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Lucio Esposito
Enrica Chiappero-Martinetti

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MULTIDIMENSIONAL POVERTY: RESTRICTED AND UNRESTRICTED HIERARCHY AMONG POVERTY DIMENSIONS

Lucio Esposito*
University of East Anglia

Enrica Chiappero-Martinetti
University of Pavia

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The increasing interest in multidimensional poverty and well-being analysis has added complexity to the way these phenomena are conceptualized and measured. When multiple attributes are considered, a criterion determining the relative importance attached to the different dimensions has to be adopted. There has not been thus far in the literature a specific attempt to conceptualize the nature of the desired hierarchy among the selected poverty dimensions. The aim of this paper is to take the first step in this direction. We envisage two simple and highly intuitive ways in which such a hierarchical system can be understood, which we label *restricted* and *unrestricted hierarchy*. The analytical conditions allowing the incorporation of these into a poverty index are derived and their implications in terms of the understanding of poverty are discussed. An empirical application shows how the choice of the hierarchical scheme for poverty dimensions can lead to opposite conclusions on the poverty trend.

JEL classification codes: D31, D63, I32
Key words: poverty measurement, multidimensionality, weighting

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* Lucio Esposito (corresponding author), School of Development Studies, University of East Anglia, NR4 7TJ Norwich, UK. lucio.esposito@uea.ac.uk; Enrica Chiappero-Martinetti, Department of Public Economics,University of Pavia, Strada Nuova 65, 27100 Pavia, Italy, and HDCP-IRC - Institute for Advanced Study (IUSS), enrica.chiappero@unipv.it. We are grateful for useful comments and encouragement to Bereket Kebede, Arjan Verschoor and the participants to the OPHI Workshop on Weighting at Oxford University, the Second GRASS Meeting at University of Bologna, the Workshop for the 10th Anniversary of IRISS, Luxemburg and the Workshop on Quantitative Methods for the analysis of Living Standard at Fondazione Ermanno Gorrieri e CAPP, Modena. A special mention is owed to two anonymous referees, whose suggestions considerably helped us refine our arguments. We also thank Willem van den Andel for helpful instructions on the Maldivian dataset. All remaining errors are ours.

I. Introduction

The assertion that poverty should be conceived and measured in a multidimensional setting has received widespread agreement in the literature and captured the attention of researchers, institutions and policymakers. At a conceptual level, Amartya Sen's seminal work in the Eighties represents a turning point regarding the desirability to extend the attention beyond income and economic resources and to conceive poverty as a condition of functioning failures or a lack of opportunities in a multidimensional setting. However, at a measurement level, it is principally in the last decade that scholars have addressed their efforts to this field of investigation and significant contributions have been offered. These include (i) multidimensional poverty indices (Tsui 2002; Bourguignon and Chakravarty 1999, 2003; Alkire and Foster 2008, among others)[1], (ii) non-aggregative strategies such as multidimensional poverty orderings and stochastic dominance (see, in particular, Bourguignon and Chakravarty 2002; Atkinson 2003; Duclos, Sahn and Younger 2006), and (iii) multidimensional poverty analysis based on the use of multivariate statistical techniques (Krishnakumar 2008; Asselin and Tuan Anh 2008, Krishnakumar and Ballon 2008). Despite the growing attention paid to multidimensionality in poverty measurement, many issues are still open to discussion and thus far most empirical work on poverty is still largely based on the unidimensional income (or consumption) space.

As in the unidimensional case, multidimensional poverty measurement involves a series of value judgements such as, *inter alia*, the choice of the poverty index and the level(s) at which the poverty threshold(s) should be set up. However, when multidimensionality comes into the picture, additional arbitrariness arises regarding the selection of the dimensions deemed relevant for poverty evaluation, the method of aggregation across dimensions and the relative importance to be assigned to each of them. This paper will focus particularly on this last issue, abstracting from the criteria for the selection of poverty dimensions and borrowing the method of aggregation from the *Set of Additive Poverty Indices* discussed by Bourguignon and Chakravarty (2003).

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[1] As Brandolini (2007) outlines, the use of multidimensional indices should be distinguished from what he classifies as "fully aggregative strategy" based on the construction of a composite index of well-being at individual level in a way that poverty evaluation can be brought back to the unidimensional space (Maasoumi and Nickelsburg 1988, Deutsch and Silber 2005, Ramos 2008). This approach entails the use of a single poverty line, while for multidimensional indexes an idiosyncratic threshold for each dimension has to be identified.

Choosing a criterion for how deprivation in different dimensions should contribute to overall poverty is a task of major importance for a convincing multidimensional evaluation. Indeed, different criteria may well lead to contrasting evaluation results, with important consequences in terms of policy implications (Brandolini, 2007). Besides, choosing such a criterion is an inescapable step because, it may definitely be argued, the act of 'not giving weights' (equivalent in fact to attributing identical weights to each dimension) is itself a subjective decision motivated by the value judgement that all dimensions are equally valuable. We therefore follow the invitation of both Sen (1973) and Atkinson (1987) not to turn the necessary awareness of the arbitrariness involved in the exercise into a nihilistic attitude leading to disregard what we can say.

In the literature there has not been thus far a specific attempt to conceptualize the nature of the desired hierarchy among the selected poverty dimensions. Theoretical and empirical work has relied on the use of dimension-specific multiplicative weights increasing as the rank of the dimension is increased and the possible meanings of the statement "dimension h is more important than dimension k" have not been searched for critically. This paper takes the first step towards this by conceptualizing two alternative, simple and highly intuitive ways in which such statement can be understood: *restricted* and *unrestricted hierarchy*. The hierarchy is *unrestricted* when even a very small deprivation in the dominating dimension is deemed more important than a large deprivation in the dominated dimension. If this ranking only holds for the same deprivation level in the two dimensions (relative to the relevant poverty line), then the hierarchy is said to be *restricted*. While the latter allows for compensation among deprivations in different poverty domains, the former informs a lexicographic representation of multidimensional poverty grounded on the view that some aspects of poverty are essential while others, though important, are of secondary importance. The conceptualization of the above types of hierarchy acquires particular significance in the light of concerns frequently exhibited by governments and international organizations –e.g., "Losses in human welfare linked to life expectancy, for example, cannot be compensated for by gains in other areas such as income or education." (UNDP 2005, p. 22).

Focusing on the class of additive multidimensional poverty indices, this paper derives the conditions on the individual deprivation functions under which the two types of hierarchy can be implemented. It is shown that restricted hierarchy is incompatible with headcount-like poverty functions, while it is not possible to implement unrestricted hierarchy when standard axioms such as focus and continuity are accepted. It follows that alternative structures are needed to account for the proposed hierarchical schemes. The standard use of dimension-specific multiplicative weights that are increasing with the importance of the poverty dimension is sufficient (but not necessary) for the implementation of restricted hierarchy. On the contrary, the unrestricted hierarchy structure requires a fixed component to be added to the poverty function of the dominating dimension, with the result that the individual poverty index takes a lexicographic form. Following Bourguignon and Fields (1997), such fixed component is directly interpretable as a 'fixed welfare loss' due to deprivation in that dimension. An empirical application using microdata for Maldives in 1997 and 2004 provides evidence that the verdict of multidimensional poverty comparisons can be completely reversed under the two types of hierarchy.

