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OF PERFECT BAYESIAN EQUILIBRIA**

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Credible signals: A refinement of perfect Bayesian equilibria

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Signals are voluntary actions a sender may use to reveal its type. When this simple insight is formalized, a bewildering plethora of perfect Bayesian equilibria arise. In particular, separating equilibria are possible when no type has an incentive to separate and pooling equilibria are possible when all types do. This motivates a refinement for signaling games. A deviation from an equilibrium is credible if and only if it forms part of an alternative equilibrium where payoffs (weakly) increase. The self-selection condition then puts zero probability, when possible, on sender types for which a deviation from equilibrium is not credible.

Las señales son acciones voluntarias del emisor para revelar su tipo. Cuando esto se formaliza, surge una plétora de equilibrios bayesianos perfectos. Son posibles equilibrios con diferenciación a pesar de que nadie tiene un incentivo para diferenciarse y equilibrios con mimetización a pesar de que todos sí lo tienen. Esto motiva un refinamiento para juegos de señales. Un desvío de un equilibrio es creíble si y solo si forma parte de un equilibrio alternativo donde los pagos aumentan (en sentido débil). La condición de autoselección pone probabilidad cero a los tipos para los cuáles un desvío no es creíble.

JEL classification codes: D8, C7

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I. Introduction

This paper was sparked by the Milgrom and Roberts (1982) entry-deterrence model where there may be both pooling and separating equilibria. If the priors are such that a pooling equilibrium is enough to deter entry, why would a separating equilibrium also exist? Insofar as signals are voluntary moves that are used to provide information that benefits the sender, there is no obvious rationale for any incumbent to bother sending any separating signal. This simple question motivates a refinement for signaling games where an informed player (the sender) sends a message to an uninformed player (the receiver).

More generally, when there are multiple perfect Bayesian equilibria (PBE), we have no obvious clue about what determines equilibrium beliefs. When there are information sets off the equilibrium path, we also are free to pull out of the analyst's hat whatever out-of-equilibrium beliefs work to support the equilibrium. John F. Nash (1950: 23) states, in regard to his "rationalistic and idealizing interpretation", that when there are multiple equilibria "good heuristic reasons can be found for narrowing down the set of equilibrium points".

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In relation to out-of-equilibrium beliefs, the literature on refinements has helped to narrow down the range of Nash equilibria since Reinhard Selten (1965) introduced subgame-perfect Nash equilibrium to rule out incredible threats on information sets off the equilibrium path, requiring equilibrium strategies to be a Nash equilibrium not only in the whole game but also when restricted to each subgame. PBE extends subgame perfection to any continuation game, since players are also required to have beliefs off the equilibrium path and to play a best response to their beliefs in every information set (David M. Kreps and Robert Wilson 1982).

Two further refinements of PBE are particularly relevant for us: the dominance requirement and the intuitive criterion. By the dominance requirement, out-of-equilibrium beliefs must not put positive probability on those types of sender for whom the strategies are strictly dominated. This makes perfect sense because rational players do not play strictly dominated strategies. The In-Koo Cho and David M. Kreps (1987) intuitive criterion places further restrictions on out-of-equilibrium beliefs by proposing the nice idea of ruling out those types of sender that could never benefit from deviating from a given equilibrium to an information set off the equilibrium path, a restriction that has a bite if at least one type of sender benefits from the deviation. This insight is behind the refinement proposed here. However, from the point of view of fully rational players considered here, the intuitive criterion goes too far, for instance when it eliminates all the pooling equilibria in the Spence (1973) job-market model even though no sender type wants to separate out. In contrast to the intuitive criterion (and refinements like the vulnerability to credible deviations in Peter Eso and James Schmmmer 2009), the refinement proposed here does not consider that any deviation works to restrict out-of-equilibrium beliefs. Rather, only deviations that form part of an alternative PBE work. The purpose of this restriction is to discard siren calls which sound attractive at first but that end up hurting the player.

The requirement that a deviation has to be part of an alternative equilibrium to be credible is suggested by the notion in the verbal communication game in Streb and Torrens (2015: 15) that a message is credible if there is an equilibrium where the literal meaning is the equilibrium meaning. Myerson (1989: 266) already introduces the notion that the literal and effective meanings of credible statements coincide, but in the context of an equilibrium refinement where credible statements are always trusted. In our setup, a credible signal is a signal that is part of a PBE, and a credible deviation is a signal that is part of an alternative PBE.

Roger Myerson (2009) points out that the analysis of the ability to commit to threats or promises in Schelling (1960) is the link between equilibrium points in Nash (1950) and the refinement of Nash equilibria in Reinhard Selten (1965). What is most relevant for us here from the Schelling (1960) experiments and thought-experiments is instead the use of unilateral communication to achieve explicit coordination. As Schelling (1960: 117) puts it, "Moves can in some way alter the game, by incurring manifest costs, risks, or a reduced range of subsequent choice; they have an information content, or evidence content, of a different character than speech. Talk can be cheap when moves are not." By burning the bridges behind us (Schelling 1960: 158), we not only commit not to retreat in the face of enemy advance, we also signal our resolve to fight. However, in the Schelling (1960: 58-59) game of a parachutist who can send but not receive verbal messages, once the sender indicates where he is, the other parachutist has no choice but to walk over there. The general point that can be extracted from Schelling (1960) is that the sender can use signals to select among the equilibria of a game.

In games of pure conflict the premium is on secrecy, in games of coordination on revelation, while in bargaining or mixed-motive games Schelling (1960: 83) adds that "though the element of conflict provides the dramatic interest, mutual dependence is part of the logical structure and demands some kind of collaboration or mutual accommodation — tacit, if not explicit — even if only in the avoidance of mutual disaster. These are also games in which, though secrecy may play a strategic role,

there is some essential need for the signaling of intentions and the meeting of minds.” Michael Spence (1973: 357), citing Robert Jervis (1970), distinguishes between *indices*, which are observable, unalterable attributes, and *signals*, which are observable, alterable attributes. Spence (2001: 407) adds that “Signals are things one does that are visible and that are in part designed to communicate.” Since signals are voluntary moves, what the sender wants to reveal about itself must play an important role in the determination of equilibrium beliefs.

