XI

Volume XI, Number 2, November 2008

Journal of Applied Economics

Luigi Landolfo

Assessing the sustainability of fiscal policies: Empirical evidence from the Euro Area and the United States

UCEMA

Edited by the Universidad del CEMA

Print ISSN 1514-0326
Online ISSN 1667-6726

Journal of Applied Economics. Vol XI, No. 2 (November 2008), 305-326

ASSESSING THE SUSTAINABILITY OF FISCAL POLICIES: EMPIRICAL EVIDENCE FROM THE EURO AREA AND THE UNITED STATES

Luigi Landolfo*
University of Warwick

Submitted June 2005; accepted July 2007

This paper provides a formal theoretical framework for analyzing the sustainability of fiscal policy based on the government intertemporal budget constraint, and derives conditions that determine whether a fiscal stance is sustainable in the medium and long term. In contrast to previous studies, it uses a log-linearization of the public debt identity and generalizes the results obtained in the literature by using a multivariate test. The analysis is applied to the fiscal position of the United States and the Euro Area. On the basis of infinite horizon-tests the broad conclusion is that both regions have a sustainable fiscal policy.

JEL classification codes: C32, E62
Key words: fiscal policy, sustainability of fiscal policy, VAR, VECM

I. Introduction

The sustainability of fiscal policy has been receiving increasing attention from economists. Several developed countries have been facing significant fiscal deficits in the last few decades. In the United States, for instance, the fiscal year 2005 ended with the highest dollar debt in the country's history. The same is true for some of the EMU countries where, despite the imposition of deficit and debt ceilings, the public debt in ratio to GDP is higher than 100% (i.e., Italy and Greece). The main question about this issue is whether or not the high public debt is becoming more and more unsustainable. This paper provides a formal theoretical framework for analyzing the sustainability of fiscal policy based on the government intertemporal budget constraint, and derives conditions that determine whether a fiscal stance is

---

* Luigi Landolfo: Department of Economics, University of Warwick, Coventry, CV4 8UW, UK. E-mail: l.landolfo@warwick.ac.uk. We would like to thank two anonymous referees of the Journal and the co-editor, Jorge M. Streb, for comments on the earlier draft of the article.

sustainable. This analysis is then applied to the fiscal position of the United States and the Euro Area.

Most of the literature on this topic uses deflated variables and is focused on the peculiarities of the process generating the ratio of public debt to GDP. In particular, fiscal policy is declared sustainable if public debt, or the first difference, is found to be stationary. The stationarity of this variable (or the first difference) is usually conditioned to the stationarity of the explanatory variables, or to the presence of a cointegrated relationship between them. Most of these studies use univariate tests. The theoretical framework provided in this paper differs from the existing literature in several important aspects. The contribution of this study is twofold. First, in contrast to previous studies, it uses a log-linearization of the discounted public debt identity, showing that, under the assumption of a stochastic interest rate, the sustainability of fiscal policy can be considered as a linear problem. Second, it generalizes the results obtained in the literature by using a multivariate test, analyzing the sustainability of fiscal policy in a multivariate framework.

The use of a multivariate test is justified by three considerations. First, by using a multivariate test, it is possible to apply specific tests for VAR models to the sustainability of fiscal policy, i.e., roots analysis. Second, a multivariate framework provides a nested model that generalizes the results of the previous literature on this issue. Third, under a Vector Error Correction representation, the VAR model permits one to apply a cointegrated analysis to the data involving all variables in the system.

The sustainability of fiscal policy is studied under two scenarios. In the first scenario, we assume both the primary balance and the interest rate are exogenous variables. In the second scenario, we assume the public debt and the primary balance are endogenous variables and the interest rate is the only exogenous variable. The analysis suggests that the condition for sustainability depends upon the stability of the VAR system. If the system is globally stationary, and thus stable, fiscal policy is said to be sustainable. If, on the other hand, the system is not globally stationary, and hence not stable, fiscal policy is said to be unsustainable.

The paper is organized as follows. Section II looks at the way in which fiscal sustainability has been assessed in the literature so far. Section III discusses the public debt identity in a VAR framework, and provides the conditions for sustainability in two scenarios for the stochastic processes generating the primary deficit and the interest rate. Section IV applies this test to the U.S. and Euro Area economies. Finally, Section V contains concluding comments.

II. Intertemporal budget constraint and literature review

The sustainability of public deficits is determined by the dynamics of the intertemporal budget constraint (IBC). In the literature there exist two approaches to study the IBC. The former uses mathematical tools, while the latter uses an econometric approach. This paper, following the second approach, relies on econometric techniques.

A short review of the fiscal deficit arithmetic is needed to set the theoretical framework for the subsequent sustainability analysis. The public debt at the beginning of fiscal year t, i.e., bt, evolves according to:

$$b_t = b_{t-1} + r_t b_{t-1} + g_{t-1} - \tau_{t-1} - s_{t-1}, \tag{1}$$

where b_{t-1} is the public debt at beginning of fiscal year $t-1$, r_t is the real interest rate adjusted for output growth rate at beginning of year t (i.e., $r_t = i_t - \pi_t - \rho_t$, where i_t , π_t and ρ_t are the nominal interest rate, the CPI inflation rate and the GDP growth rate at time t), $r_t\, b_{t-1}$ are the interest payments at the beginning of fiscal year t, g_{t-1} is the government expenditure net of interest during fiscal year $t-1$, τ_{t-1} are the tax revenues net of transfers during fiscal year $t-1$, and $s_t = (M_t - M_{t-1})/P_{t-1}$ denotes the real revenues from seigniorage, which for simplicity we assume to be constant and equal to $\overline{s}$ (note that M_t and P_t represent the money supply and the price level at time t). All fiscal variables are expressed as a proportion of nominal GDP.

The government cudget constraint (GBC) in equation (1) can be rewritten in more compact form as

$$b_t = d_{t-1} + (1 + r_t)b_{t-1}, \tag{2}$$

or equivalently as

$$\Delta b_t = d_{t-1} + r_t b_{t-1}, \tag{3}$$

where $d_{t-1} = g_{t-1} - \tau_{t-1} - s_{t-1}$ is the ratio of government primary balance to GDP generated during fiscal year $t-1$, which can either be a deficit (+) or a surplus (–).