This paper develops as follows. In Section II, we present the notation used throughout the paper and describe the value judgements informing customary poverty axioms. In Section III, we introduce our newly conceptualized types of hierarchy and derive the analytical implications for their accommodation by a poverty index. In Section IV, we first provide an illustration of how the described poverty criteria can be easily incorporated in applied works through individual deprivation functions based upon poverty dimensions' rank order. Then, we carry out our empirical application. Section V concludes.

II. Multidimensional poverty indices: basic notation and properties

We refer to $\mathbb{N}_+, \mathbb{R}_+$ and $\mathbb{R}_{++}$ as to the sets of strictly positive integers, nonnegative and strictly positive real numbers, respectively. In a society of size $n \in \mathbb{N}$, the typical i_{th} individual possesses $m \in \mathbb{N}$ attributes identified by a vector f4 whose values are drawn from $\mathbb{R}_+^m$, the m-dimensional nonnegative Euclidean space. Each vector of individual attributes can be thought of as one of the n rows of a $n \times m$ matrix $X \in M^n$, where M^n denotes the set of all conceivable a $n \times m$ matrices whose entries are nonnegative real numbers. Let $M = \bigcup_{n=1}^{\infty} M^n$, and let a vector $z = (z_1, z_2, ..., z_m) \in Z$ exhibit for each dimension the threshold below which an individual is considered poor in that dimension,[2] where $Z \subseteq \mathbb{R}_{++}^m$ is a subset of the m-dimensional nonnegative Euclidean space with the origin deleted. The multidimensional poverty level in society is obtained by means of multidimensional indices π which are real functions

[2] In this way for each dimension we follow the "weak" definition of the poor provided by Donaldson and Weymark (1986) – the "strong" definition consisting in deeming poor also those individuals which are at the poverty line.

mapping individual attributes belonging to matrix X and poverty thresholds belonging to the set Z to the nonnegative part of the real line, i.e. $\pi : M \times Z \rightarrow \mathbb{R}_+$. As the desirability of any social indicator rests on the extent to which the value judgements motivating the properties it possesses are shared, let us look at the properties met by the multidimensional poverty index:

$$\pi = \frac{1}{n}\sum_{j=1}^{m}\sum_{i \in Q_j} p_{i,j}, \tag{1}$$

where $p_{i,j} = p_j(x_{i,j}, z_j)$ is a non-increasing, positive- and real-valued deprivation function quantifying the poverty level of individual i in dimension j ($p_{i,j} = 0$ for $x_{i,j} \geq z_j$ and $p_{i,j} > 0$ otherwise) and Q_j denotes the set of individuals that are poor in dimension j.[3] The index π allows the accommodation of the *Subgroup Consistency* axiom: an increase (decrease) in the poverty level of a subset of the population induces an increase (decrease) in the aggregate poverty figure – see the results of Foster and Shorrocks (1991) and Tsui (2002) in the unidimensional and multidimensional spaces, respectively. The index π belongs to the *Set of Additive Poverty Indices* discussed by Bourguignon and Chakravarty (2003). As pointed out by the authors, those indices allow a 'two-way poverty breakdown' (p. 35) – by both population subgroups and poverty dimensions – particularly useful for the identification of the subgroup-attributes combinations exhibiting the most severe situations of deprivation and for the design of anti-poverty policies under scarcity of resources. However, the drawback of the family of (doubly) additive poverty indices is the inability to meet multidimensional versions of the principle of transfers and the insensitivity to changes in the correlation among attributes.[4] π satisfies the *Anonymity* axiom – permutations of the entire array of attributes across individuals leave the aggregated poverty level unchanged – and the *Population Invariance* axiom – if two identical societies are merged then the poverty level in the resulting society equals the poverty level in each of the original societies. Furthermore, when

[3] For an exhaustive coverage of the issue of the identification-counting of the poor in the multidimensional space, see Bourguignon and Chakravarty (2003) and Alkire and Foster (2008).

[4] In our work, we will consider only the *One Dimensional Transfer Principle* defined by Bourguignon and Chakravarty (2003) – see below. The authors discuss also a family of non-additive measures which does not allow the mentioned 'two-way poverty breakdown' but accounts for sensitivity to correlation among attributes. For an exhaustive treatment of those issues, see also Atkinson and Bourguignon (1982) and Tsui (2002).

π is insensitive to the distribution of attributes above the respective poverty thresholds the index is said to satisfy the *Focus* axiom.

Among the multidimensional indices possessing the properties mentioned above, let us consider the family $\tilde{\pi}$ where the deprivation function in the generic dimension j is $p_{i,j}$:$[0.1] \rightarrow \mathbb{R}_+$, with values $p_{i,j} = (t_{i,j})$ and argument $t_{i,j} = x_{i,j}/z_j$. The index $\tilde{\pi}_i$ yields the overall poverty value for individual i:

$$\tilde{\pi}_i = \sum_{j=1}^{m} p_j\left(t_{i,j}\right). \tag{2}$$

Since the argument of each individual deprivation function is expressed in relative terms, the resulting poverty index ensures the accommodation of the *Scale Invariance* axiom, according to which the multiplication of both the j_{th} column of attributes in the matrix X and the relevant poverty threshold by a positive scalar leaves the index unaffected. It should be noted that $p_{i,j}$ in $\tilde{\pi}$ is a transformation of the level of the j_{th} attribute possessed by the i_{th} individual expressed as a proportion of the relevant poverty threshold and, consequently, each functioning failure quantified by $\tilde{\pi}$ is a dimensionless number. Thanks to that, the comparability of functioning failures in the different dimensions as well as their combination into a unique number are significantly facilitated.

Two lines along which it is valuable to further narrow multidimensional poverty indices down concern the imposition of restrictions on (i) the *addenda* of the summation in i for a certain dimension j –i.e., the poverty levels of different poor individuals regarding dimension j– and on (ii) the *addenda* of the summation in j for a certain poor individual i –i.e., her poverty levels in different dimensions. According to the first line, identically to one-dimensional poverty analysis, the behaviour of $p_{i,j}$ as $t_{i,j}$ varies in [0,1] needs to be chosen. The further $t_{i,j}$ falls below one is typically considered as an event yielding either no variation or an increase (at either constant or increasing rates) in j-poverty. Individual deprivation functions that are either constant or linearly decreasing or convexly decreasing in $t_{i,j}$ in the interval [0,1] will be used in those three cases, respectively. All decreasing $p_{i,j}$'s satisfy the so-called *Monotonicity* axiom – a decrease in the endowment of attribute j induces an increase in j-poverty – while convexly decreasing $p_{i,j}$'s satisfy also what Bourguignon and Chakravarty (2003) call *One Dimensional Transfer Principle* since they are sensitive to mean-preserving changes in the distribution of attribute j – a transfer between two poor individuals should not increase (decrease) poverty if the recipient is poorer (richer) than the donor. Restricting the functional form of

$\tilde{\pi}$ according to the second line entails the incorporation of value judgements concerning the relative importance of different poverty dimensions. Suppose that the endowments of individual i in two different dimensions are exactly half the respective poverty thresholds. Should the poverty value in the two dimensions be the same or should one exceed the other? A rationale needs to be chosen to determine the way the different dimensions should contribute to overall poverty.