The equilibrium refinement is characterized in the second section as a self-selection condition that is inspired by the Schelling equilibrium selection argument and the Cho and Kreps intuitive criterion. The self-selection condition puts zero probability, when possible, on sender types for which a deviation from equilibrium is not credible. A deviation from equilibrium is credible if and only if it forms part of an alternative equilibrium and does not reduce equilibrium payoffs. If credible deviations lead to cycles, no unique equilibrium is selected. An iterative self-selection condition, using the George Akerlof (1970) unraveling logic, is defined for the case in which credible deviations lead to a cycle. In the third section, the self-selection condition is applied to three classic signaling games: the Cho and Kreps (1987) beer and quiche game, the Spence (1973) job-market model, and the Milgrom and Roberts (1982) entry-deterrence model. In the three games, the refinement eliminates the multiplicity of equilibria that plagues them; different sender types organize themselves, so to say, to self-select a unique PBE. The fourth section looks at cheap-talk games where the signal is payoff-irrelevant. First, the parachutist story in Schelling (1960) is formalized as a coordination game. Then, a counterexample is presented where the self-selection condition is unable to select an equilibrium, the Vincent Crawford and Joel Sobel (1982) model of an informed expert and an uninformed decision-maker; however, the iterative self-selection condition works in this case.¹ Section V states the sufficient conditions for informative or uninformative equilibria to be selected by the players. The closing section discusses pending issues.

II. Equilibrium refinement

I consider simple games with incomplete information, such as those discussed in Robert Gibbons (1992), where the sender can send a signal about its type to the receiver.

The timing of the signaling game is as follows. First, the sender type $w_i \in W$ is drawn according to a commonly known probability distribution given by the priors $p(w) = (p(w_1), \dots, p(w_I)) \in P$ about types $w = (w_1, \dots, w_I)$. Second, sender S observes its type w_i and picks an action, the signal or message $m_j \in M$. Third, receiver R observes the signal m_j , but not the sender's type w_i , forming beliefs $\mu(m_j)$ and picking an action $a_k \in A$ in response. Fourth, payoffs are given by $v^S(w_i, m_j, a_k)$ and $v^R(w_i, m_j, a_k)$. Here the sets W , M and A are here considered to be finite (however, in some of the examples below they are infinite).

Strategies and beliefs are given by $(\sigma^S(w), \sigma^R(m), \mu(m))$, where:

- (i) $\sigma^S(w) = (\sigma^S(w_1), \dots, \sigma^S(w_I))$, a strategy for the sender, is a vector of probability distributions $\sigma^S(w_i) = (\sigma^S(w_i)(m_1), \dots, \sigma^S(w_i)(m_J))$ for $i = 1, \dots, I$, where $\sigma^S(w_i)(m_j) \in [0,1]$ and $\sum_{m_j \in M} \sigma^S(w_i)(m_j) = 1$;
- (ii) $\sigma^R(m) = (\sigma^R(m_1), \dots, \sigma^R(m_J))$, a strategy for the receiver, is a vector of probability distributions $\sigma^R(m_j) = (\sigma^R(m_j)(a_1), \dots, \sigma^R(m_j)(a_K))$ for $j = 1, \dots, J$, where $\sigma^R(m_j)(a_k) \in [0,1]$ and $\sum_{a_k \in A} \sigma^R(m_j)(a_k) = 1$;

¹ A game in Farrell (1993) where neither sender type is interested in revealing its type will be analyzed later. This helps to state the sufficient conditions for informative or uninformative equilibria to be selected by the players.

(iii) $\mu = (\mu(m_1), \dots, \mu(m_J))$, a belief for the receiver, is a vector of probability distributions $\mu(m_j) = (\mu(m_j)(w_1), \dots, \mu(m_j)(w_I))$ for $j = 1, \dots, J$, where $\mu(m_j)(w_i) \in [0,1]$ and $\sum_{w_i \in W} \mu(m_j)(w_i) = 1$.

DEFINITION 1 *Perfect Bayesian equilibrium (PBE).* In a signaling game, a PBE is given by strategies and beliefs $(\tilde{\sigma}^S(w), \tilde{\sigma}^R(m), \tilde{\mu}(m))$ that satisfy conditions (1) through (4):

(1) For each $w_i \in W$,

$$\tilde{\sigma}^S(w_i) = \arg \max_{\sigma^S(w_i)} \sum_{m_j} \sigma^S(w_i)(m_j) \sum_{a_k} \tilde{\sigma}^R(m_j)(a_k) v^S(w_i, m_j, a_k).$$

(2) For each $m_j \in M$,

$$\tilde{\sigma}^R(m_j) = \arg \max_{\sigma^R(m_j)} \sum_{a_k} \sigma^R(m_j)(a_k) \sum_{w_i} \tilde{\mu}(m_j)(w_i) v^R(w_i, m_j, a_k).$$

(3) If for $m_j \in M$ there exists $w_i \in W$ such that $\tilde{\sigma}^S(w_i)(m_j) > 0$, then $\tilde{\mu}(m_j)(w_i) = \frac{\tilde{\sigma}^S(w_i)(m_j) p(w_i)}{\sum_{w_i} \tilde{\sigma}^S(w_i)(m_j) p(w_i)}$.

(4) If for $m_j \in M$, $\tilde{\sigma}^S(w_i)(m_j) = 0$ for all $w_i \in W$, then $\tilde{\mu}(m_j)(w_i) \in [0,1]$ and $\sum_{w_i} \tilde{\mu}(m_j)(w_i) = 1$.

Conditions (1) and (2) require that the sender and the receiver pick a best response to their beliefs; this is the requirement of sequential rationality. The sender's beliefs about the receiver's response to each signal are given in (1) by the receiver's actual equilibrium response. As to the receiver's beliefs for each signal in (2), they are specified by conditions (3) and (4). By (3), beliefs at information sets on the equilibrium path are derived from the sender's equilibrium signals and Bayes' rule. By (4), beliefs at information set off the equilibrium path are required to be a probability distribution over the set of sender types, a requirement due to Kreps and Wilson (1982).

Henceforth, the equilibria considered here are PBE. The refinement is inspired by the Cho and Kreps (1987) intuitive criterion. However, unlike the idea of equilibrium dominance used in the intuitive criterion, to be credible a deviation has to be to an information set that is part of an alternative equilibrium, and in which the player is not worse off.

The first requirement is that if a player deviates from an equilibrium, the deviation must be part of an alternative equilibrium to be credible. Thus, credible deviations only restrict beliefs in information sets that are part of an alternative equilibrium. Equilibrium dominance applies instead to any information set off the equilibrium path, since the deviations are not required to be themselves part of a new equilibrium in order to destroy an existing equilibrium. In summary, credible deviations drastically limit the set of possible deviations to information sets off the equilibrium path.

The second requirement introduces Schelling's equilibrium selection logic: two equilibrium payoffs are compared, the payoff $v^S(w_i, \tilde{\sigma}^S(w), \tilde{\sigma}^R(m))$ in the current equilibrium and the payoff $v^S(w_i, \tilde{\tilde{\sigma}}^S(w), \tilde{\tilde{\sigma}}^R(m))$ in the alternative equilibrium, because the most that player w_i will be able to achieve is another equilibrium payoff (the most, at any rate, if we reason in equilibrium terms). Instead, equilibrium dominance compares the current equilibrium payoff $v^S(w_i, \tilde{\sigma}^S(w), \tilde{\sigma}^R(m))$ to the highest possible payoff in an information set m_j that is off the equilibrium path.