Equation (3) states that the change of the public debt to GDP ratio must cover the primary balance to GDP ratio inclusive of interest payments. The traditional approach investigating the sustainability of countries' fiscal policy based on the intertemporal budget constraint of the government solves the equation of the debt forward n periods ahead, and then takes the expectation at time t:

$$b_t = E_t \beta_{t,n} b_{t+n} - E_t \sum_{i=1}^{n} \beta_{t,i} d_{t-1+i},$$

where $\beta_{t,n}$, the time varying real discount factor adjusted for GDP growth rate, is defined as $\beta_{t,n} = \prod_{j=1}^{n} (1 + r_{t+j})^{-1}$.

In the long term fiscal policy is said to be sustainable if the present value of public debt is identical to the present value of future primary surpluses, $b_t = -\lim_{n\to\infty} E_t \sum_{i=1}^{n} \beta_{t,i} d_{t-1+i}$. Thus, a necessary and sufficient condition for sustainability is:

$$\lim_{n\to\infty} E_t \beta_{t,n} b_{t+n} = 0.$$

This is also known as transversality condition and implies no Ponzi games, meaning no new debt is issued to meet new interest rate payments. This condition does not mean that debt should go to zero at any point in time. The debt can also grow at a positive rate. Of course a permanent positive growth rate is inconsistent with the above equation. A deficit at any point in time (or over a period of time) has to be offset by a surplus at another point in time (Uctum-Wickens 2000).

A few years ago researchers were content to assume the existence of the intertemporal budget balance (2) without worrying about whether the data generating processes were consistent with such a constraint. More recently, researchers have tried to implement tests of the intertemporal budget constraint in a variety of different contexts. In one of these contexts the main tools used to analyse the sustainability of budget deficits are stationarity tests for public debt, and cointegration tests between government expenditures and government revenues (or alternatively public debt and primary deficit). Some results of these works follow.

Hamilton and Flavin (1986) suggest that a sufficient condition for the present value of the budget constraint (PVBC) to hold is that the primary balance, and therefore that public debt, is a stationary series. It should be noted that this is a sufficient but not necessary condition for sustainability; fiscal policy could be sustainable even if debt is non-stationary. They find that non-stationarity can be rejected and that the PVBC therefore is not violated.

Trehan and Walsh (1988) argue that if debt is integrated of order 1, and if the real interest rate is constant, a necessary and sufficient condition for the PVBC to hold is that debt and the primary fiscal balance (d_{t-1}, b_{t-1}) are cointegrated. Looking at the equation (3) one can see that if b is I(1), the change in debt must be stationary by definition; the overall deficit (i.e., $d_{t-1}+rb_{t-1}$) is I(0). Thus, if r is constant, d_{t-1} and

$b_{t\text{-}1}$ are cointegrated with a cointegrating vector $(1,r)$. Three years later the same authors (Trehan and Walsh 1991) end up with the conclusion that if the (expected) real interest rate is not constant, sustainability no longer implies that $(d_{t\text{-}1}, b_{t\text{-}1})$ are cointegrated. A sufficient condition for the PVBC to hold is that the overall deficit, $d_{t\text{-}1}+rb_{t\text{-}1}$, is stationary.

The approach of Hakkio and Rush (1991) is to test the cointegrated relationship between public spending and level of taxes. Their work relies on the hypothesis that the real interest rate is a stationary variable with mean r. The authors assert that when there is no cointegration between these variables, the fiscal deficit is not sustainable; when there is cointegration with coefficient $\beta=1$ the deficit is sustainable; when there is cointegration with $\beta<1$, government expenditures are growing faster than the revenues, and the deficit may not be sustainable.

Wilcox (1989) shows that when the transversality condition holds, the present value of government debt is stationary and has an unconditional mean of zero. For the United States he finds mixed evidence on stationarity and rejects an unconditional mean of zero, thus concluding that post-war U.S. fiscal policy has been unsustainable. Uctum and Wickens (2000), by extending the results of Wilcox (1989) to the case where the discount rate is stochastic and time varying, and assuming the discounted primary deficit either exogenous or endogenous, show that a necessary and sufficient condition for sustainability is that the discounted debt-GDP ratio should be a stationary zero-mean process.

The test implemented in this paper, similarly to the approach proposed by Wilcox (1989), Trehan and Walsh (1991) and Uctum and Wickens (2000), accounts for a stochastic interest rate. However, this study differs from previous work in two important aspects. First, it uses a log-linear form of the discounted public debt identity, which allows to estimate the model in a VAR framework. Second, it generalizes the results of previous studies by applying specific tests for VAR models to the sustainability of fiscal policy.

III. A VAR framework

The intertemporal budget constraint (IBC) in equation (2) is not suitable to be studied by linear models. Under the assumption of a stochastic interest rate, the public debt is a non-linear combination of the explanatory variables. In order to express the IBC in a linear form, we use a first order Taylor approximation of the discounted public debt identity. Specifically, we take the log linear approximation of equation (2) derived in the appendix. This solves the non-linearity of the model and allows us to express the IBC in a multivariate framework. In log linear form the IBC becomes (see Appendix A for proof):

$$\ln b_t \approx \kappa + \beta^{-1} \ln b_{t-1} + \frac{1}{\overline{b}} \ln d_{t-1} + \beta^{-1} r_t + \varepsilon_t , \tag{4}$$

where β is the discount factor $(1/1+\overline{r})$ and $\overline{b}$, $\overline{r}$ are respectively the sample means of the public debt and the real interest rate adjusted for the output growth rate. The term $\ln d_{t-1}$ denotes the difference $\overline{g} \ln g_{t-1} - \overline{\tau} \ln \tau_{t-1}$ derived in the Appendix, where $\overline{g}$ and $\overline{\tau}$ represent the sample means of the ratios of public spending and taxation to GDP. In this form, equation (4) represents one of the equations of a VAR(1) model having the logarithm of the public debt to GDP ratio, the difference $\overline{g} \ln g_{t-1} - \overline{\tau} \ln \tau_{t-1}$ and the real interest rate as endogenous variables.[1]

The analysis is conducted under two scenarios. We assume the interest rate to be exogenous with respect to the remaining variables in the system. Under this hypothesis, following the approach proposed by Uctum and Wickens (2000), we leave the term $\ln d_{t-1}$ to be either exogenous or endogenous with respect to the public debt. The former case reflects a situation in which the Government, in setting the primary balance, neglects the ratio of public debt to GDP but not the level of the real interest rate. On the other hand, the latter case describes a situation in which the government, in setting the primary balance at time t, takes into account both the level of the real interest rate and the ratio of public debt to GDP at time $t-1$. For a government devoted to reducing the level of public indebtedness, negative feedbacks from discounted debt and the level of the interest rate are expected. The government reacts to an increase of the interest rate, and/or an increase of the public debt, by reducing the primary deficit or by generating primary surpluses over time.