Before addressing that issue in the next section, let us introduce the *Strong Continuity* axiom (ST), requiring a poverty index to be continuous at (also) the poverty line. Since the poverty value associated with incomes smaller than the poverty line is positive and null otherwise, ST can be accommodated only by indices decreasing towards zero in the left neighbourhood of the poverty line – i.e. $\lim_{t_{i,j} \to 1^-} p_{i,j}(\cdot) = 0$ for all $t_{i,j}$'s. ST has important implications on the way poverty is conceived. A continuous deprivation function is generally justified by the idea that "given a very small change in a poor person's income, we could not expect a huge jump in the poverty level" (Zheng, 1997: 131). However, the belief that this should happen also at the poverty line is disputable. Indeed, "the use of a poverty line to sharply demarcate the rich from the poor suggests [...] that a poverty index might be discontinuous at the poverty line" (Donaldson and Weymark, 1986, p. 674). Bourguignon and Fields (1997) conceptualize poverty indices presenting a 'jump' discontinuity at the poverty line as tools able to capture two distinct aspects of the welfare loss associated with poverty. One arises from the inability to meet what is conventionally accepted as a minimum achievement in a certain dimension and is independent of the exact magnitude of individual endowments. The other reflects the consideration that poverty becomes harsher the further the endowment in a certain dimension falls below the relevant poverty line. In their own words, those two aspects reflect "the loss from being poor and the loss from being poorer" (p. 155) – for a discussion of the poverty measurement options opened up by this approach, see Esposito and Lambert (2009).

III. Ranking poverty dimensions: restricted vs. unrestricted hierarchy

When it comes to the choice of how different poverty dimensions should contribute to overall poverty, two remarks appear necessary. Firstly, as we observed in the Introduction, any skepticism about the choice of prioritizing one dimension over another grounded merely on the arbitrary nature of such decision should bear in mind that 'not giving weights' (equivalent in fact to attributing identical weights to all dimensions) is itself a subjective decision, reflecting the value judgement that those dimensions are equally important. Secondly, when two dimensions are deemed to have different importance what primarily interests the analyst is to establish a *hierarchy* among them before raising cardinal concerns. We envisage two alternative, simple and intuitive ways of conceiving a hierarchy between poverty dimensions h and k, with dimension h deemed to be more important than dimension k. Consider the two following requirements:

Unrestricted Hierarchy (UH): $p_h(t_{i,h}) > p_k(t_{i,k})$ for all $t_{i,h}$ and $t_{i,k} < 1$;

Restricted Hierarchy (RH): $p_h(t_{i,h}) > p_k(t_{i,k})$ for all $t_{i,h}$ and $t_{i,k} < 1$ such that $t_{i,h} \leq t_{i,k}$ but $\exists\ t^*_{i,h} > t^*_{i,k}$ such that $p_h(t^*_{i,h}) < p_k(t^*_{i,k})$.

UH requires the poverty value associated with a poor achievement in dimension h to be larger than that associated with a poor achievement in dimension k independently of the magnitudes of those achievements. In other words, UH requires the poverty value associated with a $z - \varepsilon$ level of achievement in the dominating dimension, $\varepsilon > 0$, to be larger than that associated with a very small (and potentially null) level of achievement in the dominated dimension. Conversely, RH requires the hierarchy to hold only when the achievement in dimension h is no greater than the achievement in dimension k.[5] It is highly intuitive to think of requirements UH and RH in terms of percentage of failure in achieving the relevant poverty line. According to UH an X% failure in achieving z_h is harsher than a Y% failure in achieving z_k for any X and Y. Differently from UH, RH postulates that an X% (or larger) failure in achieving z_h is always harsher than an X% failure in achieving z_k, but a large enough failure in achieving z_k is harsher than a small enough failure in achieving z_h. The first kind of hierarchy will be referred to as $h \overset{UH}{\succ} k$ while the second one as $h \overset{RH}{\succ} k$. As will be shown in our empirical application, the proposed way of sorting out binary comparisons between poverty dimensions is safely extendible to the organization of $m > 2$ dimensions.

Three analytical results are offered, which concern the implications of the newly introduced types of hierarchy in terms of functional form for a poverty measure. We will show (i) the incompatibility between headcount-like individual deprivation functions and RH; (ii) the impossibility for poverty-line continuous indices to

[5] As remarked in Section II, to ensure inter-dimension comparability achievements in different dimensions are to be intended as normalised by their respective poverty lines – i.e. as quantified by the ratio $t_{i,j} = x_{i,j}/z_j$.

accommodate UH and (iii) the functional form needed for the accommodation of UH. Our Propositions 1 reads as follows.

Proposition 1. $h \overset{RH}{\succ} k$ *can be accommodated by a poverty index of the class* $\tilde{\pi}$ *only if both of* $p_h(t_{i,h})$ *and* $p_k(t_{i,k})$ *are non-constant individual deprivation functions*. Proof. See Appendix A.

Proposition 1 highlights what is a significant limitation affecting multidimensional poverty evaluation whenever only a dichotomous indicator is available for a certain dimension. The significance of this result is considerable because this circumstance is in fact rather frequent. Indeed, for a variety of non-monetary dimensions only dichotomous indicators are offered by existing datasets –e.g. the conditions of literate/illiterate, employed/unemployed, etc. In those cases the individual contribution function accounting for poverty in that dimension cannot but be constant in the poor domain, thus precluding the possibility of implementing RH in binary comparisons involving that dimension. Given Proposition 1, in what follows we shall consider only $p_j(t_{i,j})$'s which are non-constant for levels of achievement smaller than z_j.

Consider now the multidimensional extension of the Foster et al. (1984) index proposed by Bourguignon and Chakravarty (2003) within their Set of Additive Poverty Indices: $\pi^{BC} = \frac{1}{n}\sum_{j=1}^{m}\sum_{i\in Q_i} a_j\left(1-t_{i,j}\right)^{\theta_j}$. For strictly positive values of the poverty-aversion parameter θ_j, this is an example of the implementation of $\overset{RH}{\succ}$ via p_j's exhibiting a multiplicative functional form[6] such as $a_j f(t_{i,j})$, where $a_j \in \mathbb{R}_{++}$ is a weight increasing with the importance assigned to the poverty dimension j and f is a poverty-line continuous, strictly positive- and real-valued function decreasing in $t_{i,j}$.

While, as we have just seen with relation to π^{BC}, RH and ST can be jointly met by a poverty index, an impossibility theorem can be stated for what concerns the simultaneous accommodation of UH and ST. Consequently, a characterization theorem for the functional form allowing the accommodation of UH can be directly derived. Those are our Propositions 2 and 3, respectively.