DEFINITION 2 *Credible deviation.* A deviation m_j from a given equilibrium $(\tilde{\sigma}^S(w), \tilde{\sigma}^R(m), \tilde{\mu}(m))$ to an information set off the equilibrium path is credible for a sender type w_i if that deviation is part of an alternative equilibrium $(\tilde{\tilde{\sigma}}^S(w), \tilde{\tilde{\sigma}}^R(m), \tilde{\tilde{\mu}}(m))$ and the payoff $v^S(w_i, \tilde{\sigma}^S(w), \tilde{\sigma}^R(m))$ from the current equilibrium is not larger than the payoff $v^S(w_i, \tilde{\tilde{\sigma}}^S(w), \tilde{\tilde{\sigma}}^R(m))$ from the deviation to m_j in the alternative equilibrium.

In the intuitive criterion, the restriction for out-of-equilibrium beliefs in response to a deviation m_j is that if $v^S(w_i, \tilde{\sigma}^S(w), \tilde{\sigma}^R(m)) > \max_{a_k} v^S(w_i, m_j, a_k)$ for all w_i but one, then beliefs should place zero probability on all these w_i at m_j , unless of course the deviation is dominated for all types in W , in which case the restriction is moot; a more general variant is that, except for the types that benefit from deviating (which can be more than one type), beliefs should place zero probability on the rest. With the self-selection condition, the restriction for beliefs in response to a deviation m_j that is part of an alternative equilibrium is instead that if $v^S(w_i, \tilde{\sigma}^S(w), \tilde{\sigma}^R(m)) > v^S(w_i, m_j, \tilde{\sigma}^R(m))$ for w_i , then beliefs should place zero probability on w_i at m_j , unless the same holds for all other types in W , in which case the restriction is moot.

The notion of credible deviation is the heart of this refinement of perfect Bayesian equilibria. A second requirement is imposed, that if all the sender types prefer an alternative equilibrium, beliefs are given by the alternative equilibrium beliefs. Third, if credible deviations lead to cycles among equilibria (for example, a cycle where some player types prefer to deviate from $(\tilde{\sigma}^S(w), \tilde{\sigma}^R(m), \tilde{\mu}(m))$ to $(\tilde{\sigma}^S(w), \tilde{\sigma}^R(m), \tilde{\mu}(m))$, while others prefer the opposite deviation), the refinement is unable to select a unique equilibrium.

DEFINITION 3 *Self-selection condition. (i) If a deviation is not credible for certain player types, beliefs should place zero probability on these types, unless the same holds for all other types in W , in which case the restriction is moot. (ii) If credible deviations lead all sender types to pick an alternative equilibrium, beliefs are determined by the alternative equilibrium. (iii) If credible deviations lead to a cycle among equilibria, they impose no restrictions on beliefs.*

By requirement (i), if there are no credible deviations to information sets on or off the equilibrium path, the beliefs in the existing equilibrium stand. This is the opposite of requirement (ii), by which equilibrium beliefs are replaced when all types prefer to switch to an alternative equilibrium.

The self-selection condition has the flavor of the *self-signaling neologisms* in Joseph Farrell (1993), except that the deviation must lead to a new equilibrium. This is a bit like the difference between the Weimar Republic and the Federal Republic in Germany: before a vote of no confidence led to the downfall of the cabinet, now it does not lead to the fall of the incumbent unless the opposition can muster a majority vote for a successor.

In the case that (iii) cycles arise, which is what happens below in the Crawford and Sobel (1982) model of an informed expert and an uninformed decision-maker, a stronger condition can be imposed, the iterative self-selection condition where requirement (iii) is replaced by (iv).

DEFINITION 4 *Iterative self-selection condition. By the iterative self-selection condition, besides conditions (i) and (ii) above, the following condition is added: (iv) If credible deviations lead to a cycle, take into account which types actually benefit from a deviation to calculate the payoffs from the deviations from an equilibrium, iterating the process until no type wants to deviate or some types still do. If this does not lead to a unique equilibrium, then no restrictions are imposed on the remaining equilibria.*

The iterative self-selection condition (iv) can address the lack of congruence between the types that wish to deviate and the types that would actually pick the signal in an alternative equilibrium. This lack of congruence cannot happen when requirement (ii) of the self-selection condition is satisfied. The stronger self-selection condition follows the logic of the unraveling argument in Akerlof (1970), imposing the requirement that those types that actually want to deviate do so even when this fact is fully recognized by the receiver.

III. Self-selection in signaling games

I look at the role of signals in three classic games: the Cho and Kreps (1987) beer and quiche game, the Spence (1973) job-market model, and the Milgrom and Roberts (1982) entry-deterrence model which motivated this paper.

In all three games, one can first compute the Bayes Nash equilibrium when there is no signaling. This same outcome will result in an uninformative equilibrium of a signaling game when all sender types pool, as happens in beer and quiche. Informative PBE crop up in the last two models, where there are separating equilibria also.

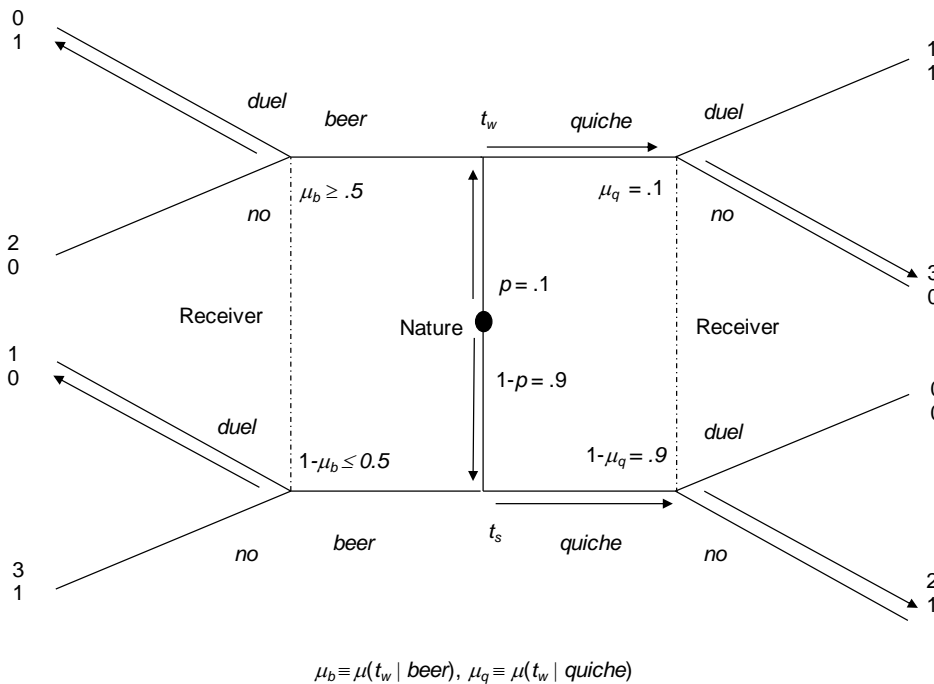
The self-selection condition leads to a unique equilibrium in the three games analyzed. The equilibrium coincides with the intuitive criterion in the first game, but in the others it differs because for certain parameter values it selects a pooling instead of a separating equilibrium.

