XIV

Volume XIV, Number 1, May 2011

Journal of Applied Economics

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UCEMA

Edited by the Universidad del CEMA
Print ISSN 1514-0326
Online ISSN 1667-6726

Journal of Applied Economics. Vol XIV, No. 1 (May 2011), 5-34

MEASURING WELFARE CHANGES IN BEHAVIOURAL MICROSIMULATION MODELLING: ACCOUNTING FOR THE RANDOM UTILITY COMPONENT

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Submitted February 2010; accepted November 2010

This paper presents a method of predicting individuals' welfare changes (compensating and equivalent variations) arising from a tax or social security policy change in the context of behavioural microsimulation modelling, where individuals can choose between a limited number of discrete hours of work. The method allows fully for the nonlinearity of the budget constraint facing each individual, the probabilistic nature of the labour supply model and the presence of unobserved heterogeneity in the estimation of preference functions. Yet it is relatively straightforward to implement. An advantage of welfare measures, compared with changes in net incomes, is that they take into account the value of leisure and home production. The method is applied to a hypothetical income tax policy change in Australia.

JEL classification codes: D63, H31, J22
Key words: welfare change measures, equivalent variation, compensating variation, labour supply modelling, nonlinear budget constraint

I. Introduction

The aim of this paper is to propose a practical method of measuring welfare changes in the special context of labour supply models, where individuals typically face highly nonlinear budget constraints. Such nonlinearities occur due to the complexity

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of the tax and social security system. For example, marginal income tax rates generally differ depending on the level of income. Special attention is given to the use of a random utility specification combined with discrete hours of labour supply, rather than allowing hours of work to vary continuously. Discrete hours models have substantial econometric advantages resulting from directly estimating the parameters of specified direct utility functions. This avoids problems concerning the endogeneity of the net wage in continuous hours models and the need to solve the first-order conditions for utility maximisation, or even to know the full budget constraint facing each individual. As no explicit labour supply function is needed, a wide range of direct utility functions can be used.[1] In addition, a discrete choice model may be more realistic since in practice individuals have limited choices over the extent to which they can vary their hours. Furthermore, this approach is valuable in estimating preference functions which allow for the effects of a range of individual characteristics and it is particularly suitable for microsimulation modelling, where emphasis is given to incorporation of population heterogeneity. Finally, the discrete approach also makes it possible to account for the full detail of the tax and social security system.

Recent developments in behavioural tax microsimulation modelling make the computation of individual welfare change measures possible, and its use in policy analyses provides a strong motivation for developing convenient methods of obtaining accurate welfare measures. The advantage of welfare measures over income-based measures in behavioural microsimulation is that they include the value of leisure and home production time.

Where welfare changes have been produced in the labour supply context, the approach has often been to adopt a minor modification of the standard expressions used to obtain welfare changes. These are usually based on linear budget lines and, using the expenditure function, involve the cost of moving along an indifference curve as the price of leisure changes.[2] However, Creedy and Kalb (2005a) showed that the standard approach does not allow sufficiently for the nonlinearity of the

[1] For a general discussion of alternative approaches to labour supply modelling, see Creedy and Duncan (2002).

[2] See for example Hausman (1981, p. 672; 1985, pp. 243-245), Blomquist (1983, pp. 187-190), Blundell, Preston and Walker (1994, pp. 4-8), and Creedy (2000, 2001). In the context of commodity demands, including situations in which there may be quantity constraints, a general approach was suggested by Neary and Roberts (1980), exploiting the Hicksian concept of virtual price; see also Latham (1980) or Johansson (1987). In these cases it is usual to define a modified expenditure function conditional on the rationed levels of consumption.

budget constraint facing an individual and may not be consistent with the discrete hours framework. The proposed method allows for the full detail of budget constraints and can be applied to a wide range of deterministic utility specifications, independent of whether the expenditure function can be written down explicitly.[3]

The present paper extends Creedy and Kalb's approach in several important respects. First, it deals with the case where a random term is added to the deterministic component of the utility function, giving rise to a probability distribution over the discrete hours levels available for work. The method proposed here involves simulation, based on drawing sets of random utility components, instead of an analytic approach.[4] This makes the method relatively easy to apply. As a result, it is equally easy to use the approach when basing labour supply responses on expected labour supply outcomes as when using calibration to fix labour supply in the pre-reform system to observed hours of work (as described in Section III.A). In addition, there is no need to specify the indirect utility function in terms of income and prices. This is a major advantage, for example, when using the quadratic translog utility function (for which no indirect utility function can be derived analytically) or even the quadratic utility function, which is popular in discrete choice labour supply modelling and for which the indirect utility function is cumbersome. In the case of the compensating variation, the present method searches for the minimum welfare change at all possible discrete labour supply points, instead of assuming that the minimum value is always found at the optimal labour supply point after the reform.[5] In a discrete choice framework, this assumption is not always valid, as shown in the empirical example reported in this paper where, even for a small tax change, the minimum welfare change occurs for an hours level different from the post-reform optimum in a number of cases.

Second, in the context of microsimulation modelling, it is important to be able to allow for observed and unobserved individual heterogeneity. The use of quadratic direct utility functions allowing for such heterogeneity, along with a random utility component, is examined. The simulation approach can be used with all distributions

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[3] In allowing for the nonlinearity it differs from the linear approximation in Hanemann (1983) and Apps and Rees (1999), and in its simple and wide applicability it differs from Preston and Walker (1999).

[4] The simulation approach is similar to the one described in McFadden (1999) and Herriges and Kling (1999). An analytical approach was proposed by Dagsvik and Karlström (2005), who also include unobserved heterogeneity.