In the presence of exogenous variables, equation (4) can be re-arranged in VAR(1)-X form as follows:

$$Y_t = k + A_y Y_{t-1} + \Pi_{yx} X_{t-1} + \varepsilon_{Yt} , \tag{5}$$

where Y_t is the p_y dimensional endogenous vector and X_t is the p_x dimensional vector of the exogenous variables. Following the approach of Mosconi-Rahbek (1997), we assume X_t to be completely and independently generated from the following VAR model:

$$X_t = A_x X_{t-i} + \varepsilon_{Xt} . \tag{6}$$

[1] A stochastic error has been added in order to capture the error due to the log linear approximation.

The sustainability of fiscal policy depends upon the stability of model (5), which ultimately depends on the stability of model (6). In particular, if model (5) is globally stationary, fiscal policy is said to be sustainable. If model (5) is not globally stationary, fiscal policy is said to be unsustainable.

In the presence of unit roots, the system (5)-(6) may be reparameterized in the VECM form as follows:

$$\Delta Y_t = k + \Pi_y Y_{t-1} + \Pi_{yx} X_{t-1} + \varepsilon_{Yt}, \tag{7}$$

$$\Delta X_t = \Pi_x X_{t-i} + \varepsilon_{Xt}, \tag{8}$$

where Π_y and Π_x are obtained respectively from A_y and A_x by solving $\Pi_y = I_{p_y} - A_y$, $\Pi_x = I_{p_x} - A_x$.

In a more explicit form the VECM form can be rewritten as:

$$\Delta Y_t = \alpha_y \beta_y^{'} Y_{t-1} + \Pi_{yx} X_{t-1} + \varepsilon_{Yt}, \tag{9}$$

$$\Delta X_t = \alpha_x \beta_x^{'} X_{t-1} + \varepsilon_{Xt}, \tag{10}$$

where α and β represent respectively the loading matrix and the cointegrating vectors.

We assume that the Mosconi-Giannini (1992) condition is verified:

$$\Pi_{yx} = \alpha_y \varphi^{'} \gamma^{'}_x Y_t + \alpha_{yx} \beta_x^{'}, \tag{11}$$

where φ is a $(p_x - r_x) \times r_y$ matrix and γ_x is any $r_x \times (p_x - r_x)$ matrix such that $rank[\beta_x, \gamma_x] = p_x$, r_x denotes the rank of β_x and r_y the rank of β_y.

When this condition is satisfied the loading $\gamma^{'}_x X_t$ lies in the $S_p(\alpha_y)$ preventing the possibility of I(2) processes in Y_t. The level of the possibly I(1) processes Xt does not generate a higher order of integration in Y_t via the unit roots in the autoregressive part of Y_t. If this condition was not satisfied fiscal policy would surely be said to be unsustainable. Based on equation (10), equation (9) may be re-arranged as follows:

$$\Delta Y_t = \alpha_Y \beta_Y^{*} \begin{bmatrix} Y_{t-1} \\ \gamma^{'}_x X_{t-1} \end{bmatrix} + \alpha_{YX}(\beta_x^{'} X_{t-1}) + \varepsilon_{Yt}, \text{ where: } \beta_Y^{*} = \begin{bmatrix} \beta_Y^{'} & \varphi^{'} \end{bmatrix}^{'}. \tag{12}$$

The model (12)-(10) can be rewritten in compact notation as:

$$\Delta Z_t = \alpha_z \beta_z^{'} Z_{t-1} + \varepsilon_{z_t}, \text{ where } Z_t = \begin{bmatrix} Y_t \\ X_t \end{bmatrix}, \ \alpha_z = \begin{bmatrix} \alpha_Y & \alpha_{YX} \\ 0 & \alpha_X \end{bmatrix}, \ \beta_z = \begin{bmatrix} \beta_Y & 0 \\ \gamma_x \varphi & \beta_X \end{bmatrix}. \quad (13)$$

We will refer to the model (13) as the full model and to the systems (9) and (10) as the conditional and the marginal systems respectively. Although we are interested in analyzing the conditional system only, the dynamic properties of the Y's obviously depend on the full system. For example, by analyzing (9) it is easily shown that r_y, the rank of β_Y, cannot be regarded in general as the cointegration rank for the Y's, since, if $r_x < p_x$, r_y does not represent the number of stationary linear combinations of the Y's. In fact, when $r_x < p_x$, $\beta_Y^{'} Y_t$ will not be stationary unless $\varphi = 0$; stationary linear combinations of $\beta_Y^{'} Y_t$ exist if and only if the rank of φ is less than r_y. Intuitively, given the triangular structure of (13), the stationarity $\beta_Y^{'} Y_t + \varphi^{'} \gamma^{'} X_t$ is due to the reaction of $\beta_Y^{'} Y_t$ to the non stationary behaviour $\gamma^{'} X_t$: since $\gamma^{'} X_t$ is I(1), $\beta_Y^{'} Y_t$ has also to be I(1) to achieve stationarity. Knowledge of the full system is essential for giving an economic interpretation to the conditional subsystem: $\beta_x^{'} Y_t$ represents the disequilibria in the exogenous variables, which are stationary and affect the dynamics of Y_t through the loadings α_{YX}. Conversely, $\gamma^{'} X_t$ represent the common trends in the exogenous variables, which are I(1) and possibly cointegrated with Y_t through the loading α_{YX} with cointegration vectors $\beta_Y^{*} = \begin{bmatrix} \beta_Y^{'} & \varphi^{'} \end{bmatrix}^{'}$.

Based on Mosconi-Rahbeck (1997), we propose a two step approach of estimating r_X and the parameters in the marginal system first, and then r_y and the parameters in the conditional system, as if r_X and β_X were known. Depending on the value of r_X, we can distinguish three cases:

Case A. $r_X = 0$: the exogenous variables are I(1) processes with no cointegration, $\alpha_X = \beta_X = 0$ and $\beta_{X\perp} = I_{p_x}$. Model (9) becomes:

$$\Delta Y_t = \alpha_Y \beta_Y^{*'} \begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} + \varepsilon_{Yt}. \quad (14)$$

The conditional system is stationary if the I(1) exogenous variables cointegrate with the I(1) endogenous variable. This implies that Y must be necessarily an I(1) process in order for the conditional system to be stationary.

