NOTES ON INTERTEMPORAL TRADE IN
GOODS AND MONEY [1]

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Ever since Svensson and Razin (1983) pioneered intertemporal analysis in a cutting edge analysis of a microeconomic conundrum—the Laursen-Metzler-Harberger problem of the terms of trade effect on spending—, intertemporal approaches to the current account have flourished. The state of the art is codified in Obstfeld and Rogoff (1995, 1996); the range of applications has widened to analyses involving risk, as in the most recent Obstfeld-Rogoff work or macroeconomic interdependence as in the ambitious paper of Corsetti and Pesenti (1998). The literature is in full swing. But it also suffers the difficulty that it quickly runs into diminishing returns—with many goods, assets and periods the range of outcomes quickly widens and models lose their leverage.

The present paper takes the opposite tack of the literature in choosing heuristically to go backward toward the very simplest model and the most basic structure, namely trade in a single good and a common money. This simplification highlights intertemporal trade and focuses on Patinkin-Ricardo style monetary economics in the open economy. Imagine the simple question: there is one time increase in the home money supply, what happens to world real interest rate and the path of prices? More broadly, when does money have an effect on real interest rates and how does a fiscal policy disturbance affect price? As it turns out, in this very simple world of real interest rates, current and future price levels (even with functional forms that are rock-

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bottom simple) there is a tight structure of what determines real interest rates and prices. This can serve as a background for more ambitious questions.

The Set-Up

The model considers two countries, two periods, a single good and one money in the world. The representative agent, identical across countries, maximizes welfare subject to the intertemporal budget constraint.

$$V = U(C_t, m_t) + d \, U(C_{t+1}, m_{t+1}); \qquad d = 1/(1+\delta) \qquad (1)$$

$$C_t = y_t + qb + (M_o - M_t)/P_t, \qquad q = 1/(1+r) \qquad (2)$$

$$C_{t+1} = y_{t+1} - b - (M_{t+1} - M_t)/P_{t+1}$$

where C, M and m denote consumption, nominal and real balances, y is the output endowment, r is the real interest rate and δ the discount rate. Bonds in this model pay off a unit of output with b the number of bonds and q the current bond price. Agents have an initial endowment of money, M_o as well as endowments of non-storable goods in each period.

Optimization

The first step is optimization. The consumer must chose a path of consumption, debt and real balances that maximize intertemporal welfare. One way to proceed is to select the optimal level of debt (or lending, depending on endowments), b, and the optimal initial and second period money holdings, M_t and M_{t+1}. As seen from the budget constraints in (2), this will imply levels of consumption C_t and C_{t+1}. The first order conditions are shown in (3) to (5).

In (3) debt accumulation involves, for given money holdings, shifting consumption between periods. Accordingly, we see the marginal rate of substitution today and tomorrow as well as the expression $d/q = (1+r)/(1+\delta)$

which represents the real interest rate relative to the discount rate which is the traditional determinant of the intertemporal consumption profile.

$$qU_c(C_t, m_t) = dU_c(C_{t+1}, m_{t+1}) \tag{3}$$

$$[-U_c(C_t, m_t) + U_m(C_t, m_t)]/P_t + dU_c(C_{t+1}, m_{t+1})/P_{t+1} = 0 \tag{4}$$

$$U_c(C_{t+1}, m_{t+1}) = U_m(C_{t+1}, m_{t+1}) \tag{5}$$

In (4) we see how increasing current period money holdings, for given debt, the resource endowment as well as initial nominal balances, implies a reduction in current consumption as a tradeoff. In return, higher money balances yield both current utility services as well as higher future consumption opportunities since money holdings are one way of transferring consumption to the future.

Equation (5) finally shows the tradeoff involved in selecting second period money holdings: increased money holdings come at the expense of consumption in that period.