[5] Compensating variation is defined as the minimum amount of money necessary to return the individual to the same utility level as in system 0 after a policy change from system 0 to system 1.

of the random utility component from which random values can be drawn, including the most popular case of the Extreme Value type I distribution, which leads to a multinomial logit specification of the discrete choice labour supply model.[6] In addition, it is straightforward to allow for unobserved heterogeneity in the preference terms of the utility function. Given that the proposed use of the present approach is in the context of behavioural microsimulation modelling, in which labour supply changes are usually computed by drawing repeatedly from the distribution of the random utility term (and from the distribution of any unobserved heterogeneity terms if required), it is widely applicable. That is, if the relevant structural labour supply specification can be used in a microsimulation setting, the welfare change measurement approach proposed in this paper can also be used.

Third, after obtaining individual welfare change measures, an overall evaluation of a tax policy change is often required. Here, while bearing in mind the theoretical difficulties involved, the present paper explores the use of money metric welfare measures in a social welfare function which aggregates the individual effects. Comparisons are made with the simple use of net income.

The measurement of welfare changes when a discrete number of hours levels are allowed, but utility is deterministic, is summarised briefly in Section II. The implementation of the simulation approach is described in Section III, concentrating on the compensating variation, since no different principles are involved in obtaining the equivalent variation. In Section IV, the method is illustrated by using it to evaluate a simple policy reform involving changes in income taxation rates. The relevance of assessing welfare changes in addition to net income changes is shown by comparing results for the two measures. An Australian microsimulation model, the Melbourne Institute Tax and Transfer Simulator (MITTS), is used to generate results. A concise explanation of the model is given in Appendix A. Conclusions are in Section V.

II. Measuring welfare changes

This section shows how welfare measures can be obtained in labour supply models. The basic framework with continuous hours is described in subsection A, which

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[6] The choice of this thin-tailed distribution has the advantage that, besides the preference parameters in the utility function, no further parameters need to be estimated. Its name arises because it has been found useful in many applications involving extreme values. It is highly tractable in the present context. These qualities have generally been taken as sufficient justification for its use in discrete choice labour supply modelling.

shows how the standard approach gives rise to problems when applied in situations with nonlinear budget constraints. Subsection B presents a method of computing welfare changes in the discrete hours framework.

A. The standard approach

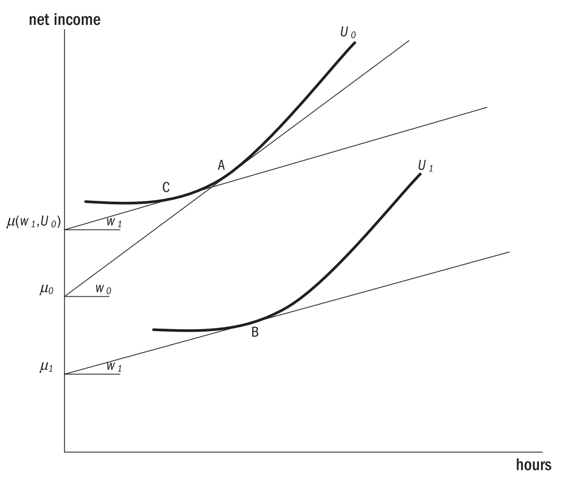
Let h denote the number of hours devoted to labour supply, which may be varied continuously, and let c denote net income. In a static framework, with the price index normalised to unity, net income and consumption are equal. The direct utility function is written as $U(c, h)$. Leisure is $T - h$ where T is the total number of hours available for work and leisure. The tax and transfer system is characterised by a piecewise-linear budget constraint.

Any optimal position can be regarded as being generated by a virtual linear constraint of the form:

$$c = wh + \mu. \tag{1}$$

For tangency solutions, w and μ represent the net wage rate along the relevant segment of the piecewise linear constraint and virtual income respectively, where the latter is the non-wage income (the intercept of the extended segment on the consumption axis) that would apply if the extended segment were the full constraint. With a corner solution, the virtual wage is the slope of the indifference curve at the

Figure 1. Compensating variation under a linear budget constraint



kink and virtual income is the value generated by a linear constraint having a net wage equal to the virtual wage. This is illustrated in Figure 1.

The evaluation of welfare changes requires the expenditure function, giving the minimum expenditure needed to reach a specified indifference curve at a given net wage rate. The expenditure function can be written in terms of virtual income, using $\mu(w, U)$. Virtual income $\mu(w, U)$ is obtained by first obtaining the indirect utility function. Substitute $c = wh + \mu$ into $U(c, h)$ and substitute the solution for optimal h, from $\left.\frac{dc}{dh}\right|_U = w$ and $c = wh + \mu$, into U. Then invert the indirect utility function by solving U for μ. Welfare changes can also be expressed in terms of full income, M, which is equal to $\mu + wT$.

Suppose there is a change in taxes and transfers from system 0 to system 1. The compensating variation is the minimum amount of money necessary to return the individual to the same utility level as in system 0 after the change to system 1. A tax change has a price (of leisure) effect of $\mu(w_1, U_0) - \mu_0$ and a (virtual) income effect of $\mu_0 - \mu_1$, so that the standard expression for the compensating variation is:

$$CV = \mu(w_1, U_0) - \mu_1, \tag{2}$$

where w_i is either the virtual or actual net wage rate at the optimum position under system i, U_i is the maximum utility that can be reached under tax system i, and $\mu_i = \mu(w_i, U_i)$. This welfare change is defined to be positive for a loss. The equivalent variation is defined as: $EV = \mu_0 - \mu(w_0, U_1)$.