Case B. $r_X = p_X$: the exogenous variables are I(0) process, $\beta_X = I_{p_X}$, and $\beta_{X\perp} = 0$. Therefore, model (9) reduces to:

$$\Delta Y_t = \alpha_Y \beta_Y^{'} Y_{t-1} + \alpha_{YX} X_{t-1} + \varepsilon_{Yt}. \quad (15)$$

The conditional model is stationary if the endogenous variables are I(0), or they are I(1) and cointegrate.

Case C. $0 < r_X < p_X$: the conditional subsystem is a VAR-X model with both non-stationary $(\beta'_{X\perp}X_t)$ and stationary $(\beta'_X X_t)$ exogenous regressor. Model (9) becomes:

$$\Delta Y_t = \alpha_Y \begin{bmatrix} \beta_Y \\ \varphi \\ \vartheta \end{bmatrix}' \begin{bmatrix} Y_{t-1} \\ \beta'_{X\perp}X_{t-1} \\ \beta'_X \sum_{i=1}^{t-1} X_{t-1} \end{bmatrix} + \alpha_{YX}\beta'_X X_{t-1} + \varepsilon_{Yt}. \tag{16}$$

The conditional model is stationary, if the common trend in the exogenous variables is cointegrated with the endogenous variables.

In the following, we analyze the sustainability of fiscal policy under two scenarios. Assuming the time series properties of the variables, we discuss the three cases above described for each scenario.

A. Scenario I: exogenous interest rate and exogenous primary deficit

Under the assumption of an exogenous interest rate and an exogenous primary deficit, model (10) includes only an endogenous variable: Π_y is just a scalar, $Y_t = [b_t]$ and $X_t = [d_t\ , r_t]'$. The VAR-X model is reduced to be an univariate process with the exogenous variables generated by a VAR process.

Case A. $r_X = 0$: the primary deficit and the real interest rate are I(1) variables with no cointegration. The non-stationarity of these variables generate I(1) processes in the autoregressive part of b_t. Fiscal policy is said to be sustainable if there is a cointegrated relation between endogenous and exogenous variables.

Case B. $r_X = p_X$: the primary balance and real interest rate are both stationary variables. The stability of the system depends only upon the time series properties of the public debt to GDP ratio. Specifically, if Π_y is smaller than one, the system is said to be stable. On the other hand, if Π_y is greater than one, the system is said to be unstable. Calibrating Π_y with $(1+\overline{r})$ (i.e., β^{-1}) from the IBC in log-linear form, the analysis gives further insights. The root of the public debt depends upon the sample mean of the interest rate. If the sample mean of the real interest rate adjusted for output growth rate is negative, Π_y is less than one and the fiscal policy is said to be sustainable. If the sample mean $r\,\overline{r} \geq 0$, the coefficient Π_y is equal or greater than one, the system is either unstable or explosive and the fiscal policy is said to be unsustainable.

Case C. $0< r_X < p_X$: the real interest rate or the primary balance is not stationary. The conditional system is said to be stationary if the only endogenous variable (i.e., b_t) and the I(1) exogenous process in the system are cointegrated. If the interest rate is assumed to be a stationary process, the cointegrated vector is between the public debt and the primary deficit:

$$\beta_Y^{*'} \begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} = (1,\ \beta_1,\ 0) \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}.$$

If the primary deficit is a stationary process the cointegrated vector is between the public debt and the interest rate:

$$\beta_Y^{*'} \begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} = (1,\ 0,\ \beta_1) \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}.$$

The theoretical analysis stresses the importance of the real interest rate adjusted for output growth rate in the assessment of the fiscal policy sustainability. In case B, fiscal policy is said to be sustainable, if the real interest rate is a stationary process around a negative mean. In cases A and C, a cointegrated relation involving the public debt and real interest rate, when they are assumed to be both not stationary, is absolutely necessary to make any clear statement regarding fiscal policy sustainability. The crucial role of the real interest rate ultimately highlights the importance of another macroeconomic variable, namely the consumer price index (CPI). The stationarity of the real interest rate, in fact, depends on the time series properties of the CPI. If the CPI is an I(1) variable, the real interest rate is not stationary as the inflation rate is I(0). But if the CPI is I(2), the inflation rate is I(1) and cointegrated with the nominal interest rate with cointegrating vector (1,1). There is a strand of literature (St-Amant 1996) arguing that the CPI is actually I(2), so the real interest rate is a stationary process in the long run. Even if we take this argument as true, in case B, still the sustainability of fiscal policy depends on the sample mean of the real interest rate. Specifically, fiscal policy is said to be sustainable if the sample mean of the real interest rate adjusted for the output growth rate is negative. Therefore, the peculiarities of this variable cannot be neglected in the assessment of fiscal policy sustainability.

B. Scenario II: endogenous primary deficit and exogenous interest rate

Under the assumption of an endogenous primary deficit, system (5) includes two endogenous variables and one exogenous variable. In this case, Π_y is a (2×2) matrix, Π_{yx} is a (2×1) vector and $Y_t = [\ln b_t \text{ , } \ln d_t]'$ is a (2×1) vector of the endogenous variables. The marginal system is reduced to be an AR(1) process. Calibrating the coefficients in the first equation with the weights from the IBC in log-linear form, the model (5) can be expressed as follows:

$$\begin{bmatrix} \ln b_t \\ \ln d_t \end{bmatrix} = \begin{bmatrix} (1+\bar{r}) & \frac{1}{b} \\ \pi_{21} & \pi_{22} \end{bmatrix} \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \end{bmatrix} + \begin{bmatrix} (1+\bar{r}) \\ \pi_{23} \end{bmatrix} r_t + \begin{bmatrix} \varepsilon_{t1} \\ \varepsilon_{t2} \end{bmatrix}. \tag{17}$$

Under the assumption of an endogenous primary deficit, the roots of the model are more complicated, equaling $\upsilon_{1,2}=(1/2)(1+\bar{r})+(1/2)\pi_{22} \pm (1/2)[((1+\bar{r})-\pi_{22}))^2 +4(\pi_{21}/\bar{b})]^{1/2}$, and depend on the sample mean of the interest rate, as well as the coefficients π_{21} and π_{22} reflecting the sensitivity of the primary balance to the changes in the public debt and the sensitivity of the primary balance to its value in the previous period. The roots analysis under this scenario gives interesting insights. In particular, it shows that the system can be stable even if the sample mean of the real interest rate adjusted for output growth rate is positive. For $\pi_{21} \geq (1+\bar{r})\pi_{22}\bar{b}$, the first derivative of the eigenvalues (in absolute value) with respect to π_{21} is positive (i.e., $\partial \mid \upsilon_{1,} \upsilon_2 \mid / \partial \pi_{21} > 0$). As π_{21} decreases, the roots of the model decrease and the system becomes more and more stable. The economic intuition is straightforward. A government which is very sensitive to increases in public debt (i.e. π_{21} is very low, possibly negative) runs primary surpluses to more than compensate the interest payments. This makes the ratio of public debt to GDP convergent towards a sustainable path and the system globally stable.