A. Beer and quiche game

In the Cho and Kreps (1987) beer and quiche game, there is a wimpy type that likes quiche and a surly type that likes beer for breakfast. Neither type likes to fight. The bully would like to pick on the wimpy type because he can beat him, but prefers to avoid the surly type. In the game without signals, the Bayes Nash equilibrium is for the bully not to fight when the proportion of wimpy and surly types is 0.1 and 0.9, since the expected payoff from fighting is 0.1 and that of not fighting is 0.9.

The signaling game, on the other hand, has two perfect Bayesian equilibria, in both of which there is no fighting: in one both types of sender pick beer, in the other both pick quiche. Figure 1 represents the second of the two equilibria. The bully decides not to fight because there is no revelation of information in the pooling equilibrium.

Figure 1. Beer and quiche game: pooling on quiche



Cho and Kreps (1987) question this second equilibrium because the beliefs off the equilibrium path, when the sender orders beer for breakfast, require putting probability

one half or more on the wimpy type that prefers quiche. Since deviating to that strategy is dominated in equilibrium for the wimpy type (it can get at most a payoff of 2, less than the equilibrium payoff of 3), but not for the surly type (it can get a payoff of 3, more than the equilibrium payoff of 2), the intuitive criterion rules out these out-of-equilibrium beliefs, placing instead a probability of one for the surly type and probability of zero for the wimpy type. This destroys the pooling equilibrium where both types order quiche.

The same outcome results with the self-selection condition. If the pooling signal in a PBE is quiche, instead of beer, for breakfast, the surly guy has a temptation to order beer because it would be better off in the alternative equilibrium (it would get 3 instead of 2), while the wimpy guy would be worse off (it would get 2 instead of 3). Hence, the deviation to beer is credible only for the surly type. The unique pooling equilibrium is that where both types order beer for breakfast.

B. Job market signaling

We consider the Michael Spence (1973) job-market model where productivity is not observed by the firms. With two types of productivity, θ_1 (low) and θ_2 (high), where $\theta_1 < \theta_2$, and a probability λ that workers have type θ_2 , expected productivity is given by $E[\theta] = (1 - \lambda)\theta_1 + \lambda\theta_2$. In a competitive market, the expected profits of firms $E[\pi] = E[\theta] - w$ is zero, so workers are paid a wage that equals expected productivity: $w = E[\theta]$.²

If higher-productivity workers have a lower cost of education, education becomes a signal of underlying productivity. Specifically, let the utility of workers be given by the wage w minus the costs of education e , and let the costs of education be inversely related to productivity: $U(w, e; \theta) = w - \frac{e^2}{\theta}$.

When education is introduced as a signal, the model has both pooling PBE where both players receive a wage $w = E[\theta]$ and separating PBE where each type is paid its productivity, either $w_1 = \theta_1$ or $w_2 = \theta_2$. The pooling PBE are supported by any signal $e \in [0, e_p]$, where highest pooling signal e_p is such that type θ_1 is just indifferent between the education-wage pairs $(0, \theta_1)$ and $(e_p, E[\theta])$. These equilibrium signals can be supported by the out-of-equilibrium beliefs that less education implies a low productivity worker and more education implies average productivity.

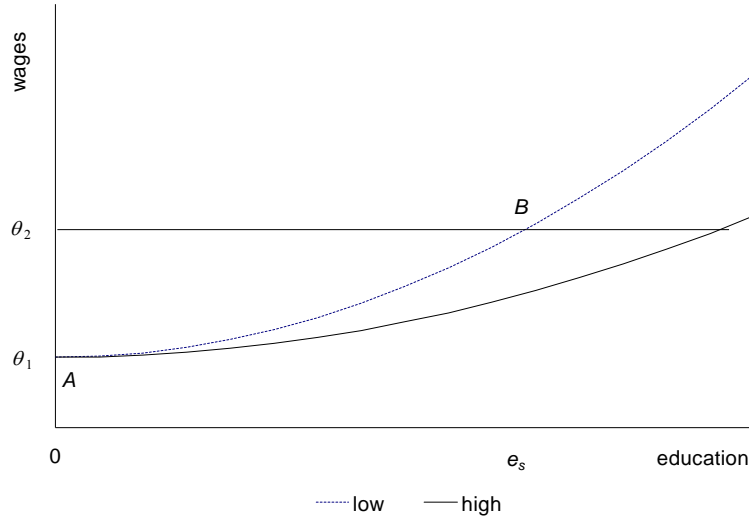
The separating PBE are supported by any separating signal $e \in [e_s, e_s']$, where the smallest separating signal e_s is such that type θ_1 is just indifferent between the education-wage pairs $(0, \theta_1)$ and (e_s, θ_2) ; $e_s > e_p$ because the minimal separating signal leads to a higher wage, while the largest separating signal e_s' is such that type θ_2 is just indifferent between the education-wage pairs $(0, \theta_1)$ and (e_s', θ_2) . The separating signal can be supported by the belief that less education implies a low productivity worker and more education implies high productivity.

The intuitive criterion reduces the bewildering multiplicity of PBE in the Spence job-market model to just one, the most efficient separating equilibrium e_s . In regard to separating equilibria, since type θ_1 is always better off with $e = 0$ and $w = \theta_1$ than with $e \in (e_s, e_s']$ and wage $w = \theta_2$ (and is just indifferent with $e = e_s$ and $w = \theta_2$), out-of-equilibrium beliefs have to put probability 1 on type θ_2 for $e \in [e_s, e_s']$. Of these, type θ_2 prefers the minimal separating signal e_s . All pooling equilibria are eliminated because type θ_2 is always willing to deviate to larger levels of education than type θ_1 .

² The approach is motivated by signaling games where there is a single sender, but it might apply to other contexts. For example, the Spence (1973) job-market model typically refers to a market setup where there are multiple senders and multiple receivers (see, e.g., Jack Hirshleifer and John Riley 1992: 405–430). In that setup, some kind of dynamics is often implicit when the intuitive criterion is applied to ask about what happens when a worker deviates from the prevailing market equilibrium by picking an out-of-equilibrium signal that may actually affects its payoffs: this works if a new equilibrium is only reached after time elapses. In those circumstances, it may be individually rational for a player to pick a disequilibrium move when the adjustment process is slow enough.

By the self-selection condition, for any separating signal $e \in (e_s, e_s^*]$, e_s is a credible deviation for type θ_2 , but not for type θ_1 . This is described in Figure 2, where points $A = (0, \theta_1)$ and $B = (e_s, \theta_2)$ are on the same indifference curve for the low productivity type, but point B is on a higher indifference curve for the high productivity type.