Proposition 2. *There exists no poverty index of the class* $\tilde{\pi}$ *jointly accommodating* UH *and* ST. Proof. See Appendix A.

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[6] Such a functional form is also used by Chakravarty and Silber (2008, p.199).

Proposition 3. *A poverty index of the class $\tilde{\pi}$ satisfies* UH *between the two dimensions h and k (that is $h \overset{UH}{\succ} k$) if and only if $p_h(t_{i,h}) = g_h(t_{i,h}) + \Delta_h$ where g_h is a strictly positive, continuous and decreasing function and Δ_h is an interval function, with constant value $\Delta_h \geq \max p_k(t_{i,k})$ when $t_{i,h} \in [0,1)$ and $\Delta_h = 0$ for $t_{i,h} = 1$.* Proof. See Appendix A.

As a corollary of our Proposition 2, it follows that $h \overset{RH}{\succ} k$ is implemented whenever $p_h(t_{i,h})$ lies above $p_k(t_{i,k})$ and the magnitude of the jump-discontinuity at the poverty line for $p_h(t_{i,h})$ (if any) is smaller than $\max_{t_{i,k}} p_k(t_{i,k})$.

Propositions 2 and 3 show that the desired type of hierarchy among poverty dimensions has considerable implications on the way poverty is conceptualized. Proposition 2 shows that the implementation of $h \overset{UH}{\succ} k$ cannot get away from an understanding of poverty which encompasses a 'fixed' welfare loss in dimension h, along the lines of the interpretation suggested by Bourguignon and Fields (1997) and described in Section II. More specifically, the $h \overset{UH}{\succ} k$ type of hierarchy requires not only the existence for dimension h of a 'fixed' welfare loss, but also the magnitude of such a loss to be at least as large as the poverty value associated with a complete functioning failure in dimension k –as postulated by Proposition 3.

Here is a clarifying example. Take shelter (*sh*) and education (*ed*) as dominating and dominated dimensions, respectively. Suppose that being poor in *sh* means not to have a roof over your head every night, while being poor in *ed* means having less than z_{ed} years of schooling. One may reasonably argue that the more nights you go without a roof over your head the poorer you are, but the 'homelessness' condition of not having a guaranteed shelter is per se worse than having less than z_{ed} years of schooling –even worse than never having gone to school. Such a value judgement would be reflected in a $sh \overset{UH}{\succ} ed$ type of hierarchy, which would require $p_{sh}(t_{i,sh})$ exhibiting a jump-discontinuity at least as large as $p_{ed}(t_{i,ed} = 0)$. It follows that no improvement in the education dimension (not even escaping education poverty) can compensate for a condition of deprivation in the shelter dimension. The assumption of no compensation between dimensions is inherent in a lexicographic representation of multidimensional poverty, according to which some attributes are essential while others, though important, are of secondary importance.[7] It should be noted that the lack of compensation at the individual level does not extend to the aggregate level.

[7] This is surely a strong assumption in those cases where variables are continuous and the possibility is open for an individual to be only an epsilon below the poverty line in the dominating dimension. It is less questionable when variables are dichotomous or categorical.

In our framework the contribution to overall poverty of an individual who is poor in the shelter dimension can be outweighed by the contributions of, say, two individuals lacking adequate levels of education.

IV. Restricted vs. unrestricted hierarchy: an empirical application through a simple ordinal approach

A. Multidimensional poverty: an ordinal approach to measurement

In the poverty function $p_j(t_{i,j}) = g_j(t_{i,j}) + \Delta_j$ discussed in our Proposition 3, the first addendum $g_j(t_{i,j})$ accounts for the 'variable loss', while the second addendum Δ_j represents the 'jump' discontinuity at the poverty line and hence the magnitude of the 'fixed loss'. In order to provide an illustration of the proposed approach let us choose $g_j(t_{i,j}) = \alpha_j \varphi(t_{i,j})$ and $\Delta_j = \omega_j r_j$, leading to the following parametric formulation:

$$\breve{p}_j(t_{i,j}) = \alpha_j \varphi(t_{i,j}) + \omega_j r_j, \tag{3}$$

where $0 < \alpha_j < 1$ is a weight increasing with the importance of dimension j, φ is a poverty-line continuous, normalized, strictly positive, and real-valued function decreasing in $t_{i,j}$,[8] $\omega_j \in \mathbb{R}_+$, and $r_j \in \mathbb{N}$ denotes dimension j's rank order –poverty dimensions being ranked in increasing order of importance. The rationale for choosing (3) in our application is that, for $\omega_j = \omega \geq 1$, a hierarchical scheme of the kind $\overset{UH}{\succ}$ is implemented simply by building upon poverty dimensions' rank order. The reason is that the difference between the rank order of two poverty dimensions that are subsequent in the ranking is nothing but the difference between two subsequent positive integers, which equals one and hence is able to offset any possible difference between the variable losses of two poverty functions – since $\max\limits_{t_{i,j}} \varphi(t_{i,j}) = \varphi(t_{i,j} = 0) = 1$ then $0 < \alpha_j \varphi(t_{i,j}) < 1$ for all j and $\left|\alpha_k \varphi(t_{i,k}) - \alpha_h \varphi(t_{i,h})\right| < 1$ for all h and k. Differently, when ω equals zero for all j's we have poverty-line continuous functions accounting only for a 'variable loss' and a hierarchy of the kind $\overset{RH}{\succ}$ is implemented among poverty dimensions. The overall poverty level of individual i is given by:

$$\begin{aligned}\breve{\mathrm{P}}_i &= \sum_{j=1}^{m} \left[\alpha_j \varphi(t_{i,j}) + \omega_j r_j\right] = \left[\alpha_1 \varphi(t_{i,1}) + \omega_1\right] + \left[\alpha_2 \varphi(t_{i,2}) + 2\omega_2\right] \\ &+ \ldots + \left[\alpha_m \varphi(t_{i,m}) + m\omega_m\right].\end{aligned} \tag{4}$$

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[8] Possible φ's are the individual deprivation functions of well-known indices such as those in Foster et al. (1984) and Chakravarty (1983).