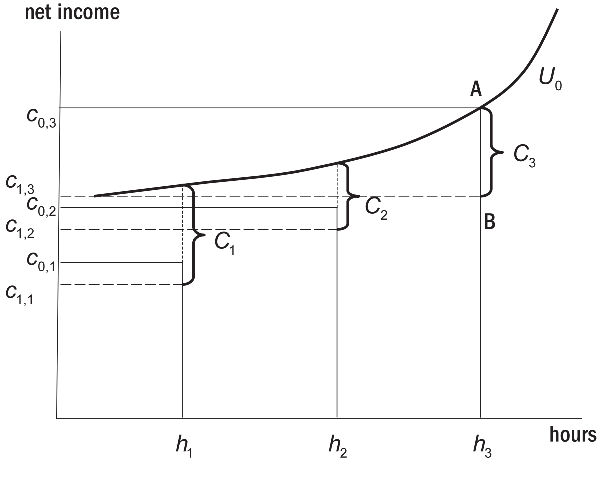
Payment of $\mu(w_1, U_0) - \mu_1$ allows the individual to reach indifference curve U_0 at point C, while in receipt of net wage w_1 and working fewer hours than at point B. This assumes that the virtual budget line in Figure 1, given by the tangent to U_1 at point B with associated virtual income of μ_1, does apply over the relevant range. That is, the individual can move to the left of B (and therefore increase consumption of leisure) along the linear budget line until hours worked correspond to those at point C. The addition of the compensated variation to net income allows consumption to increase so that point C can be reached. However, Creedy and Kalb (2005a) showed that nonlinearity of budget constraints can imply that the compensating variation, as defined above, may be insufficient or more than the minimum required to restore the individual to U_0. Moreover, point C cannot be guaranteed to avoid negative hours of work or an implausibly high level of hours. An appropriate method allowing for nonlinearity in the budget constraint in the context of discrete hours is described in the following subsection.

B. The discrete choice framework

In a discrete hours model, individuals are restricted to a limited number of hours levels, $h_1,\ldots,h_H$. The optimal number of hours is obtained by calculating utilities at a relatively small number of points, each of which is treated as a corner solution. Figure 2 shows an example with three hours levels: h_1, h_2, h_3. The initial tax system is such that the individual has net incomes of $c_{0,1}, c_{0,2}, c_{0,3}$ associated with the hours levels. Utility is maximized at A on indifference curve U_0. A policy reform changes the net incomes associated with the three hours levels to $c_{1,1},\ldots,c_{1,3}$. The new optimal position is at point B, so that labour supply does not change (the new optimal indifference curve is not shown, to avoid cluttering the diagram). Instead of using virtual (linear) budget lines and the standard approach described in Section II.A, the compensating variation is C_2, which is the minimum compensation required (across all labour supply points) to regain the old indifference curve U_0. Hence the individual could work h_2 hours, receive a CV of C_2, and regain the original indifference curve. This approach has the advantage of being fully consistent with the discrete choice framework. By contrast, the use of the virtual wage in the standard approach implies that the individual can move along a linear budget line, which is not consistent with a discrete choice framework.

The procedure outlined above requires only the calculation of the net income corresponding to a specified hours level and indifference curve. It is another advantage of this approach since it may not always be possible to obtain an analytical expression

Figure 2. Compensating variation in a discrete hours labour supply context



for the virtual income. This is the case if a utility function is used for which the labour supply and expenditure functions cannot be derived explicitly. Here, it is only necessary to compute net income, c, corresponding to a specified hours level, h, along an indifference curve with known utility level U. The minimum difference between this net income and the net income on the post-reform budget constraint is obtained by comparing all discrete labour supply points.

III. Implementation of the welfare measure

This section discusses the implementation of the procedure described in Section II.B in the context of behavioural microsimulation modelling where there is a random component of utility. Combined with the discrete hours approach, this gives rise to a probability distribution of hours worked for each individual, rather than a single deterministic hours level. The corresponding probability distributions of welfare and net income changes by hours worked are used for social welfare evaluations.[7] The MITTS labour responses are based on quadratic direct utility functions, and this specific case is discussed in subsection B. Subsection C considers the overall evaluation of a policy change using explicit value judgements and probability distributions. A more detailed discussion of the approach (with the exception of the inclusion of a random component) can be found in Creedy and Kalb (2005a).

A. Random utility component

In the discrete hours approach to specification and estimation, direct utility functions are assumed to consist of a deterministic and a random component, which implies that each individual has a probability distribution over the available hours levels. Before describing the calculation of expected welfare changes, it is necessary to describe the simulation method, referred to as 'calibration'. This uses a numerical approach designed to ensure that each individual in the sample has an optimal labour supply, under the actual tax structure, that corresponds to observed (discretised) hours of work. This means that the post-reform distribution of hours worked, and

[7] However, the use of probability distributions is not required to examine aggregate welfare changes. When results are aggregated (or averaged) over income units, the expected value of the welfare change for each individual can be used without loss of information (see Section III.A). By contrast, the use of the expected welfare (or net income) change rather than its probability distribution would lead to different results for social welfare evaluations and other inequality and poverty measures. This is because the use of expected values ignores the dispersion in predicted individual changes.

of welfare changes, is effectively a conditional distribution.[8] An alternative method would be to use the analytical expression for the unconditional distribution of hours worked and a corresponding analytical method to derive expected welfare changes (the numerical approach could be used to obtain the unconditional distribution as well). Preston and Walker (1999) and Dagsvik and Karlström (2005) use the unconditional distribution. The present approach has the advantage of simplicity combined with the benefit of being able to exploit observed behaviour in placing individuals at their pre-reform observed hours. In addition, the approach works equally well using the unconditional pre-reform expected labour supply.