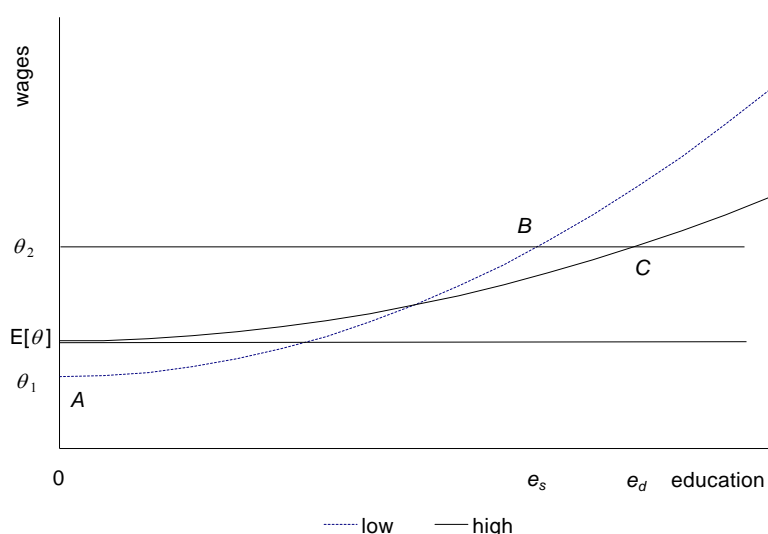
Figure 2. Job market signaling: efficient separating equilibrium



By requirement (iii) of the self-selection condition, for any positive equilibrium pooling signal $e \in (0, e_p]$, out-of-equilibrium beliefs have to assign the prior probabilities to both types for any smaller educational level: these are credible deviations for both types, so only the most efficient pooling equilibrium $(0, E[\theta])$ survives.

Finally, type θ_2 has to compare the utility of (e_s, θ_2) in a separating equilibrium with that of $(0, E[\theta])$ in a pooling equilibrium. The high productivity type only has an incentive to pick the separating signal $e = e_s$ if it is better off than with the payoffs from the pooling equilibrium with $e = 0$. This depends on whether the deviation e_d that leaves the high-productivity type just indifferent between (e_d, θ_2) and $(0, E[\theta])$ falls short or long of e_s . If $e_d \leq e_s$, then the pooling equilibrium stands because the smallest credible deviation leaves the high-productivity type worse off; otherwise, the separating equilibrium holds because it is a credible deviation, as described in Figure 3.

Figure 3. Job market signaling: maximum deviation from pooling equilibrium



Another way of seeing this is that there is a separating equilibrium when the probability λ that workers have type θ_2 is small. As probability $p\lambda$ grows, there is a point λ_s where $e_d = e_s$ so the gain in wages $(1 - \lambda p_s)(\theta_2 - \theta_1)$ just compensates the utility costs of switching from $e = 0$ to $e = e_s$ for type θ_2 . For a larger probability λ , there is instead a pooling equilibrium. For these parameter values, the prediction differs from the intuitive criterion.

When there are three types of workers, the intuitive criterion still eliminates all pooling equilibria, but it only eliminates the separating equilibria where the lowest productivity type is unwilling to deviate even if it earns the highest possible wage. With the self-selection condition, the relevant payoff to consider is whether a deviation is worthwhile given the alternative equilibrium payoffs, so it eliminates all separating equilibria except the one that corresponds to the Riley outcome where each type selects the most efficient signal to separate. To determine whether there is a separating or a (partially) pooling equilibrium will depend on the parameter values, which will determine whether the higher productivity types prefer to pool or not with the lower productivity types.

C. Entry-deterrence game

The third example, the Milgrom and Roberts (1982) entry-deterrence game with an incumbent firm and a potential entrant, motivates this paper. The Milgrom and Roberts (1982) entry deterrence game has multiple PBE. This holds also in dynamic versions of the signaling game (Flavio Toxvaerd 2013).

I follow the version of the entry-deterrence game in Christopher Avery (2014).³ The demand function is $p(q) = 9 - q$. The incumbent i has a low marginal cost $c_L = 1$ with probability λ and a high marginal cost $c_H = 3$ with probability $1 - \lambda$. The potential entrant e has a known marginal cost of 3 and incurs a fixed cost of 3 if it enters. Both firms maximize profits π .

³ Though quite similar to educational signaling, there are differences. Avery (2014: 457) points out two: first, entry deterrence resembles educational signaling when education is productive so each sender type has a different preferred outcome (if education is unproductive, both sender types prefer zero education); second, the incumbent firm's payoffs do not vary continuously with the beliefs of the entrant, as wages vary with the beliefs of employers.

The timing is as follows. There are two periods. There is no discounting. In the first period, the incumbent is a monopolist. The entrant observes the price that the incumbent charges. In the second period, the entrant decides whether to enter or not. After that, the incumbent's marginal costs are revealed. If the entrant decided to enter, the firms engage in Cournot competition, otherwise the incumbent acts like a monopolist.

In the second period, the outcomes depend on the entry decision as well as on the incumbent's type. Entry is beneficial if the incumbent has a high marginal cost, but not if it has a low marginal cost:

(i) no entry: with low-cost incumbent, $q_2 = 4$, $p_2 = 5$, $(\pi_{2i}, \pi_{2e}) = (16, 0)$; with high-cost incumbent, $q_2 = 3$, $p_2 = 6$, $(\pi_{2i}, \pi_{2e}) = (9, 0)$.

(ii) entry: with low-cost incumbent, $(q_{2i}, q_{2e}) = (\frac{10}{3}, \frac{4}{3})$, $p_2 = 13/3$, $(\pi_{2i}, \pi_{2e}) = (\frac{100}{9}, \frac{16}{9} - 3)$; with high-cost incumbent, $(q_{2i}, q_{2e}) = (2, 2)$, $p_2 = 5$, $(\pi_{2i}, \pi_{2e}) = (4, 4 - 3)$.

In the first period, there are both separating and pooling PBE. In a separating equilibrium, the high-cost incumbent chooses its monopoly price since its action reveals its type. The low-cost firm must choose a sufficiently low price such that the high-cost firm does not want to mimic it. Hence, the separating signal in the first period, which is given by the limit price p_l , has to be such that the profits of the high-cost firm are $(9 - p_l)(p_l - 3) \leq 4$.⁴ The low-cost firm has to be willing to pick the separating signal, so $(9 - p_l)(p_l - 1) \geq \frac{100}{9}$. These restrictions imply that the range for a separating signal is $2.79 \leq p_l \leq 3.76$ (Avery 2014: 453).

The intuitive criterion requires out-of-equilibrium beliefs to put probability zero on the high-cost type in this range. Hence, the only separating equilibrium that remains is with $p_l = 3.76$. The self-selection condition also restricts separating equilibria to at most that one.