Note that (4) allows to choose idiosyncratic ω_j's for different poverty dimensions. This opens up the possibility to implement alternative strategies for what concerns the type of hierarchical schemes among dimensions. For example, suppose that the analyst assessing multidimensional child poverty identifies as relevant dimensions 'calories intake' (*ci*), 'housing' (*ho*), 'education' (*ed*) and 'play time' (*pt*), arranged in decreasing order of importance. By setting $\alpha_j = 0.1r_j$ a hierarchy of type $\overset{RH}{\succ}$ will be implemented and the functional form in (4) yields:

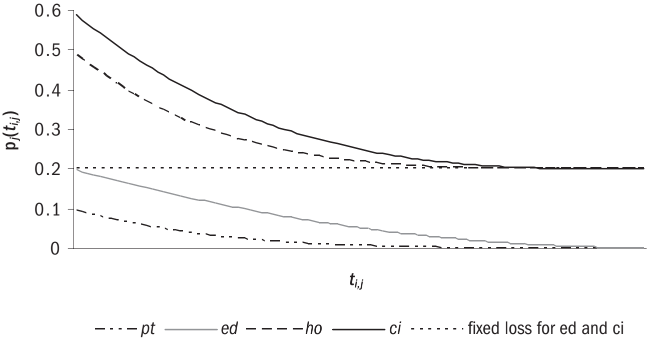
$$\mathrm{P}_i = \sum_{j=1}^{4}\left[\alpha_j\varphi(t_{ij}) + \omega_j r_j\right] = \left[0.1\varphi(t_{i,pt}) + \omega_{pt}\right] + \left[0.2\varphi(t_{i,ed}) + 2\omega_{ed}\right]$$
$$+\left[0.3\varphi(t_{i,ho}) + 3\omega_{ho}\right] + \left[0.4\varphi(t_{i,ci}) + 4\omega_{ci}\right]$$

Suppose now that the analyst deems it appropriate to require $ci \overset{RH}{\succ} ho \overset{UH}{\succ} ed \overset{RH}{\succ} pt$. This may be justified by the willingness to implement a $\overset{RH}{\succ}$ type of hierarchy both between the two poverty dimensions related to physical integrity (*ci* and *ho*) and between the two poverty dimensions related to psychological development (*ed* and *pt*), but an $\overset{UH}{\succ}$ hierarchy type between the two groups. Among the functional forms for individual poverty functions granting the implementation of the hierarchical scheme $ci \overset{RH}{\succ} ho \overset{UH}{\succ} ed \overset{RH}{\succ} pt$, for dimensions pt, ed, ho and ci we consider, respectively:

$$\breve{p}_{pt}(t_{i,pt}) = 0.1\varphi(t_{i,pt}),\ \breve{p}_{ed}(t_{i,ed}) = 0.2\varphi(t_{i,ed}),\ \breve{p}_{ho}(t_{i,ho}) = 0.3\varphi(t_{i,ho}) + 0.2,$$
$$\breve{p}_{ci}(t_{i,ci}) = 0.4\varphi(t_{i,ci}) + 0.2.$$

For all dimensions pt, ed, ho and ci those functional forms relate to $x_{i,j} < z_j$, while for $x_{i,j} \geq z_j$ the accommodation of the *Focus* axiom clearly requires that $\breve{p}_j(t_{i,j}) = 0$. See first case in Appendix B for the derivation of the conditions on ω_j. In Figure 1

Figure 1. Individual poverty functions for dimensions *pt*, *ed*, *ho* and *ci*



those poverty functions are plotted for $\varphi = (1 - t_{i,j})^2$ – squared poverty gap. As can be noted, under both UH and RH the dominant deprivation function is always required to lie above the dominated one, while the existence of intersection points between them would imply switches in the hierarchy.

B. An application to Maldives 1997-2004

The analysis is carried out using data from the Vulnerability and Poverty Assessments, two household surveys run in The Maldives Republic in 1997 and 2004 (hereby VPA-1 and VPA-2) by the Minister of Planning and National Development (MPND) with the UNDP collaboration. Basically the same questionnaire and definitions are used in the two waves, making the data fully consistent between the two surveys. All 200 inhabited islands are covered and the sample size is over 2,700 households (2,400 from all Atolls, the remaining from Malé).[9]

These surveys provide a wide range of variables regarding living conditions and socio-economic characteristics at both household and individual level, allowing the performance of multidimensional poverty and well-being assessments. Moreover, the surveys gather information on the importance attached to a pre-defined list of living standard dimensions by both individuals and the Island Communities – the latter being represented by Island Development Committees and Women's Development Committees.[10]

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[9] Both datasets are freely downloadable on the MPND website (www.frdp-maldives.gov.mv/hies/VPA.htm). See also de Kruijk and Rutten (2007).

[10] Both heads of households and their spouses were asked their views. In particular, in VPA-1 individuals were asked to reply to the following question: "Some of these problems are listed below, in different groups. We would like to ask you to tell us which are the important ones in your life. Please give the most pressing problems first, and then the next most important, and so on, until all areas have been covered". The list included the following dimensions: 1. Quality of house; 2. Availability of transport service; 3. Availability of electricity; 4. Communication facilities; 5. Employment opportunity; 6. Possibilities to earn a good income; 7. Food security all year around; 8. Environmental security; 9. Availability of drinking water; 10. Access to consumer goods; 11. Access to health services/improvements; 12. Access to quality education; 13. Community participation; 14. TV/entertainment facilities; 15. Availability of recreational facilities. VPA-2 posed the same question but adopted a shorter list dropping the last three dimensions. In both surveys, the same list of dimensions and a similar question were submitted to the Island Committees ("Some of the problems faced by the islanders are listed below. We would like you to tell us which are the most important ones for your community. Please give the most pressing problems first, and then the next most important, and so on, until all areas listed have been covered").

Since our exercise has primarily illustrative purposes, we restrict our poverty analysis to the four top-ranking dimensions according to the aggregate preferences expressed by the population. In decreasing order of importance, those are education (*ed*), health (*he*), housing (*ho*) and employment (*em*). For each of these four dimensions, an indicator of achievement is built upon the information contained in the survey.[11] In our application, an individual is deemed poor in a certain dimension if she is unable to fully function in that dimension – see Appendix C for a more detailed description of the indicators, the corresponding poverty gaps and the relevant aggregation procedure. It is worthwhile to stress that we build polytomous indicators for each of the four dimensions so that we are free from the restrictions pointed out by Proposition 1.

We assess poverty in the Maldives according to three different specifications of the ranking $ed \succ he \succ ho \succ em$. The first two applications are based on the belief that the hierarchy among all poverty dimensions is uniquely of a $\overset{RH}{\succ}$ type or of an $\overset{UH}{\succ}$ type, respectively. In the third case, we let the type of hierarchy depend on the concurrence between the opinion expressed by the population and the one formulated by the Islands' Development Committees. In other words, if people's value judgement that $h \succ k$ is in line with the proclamation of the Committees then we consider that $h \overset{UH}{\succ} k$; if instead the Committees do not agree and deem k more important than h then we opt for $h \overset{RH}{\succ} k$. Since the Committees' ranking is $ed \succ he \succ em \succ ho$, we see that the dominance relations $ed \succ he$ and $he \succ ho$ indeed match those of the population but the ordering of *ho* and *em* is reversed.[12] Hence, in our third empirical application we implement a 'mixed strategy' where $ed \overset{UH}{\succ} he \overset{UH}{\succ} ho \overset{RH}{\succ} em$. Needlessly to say, the criterion we adopt here for the choice of a $\overset{RH}{\succ}$ rather than of an $\overset{UH}{\succ}$ type of hierarchy is arbitrarily set up to take advantage of the existence of two sources of opinion on the ranking of poverty dimensions. Different types of secondary data may suggest different criteria, while the opportunity to collect primary data can offer *ad hoc* information; alternatively, the analyst can rely on her own value judgements or on experts' opinions.

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[11] This indicator assumes discrete values ranging from 0 (null achievement) to 4 (full achievement) for all dimensions except for health, where levels of achievement equal to 0, 1, 2 are used. See Appendix C for further details.