The calibration approach proceeds as follows. If a set of random utility terms (one for each hours level) is drawn from the stochastic component of utility for an individual, the resulting utility levels (combining point estimates of the utility parameters and observed characteristics for the deterministic utility component with the random component) can be computed. Comparing the utility values at each labour supply point, optimal labour supply can be determined. In the calibration approach, only sets of error terms are drawn which result in the implied optimal hours being equal to the observed labour supply in the pre-reform tax system. Using the method of generating conditional draws described in Bourguignon, Fournier and Gurgand (1998), the error terms are drawn from the set of error terms which all result in observed labour supply.

A new tax and transfer system gives rise to a new set of net incomes for each hours level, and each set of draws produces a single optimal post-reform hours level. Pre-reform hours are always equal to observed hours but, using the set of error terms, a frequency distribution of post-reform hours arises.

The calibration approach is preferred in this case as it uses important information in the sample about each individuals' actual labour supply in a given tax structure. The resulting expected welfare change is also easily interpreted, as starting from a single hours level.[9]

It is straightforward to embed the calculation of welfare changes in this process of simulating labour supply responses arising from policy reforms, independent of whether conditional or unconditional distributions are used. The compensating

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[8] For a detailed introduction to modelling and estimation of labour supply, and the use of these models in behavioural microsimulation, see Creedy and Kalb (2005b).

[9] This contrasts with any approach using a set of random values from the error distribution, giving a frequency distribution over hours for the pre-reform system, as described for example by Small and Rosen (1981).

variation, for any set of random values, is obtained as follows. Utility for hours level h_i, giving rise to net income of c_i under tax structure T_k, is equal to a deterministic component $U(c_i|T_k)$ plus a random component v_i, so that:

$$U_{i,k} = U\left(c_i \middle| T_k\right) + v_i . \tag{3}$$

In the application in Section IV, an Extreme Value type I distribution is used for this random component. The usual indifference curve diagram is no longer helpful because indifference curves through any particular combination of hours of leisure and net income for different values of v_i are identical in a two-dimensional graph, although they represent different levels of utility. Hence indifference curves which are located further away from the origin (zero leisure and consumption) no longer necessarily represent higher utility than indifference curves located closer to the origin. A three-dimensional graph would be needed to present all relevant information. However, it is straightforward to explain the approach in terms of equation (3).

Suppose there are just two discrete hours levels, and consider a single set of random draws from the distribution of random utility components. Under tax structure 0, hours level 2 is chosen if: $U_{2,0} = U\left(c_2 \middle| T_0\right) + v_2 > U_{1,0} = U\left(c_1 \middle| T_0\right) + v_1$. Suppose the tax structure changes to structure 1. Using the calibration approach (or the expected labour supply approach), the random terms are the same before and after the change. Hours level 2 continues to be chosen if $U_{2,1} = U\left(c_2 \middle| T_1\right) + v_2 > U_{1,1} = U\left(c_1 \middle| T_1\right) + v_1$, but hours level 1 is chosen if the inequality is reversed.

For this set of draws, conditional on hours level 2 being chosen under the initial tax structure and irrespective of the hours chosen after the policy change, the compensating variation is the smaller of two values of CV, obtained by solving each of the two equations:

$$U\left(c_2 + CV_2 \middle| T_1\right) + v_2 = U\left(c_2 \middle| T_0\right) + v_2 \tag{4}$$

and

$$U\left(c_1 + CV_1 \middle| T_1\right) + v_1 = U\left(c_2 \middle| T_0\right) + v_2 . \tag{5}$$

If, under the initial tax structure, hours level 1 had been chosen instead of level 2, the compensating variation would be the smaller of two values of CV, obtained by solving each of the two equations:

$$U\left(c_2 + CV_2 \middle| T_1\right) + v_2 = U\left(c_1 \middle| T_0\right) + v_1 \tag{6}$$

and

$$U\left(c_1 + CV_1 \middle| T_1\right) + v_1 = U\left(c_1 \middle| T_0\right) + v_1. \tag{7}$$

Thus in general, where there are H hours levels, suppose that level h_m is chosen in the base tax structure 0. After the shift to tax structure 1, calculate the H values of CV_j. where

$$U\left(c_j + CV_j \middle| T_1\right) + v_j = U\left(c_m \middle| T_0\right) + v_m, \tag{8}$$

and the CV is given by $Min(CV_1,\ldots,CV_H)$. These calculations can be made for all sets of draws, giving rise to a probability distribution, after which the expected value of CV for this particular individual can be calculated by averaging across all sets of draws.

B. Quadratic utility functions

The general procedure described in Section III.A can be applied in a straightforward manner using quadratic direct utility functions, which are a fairly flexible specification. These have been frequently used in empirical analyses of discrete hours labour supply, and the labour responses in the MITTS model used in this paper are based on this utility function. Other examples include Keane and Moffitt (1998) and Duncan and Weeks (1997, 1998). Formulation of the likelihood function in this case is tractable, even though it becomes quite complex. A more flexible basic functional form for utility functions would make the likelihood function highly complex and therefore would make estimation more difficult. The quadratic direct utility function takes the form:

$$U = \alpha c^2 + \beta h^2 + \gamma ch + \delta c + \varepsilon h. \tag{9}$$

The parameters can be specified as functions of a range of individuals' characteristics, so that substantial heterogeneity in preferences can be captured. In order to obtain the welfare change measures, it is required to compute net income, c, corresponding to a specified hours level, h, along an indifference curve with known utility level, U_i^0 (computed from net income and hours at the optimal position i). This is obtained as the appropriate root of the quadratic[10]

$$Ac^2 + Bc + D = 0, \tag{10}$$

with

$$\begin{aligned} A &= \alpha, \\ B &= \gamma h + \delta, \\ D &= \beta h^2 + \varepsilon h - U_i^0. \end{aligned} \tag{11}$$