In a pooling equilibrium, there will be no entry if the entrant's expected profits $\lambda(\frac{16}{9} - 3) + (1 - \lambda)(4 - 3) \leq 0$. For $\lambda < \frac{9}{20}$, the entrant's expected profits are positive, so no pooling equilibrium is possible: entry would occur anyways, so each type of incumbent would prefer to charge its monopoly price in the first period. For $\lambda \geq \frac{9}{20}$, the entrant's expected profits are non-positive, so there is no entry with the pooling signal (the entrant is just indifferent when expected profits are zero). If the expectations for prices other than the pooling signal are that the type is high-cost, which would spur entry, the pooling signal has to satisfy the restrictions $(9 - p_p)(p_p - 3) \geq 4$ and $(9 - p_p)(p_p - 1) \geq \frac{100}{9}$ for both firms to be willing to pick it. This leads to a range of pooling signals given by $3.76 \leq p_p \leq 7.21$ (Avery 2014: 455).

The intuitive criterion restricts pooling equilibria to the range $3.76 \leq p_p \leq 5.00$. For any price between 5 and 6, only the low-cost incumbent has an incentive to pick 5; for any price between 6 and 7.21, the low-cost incumbent always has a profitable deviation to a lower price than the high-cost incumbent. On the other hand, the self-selection condition restricts pooling equilibria to $p_p = 5$. For prices lower than 5, both type of incumbents prefer to deviate to 5, and for prices higher than 6, both type of incumbents prefer to deviate to 6; for prices between 5 and 6, only the low-cost incumbent prefers to deviate to 5, so beliefs would put probability one on the low-cost incumbent.

Finally, if $\lambda \geq \frac{9}{20}$, the self-selection condition indicates that only a pooling equilibrium exists. Since the low-cost incumbent prefers to charge its monopoly price if that deters entry, it prefers not to pick a separating signal with lower prices. Hence there will either be a pooling equilibrium, when the priors are enough to deter entrance, or a separating

⁴ By signaling with a low price, the high-cost firm foregoes a profit of 9 in the first period for a potential profit of 9 in the second period. The high-cost firm does not mimic that separating signal if it cannot earn more profits in the first period than the profit of 4 in the second period that it would make anyways.

equilibrium, when not. The self-selection condition thus leads to a separating equilibrium only if the priors are not enough to deter entrance.

IV. Self-selection in cheap-talk games

An especially important class of signaling games are cheap-talk games, where signals are payoff irrelevant so payoffs are given by $v^S(w_i, a_k)$ and $v^R(w_i, a_k)$. An equilibrium is *informative* if the receiver changes beliefs after some message on the equilibrium path (Joel Sobel 2011: 5).

In cheap-talk games, an uninformative equilibrium always exists. And once there is an informative equilibrium, there are infinitely many since the equilibrium messages can be permuted at will (Farrell 1993). To abstract from this second feature, I concentrate on the informative equilibrium where the literal and effective meanings coincide. This informative equilibrium where words are used in their ordinary sense can be justified, for example, resorting to the focal point argument in Schelling (1960) by which the players tacitly select an equilibrium without explicit communication.

I first formalize a game in Schelling (1960) with a parachutist that does not like to walk, nor wait. Then I discuss the game in Crawford and Sobel (1982) where there is an informed expert and an uninformed decision maker. In both games there is an infinity of types, unlike the previous section that discusses a finite number of types.

A. Schelling's parachutist

Schelling (1960) considers several games of verbal communication. Schelling (1960:58-59) presents two parachutists, one of which has “the power to send but not to receive messages” because his receiver is broken. Neither parachutist likes to walk.

This can be framed as a simple cheap-talk game where the sender can provide a verbal message about its location. We add the assumption that the parachutists prefer to wait as little as possible. If the locations are on the unit circle, the second parachutist will find the first parachutist sooner or later, so the basic decision is which way to walk.

There is an informative equilibrium where the parachutist uses the transmitter to state his exact location, which is indeed the answer that Schelling (1960: 59) provides: “if one can announce his position and state that his transmitter works but not his receiver, saying that he will wait where he is until the other arrives, the latter has no choice. He can make no counteroffer, since no counteroffer could be heard.” A myriad of other informative PBE are also possible. Any possible partition of the unit circle can provide an informative equilibrium that is partially pooling. An uninformative equilibrium is also possible where the parachutist just babbles and the other parachutist ignores the message.

The self-selection condition captures the Schelling (1960) intuition that the different sender types self-select their preferred equilibrium in this game. The self-selection condition selects a unique equilibrium: that where the parachutist states his exact location. Except at the most informative equilibrium where each sender type reveals itself fully, a credible deviation is available to all sender types. While the sender can stay put, the receiver has an incentive to heed the sender's message, knowing that full revelation is the sender's preferred signal. That way the receiver can minimize search costs, walking as little as possible and getting there as fast as possible.

B. Expert advice

The refinement inspired by Schelling's approach not always works. As a counterexample, the equilibrium refinement does not select any equilibrium in the uniform-quadratic model in Crawford and Sobel (1982) if the expert has a positive bias,

because there is no equilibrium partition that is commonly preferred by all sender types.

In the Crawford and Sobel (1982) model, there is a continuum of expert types $m \in [0,1]$ that are uniformly distributed over the unit interval. The sender (an informed expert) transmits a message "m" about its type m , and the receiver (an uninformed decision-maker) takes an action $y \in [0,1]$. The expert has a positive bias $b > 0$. The payoffs of sender and receiver are given by

$$u_S(m, y) = -(y - (m + b))^2,$$

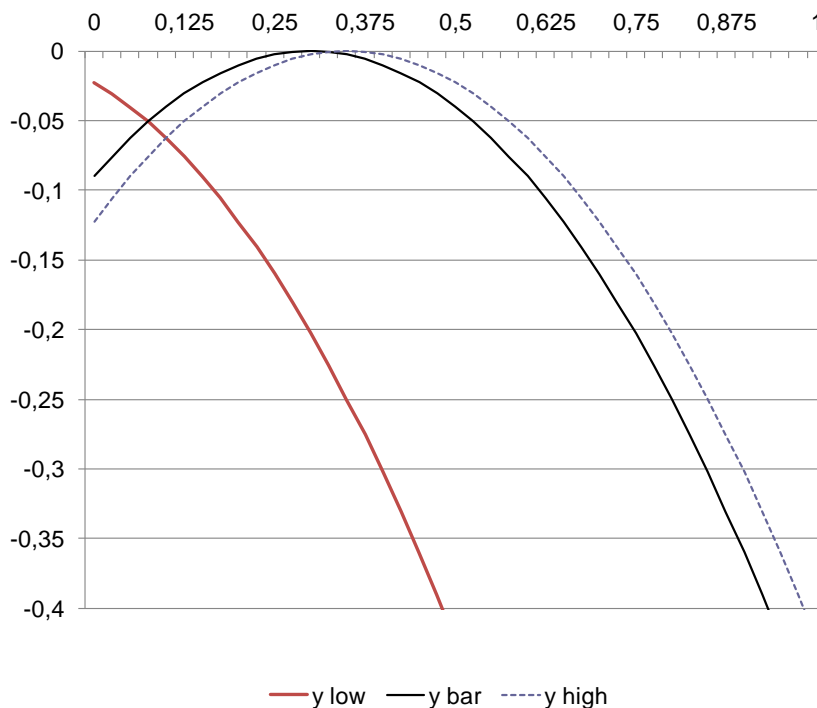
$$u_R(m, y) = -(y - m)^2.$$

Crawford and Sobel (1982) demonstrate that there are partially informative PBE if the bias $b < 1/4$. Take $b = 1/5$, where there is either an uninformative equilibrium or a partially informative equilibrium that partitions the unit interval in two.