Consider now a particularly simple functional representation for U(.), namely a log form which is both separable and implies a unit intertemporal elasticity of substitution:

$$U_t = \alpha \log C_t + \log m_t; \quad U_c = \alpha/C; \quad U_m = 1/m; \quad \alpha > 0 \tag{6}$$

The first order conditions above now become:

$$C_{t+1}/C_t = d/q \tag{3a}$$

This is the conventional Euler equation stating that future consumption is higher relative to current consumption the higher is the real interest rate relative to the discount rate. Thus the consumption profile does not depend on the inflation rate.

Next,

$$[-\alpha/C_t + 1/m_t]/P_t + (d\alpha/C_{t+1})/P_{t+1} = 0 \qquad (4a)$$

or, using (3a) and denoting by λ the nominal interest rate factor $\lambda \equiv q/(P_{t+1}/P_t) = 1/(1+r)(1+\pi)$ where π is the actual and expected rate of inflation, we have:

$$C_t = \alpha(1 - \lambda) m_t; \quad 1 \geq \lambda > 0 \qquad (4b)$$

where the restriction on λ derives from the assumption that the nominal interest rate is nonnegative. Thus, with extreme inflation (given real interest rates), λ will tend to zero and conversely with large deflation the term will tend toward unity.

In (4b) monetary choices obviously depend on inflation. Specifically, with low nominal interest rates the ratio of consumption to real balances is low and conversely when nominal rates are high. The term $\alpha(1-\lambda)$ can be viewed as the (first period) consumption velocity of money. Finally, the second period choice of money only involves consumption foregone in that period since inflation and interest apply only between periods:

$$C_{t+1} = \alpha m_{t+1} \qquad (5a)$$

Using the budget constraint in (2) we can solve for initial consumption plans and borrowing:

$$C_t = \theta \ [y_t + qy_{t+1} + M_o/P_t] ; \quad \theta \equiv \alpha/[(1+\alpha)(1+d)] < 1 \qquad (7)$$

where θ is the share of first period consumption in wealth.[2]

In (7), the share of optimal present consumption in wealth is a declining function of the real interest rate and of the current price level.

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[2] Using (3a) second period consumption is given by $C_{t+1} = (d/q)C_t$

Optimal current borrowing, qb, is a declining function of real interest rates and of inflation but an increasing function of the first period price level.

$$qb = \theta\beta \{y_t + qy_{t+1} + M_o/P_t\} - (y_t + M_o/P_t); \quad \beta \equiv [1 + 1/\alpha(1-\lambda)] \quad (8)$$

Thus current borrowing depends on real interest rates, inflation (via the term λ) and the initial price level. This borrowing may be positive or negative and it may finance current consumption, additions to initial nominal balances or both.

The effect of current resources on lending is critical to the capital market. We would, without much reflection, expect an increase in present resources to reduce borrowing. This is not necessarily the case, except when inflation is high. Specifically, the condition for borrowing to decline in response to current resources is: $\lambda/(1-\lambda) < d(1+\alpha)$. As nominal interest rates tend toward zero (and λ toward unity), the condition I bound to be violated. In what follows, we concentrate on the case where the restriction holds so that an increase in current resources or initial money balances reduces borrowing.

Market Equilibrium

Consider now two countries and assume that the representative agents in each country have identical preferences. As usual, a star denotes the foreign country (*). Their endowments of goods and initial money holdings may differ. Let M^w and y^w denote world money and world output.

Equilibrium in the current goods market requires that world consumption equal world output:

$$C_t + C^*_t = y^w \quad (9)$$

and the capital market must clear:

$$b + b* = 0 \quad (10)$$

In addition, in the second period, the goods market must clear:

$$C_{t+1} + C^*_{t+1} = y^w_{t+1} \tag{11}$$

Since distribution effects are assumed absence— θ is common to the two countries—equilibrium interest rates and prices will only depend on the world aggregates of endowments and the path of money. Just as economic agents forming expectations would do, the model can be solved starting with the second period:

$$\alpha M^w_{t+1} = P_{t+1}(C_{t+1} + C^*_{t+1}) = P_{t+1}y^w_{t+1} \tag{11a}$$

which yields the equilibrium second period price level denoted simply by $P_{t+1} = (M/y)^w_{t+1}$. We make further headway by noting that the consumption profile over time is constrained by resource endowments. Accordingly, from (3a) we obtain the equilibrium real interest rate as

$$q^{-1} \equiv 1+r = (1+g)(1+\delta) \tag{3b}$$

with g the growth rate of the endowment.