In practice the appropriate root is obvious. If $\alpha < 0$, it is equal to the smaller root, since this places the solution on the section of the utility function which increases with net income, c. The other solution is located on the downward sloping section of the utility function. The fact that the quadratic utility function can be downward sloping over a range of c values may give rise to difficulties when computing welfare measures. That is, in some cases there may be no solution corresponding to the optimal utility before the policy change (U_i^0). The utility function can become downward sloping before reaching utility level U_i^0; this occurs in particular for some of the higher discrete hours levels which may require very high net income to compensate for the high labour supply levels. Comparisons between net incomes on the post-reform budget constraint and the net incomes required to reach the pre-reform optimal utility curve need to be made at all levels of labour supply, not only at the observed hours level where the no-solution problem is unlikely to arise. This is discussed in the following paragraphs, along with other details regarding the practical implementation of the approach.

For all income units, the quadratic utility function is increasing with income at the observed labour supply, and for virtually all labour supply points the utility function is increasing with income under the relevant budget constraint.[11] However, further increases in net income are not guaranteed to remain below the income at which the utility function turns from increasing to decreasing with income.

Therefore, in obtaining welfare changes, a check is made at each labour supply point to ensure that the relevant range of the utility function implies increasing

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[10] See Creedy (2001) for derivation and further details of the quadratic utility function.

[11] Only for two income units was the income associated with the observed labour supply point located on the downward slope of the utility function. Labour supply was fixed at the observed level for these income units.

utility when net income increases. The results of these checks are presented in Appendix B. If the condition is violated before the desired utility is reached, the particular point is ignored. These points tend to represent the higher levels of hours worked. This could be interpreted as an indication that working too many hours may prevent some individuals from reaching utility levels above a particular threshold, independent of the income they receive. In those cases, the disutility due to limited leisure time may no longer be compensated by more net income.

C. Social evaluations

The method described in Section III.B can be used to obtain the probability distributions of welfare changes for each individual or couple in the database used in a microsimulation model. These can then be employed to obtain excess tax burdens and marginal welfare costs for each income unit. However, it is often desired to evaluate tax reforms in terms of their effects on specified demographic groups or for the population as a whole.

Here a fundamental difficulty arises in specifying a suitable welfare metric and social welfare function. Theoretical work on understanding interpersonal comparability problems and the aggregation problems involved in evaluating alternative structures, along with the conditions under which price-independent welfare comparisons can be made, has shown that the conditions required turn out to be highly restrictive: see, for example, Donaldson (1992) and Blackorby, Laisney and Schmachtenberg (1993).[12] Even a minimum requirement of homotheticity is not satisfied by the types of direct utility function used in microsimulation models and, in particular, complications are raised by the existence of differences between individuals in their preference for leisure.

In view of these problems, it is perhaps not surprising that few attempts have been made to compute social welfare functions using a microsimulation model. One approach has recently been suggested by Aaberge and Colombino (2008), and Ericson and Flood (2009). Both studies, as in the present context, use a discrete hours structural approach to model labour supply, allowing for a substantial amount of population heterogeneity. However, the welfare metric used in their social welfare function is a value of utility based on an independently estimated utility (or 'welfare')

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[12] This has led to the adoption of non-welfarist approaches, such as that proposed by Fleurbaey and Maniquet (1999), which abandons the requirement of independence of irrelevant alternatives in social choices.

function which is considered to be the same for all individuals. This overcomes one problem at the cost of assuming a type of schizophrenic behaviour by individuals, whereby their welfare metric may differ from the utility levels determining labour supply behaviour. In contrast, Blundell and Shephard (2009) use utility (as in their labour supply specification) as the welfare metric, and adopt a social welfare function based on an iso-elastic transformation of utility. They simplify the resulting expression for aggregate welfare, allowing for the stochastic utility component.

The welfare metric investigated here is money metric utility per adult equivalent person, allowing fully for the fact that individuals have a probability distribution in the context of a discrete hours approach with a random utility component. While the difficulties associated with this metric are acknowledged, as Donaldson (1992, p. 89) stressed, 'no methodology in applied welfare economics is perfect. Practical work is always limited by the availability of data and the problem of estimating the economic consequences of projects. Different evaluation procedures are, therefore, bound to be differentially useful in different situations'.[13]

Value judgements in social evaluations also concern the definition of the unit of analysis and the form of the social welfare function to be used. The empirical section reports results based on the use of money metric utility per adult equivalent, using the Whiteford equivalence scales reported by Binh and Whiteford (1990), and using the individual as the unit of analysis.[14]

The steps in the social evaluation are as follows. For each income unit, the initial money metric utility, M_0, is obtained using pre-reform taxes as 'reference prices' (this is equal to full income under the pre-reform system). For each income unit, the net income at 80 hours of work by all adult members of the income unit under pre-reform taxes is calculated. Assuming that 80 hours is the maximum number of hours that can be worked per week, this net income represents full income for the income unit. Then, given the equivalent variation, EV, resulting from the reform, post-reform money metric utility is computed as $M_1 = M_0 - EV$ for each set of error terms. For each income unit, the adult equivalent size, s, is obtained and used to compute money metric utility per adult equivalent, m_{ji}, where j refers to the tax

[13] In empirical cost-benefit analyses, consumer's surplus is still often used (without testing for the absence of income effects), and often policies are evaluated using simple aggregates of, say, equivalent variations.