As to the uninformative equilibrium, let the equilibrium message be "My type is in the unit interval", and let beliefs in response to out-of-equilibrium messages be that the type is $m = 50/100$. The equilibrium outcome is $y = 50/100$.

As to the partially informative equilibrium, it partitions the unit interval into low types $m \in [0, 1/10]$ and high types $m \in [1/10, 1]$. Let the equilibrium messages be "My type is between 0 and 1/10" and "My type is between 1/10 and 1", while any out-of-equilibrium messages lead to the beliefs that the type is $m = 0$. At the cutoff point $1/10$, type $m = 10/100$ is just indifferent between an action of $y = 5/100$ that corresponds to the low interval and an action of $y = 55/100$ that corresponds to the high interval.

Figure 4. Preferred outcome for different types of sender



The self-selection condition is unable to select a unique equilibrium. On the one hand, types $m \in [0, \frac{15}{200}]$ prefer the outcome $y = 5/100$ to $y = 50/100$, and types $m \in [\frac{65}{200}, 1]$ prefer the outcome $y = 55/100$ to $y = 50/100$. Hence, they are willing to deviate from the uninformative equilibrium to the informative equilibrium. On the other

hand, types $m \in \left[\frac{15}{200}, \frac{65}{200}\right]$ prefer the outcome $y = 50/100$ to any of the alternatives, so they are willing to deviate from the informative equilibrium to the uninformative equilibrium. This conflict of interests is depicted in Figure 4. This leads to cycles among equilibria.

This is a generic problem because different sender types have different preferences over the possible partitions of the unit interval. Crawford and Sobel (1982) already show this in their Figure 1, with $b = 1/20$ for which the most informative equilibrium partitions the unit interval in three intervals. Crawford and Sobel (1982: 1448) demonstrate that the receiver always prefers the most informative partition, so giving the receiver a say is one way out of this problem.

What happens if we look at this process iteratively instead? Consider the informative equilibrium that divides the unit interval into two subintervals. Since the types that actually want to deviate to the out-of-equilibrium message “My type is in the unit interval” are $m \in \left[\frac{15}{200}, \frac{65}{200}\right]$, the expected value of those who deviate is $y = \frac{40}{200}$. If the receiver takes this into account, only the types $m \in \left[0, \frac{35}{200}\right]$ will actually want to deviate, which gives an expected value of $y = \frac{17.5}{200}$. But then only the types $m \in \left[0, \frac{23.75}{200}\right]$ will actually want to deviate. Iterating this process, in the limit we reach the set $m \in \left[0, \frac{20}{200}\right]$. The iterated application of the self-selection condition should lead the receiver to offer the types that say “My type is in the unit interval” at most $y = \frac{10}{200}$, the same as those that say “My type is between 0 and 1/10”. This resembles the unraveling argument in Akerlof (1970) where higher types go dropping out until only the lowest types are left. The informative equilibrium survives the iterative self-selection condition.

On the contrary, the iterative self-selection condition destroys the uninformative equilibrium. This is not because of the types $m \in \left[0, \frac{15}{200}\right]$, because the iteration of the process leads to expect $y = 0$ for the out-of-equilibrium message “My type is between 0 and 1/10”. Rather, it is because of the types that are willing to say “My type is between 1/10 and 1”: this process starts with $m \in \left[\frac{65}{200}, 1\right]$. Though the interval shrinks with the iterations, the process stops at $m \in \left[\frac{80}{200}, 1\right]$, an irreducible group that is willing to deviate even if it is correctly identified as such. This irreducible group has the flavor of the “self-signaling neologisms” in Farrell (1993), because it is willing to deviate when it is identified as such.

The self-selection condition can also be expected to select the most informative partition for lower values of the bias \square because there will always be an irreducible group of high types that are willing to be identified as such in the Crawford and Sobel (1982) model. However, it is an open question whether the self-selection condition will also work in other games where the self-selection condition leads to cycles.

V. Implications of the self-selection condition

The key idea is to let the players self-organize from within, instead of imposing a structure from without, given that the sender voluntarily decides whether to pick a signal or not. We now look at two general implications of the self-selection condition.

A. Necessary condition for informative equilibria

After stating a simple implication of the self-selection condition, a game in Farrell (1993) where neither sender type is interested in revealing its type will be analyzed to illustrate this.

LEMMA 1: *Informative equilibria of signaling games pass the self-selection condition only if in each informative equilibrium not all sender types are worse off than in an uninformative equilibrium where the priors hold.*

PROOF: *Suppose not, so all types of sender are worse off in any of the possible informative equilibria when compared to an uninformative equilibrium where the priors hold. But if no sender type has an incentive to disclose information, they have no incentive to voluntarily pick a (semi) separating signal, whether that signal is costly or not. Hence, the receiver will have to rely on the priors of the game without communication.*

By Lemma 1, informative equilibria must make sense for at least one type of sender. Else, an uninformative equilibrium obtains. We already saw an example with the Milgrom and Roberts (1982) entry deterrence game. But that also holds for cheap-talk games where signaling costs nothing to the sender.

Take an example from the cheap-talk game “I won’t tell,” Example 2 in Farrell (1993), where the receiver has best responses $a(A)$ and $a(B)$ for each sender type, but prefers a safe strategy $a(T)$ if it must act on its priors ($p(A), p(B)$). There are informative and uninformative cheap-talk equilibria.

While the two sender types prefer the uninformative equilibrium, Figure 5 depicts the informative cheap-talk equilibrium where the receiver plays $a(B)$ unless the message “A” is uttered.⁵ This out-of-equilibrium message, of course, comes out of the analyst’s hat to support the informative equilibrium.