Finally, from goods market equilibrium, using (7) we have:

$$\theta(y^w_t + qy^w_{t+1} + M^w_o/P_t) = y^w_t \tag{7a}$$

or, using (3b)

$$P_t = \alpha (M_o/y)^w/(1+d) \tag{12}$$

We thus have a set of strong results: The real interest rate only depends on the growth rate of the endowment, the present price level only depends on current money adjusted for present output, time preference and the coefficient α which captures the relative importance of consumption in utility.

Finally, inflation is given by:

$$P_{t+1}/P_t = (1+\mu)(1+d)/(1+g) \tag{13}$$

Which depends on time preference, money growth and real growth. Note that money growth neither real growth nor time preference affect current prices; their incidence is fully shifted forward into future prices.

Finally, the equilibrium nominal interest rate φ only depends on time preference and money growth, not on the growth rate of endowments.

$$\varphi = \mu + (1+\mu)(1+\delta) \tag{14}$$

Figure 1 offers a way of looking at the equilibrium of the model. The schedule GG shows the present goods market shown in Eq. (11a).

Figure 1
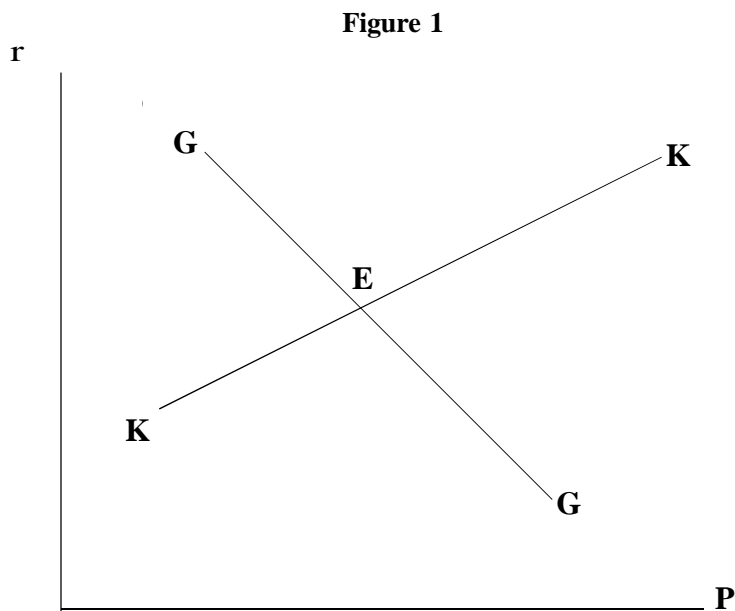
Using (7), higher real rates create an excess supply of present goods while a lower price level, via the effect on the endowment raises demand. Accordingly, GG is without ambiguity negatively sloped.

The schedule KK shows capital market equilibrium.

$$(\theta\beta - 1) (y^w_t + M^w_o /P_t) + \theta\beta q y^w_{t+1} = 0 \qquad (10a)$$

Higher real interest rates reduce borrowing while a higher price level, by reducing real balances, will increase borrowing (remember $\theta\beta < 1$ by assumption). Accordingly, KK is positively sloped. Thus a fall in interest rates which encourages borrowing is offset by higher prices that reduce wealth and spending.

The first period equilibrium is at point *E* where goods and capital markets clear. Whether there is inflation or deflation and whether real interest rates exceed or fall short of the discount rate depends on the patterns of endowments across countries and across time. We next turn to a simple no-trade equilibrium.