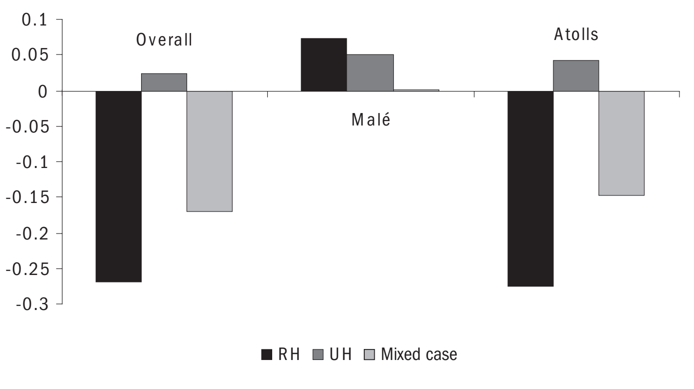
[12] The priorities of both islanders and the Island Committees do not seem to reflect the extent of deprivation in the country. According to the poverty indicators we constructed, the highest poverty level is found for the housing dimension followed by education, health and employment. Headcount ratio (H) and Poverty Gap (PG) in the four dimensions in 2004 are the following: housing, H=0.57 and PG=0.262; education, H=0.41 and PG=0.23; health, H=0.23 and PG=0,13; employment, H=0.15 and PG=0.08.

In all of our three exercises we set $\alpha_j = 0.1r_j$.[13] For what concerns the choice of ω_j, in our first and second applications we set $\omega_j = 0$ and $\omega_j = 1$ for all j's, respectively. As to the third application, the poverty functions we adopt for poor achievements in dimensions *em*, *ho*, *he* and *ed* are, respectively (see second case in Appendix B for the derivation of the conditions on ω_j):

$$\breve{p}_{em}(t_{i,em}) = 0.1\varphi(t_{i,em}),\ \breve{p}_{ho}(t_{i,ho}) = 0.2\varphi(t_{i,ho}) + 0.04,$$
$$\breve{p}_{he}(t_{i,he}) = 0.3\varphi(t_{i,he}) + 0.24,\ \breve{p}_{ed}(t_{i,ed}) = 0.4\varphi(t_{i,ed}) + 0.54.$$

According to our results, the choice of the type of hierarchy is crucial in the assessment of the multidimensional poverty trend in the Maldives. While a poverty increase is the response of all of the three applications for what concerns Malé, for the Atolls as well as for the country overall this conclusion is suggested by UH while both RH and the mixed strategy signal a poverty decrease. This pattern is robust to the use of the two most commonly used functional forms for the deprivation function $\varphi(\cdot)$– i.e. the poverty gap (PG) $\varphi = (1 - t_{i,j})$ and the squared poverty gap (SPG) $\varphi = (1 - t_{i,j})^2$. Further, the results for the country overall also hold for gender subgroups. All figures are presented in Table 1, while Figure 2 provides a graphical illustration of the pattern by area for φ = PG.

Figure 2. Variation in poverty in Maldives (%) by area, 1997-2004, φ = PG



Notes: our elaboration on VPA1-1997 and VPA2-2004 datasets. φ = PG denotes that the poverty gap is used as a poverty function.

[13] Note that in our exercise $\sum_{j=1}^{m} \alpha_j = 1$ but the analyst can abstract from this restriction.

Table 1. Poverty in Maldives, 1997 and 2004: as PG and SPG for RH, UH and mixed case

			Overall			
RH	PG^{97}=.2364	>	PG^{04}=.1867	SPG^{97}=.1447	>	SPG^{04}=.0966
UH	PG^{97}=4.4992	<	PG^{04}=4.6200	SPG^{97}=4.4075	<	SPG^{04}=4.5299
Mixed case	PG^{97}=.6947	>	PG^{04}=.5946	SPG^{97}=.6030	>	SPG^{04}=.5045
			Decomposition by area			
			Malé			
RH	PG^{97}=.1615	<	PG^{04}=.1745	SPG^{97}=.0975	<	SPG^{04}=.1142
UH	PG^{97}=3.3740	<	PG^{04}=3.565	SPG^{97}=3.310	<	SPG^{04}=3.5047
Mixed case	PG^{97}=.4148	<	PG^{04}=.4159	SPG^{97}=.3508	<	SPG^{04}=.3556
			Atolls			
RH	PG^{97}=.2396	>	PG^{04}=.1882	SPG^{97}=.1467	>	SPG^{04}=.0943
UH	PG^{97}=4.5478	<	PG^{04}=4.756	SPG^{97}=4.4548	<	SPG^{04}=4.6621
Mixed case	PG^{97}=.7068	>	PG^{04}=.6176	SPG^{97}=.6139	>	SPG^{04}=.5237
			Decomposition by gender			
			Female			
RH	PG^{97}=.2356	>	PG^{04}=.1841	SPG^{97}=.1454	>	SPG^{04}=.0964
UH	PG^{97}=4.423	<	PG^{04}=4.5265	SPG^{97}=4.3336	<	SPG^{04}=4.4388
Mixed case	PG^{97}=.6833	>	PG^{04}=.5812	SPG^{97}=.5931	>	SPG^{04}=.4935
			Malé			
RH	PG^{97}=.2374	>	PG^{04}=.1909	SPG^{97}=.1436	>	SPG^{04}=.0969
UH	PG^{97}=4.5955	<	PG^{04}=4.7751	SPG^{97}=4.5017	<	SPG^{04}=4.6811
Mixed case	PG^{97}=.7093	>	PG^{04}=.6168	SPG^{97}=.6155	>	SPG^{04}=.5228

Notes: our elaboration on VPA1-1997 and VPA2-2004 datasets. PG stands for poverty gap, SPG for squared poverty gap.

Table 2 helps identify the origins of the dissonance in poverty appraisals under UH and RH. The basic indicators used in our empirical application, headcount ratio (H) and PG in 1997 and 2004, are concisely presented for the overall country with and without the application of preference-based weights. Focusing on the aggregate figures (last row of the Table), it is possible to see how the introduction of unequal weights does not suffice to explain the conflicting results obtained under UH and RH: whether preferences are accounted for or not, in the period 1997-2004 H increases and PG decreases. It is precisely in this opposite trend of H and PG that our results are rooted. By including a fixed loss from poverty at the individual level, UH enhances the role of H in the aggregate poverty figure. Hence, in poverty evaluation under UH more importance is attached to the number of people living in poverty and less to the distance from the poverty threshold (accounted for by PG) than is the case under RH – as well as under the mixed strategy, which is somehow an intermediate position.

Table 2. Summary of basic indicators for the overall country, 1997 and 2004

	Equal weights				Preference-based weights[a]			
	H_{1997}	H_{2004}	PG_{1997}	PG_{2004}	H_{1997}	H_{2004}	PG_{1997}	PG_{2004}
education	0.55	0.41	0.43	0.23	2.2	1.64	0.17	0.09
health	0.12	0.23	0.06	0.13	0.36	0.69	0.02	0.04
housing	0.29	0.56	0.15	0.26	0.58	1.12	0.03	0.05
employment	0.19	0.15	0.09	0.08	0.19	0.15	0.01	0.01
Aggregate (sum)	1.15	1.35	0.73	0.70	3.33	3.6	0.23	0.19

Notes: H stands for the headcount ratio, PG for the poverty gap. [a]: in conformity with our application, weights for H are 4, 3, 2 and 1 while for PG they are 0.4, 0.3, 0.2 and 0.1.