As regards the cointegration analysis, the analysis of the marginal system is reduced to checking whether the real interest rate adjusted for output growth rate is a stationary process. This means that we have just two cases, specifically, cases A and B.

Case A. $r_X = 0$: the real interest rate is not stationary. If the endogenous system contains one unit root, the stability of the conditional system depends on the existence of a cointegrated relationship between the I(1) endogenous variable and the I(1) real interest rate.

If the public debt is I(1) and the primary deficit is I(0), the cointegrated vector is between the public debt and the real interest rate:

$$\beta_Y^{*'}\begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} = (1,\ 0,\ \beta_1)\begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}.$$

If the public debt is I(0) and the primary deficit is I(1), the cointegrated vector is between the primary deficit and the real interest rate:

$$\beta_Y^{*'}\begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} = (0,\ 1,\ \beta_1)\begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}.$$

When the endogenous system (11) contains two unit roots there must be two cointegrated relationships in order for the VAR-X system to be globally stationary. The first cointegrated vector is between public debt and interest rate:[2]

$$\beta_Y^{*'}\begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} = (1,\ 0,\ \beta_1)\begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}.$$

The second cointegrated vector is between primary deficit and real interest rate adjusted for output growth rate:

$$\beta_Y^{*'}\begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} = (0,\ 1,\ \beta_1)\begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}.$$

Alternatively, the two cointegrated vectors are given by one of these cointegrated vectors and the cointegrated vector between public debt and primary deficit, that is:

[2] The cointegrated vector has been normalized with respect to public debt.

$$\beta_Y^{*'} \begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} = (1,\ \beta_1,\ 0) \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}.$$

Case B. $r_X = p_X$: the real interest rate is stationary. The stability of the system depends on the existence of a cointegrated relationship between the endogenous variables. Fiscal policy is said to be sustainable if the public debt and the primary deficit are cointegrated. The cointegrated vector is:

$$\beta_Y^{*'} \begin{bmatrix} Y_{t-1} \\ X_{t-1} \end{bmatrix} = (1,\ \beta_1,\ 0) \begin{bmatrix} \ln b_{t-1} \\ \ln d_{t-1} \\ r_{t-1} \end{bmatrix}.$$

Similar to the previous scenario, the theoretical analysis confirms the importance of the time series properties of the real interest rate. In particular, it suggests that, in the presence of a non-stationary interest rate, the sustainability condition provided by Trehan and Walsh (1988) is not sufficient. If the public debt and primary deficit are I(1) and cointegrated, but the real interest rate is I(1), fiscal policy cannot be said to be sustainable. As stated in case A, another cointegrated relation involving the real interest rate is absolutely necessary to make any clear statement about the sustainability of fiscal policy. Actually, the sustainability condition provided by Trehan and Walsh (1988) is not even necessary if public debt and primary balance cointegrate respectively with the real interest rate. It becomes a necessary and sufficient condition on the base of an infinite horizon only under the assumption of a stationary interest rate (i.e., case B). In this case, and only in this case, fiscal policy is said to be sustainable if public debt and primary deficit cointegrate.

C. Empirical procedure to analyze the sustainability of fiscal policy

Given the theoretical framework, an empirical procedure to analyze the sustainability of fiscal policy can be articulated as follows:

a. Unit root analysis to determine the number of unit roots in the system;
b. Exogeneity test to determine the scenario;
c. A cointegration analysis to determine the time series properties of the marginal system;
d. A cointegration analysis of the conditional system to determine the sustainability of fiscal policy.

It should be borne in mind that the sustainability procedure just described is backward looking. The outcome of the sustainability test is conditional on the fact that the processes generating the fiscal variables will continue into the future. To assert that fiscal policy is on a sustainable path over a given period does not exclude that it could revert in the future. A period of persistent primary deficits or a persistent increase in the real interest rate could lead the fiscal stance on an unsustainable path. Conversely, a fiscal policy that is said to be unsustainable could become sustainable if the government runs primary surpluses exceeding the interest payments on the existing debt over a sufficient period of time. Therefore, the outcome of the test should not be thought as definitive. Instead it should be seen as a warning message concerning the need to undertake corrective measures. As long as the government is able to generate primary surpluses leading the fiscal stance on a sustainable path, any fiscal policy is said to be sustainable.

Overall, this section provides a theoretical framework to analyze the sustainability of fiscal policy under different scenarios and different hypotheses regarding the dynamic properties of the variables. In addition, it suggests a procedure to test the sustainability of fiscal policy on empirical grounds. In the following section, we apply this procedure to the United States and the Euro Area.

IV. Empirical evidence

The data used in the analysis are annual and are taken respectively from the EconStats and Euro Area Wide Model dataset. The U.S. sample covers the period 1966 to 2004. The Euro Area sample, due to unavailability of aggregate data, covers the shorter period 1977 to 2003.

The econometric methodology follows in four steps: in the first step, we study the stationarity properties of the time series by using unit root tests; in the second step, we conduct classic tests aiming to test the specification of the model (i.e. lag length, exogeneity test); in the third step, we analyze the marginal system. In the final step, conditional on the previous results, we apply a cointegration analysis. In the following we briefly describe the outcome of the unit root tests and the diagnostic tests. The tables with the results will be available from the author upon request.

A. Univariate analysis

We checked the unit roots of the model with the Augmented Dickey Fuller (ADF) test. The null hypotheses have been tested under joint hypotheses concerning the coefficient (ρ), the mean (μ), and the trend (β).[3] The reason is clearly explained in Hamilton (1994). Overall, the analysis suggests that variables are non-stationary.