An Equilibrium without Trade

To see some properties of the equilibrium, consider the case of a fully symmetric world. Assume endowments of goods are equal across time and across countries and let initial money endowments be equal and money constant across time. Clearly, there is no ground for international trade, neither of goods for present money nor for goods today in exchange for goods tomorrow. Since the endowment is flat over time, so must be equilibrium consumption. Hence, according to (3a) the real interest rate will be equal to the discount rate.

Furthermore, from (5a) and (4b) we have: $P_{t+1} /P_t = 1+d$. Accordingly, even though endowments are constant as is nominal money, the price level is rising. Moreover, the lower the discount rate the higher inflation.

Consider next the simple extension to a case where the equally distributed

endowments of goods increase at the rate g and those of nominal money (delivered by helicopter) at the rate μ. It is readily shown that the real interest rate is higher, the higher is the growth rates of endowments, $(1+r) = (1+g)(1+\delta)$. Monetary disturbances thus do not influence the real interest rate. As to inflation, the equilibrium rate is determined by both money growth and endowment growth. There is no surprise in higher endowment growth reducing inflation and conversely for money growth.

Figure 2 shows the adjustment to increased endowment growth. In the goods market higher wealth leads to increased demand for present output thus shifting GG out and to the right. In the capital market the financing of current consumption and extra real balances raises bond issues shifting KK up and to the left. Both the price level and the real interest rates are mechanisms for crowding out current consumption. The equilibrium real interest rate unambiguously rises. As to the price level, there seems to be some ambiguity. But we have already seen above that the current price level is independent of money growth and endowment growth.

Figure 2
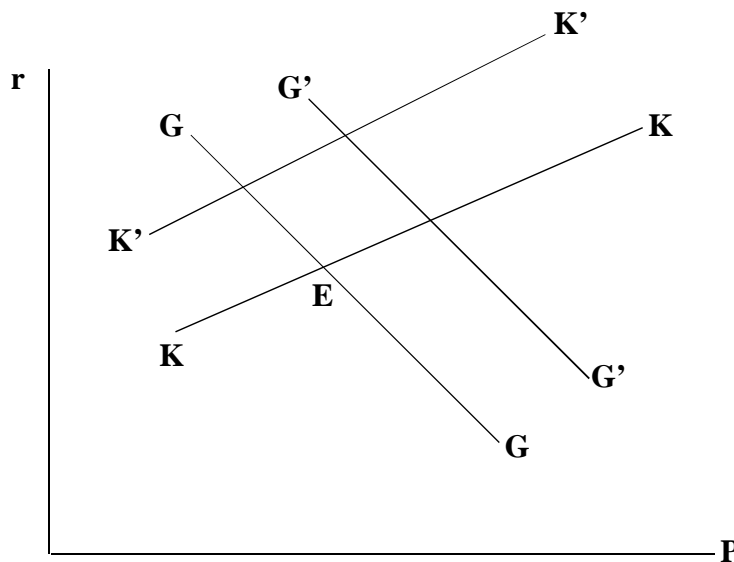
Welfare

Corsetti and Pesenti (1998) presented an analysis of the welfare economics of macroeconomic disturbances. This is readily done in the present structure by looking at the determinants of changes in welfare. The change in welfare can be decomposed into three factors:[3]

$$dV = \; bdq + \; (\; dy_t + q \; dy_{t+1} \;) + d(M_o/P_t) - (\lambda/(1+\pi)) \; m_t \; d\pi \qquad (15)$$

- Changes in the intertemporal terms of trade which are proportional to the debt position
- Changes in the value of the endowment (measured at initial real interest rates) or changes in initial real balances.
- The adverse effect of an increase in inflation.

Thus, for example, endowment growth in the world raises welfare directly and by reducing inflation. An increase in initial money elsewhere raises current (and future) prices and hence deteriorates welfare in the country left out.