[14] To allow for economies of scale, in calculating equivalised income, the Whiteford equivalence scale counts the first adult as 1 person, subsequent adults as 0.56 persons and any children as 0.32 persons. The weighted sum of persons produces the adult equivalent size of the income unit.

structure and i refers to the income unit. The probability distributions of m_{0i} and m_{1i} can be used to make social evaluations.

With the individual as the unit of analysis, in computing inequality measures each value of m_{ji} is weighted by the unadjusted number of persons in the income unit, n_i.[15] This paper uses Atkinson's inequality measure, $A(\varepsilon)$, where ε is the degree of relative inequality aversion. The inequality measure is expressed as 1 minus the ratio of the equally distributed equivalent value to the arithmetic mean (Atkinson 1970). The equally distributed equivalent value is the value which, if obtained by everyone, gives the same social welfare as the actual distribution. Using an additive welfare function based on constant relative aversion, the equally distributed equivalent value is in general, for a set of values y_i (for i=1,…,n) equal to:

$$y_{ede} = \left(\frac{1}{n} \sum_{i=1}^{n} y_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)}. \tag{12}$$

In the present context, an adjustment must of course be made for the weighting by the number of persons in each household and for the weighting required to obtain population-level results. Results can be obtained for a range of inequality aversion parameters, ε. To illustrate the sensitivity of the results to the choice of this parameter, different values of inequality aversion are used in the application (a low value of 0.2 and a high value of 1.4). Finally, social welfare in each system is obtained using the abbreviated welfare function, $W = \bar{m}(1 - A(\varepsilon))$, associated with the Atkinson inequality measure (and where $\bar{m}$ is the arithmetic mean value of m_i). This means that social welfare W is the equally distributed equivalent level of money metric y_{ede}.

IV. The approach applied to income tax increases

To demonstrate the approach outlined in the previous two sections, a hypothetical policy change involving increasing all non-zero income taxation rates by 5 percentage points is investigated.[16] The starting point is the social security and income tax

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[15] In addition, income unit weights provided by the Australian Bureau of Statistics with the Survey of Income and Housing Cost data are used in the empirical analysis to obtain aggregate measures at the population level.

[16] This policy change is not designed to be revenue neutral. That is, there is no allowance for additional expenditure potentially arising from the additional tax revenue, which could affect households' utility levels.

system in place in Australia in January 2001. This system involved no tax up to AU$6,000, 17% tax between AU$6,001 and AU$20,000, 30% tax between AU$20,001 and AU$50,000, 42% tax between AU$50,001 and AU$60,000, and 47% tax from AU$60,001 onwards.

The policy change is evaluated using the Melbourne Institute Tax and Transfer Simulator (MITTS), a behavioural microsimulation model for Australian households: see Appendix A for further details. Survey of Income and Housing Cost 2000/2001 data are used in the model. Examples of the effect on specific income units are provided in subsection A. Overall results aggregated to the demographic group level are reported in subsection B. Subsection C presents results by different subgroups, for individuals with positive *EV* only.

A. Individual results

To illustrate potential orders of magnitude, Table 1 shows outcomes for typical income units taken from each of the household groups representing low, medium and high income levels. In each case the higher tax rates imply either reductions or no change in expected hours worked and net incomes. The table shows large variations in the marginal welfare cost of taxation, defined as the marginal excess burden (in terms of the equivalent variation) per dollar of extra tax paid.[17] Furthermore the marginal welfare costs are substantial, the efficiency cost per extra dollar in some cases exceeding one dollar. For example, the marginal welfare cost for the low-income single parent shown is $2.70 per extra $1 of tax raised, and is as high as $5.60 for the medium-income single parent shown. This arises despite small expected reductions in labour supply.[18]

The difference between the standard compensating variation (that is, using linear virtual budget lines starting from the post-reform optimal point) and the compensating variation varies from very small amounts to more substantial differences. For example, the standard compensating variation is $669 for the medium-income single

[17] Here, the marginal excess burden is calculated as the equivalent variation from the policy change less the extra tax paid plus the income support received. This is not, strictly speaking, the accurate form of the excess burden. This is because the difference between the new revenue and the tax minus income support which would be paid under the old rates but at the new utility level should be used instead of simply the extra tax minus the income support. However, this is difficult to calculate since gross income at the new utility level is required.

[18] In one case in Table 1 the additional revenue raised is negative (that is, there is additional government expenditure). For this case the calculation of the marginal welfare cost is not relevant.

Table 1. Examples of individual results (in $ per year, except age and hours worked)[a]

	Couples	Single men	Single women	Single parents
Low income				
Age	60, 50	30	20	40
Hourly wage rate	11.59, 7.65	5.20	12.18	14.14
Net income (pre-reform)	25,027	11,705	11,730	13,409
Net income (post-reform)	24,023	11,348	11,295	12,852
Change in net income	-1,004	-357	-435	-557
Net government revenue (labour supply fixed)	647	335	336	437
Net government revenue (including labour response)	-77	290	182	116
Hours worked per week	21, 13	25	21	14
Hours change in hours per week	-1.2, -0.9	-0.25	-0.40	-0.60
Compensating variation	628	332	332	431
Standard compensating variation	628	327	330	430
Equivalent variation (*EV*)	627	332	333	430
Marginal welfare cost	-[b]	0.1	0.8	2.7
Difference between net income change and *EV* (in %)	-37.6	-7.1	-23.5	-22.7
Medium income				
Age	35, 35	25	45	40
Hourly wage rate	16.54, 11.74	20.00	14.86	13.84
Net income (pre-reform)	39,457	17,986	16,913	24,328
Net income (post-reform)	37,217	17,165	16,033	23,401
Change in net income	-2,240	-821	-880	-926
Net government revenue (labour supply fixed)	1,850	740	666	431
Net government revenue (including labour response)	1,258	613	300	63
Hours worked per week	46, 6	18	26	19
Hours change in hours per week	-1, -0.2	-0.20	-0.75	-1.20
Compensating variation	1,831	737	653	416
Standard compensating variation	1,830	669	650	415
Equivalent variation (*EV*)	1,836	741	667	417
Marginal welfare cost	0.5	0.2	1.2	5.6
Difference between net income change and *EV* (in %)[a]	-18.1	-9.8	-24.2	-55.0