Figure 5. Cheap talk: not withholding information

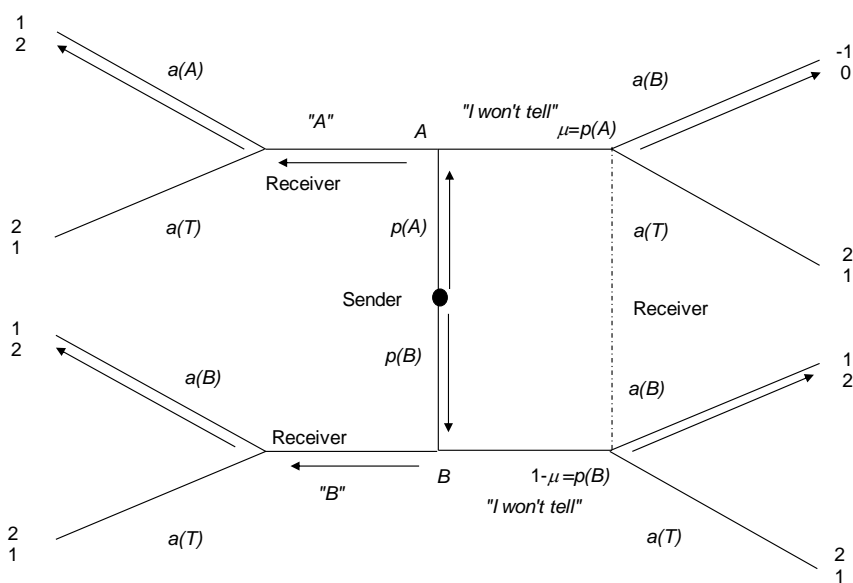
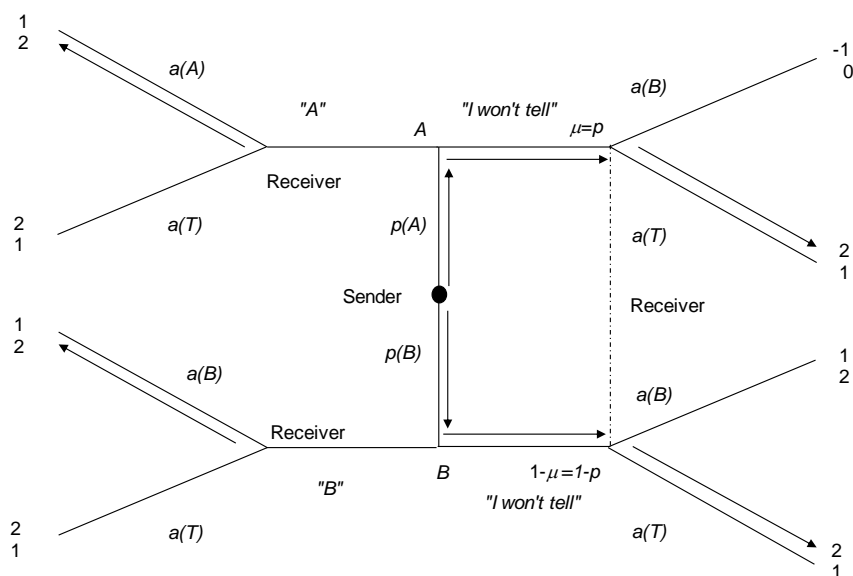


Figure 6 shows that this informative equilibrium is impossible once we apply the equilibrium refinement. Since neither type of sender has an incentive to reveal itself, they will select the message “I won’t tell”, which is equivalent to “I am either type A or

⁵ To not clutter the figure, I simplify the game. First, I eliminate the options where A can send message “B” and B the message “A”; hence, for the receiver it is never a best response to pick $a(A)$ in response to “B” nor $a(B)$ in response to “A”. Second, I eliminate the option where the receiver may play $a(A)$ in response to “I won't tell”, since the option $a(B)$ suffices to sustain the informative equilibrium.

B". In the absence of new information, the best strategy for the receiver is to play a best response to its priors.

Figure 6. Self-selection condition: withholding information



B. Necessary condition for uninformative equilibria

In Schelling's parachutist game, an uninformative cheap-talk equilibrium always exists. However, all sender types have a preference for the most informative equilibrium, so they have an incentive to choose messages that point to their exact type. Since the receiver knows that no sender type has an incentive to distort the information, if it expects all sender types to coordinate on this equilibrium, it makes sense to trust the messages. More generally:

LEMMA 2: *Uninformative equilibria of signaling games pass the self-selection condition only if in each uninformative equilibrium not all sender types are worse off than in an informative equilibrium.*

PROOF: *Suppose not, so all types of senders are worse off in any of the uninformative equilibria when compared to a given informative equilibrium. But then all sender types have an incentive to pick a (semi) separating signal. They can also point out to the receiver that they prefer to select a more informative equilibrium where they reveal, in part or in whole, their type.*

This equilibrium refinement provides a sufficient condition for signaling to be informative: all types prefer to reveal their type. Only having some types willing to reveal information about their type is not a sufficient condition for an informative equilibrium to be selected, as the cycles in the linear-quadratic game in Crawford and Sobel (1982) attest.

VI. Conclusions

As Myerson (2009) puts it, Schelling (1960) studies "problems of conflict and cooperation among people who are intelligent and rationally motivated by individual self-interest". Schelling's analysis is in line with the rational interpretation of equilibrium points in Nash (1950) where the players know the model and use that information to

find the equilibrium. In the formation of mutually consistent expectations, Schelling (1960:74) points out that “the role of signals that have the power to coordinate expectations may be powerful.”

For signals to work, they have to be credible, i.e., part of an equilibrium. In the case of multiple equilibria, what signal will the sender actually pick? The Cho and Kreps (1987) intuitive criterion is adapted so that only deviations to signals that are part of an alternative PBE are credible and thus affect beliefs. Incorporating the ideas on equilibrium selection in Schelling (1960), the only sender types that deviate are those for which the alternative equilibrium payoffs are larger (or equal). This refinement can capture the role of signals as an explicit equilibrium-selection device. Schelling (1960) had this in mind and gives an example where the sender’s preferences pinpoint equilibrium beliefs, pointing to the most informative equilibrium that is in the mutual interest of both players.

The self-selection condition may lead to cycles, as shown in the Crawford and Sobel (1982) cheap-talk game of an informed expert and an uninformed decision-maker. An iterative self-selection condition based on the logic of the Akerlof (1970) unraveling argument, which looks at what types effectively wish to deviate if the deviation is correctly anticipated by the receiver, selects a unique equilibrium in this specific instance. However, their general implications have to be explored to determine the class of signaling games for which they address the problem of multiplicity of equilibria.

The question of equilibrium selection posed by John C. Harsanyi and Reinhard Selten (1988) is thus recast here as the question of which is the class of signaling games for which a unique equilibrium will be selected by the players themselves, thus determining mutually consistent expectations. The implications of different sender types that self-select signals might also be explored under the first interpretation of equilibrium points in Nash (1950), that of players that do not know the model and optimize adaptively, perhaps building on the ideas of proper equilibrium in Roger Myerson (1978) and divine equilibrium in Jeffrey S. Banks and Joel Sobel (1987).

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