B. Specification analysis

To test the exoegeneity of the real interest rate with respect to the public debt and primary deficit, we use three consecutive F-tests. First, we tested whether the lag of the real interest rate enters into both public debt and primary deficit equations. Then, we tested whether the lags on public debt and primary deficit are jointly equal to zero in the real interest rate equation. F-tests reject the hypothesis that the lagged coefficient on the interest rate is zero for the first two equations. On the other hand, the F-test for the interest rate equation does not reject the hypothesis that the lagged coefficients on the public debt and primary deficit are jointly equal to zero. Overall, the empirical evidence does not reject the exogeneity of the real interest rate for both regions. The real interest rate affects both the primary deficit and public debt but these variables do not affect the real interest rate.

The optimal lag length has been checked by the Information criteria (i.e.,the Akaike (AIC), the Schwarz (BIC), and the Hannan-Quinn (HQ) tests), and by the Godfrey-Portmanteau test.[4] The Schwarz criterion and the Godfrey-Portmanteau test select one (our choice) as the optimal lag length for both regions. This choice is also supported by Hannan-Quinn criterion for the Euro Area.

The normality and whiteness of the residuals has been checked looking at the autocorrelation function and Normality tests. The autocorrelation function shows no correlation of the residuals. The Jarque-Bera Normality test and Mardia Multivariate normality test do not reject normality for U.S. as well as the Euro Area.

The presence of a trend polynomial is tested with the likelihood ratio test. We considered different restrictions on the deterministic trend coefficients. The test clearly supports the model with constant and no trend for the United States, while for the Euro Area the same hypothesis is just accepted at the 1% significance level.

The roots analysis is conducted in terms of the characteristic polynomial. The stability condition is that all eigenvalues (roots) have modulus less than one. For

---

[3] For a description of the Dickey-Fuller test (DF) and the Augmented Dickey-Fuller test (ADF) see respectively Dickey and Fuller(1979) and Said and Dickey(1984).

[4] Maximum lag analysis in VAR models is discussed at length in Lütkepohl (1985) and Reimers (1993). The formulae for the information criteria, i.e., Akaike (AIC), Schwarz (BIC), Hannan and Quinn (HQ) may be found in Lütkepohl (1991).

both regions the roots of the companion matrix are lower than one, which signals that systems are not explosive. However, some roots are quite close to one, which suggests that the data are I(1).

C. Marginal system analysis

Conditional on the results of the exogeneity test, the marginal system is reduced to be a univariate process with unit root. In this case, the cointegration analysis is not required. For both regions, the rank of the marginal system is clearly equal to zero: $r_X = 0$.

D. Cointegration analysis of the full system

Having established the empirical model and the dynamic properties of the marginal system, the next stage is to determine the cointegration rank corresponding to the number of equilibrium relationships among the variables in the VAR-X model. The procedure for assessing the cointegration rank of the I(1) model is represented by a sequence of likelihood ratio tests, as shown in Johansen (1996). Specifically, only the so-called trace test is computed since the lambda-max test, although easily computable, does not give rise to a coherent testing strategy as illustrated in Johansen (1992) based on Pantula (1989).

The tests are reported, together with the critical values in Table 1. The test statistics h_p for H (rank $\leq p$) are listed with p-values based on Doornik (1998). Testing commences at H (rank = 0), and stops at the first insignificant statistics. The table shows that, when the model with unrestricted constant is used, the selected rank is equal to 2 for both regions at the 99% level of confidence. Notice that in a model with exogenous variables, accepting $r = p_y$ does not mean that the endogenous variables are stationary. Assuming Granger non causality, the order of integration of the endogenous variables, as well as the distribution of the trace test, will depend on the degree of non-stationarity of the exogenous variables, which in this case is represented by the real interest rate.

As suggested in Hansen and Johansen (1999), the stability tests of the cointegrated rank and cointegrated space $Sp(\beta)$ are carried out in two different ways: within the Z-model and the R-model. For the United States, the length of the first sub-sample is fixed at 26. At 95% and 99% confidence, we found evidence in favour of the stability of the cointegrated rank. The hypotheses r_y=0 and $r_y \leq 1$ are clearly rejected for any sample size by both models since the ratio of the test to both critical values is larger than one. Turning to the stability of the cointegrated space, only the Z-

Table 1. Rank determination

		United States	
Eigenvalue		Loglik for rank	
		19.95408	0
0.79478		55.58705	1
0.1473		59.17237	2
H_0: rank<=	Trace test		[Prob]
0	78.437		[0.000]**
1	7.1706		[0.007]**
		Euro Area	
Eigenvalue		Loglik for rank	
		25.7737	0
0.85531		50.90448	1
0.38282		57.17817	2
H_0: rank<=	Trace test		[Prob]
0	62.809		[0.000]**
1	12.547		[0.000]**

Notes: * and ** denote significance at 95% and 99%.

model shows clear evidence in favour of the stability of $Sp(\beta)$. The R-model instead, rejects the hypothesis of the stability of $Sp(\beta)$ at the 95% significance level for the sample period 1988-1995, but it accepts the hypothesis of stability at the 99% significance level for any sample size.

As regards the Euro Area, the length of the first sub-sample is fixed at 18. Also for this region we found evidence in favour of the stability of the cointegrated rank and cointegrated space $Sp(\beta)$. For any sample size the hypotheses r_y=0 and $r_y \leq 1$ are clearly rejected, and the stability space test shows that the normalized test is well below one in both models.

Overall, the stability parameters analysis supports our choice of two cointegrated vectors for the United States and two cointegrated vectors for the Euro Area. The estimated long run relationships are reported in Table 2.

The lower part of the table reports the routine tests of each time series within the cointegrated space. Weak exogeneity is rejected for public debt and primary deficit. Excludability tests indicate that none of the variables can be excluded from the cointegration space without loss of useful information.

Table 2. Long run relationships

	United States			Euro Area		
	ln*b*	ln*d*	*r*	ln*b*	ln*d*	*r*
First Vector	1	-3.897 (0.4)	-	1	0.31 (0.0295)	-
Second Vector	-	1	11.819 (1.7)	1	0	-0.34 (0.05)
Weak exogeneity	χ^2 (2) [0.00]	χ^2 (2) [0.00]	-	χ^2 (2) [0.00]	χ^2 (2) [0.00]	-
Exclusion	χ^2 (2) [0.01]	χ^2 (2) [0.00]	χ^2 (2) [0.00]	χ^2 (2) [0.01]	χ^2 (2) [0.01]	χ^2 (2) [0.00]

Notes: standard errors and significance level are reported respectively in parenthesis and square brackets.

