear	0.30 ***	0.52	481,254	843,101
Iceland	Box-Cox		0.41 2.36	-1.09	2,009 7,403
New Zealand	Box-Cox		1.04 1.59 **	-0.62	25,771 76,857
Norway	Log-Log	0.11 ***	0.54	33,542	164,941
Switzerland	Box-Cox		0.27 2.38 ***	-1.75 ***	43,227 234,585
<b>Developing</b>					
Antigua and Barbuda	Box-Cox		0.74 3.15 ***	1.50 ***	305 1,071
Bhutan	Box-Cox		1.46 0.06	-0.49 *	272 1,287
China	Log-Log	0.69 ***	0.56	3,211,251	2,607,619
Israel	Box-Cox		0.92 -0.29	-1.35 ***	44,089 110,824
Maldives	Box-Cox		1.67 0.67	-0.28	343 1,412
Mexico	Linear	0.28 ***	0.85	325,068	989,133

Notes: \*, \*\*, \*\*\* denote 10%, 5% and 1% of significance. The elasticities have been evaluated at the mean values of CO<sub>2</sub> and GDP. Papua and New Guinea was not included in the Table because the relationship between the emissions and the GDP shows more complex patterns than the ones observed for the other countries.

**Figure 1. Relationship between emissions and GDP by country: functional forms**



*Note: The Figure includes those countries whose emissions and GDP series are stationary or non stationary of the same order and cointegrated.*

Finally, as shown in Table 4, the Emissions-GDP elasticities vary greatly among countries. This result is consistent with the literature that analyzes dynamic targets (Höhne and Harnisch 2002, for example, find quite different elasticities depending on the country analyzed). Average elasticities are lower than 1 (emissions increases in a lower % when GDP increases) in most nations (8 out of 11). As seen in Table 1, only India and China chose for their Copenhagen submission a GDP related reduction target. The Emissions-GDP elasticity cannot be calculated for India since transformed Emissions and GDP are not cointegrated, but the result for China (an elasticity of 0.56 on average for the 1980-2008 period) would not support the choice of a linearly adjusted target to GDP. Emissions in China do not seem to be moving linearly with GDP over the whole period as shown in Figure 1.A. of the Appendix, which describes the path of emissions, GDP and emissions intensity. However, it is fair to acknowledge that the few last years of the period seem to show a different path.

#### IV. Conclusions

With CAIT WRI data for those countries which submitted quantifiable reduction limits under the Copenhagen Agreement, this note supports the existence of a long run relationship between CO<sub>2</sub> emissions and GDP in 11 of the 26 countries in our sample over the period 1980-2008. However, the functional specification of that relationship is not homogenous among nations, being linear for 2 countries, log-log for 2 other cases, while the relationship follows a Box-Cox functional form for 7 nations. Moreover, mean elasticities of the emissions-income relationship differ among

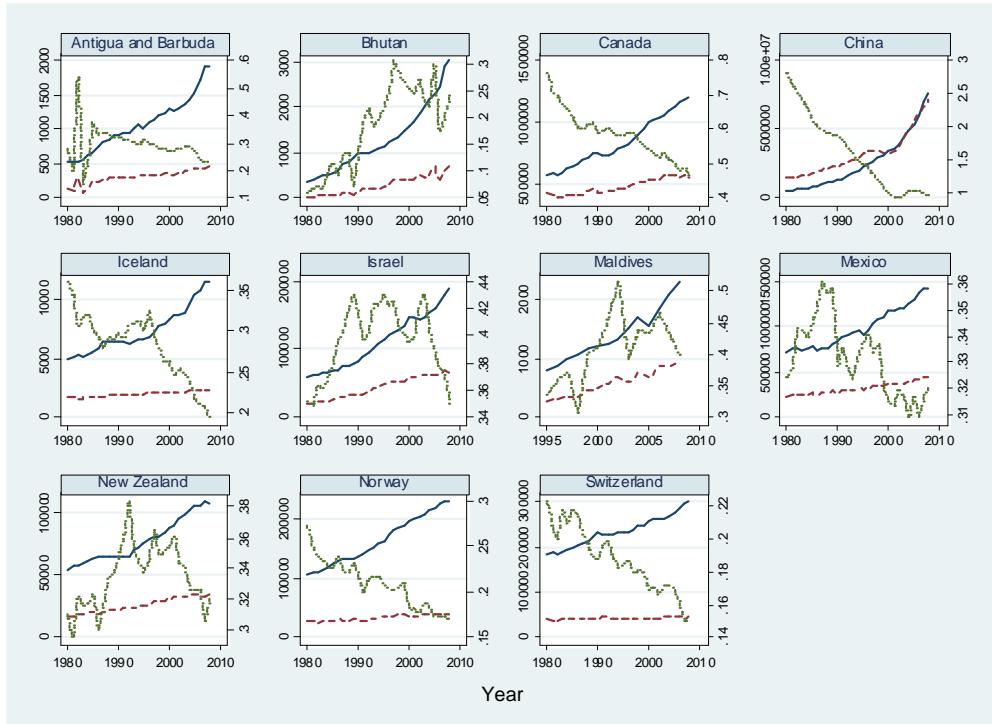
countries, independently of the shape of the relationship. But in most cases (8 out of 11), the magnitude of the average elasticity is lower than 1, which means that emissions increase less than the GDP.

The contributions of our findings are twofold. First, this note reinforces the fact that the analysis of the time series properties of the variables is crucial not only in the study of the EKC, but also in the analysis of the relationship between emissions and GDP (in levels, not in per capita terms). We reject 15 of the 26 individual countries regressions between CO<sub>2</sub> emissions and countries' income level for considering them spurious.

Secondly, this note highlights the importance of considering an appropriate functional form for each individual country. This fact is already acknowledged in the EKC related literature (for example, by Piaggio and Padilla 2012, who reject the assumption of equality in countries' functional forms). But, this note goes a step further since instead of hypothesize quadratic or cubic transformations for income, use different exponentials for income from 0 to 2 (as in Wang 2013) where 0 is the specific case of a linear EKC function and 2 the case of a quadratic EKC function, this note introduces the Box-Cox functional form to deal with the different shape of the CO<sub>2</sub> emissions and GDP relationship in different individual countries. As a result, we conclude that in most of the countries for which a non spurious relation is possible (8 of the 11 cases), the Box-Cox shape is more appropriate than other more traditional shapes (e.g., linear or log-log).

## Appendix

**Figure 1.A. GDP, CO<sub>2</sub> emissions and emissions intensity by country**



*Note: The solid lines denote GDP in millions, PPP (constant 2005 international \$), The Dashes denote Thousands of metric tons (tonnes) CO<sub>2</sub> equivalent (MtCO<sub>2</sub>e) and the Dots denote the Emissions Intensity defined as the ratio of the previous variables.*

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