Table 1 (continued). Examples of individual results (in $ per year, except age and hours worked)[a]

	Couples	Single men	Single women	Single parents
High income				
Age	56, 40	45	60	35
Hourly wage rate	66.84, 43.50	28.25	14.29	38.85
Net income (pre-reform)	188,417	38,669	45,884	61,249
Net income (post-reform)	176,521	35,490	42,891	56,237
Change in net income	-11,896	-3,179	-2,993	-5,012
Net government revenue (labour supply fixed)	11,896	2,271	2,993	4,245
Net government revenue (including labour response)	11,896	1,490	2,993	3,194
Hours worked per week	50, 50	35	46	45
Hours change in hours per week	0, 0	-1.15	0.0	-0.90
Compensating variation	11,193	2,225	2,776	4,116
Standard compensating variation	11,231	2,212	2,924	3,831
Equivalent variation (*EV*)	11,896	2,260	2,993	4,187
Marginal welfare cost	0.0	0.5	0.0	0.3
Difference between net income change and *EV* (in %)[a]	0.0	-28.9	0.0	-16.5

Notes: a) Post-reform values and welfare changes are expected values b) The marginal welfare cost is computed to be negative for this income unit.

man, whereas the compensating variation using the approach proposed here is computed to be $737.

A further observation is that the differences between net income changes and equivalent variations are substantial and vary considerably among units. The biggest difference, of 55%, is for the medium-income single parent while the smallest difference is for the low-income single man. There is no difference for the high-income couple and single woman, who have unchanged labour supply after the tax increase. However, there is a difference between the standard compensating variation and our compensating variation for these two cases despite the unchanged hours of work.

Not surprisingly, there are some units for which the expected reduction in labour supply is such that there is a reduction in tax paid as a result of the tax rate increase. For example, it is the case of the low-income couple in Table 1. This means that the income unit is on the ‘downward sloping’ or ‘wrong’ side of the Laffer curve.

B. Aggregate results

Table 2 presents simple aggregate effects, in millions of dollars, of the policy change, obtained by adding all expected *EV*, *CV* and net incomes across all income units without equivalising the amounts. The policy change would increase net government revenue by just under AU$14 billion with no labour supply responses, or just under AU$11 billion after allowing for labour supply responses. The reduction in labour supply reduces the additional government revenue generated by the higher tax rates. Relatively speaking, the reduction in net revenue on single parents, after allowing for labour supply changes is particularly large. This is due to the increase in government expenditure on family payments and social security payments arising from the reduced labour supply by this group.

Although the previous section showed differences at the individual level between the standard compensating variation and our approach to the compensating variation, Table 2 shows that the differences are quite small in aggregate. However, the difference depends on the policy change analysed, and can vary for particular subgroups in the population. In particular, the difference depends on the number of cases for which the minimum *CV* is found at a labour supply point different from the optimal post-reform labour supply point. It is for these cases that our *CV* is most likely to differ from the standard *CV* since the latter is computed at hours levels in

Table 2. The aggregate effects of increases in income taxation rates (in $m per year)

	Couples	Single men	Single women	Single parents	Total
Net government revenue change (labour supply fixed)	9,704	2,308	1,338	326	13,677
Net government revenue change (incl. labour response)	7,780	1,859	1,157	78	10,874
Average hours change in hours per week	-0.52, -0.47	-0.41	-0.21	-0.88	-0.45
Compensating variation (*CV*)	9,573	2,266	1,312	312	13,463
Standard compensating variation	9,564	2,266	1,316	310	13,456
Equivalent variation (*EV*)	9,651	2,293	1,336	318	13,598
Marginal Welfare Cost	0.24	0.23	0.15	3.09	0.25
EV-CV gap (in %)	0.8	1.2	1.8	2.1	1.0
Aggregate net income change	-11,536	-2,724	-1,489	-497	-16,246
Difference between net income change and *EV* (in %)	-16.3	-15.8	-10.3	-36.0	-16.3

the neighbourhood of the optimal post-reform labour supply point. For this particular policy change, on average, 1.6% of the 100 values of *CV* computed for each income unit were found at a labour supply point different from the post-reform optimum.[19]

The final line of Table 2 compares equivalent variation and average net income changes. The change in net income clearly exceeds the welfare change measure. The average gap between the two sets of changes is expressed as a percentage of the aggregate net income change. The relative differences vary among unit types and are typically large, particularly for single parents. This means that potentially different conclusions could be drawn with regard to how the different groups and individuals in the population are affected, depending on whether net income changes or welfare changes are considered.