Figure 1. United States: Estimated cointegrated vectors

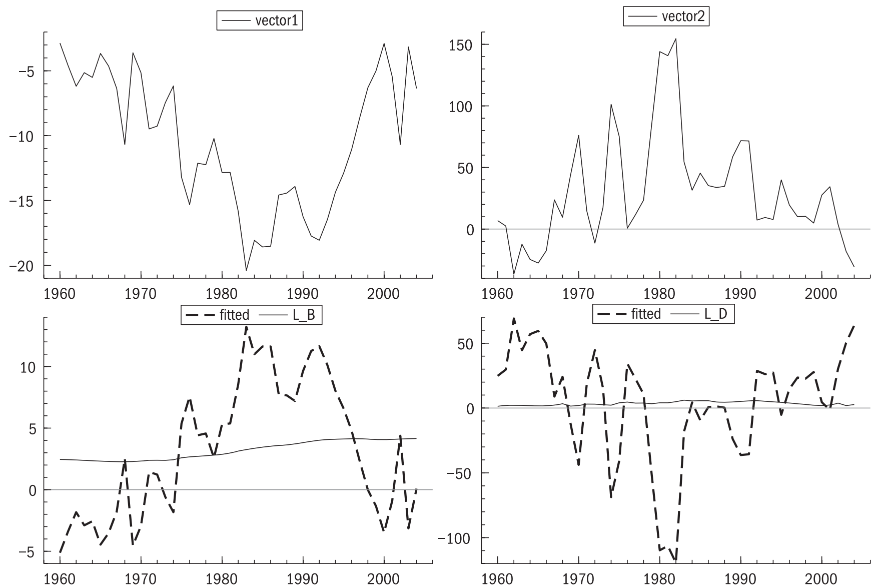
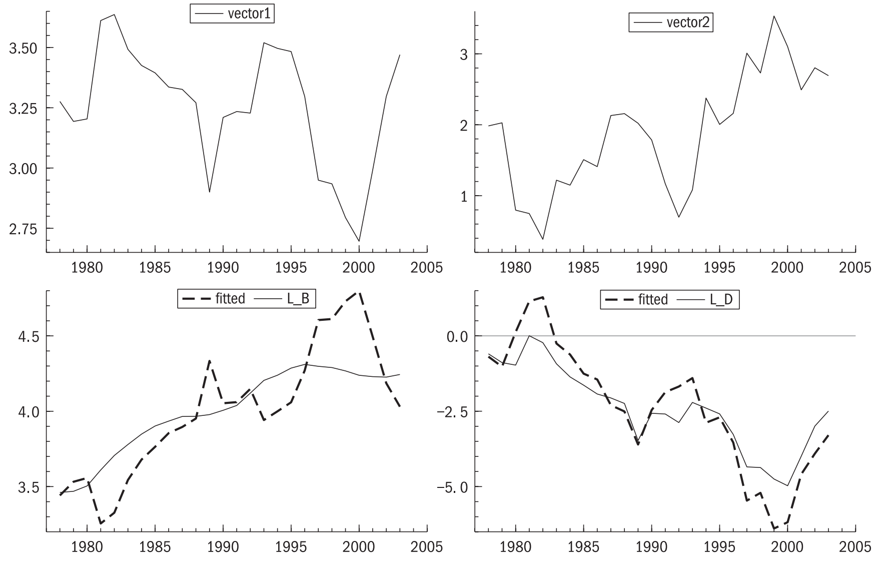


Figure 1 illustrates the two linear combination for the US. The graphs below the cointegrated vectors show the sum of the non normalized coefficients (with opposite sign) against the normalized variable (that is, long run fitted and actual). The long run relationships look fairly stationary but with not much relation between

Figure 2. Euro Area: Estimated cointegrated vectors



fitted and actual values. Figure 2 shows the two long run relationships and the sum of the non normalized coefficients against the normalized variable for the Euro Area. For this region fitted and actual values track each other reasonably closely.

Overall, the empirical analysis suggests the stability of the systems and the sustainability of fiscal policy for both regions. Moreover, it brings out the importance of the real interest rate. Evidence from our sample shows that the real interest rate is not stationary but cointegrates with the other variables in the system, making the full system globally stationary and thus fiscal policy sustainable.

V. Conclusion

In this paper, we have suggested a procedure to assess the sustainability of fiscal policy under two different scenarios. To this end, we have log-linearized the public debt identity and re-arranged it in terms of a multivariate system. Conditional on given assumptions regarding the time properties of the variables, we have discussed three cases for each scenario and generalized the results provided by other studies. By using the Vector Error Correction Model (VECM), we derived the conditions for the sustainability in the long run for each case. We stated that the condition for sustainability depends upon the stationarity of the VAR-X system. In particular, if the VAR-X model is globally stationary, fiscal policy is said to be sustainable. If the VAR-X model is not globally stationary, fiscal policy is said to be unsustainable. We stressed that the stationarity of the system strongly depends on the real interest rate and its dynamic properties are crucial to assess the sustainability of fiscal policy. Under the assumption of an exogenous interest rate and an endogenous primary deficit, we showed that the sustainability condition provided by Trehan and Walsh only holds if the stochastic interest rate is a stationary process.

In the empirical analysis we have applied the fiscal sustainability procedure to the fiscal stances of the United States and the Euro Area. The ADF test showed that the public debt, the primary deficit and the interest rate are integrated processes of order one in both regions. The specification analysis suggested that the real interest rate is the only exogenous variable. The following cointegration analysis estimated two cointegrated relationships for the Euro Area and two cointegrated relationships for the United States. The broad conclusion is that the Euro Area and the United States are still on a sustainable fiscal policy path. However, these conclusions have to be taken with caution. It should be borne in mind that the sustainability procedure implemented in this paper, as in most of the literature on this issue, is backward looking. The statement that fiscal policy is on a sustainable path over a given period does not exclude that it could revert in the future. A period of persistent primary deficits or a persistent increase in the real interest rate could lead the fiscal stance of these regions on an unsustainable path.

Appendix: the log linear approximation of the discounted public debt identity

Assuming a constant level of real revenues from seigniorage equal to $\bar{s}$, the public debt identity at time t is as follows:

$$b_t = g_{t-1} - \tau_{t-1} - \bar{s} + (1 + r_t)b_{t-1}, \tag{A1}$$

or alternatively it is $e^z = -e^{\ln \bar{s}} + e^x - e^\gamma + e^y$, where $z = \ln b_t$, $x = \ln g_{t-1}$, $\gamma = \ln \tau_{t-1}$, $y = \ln[(1 + r_t)b_{t-1}]$.