V. Conclusions

Despite the growing interest in multidimensional poverty and well-being assessment, thus far very little attention has been paid to how the incorporation of different criteria for the relative importance of poverty dimensions can respond to conflicting value judgements and lead to contradictory empirical results. In this paper we have presented two alternative, simple and highly intuitive ways in which a hierarchy among poverty dimensions can be conceptualized: *restricted* and *unrestricted hierarchy*. While the former allows for compensation among poverty dimensions, inherent in the latter are a lexicographic representation of multidimensional poverty and a conceptualization of poverty as entailing a fixed welfare loss due to deprivation.

In the realm of additive multidimensional poverty indices, we have derived and fully explained a methodology based on dimensions' rank order allowing the implementation of alternative hierarchical schemes. An application of the proposed methodology to the evaluation of multidimensional poverty for Maldives in 1997 and 2004 shows that the choice of the hierarchical scheme for poverty dimensions can lead to opposite conclusions on the poverty trend. The observed reversal of the poverty ordering is evidently a possible result in all those situations where the condition of first order stochastic dominance between the distributions to be compared is not met. Hence, it is of primary importance to be aware of the potential implications that the choice of the hierarchical scheme among poverty dimensions may have on poverty appraisals. Further research is needed on both the theoretical and empirical side to increase the options open to the analyst and to test their empirical robustness.

Appendix

A. Proofs

Proof of Proposition 1

To see the impossibility for RH to be implemented by constant individual poverty functions, three cases have to be investigated: (i) $p_h(t_{i,h})$ is constant while $p_k(t_{i,k})$ is not, (ii) $p_k(t_{i,k})$ is constant while $p_h(t_{i,h})$ is not and (iii) both $p_h(t_{i,h})$ and $p_k(t_{i,k})$ are constant individual deprivation functions. Consider that the accommodation of $h \overset{RH}{\succ} k$ requires $\max_{t_{i,h}} p_h(t_{i,h}) > \max_{t_{i,k}} p_k(t_{i,k})$ and $\min_{t_{i,h}} p_h(t_{i,h}) > \min_{t_{i,k}} p_k(t_{i,k})$ but $\max_{t_{i,k}} p_k(t_{i,k}) > \min_{t_{i,h}} p_h(t_{i,h})$. If the generic individual deprivation function $p_j(t_{i,j})$ is constant then $p_j(t_{i,j}) = \lambda_j$ for all $t_{i,j}$. By substituting in the above condition for the accommodation of $h \overset{RH}{\succ} k$ we obtain a contradiction for each of the three cases: (i) $\lambda_h > \max_{t_{i,k}} p_k(t_{i,k})$, $\lambda_h > \min_{t_{i,k}} p_k(t_{i,k})$ and $\max_{t_{i,k}} p_k(t_{i,k}) > \lambda_h$; (ii) $\max_{t_{i,h}} p_h(t_{i,h}) > \lambda_k$, $\min_{t_{i,h}} p_h(t_{i,h}) > \lambda_k$ and $\lambda_k > \min_{t_{i,h}} p_h(t_{i,h})$; (iii) if both $p_h(t_{i,h})$ and $p_k(t_{i,k})$ are constant then the contradiction is evident. Q.E.D.

Proof of Proposition 2

We shall show that a hierarchy of the kind $h \overset{UH}{\succ} k$ between dimensions h and k is incompatible with poverty-line continuous $p_h(t_{i,h})$'s. First, note that $h \overset{UH}{\succ} k$ requires the inequality $p_h(t_{i,h}) > p_k(t_{i,k})$ to be verified for all possible $t_{i,h}$ and $t_{i,k}$. Rearranging the above inequality as $p_h(t_{i,h}) - p_k(t_{i,k}) > 0$, we see that zero is the greatest lower bound of the set whose elements are the difference of the generally specified poverty functions for dimensions h and k, independently of the values of possible $t_{i,h}$ and $t_{i,k}$. For decreasing poverty functions in the poor domain [0,1) for possible $t_{i,h}$ and $t_{i,k}$, the condition for $h \overset{UH}{\succ} k$ is equivalent to $\inf_{t_{i,h}} p_h(t_{i,h}) - \max_{t_{i,k}} p_k(t_{i,k}) \geq 0$. The proof of Proposition 1 follows easily by contradiction. Suppose that p_h satisfies ST. Then, given continuity, positivity and monotonic decreasingness of p_h it must be that $\lim_{t_{i,h} \to 1} p_h = 0$ equals also $\inf_{t_{i,h}} p_h(t_{i,h})$. Now, recalling that p_k is a positive-valued function we are sure that $\max_{t_{i,k}} p_k(t_{i,k}) = \lambda > 0$. Substituting in the above condition for $h \overset{UH}{\succ} k$, the contradiction inherent in $0 - \lambda \geq 0$ becomes evident. This proves that $h \overset{UH}{\succ} k$ implies poverty-line discontinuous contribution functions for dimension h, hence, that a poverty index cannot meet both UH and ST. Q.E.D.

Proof of Proposition 3

The sufficiency side of the proposition is obvious. As to the necessity side, it is easy to see that the discontinuity necessarily required by Proposition 2 must be a discontinuity of the first kind – also known as 'jump' discontinuity. Hence $p_h(t_{i,h})$ must be of the form $g_h(t_{i,h}) + \Delta_h$, where g_h is a strictly positive, continuous and decreasing function and Δ_h is the magnitude of the upward translation. We now prove that, for UH to be granted, Δ_h must be at least as large as $\max_{t_{i,k}} p_k(t_{i,k})$ when $t_{i,h} \in [0,1)$. Since in the poverty domain Δ_h is invariant in $t_{i,h}$ we can substitute $\inf_{t_{i,h}} p_h(t_{i,h})$ in the general condition for the accommodation of UH derived in the proof of Proposition 2 – i.e. $\inf_{t_{i,h}} p_h(t_{i,h}) - \max_{t_{i,k}} p_k(t_{i,k}) \geq 0$. Doing so, we obtain $[\inf_{t_{i,h}} g_h(t_{i,h}) + \Delta_h] - \max_{t_{i,k}} p_k(t_{i,k}) \geq 0$, which given continuity of g_h becomes $[0 + \Delta_h] - \max_{t_{i,k}} p_k(t_{i,k}) \geq 0$, yielding the desired condition $\Delta_h \geq \max_{t_{i,k}} p_k(t_{i,k})$. Finally, given the weak definition of the poor adopted here, $\Delta_h = 0$ when $t_{i,h} = 1$. Q.E.D.