Borrowing and Lending

Consider now international issues raised in this framework. Starting from a symmetric equilibrium without trade, suppose that home endowments are expected to rise. Just as in Figure 2, real interest rates will rise and the current price level will remain unchanged. These results simply depend on the world increase in planned consumption and borrowing.

Of course, world current consumption cannot rise since the present endowment has not changed. Accordingly, crowding out means that the home

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[3] The welfare change is derived from the utility function, the first order conditions and the budget constraint.

country consumes more and the foreign country less. The home country borrows and the foreign country lends. Moreover, in the home country real balances will rise relative to those abroad. With a common price level, this means that the home country's borrowing finances both higher consumption and a build-up of real balances relative to the partner country.

Accordingly the model predicts current account deficits more than offset by capital account surpluses for countries anticipating relative growth.

Assuming a zero net debt situation to start, the home country gains in welfare from a higher endowment and reduced inflation. The foreign country also gains, not directly from a higher endowment but at least from reduced inflation.

Suppose, in contrast, that the world money supply is expected to rise in the second period but the money transfer accrues entirely to the home country.[4] Using again Figure 2, in the home country current consumption rises as does borrowing by the domestic country as agents adjust spending plans to the increased resources. We already know that the price level will remain unchanged. Equilibrium real interest rates increase. The home country runs a current account deficit. Not surprisingly, foreign welfare declines because inflation rises.

Government

Consider now the introduction of government spending and tax disturbances. Let the government be financed by lump sum taxes and assume that government spending is not at all valued. In this world, Ricardian equivalence of course applies fully and the present value of spending is fully discounted as an equal reduction in private sector resources. But note, too, that goods cannot be carried forward although real claims can. Also note that time preference as well as the allocation of wealth to money holdings

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[4] In terms of the budget constraint in (2) we add the term T/P_{t+1} to denote the real value of transfers where $T = \mu M_0^W$. According this term appears as qT/P_{t+1} in first period consumption.

complicate the analysis. There is, of course, also the open economy aspect which allows to distribute the crowding out.

We concentrate on the case where the home government spends and taxes rather than prints money. The various possibilities are present spending or future spending, present taxes and future taxes in all possible permutations. Consider first a current increase in both spending and taxes. Households will reduce spending only by a portion of the current tax increase while government spending rises fully. Accordingly, at the initial price level and interest rates there is an excess demand for present goods and an excess supply of bonds. In terms of the diagram, GG shifts up as does KK.

World real interest rates clearly rise and the home country runs a current account deficit. Suppose taxation had been spread out, what difference would there be? In terms of spending decisions, Ricardian equivalence would leave the impact on demand unchanged. But now the government would borrow in place of the private sector. There would still be a rise in interest rates and a current account deficit.

Reverse next the order of things and consider future government spending. In this case crowding *in* is required in the current period. The incidence of taxes, now or later, is to reduce private spending today and to lend. As a result, real interest rates decline and the home country runs a current account surplus.

In terms of welfare, increased government spending of course acts like a reduction in the endowment. Moreover, although taxation is lump sum and as such has no "collection cost", extra effects on the price level and the rate of inflation do affect welfare. Thus, to the extent that the home country's spending program raises present prices and/or inflation, it will reduce welfare abroad. Thus suppose the home government increases future spending by a fraction σ of the world endowment and taxes in the same period by the same amount. It is readily shown that the present price level is unchanged, real interest rates rise and the future price level increases. Unambiguously, the home country is worse off because of taxes and inflation; the foreign country experiences a deteriorate in welfare due to inflation.

Extensions

The simple framework offered here has obvious extensions. One is a technology that allows carrying goods between periods—investment. Another is to introduce uncertainty. Yet another one is to revisit the way money is introduced in the model and explore in a deeper fashion the price level impact of disturbances.

The basic point of the model is to organize our intuition about the implications for world real interest rates and prices of standard disturbances as well as for international lending. The simplicity of the model meets exactly that ambition.

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