The first order Taylor polynomial approximation of $f(z,x,\gamma,y) = e^z - [-e^{\ln \bar{s}} + e^x - e^\gamma + e^y] = 0$ about $x = \bar{x} = \ln \bar{g}$, $\gamma = \bar{\gamma} = \ln \bar{\tau}$, $y = \bar{y} = \ln[(1 + \bar{r})\bar{b}]$ and $z = \bar{z} = \ln \bar{b}$ follows:

$$e^{\bar{z}} + e^{\bar{z}}(z - \bar{z}) - [-e^{\ln \bar{s}} + e^{\bar{x}} + e^{\bar{x}}(x - \bar{x}) - \left(e^{\bar{\gamma}} + e^{\bar{\gamma}}(\gamma - \bar{\gamma})\right) + e^{\bar{y}} + e^{\bar{y}}(y - \bar{y})] = 0. \tag{A2}$$

By substituting z, x, γ and y with their respective values, equation (A2) becomes

$$\begin{aligned} e^{\ln \bar{b}} + e^{\ln \bar{b}}(\ln b_t - \ln \bar{b}) = -e^{\ln \bar{s}} + e^{\ln \bar{g}} + e^{\ln \bar{g}}(\ln g_{t-1} - \ln \bar{g}) - e^{\ln \bar{\tau}} - \left(e^{\ln \bar{\tau}}(\ln \tau_{t-1} - \ln \bar{\tau})\right) \\ + e^{\ln(1+\bar{r})\bar{b}} + e^{\ln(1+\bar{r})\bar{b}}(\ln(1+r_t)b_{t-1} - \ln(1+\bar{r})\bar{b}) \end{aligned} . \quad \text{(A3)}$$

Rearranging equation (A3), then dividing both sides by $\bar{b}(1+\bar{r})$, and solving with respect to $\beta \ln b_t$, we obtain:

$$\begin{aligned} \beta \ln b_t = -\frac{\bar{s}}{\bar{b}}\beta - \beta + \beta \ln \bar{b} + \beta[\frac{\bar{g}}{\bar{b}}(1 - \ln \bar{g}) - \frac{\bar{\tau}}{\bar{b}}(1 - \ln \bar{\tau})] \\ +1 - \ln((1+\bar{r})\bar{b}) + \ln b_{t-1} + \frac{1}{\bar{b}}\beta \ln d_{t-1} + \ln(1+r_t), \end{aligned} \quad \text{(A4)}$$

where $\ln d_{t-1}$ denotes the difference $(\bar{g} \ln g_{t-1} - \bar{\tau} \ln \tau_{t-1})$ and β is the discount factor $(1+\bar{r})^{-1}$.

The log linear approximation of the discounted public debt identity is obtained by dividing both sides of (A4) by the discount factor β and using the fact that $\ln(1+r_t)$ is approximately equal to rt:

$$\ln b_t \approx \kappa + \beta^{-1} \ln b_{t-1} + \frac{1}{\bar{b}} \ln d_{t-1} + \beta^{-1} r_t . \quad \text{(A5)}$$

The parameter k is a constant equal to:

$$\kappa = -\frac{\bar{s}}{\bar{b}} - 1 + \ln \bar{b} + [\frac{\bar{g}}{\bar{b}}(1 - \ln \bar{g}) - \frac{\bar{\tau}}{\bar{b}}(1 - \ln \bar{\tau})] + \beta^{-1} - \beta^{-1} \ln((1+\bar{r})\bar{b}).$$

References

Dickey, David A., and Wayne A. Fuller (1979), Distribution of the estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association* **74**: 427–431.

Doornik, Jurgen A. (1998), Approximations of asymptotic distribution of cointegration tests, *Journal of Economic Surveys* **12**: 573-593.

Hamilton, James D., and Marjorie A. Flavin (1986), On the limitations of government borrowing: A framework for empirical testing, *American Economic Review* **76**: 809-819.

Hamilton James D. (1994), *Time series analysis*, Princeton, Princeton University Press.

Hansen, Henrik and Søren Johansen (1999), Some Tests for parameter constancy in cointegrated VAR-models, *Econometrics Journal* **2**: 306-333.

Hakkio, Craig S., and Mark Rush (1991), Is the budget deficit too large?, *Economic Inquiry* **29**: 429-45.

Johansen, Søren (1992), Cointegration in partial systems and the efficiency of single equation analysis, *Journal of Econometrics* **52**: 389-402.

Johansen, Søren (1996), *Likelihood-based inference in cointegrated vector autoregressive models*, 2nd ed, Oxford, Oxford University Press.

Lütkepohl, Helmut (1985), Comparison of criteria for estimating the order of a vector autoregressive process, *Journal of Time Series Analysis* **6**: 35-52 (Correction 1987, **8**:373).

Lütkepohl, Helmut (1991), *Introduction to multiple time series analysis*, Berlin, Springer-Verlag.

Mosconi, Rocco, and Anders Rahbek (1997), Cointegrated VAR-X models, Working paper 5 (Quaderni DEP), Politecnico di Milano.

Mosconi, Rocco, and Carlo Giannini (1992), Non-causality in cointegrated systems: Representation, estimation and testing, *Oxford Bulletin Economics Statistics* **54**: 399–417.

Pantula, Sastri G. (1989), Testing for unit roots in time series data, Econometric Theory **5**: 256-271

Reimers, Hans E. (1993), Lag order determination in cointegrated VAR systems with application to small German macro-models, Paper presented at ESEM '93, Uppsala.

Said, Said E., and David A. Dickey (1984), Testing for unit roots in autoregressive moving average models of unknown order, *Biometrika* **71**: 599–607.

St-Amant, Pierre (1996), Decomposing US nominal interest rates into expected Inflation and ex-ante real interest rates using structural VAR methodology, Working paper 96-2, Bank of Canada.

Trehan, Bharat, and Carl E. Walsh (1988), Common trends, the government budget constraint, and revenue smoothing, *Journal of Economics and Dynamics and Control* **12**: 425-444.

Trehan, Bharat, and Carl E., Walsh (1991), Testing intertemporal budget constraints: Theory and applications to US federal budget and current account deficits, *Journal of Money, Credit and Banking,* **23**: 206-23.

Uctum, Merih, and Michael R. Wickens (2000), Debt and deficit ceilings, and sustainability of fiscal policies: An intertemporal analysis, *Oxford Bulletin of Economics and Statistics* **62**: 197-222.

Wilcox, David W. (1989), Sustainability of government deficits: Implications of the present value borrowing constraint, *Journal of Money Credit and Banking* **21**: 291-306.