B. Orderings

First mixed case

Derivation of the conditions on ω_j's for the implementation of $ci \overset{RH}{\succ} ho \overset{UH}{\succ} ed \overset{RH}{\succ} pt$:

$$\text{for } ed \overset{RH}{\succ} pt: \quad \underbrace{\omega_{pt}}_{\textit{fixed component for pt}} \leq \underbrace{2\omega_{ed}}_{\textit{fixed component for ed}} < \underbrace{\max_{t_{i,pt}} \breve{p}_{pt}\left(t_{i,pt}\right)}_{\text{max } \textit{poverty value for pt}}$$

$$\frac{\omega_{pt}}{2} \leq \omega_{ed} < \frac{0.1\varphi(t_{i,pt} = 0) + \omega_{pt}}{2}$$

$$\text{for } ho \overset{UH}{\succ} ed: \quad \underbrace{3\omega_{ho}}_{\textit{fixed component for ho}} \geq \underbrace{\max_{t_{i,ed}} \breve{p}_{ed}\left(t_{i,ed}\right)}_{\text{max } \textit{poverty value for ed}}$$

$$\omega_{ho} \geq \frac{0.2\varphi(t_{i,ed} = 0) + 2\omega_{ed}}{3}$$

$$\text{for } ci \overset{RH}{\succ} ho: \quad \underbrace{3\omega_{ho}}_{fixed\ component\ for\ ho} \leq \underbrace{4\omega_{ci}}_{fixed\ component\ for\ ci} < \underbrace{\max_{t_{i,ho}} \breve{p}_{ho}\left(t_{i,ho}\right)}_{\max\ poverty\ value\ for\ ho}$$

$$\frac{3\omega_{ho}}{4} \leq \omega_{ci} < \frac{0.3\varphi(t_{i,ho}=0)+3\omega_{ho}}{4}$$

There desired ordering does not impose limitations to the admissible values of ω_{pt}. Suppose we choose $\omega_{pt} = 0$ – in order to have a poverty-line continuous deprivation function for dimension pt. The condition on ω_{ed} for the implementation of $ed \overset{RH}{\succ} pt$ becomes $0 \leq \omega_{ed} < \frac{0.1\varphi(t_{i,pt}=0)}{2} \Rightarrow 0 \leq \omega_{ed} < \frac{0.1}{2}$. Any value of ω_{ed} in the specified interval is indifferent as to the implementation of $ed \overset{RH}{\succ} pt$. Similarly to $\breve{p}_{pt}$, if we want $\breve{p}_{ed}$ to exhibit only a 'variable loss' we would go for $\omega_{ed} = 0$. The condition for $ho \overset{UH}{\succ} ed$ would then become $\omega_{ho} \geq \frac{0.2}{3}$. Finally, if we opt for $\omega_{ho} = \frac{0.2}{3}$ then $ci \overset{RH}{\succ} ho$ will require $\frac{0.2}{4} \leq \omega_{ci} < \frac{0.5}{4}$. Once also the value for ω_{ci} is selected, say $\omega_{ci} = \frac{0.2}{4}$, the individual poverty functions for the four dimensions are fully determined.

Second mixed case

Derivation of the conditions on ω_j's for the implementation of $ed \overset{UH}{\succ} he \overset{UH}{\succ} ho \overset{RH}{\succ} em$:

$$\text{for } ho \overset{RH}{\succ} em: \quad \underbrace{\omega_{em}}_{fixed\ component\ for\ em} \leq \underbrace{2\omega_{ho}}_{fixed\ component\ for\ ho} < \underbrace{\max_{t_{i,em}} \breve{p}_{em}\left(t_{i,em}\right)}_{\max\ poverty\ value\ for\ em}$$

$$\frac{\omega_{em}}{2} \leq \omega_{ho} < \frac{0.1\varphi(t_{i,em}=0)+\omega_{em}}{2}$$

$$\text{for } he \overset{UH}{\succ} ho: \quad \underbrace{3\omega_{he}}_{fixed\ component\ for\ he} \geq \underbrace{\max_{t_{i,ho}} \breve{p}_{ho}\left(t_{i,ho}\right)}_{\max\ poverty\ value\ for\ ho}$$

$$\omega_{he} \geq \frac{0.2\varphi(t_{i,ho}=0)+2\omega_{ho}}{3}$$

$$\text{for } ed \overset{UH}{\succ} he: \quad \underbrace{4\omega_{ed}}_{\textit{fixed component for ed}} \geq \underbrace{\max_{t_{i,he}} \breve{p}_{he}\left(t_{i,he}\right)}_{\max \textit{ poverty value for he}}$$

$$\omega_{ed} \geq \frac{0.3\varphi(t_{i,he}=0)+3\omega_{he}}{4}$$

Among the possible ω_j's respecting the above conditions we choose $\omega_{em} = 0$, $\omega_{ho} = 0.02$ (because, for some reasons, we want a poverty-line continuous deprivation function for *em* but a poverty-line discontinuous deprivation function for *ho*), $\omega_{he} = 0.08$ and $\omega_{ed} = 0.135$.

C. Description of variables

The four indicators have been constructed according to the following lines: (a) *housing* is a composite outcome of three variables: available living space per capita, availability of rain water storage facility and days of lack of drinking water; (b) *health* is described in terms of possibility to get medicine when necessary, their availability and affordability; (c) *education* is measured in terms of educational attainment and (d) *employment* is measured with reference to the condition of unemployment and under-employment (number of earners in the family and number of months of work per year). Normalized poverty gaps ranging from zero to one are calculated on the basis of a certain number of linearly distributed categories (three in case of health, five for the remaining cases). In the case of housing, the aggregate indicator assumes the value of the largest poverty gap across the 'living space' and the 'water availability' categories. While housing and health are determined at household level, education and employment refer to the individual. More details can be found in Table A1.

Table A1. Elementary variables, composite indicators and corresponding poverty gaps

Indicators	Variables	Variable description	Poverty gaps	
housing	Living space	Per capita square feet (SQF)	≤ 40 SQF	=1
			41-60 SQF	=.75
			61-100 SQF	=.50
			101-200 SQF	=.25
			> 200 SQF[1]	=0
	Rain water storage facility and drinking water	No water storage because cannot afford (NWS) and days of insufficient drinking water experienced (LDW)	NWS & 90 LWD	=1
			> 90 LWD	=.75
			30-90 LWD	=.50
			10-29 LWD	=.25
			< 10 LWD	=0
health	Access to medicine	Getting medicine (GM) or not getting medicine (NGM) when necessary	NGM & CA	=1
			NGM&NA	=.50
			Otherwise	=0
		Reasons for not: cannot afford (CA), not available on the island (NA)		
education	Education level	Highest level of education achieved	None/illiterate	=1
			Read/write only	=.75
			Functional literacy	=.50
			Local certificate	=.25
			Otherwise	=0
employment	Unemployment and underemployment	Unemployment (U)	U & NE	=1
		No earners in the family (NE)	U & OE	=.75
		At least one earner (OE)	EM ≤ 1	=.50
		No. of months/year of work (EM)	EM 2-11	=.25
			Otherwise	=0

Note: [1] 200 sq. feet corresponds to the median value.

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