Summary information regarding abbreviated social welfare functions is given in Table 3, based on Gini and Atkinson inequality measures. In obtaining these measures, expected welfare changes at each labour supply point after the tax change, rather than simply expected values for each person, were used reflecting the dispersion in post-reform labour supply outcomes. The approach proposed for examining distributional implications in discrete hours models by Creedy, Kalb and Scutella (2006) was followed, making use of the probability distribution of post-reform labour supply. Social welfare and inequality decrease as a result of the tax increase, but the use of net income produces much higher reductions than the use of money metric utility. This arises because of the failure to value leisure time in measures based on net income only. If the expected value for each person instead of the probabilistic approach were used, then post-reform values would change, but pre-reform values are not changed since these are calibrated to observed labour supply. The post-reform Atkinson's index based on net income increases when using the probabilistic approach, indicating higher inequality than would otherwise have been the case. Social welfare based on net income is lower when using the probabilistic approach. The measures based on the money metric utility barely change. This is probably because full income under the pre-reform system, M_0, does not vary by labour supply point and the value of *EV* is usually quite small compared to M_0 (leading to relatively small variations in post-reform money metric utility by labour supply point).

The magnitude of the reductions in the Atkinson's index is similar for low and high relative inequality aversion ε when the index is based on money metric utility

[19] For an alternative, larger, policy change in which all tax rates were increased by 15 instead of 5 percentage points, the average was above 5%.

Table 3. Social welfare function evaluations

	Mean	Atkinson's index		Social Welfare		Gini
		$\varepsilon = 0.2$	$\varepsilon = 1.4$	$\varepsilon = 0.2$	$\varepsilon = 1.4$	
Pre-reform net income	22,880	0.0281	0.1702	22,237	18,987	0.2907
Pre-reform money metric	49,382	0.0168	0.0990	48,554	44,493	0.2172
Post-reform net income	21,647	0.0262	0.1584	21,079	18,218	0.2805
% change	-5.39	-6.65	-6.89	-5.21	-4.05	-3.51
Post-reform money metric	48,353	0.0161	0.0950	47,575	43,761	0.2122
% change	-2.08	-4.05	-4.07	-2.02	-1.65	-2.30

Note: Money metric utility and net income are per adult equivalent. Social Welfare is the equally distributed equivalent level of money metric utility (or net income).

and when the index is based on net income. The effect on social welfare, independent of whether it is based on money metric utility or on net income, is larger under lower relative inequality aversion.

To explore differences in implications depending on whether money metric utility or net income is used, Table 4 reports results disaggregated by demographic group. Although inequality reductions are the highest (by a large margin) for single parents when using net income, the decreases in inequality are the smallest for this specific group when money metric measures are used.[20] Likewise, the ranking of single women and single men in terms of inequality changes is reversed if money metric utility is used instead of net income at the lower level of relative inequality aversion. Different conclusions may thus be drawn if the welfare implications of changes in leisure and home production time are ignored. Table 4 also shows that although all welfare changes are negative, the ranking of the demographic groups can change with the relative inequality aversion index.

Two reasons can be given to explain why the relative reductions in social welfare are consistently higher when using net income. First, absolute net income changes are on average higher than welfare changes because the latter take into account the increase in leisure and home production time. Second, initial social welfare values are lower when using net income because, unlike money metric utility, it does not attribute value to leisure and home production time. As a result, higher relative changes are obtained for net income based welfare measures than for money metric utility based welfare measures even if absolute changes were similar in size.

[20] The increase in leisure and home production time partly compensates for the reduced net income.

Table 4. Social welfare function evaluations disaggregated by demographic group

	Mean	Atkinson's index		Social Welfare		Gini
		$\varepsilon = 0.2$	$\varepsilon = 1.4$	$\varepsilon = 0.2$	$\varepsilon = 1.4$	
Couples						
Pre-reform net income	24,590	0.0268	0.1588	23,931	20,686	0.2829
Post-reform net income	23,173	0.0252	0.1489	22,588	19,723	0.2742
% change	-5.76	-5.85	-6.24	-5.61	-4.65	-3.08
Pre-reform money metric	52,971	0.0171	0.0989	52,064	47,734	0.2183
Post-reform money metric	51,784	0.0165	0.0951	50,930	46,861	0.2136
% change	-2.24	-3.67	-3.84	-2.18	-1.83	-2.14
Single men						
Pre-reform net income	21,649	0.0325	0.2104	20,946	17,095	0.3164
Post-reform net income	20,495	0.0304	0.1968	19,872	16,462	0.3063
% change	-5.33	-6.42	-6.46	-5.12	-3.70	-3.17
Pre-reform money metric	44,593	0.0115	0.0759	44,079	41,207	0.1810
Post-reform money metric	43,622	0.0108	0.0721	43,149	40,477	0.1754
% change	-2.18	-5.90	-5.07	-2.11	-1.77	-3.07
Single women						
Pre-reform net income	18,091	0.0274	0.1743	17,596	14,938	0.2915
Post-reform net income	17,374	0.0252	0.1615	16,935	14,567	0.2795
% change	-3.96	-7.75	-7.32	-3.75	-2.48	-4.11
Pre-reform money metric	41,103	0.0095	0.0619	40,714	38,559	0.1641
Post-reform money metric	40,460	0.0089	0.0587	40,099	38,086	0.1588
% change	-1.57	-5.62	-5.20	-1.51	-1.23	-3.27
Single parents						
Pre-reform net income	17,141	0.0129	0.0788	16,921	15,790	0.1958
Post-reform net income	16,609	0.0115	0.0704	16,418	15,440	0.1841
% change	-3.10	-10.78	-10.65	-2.97	-2.22	-5.99
Pre-reform money metric	38,318	0.0141	0.0873	37,779	34,974	0.2084
Post-reform money metric	37,980	0.0137	0.0854	37,458	34,738	0.2060
% change	-0.88	-2.35	-2.18	-0.85	-0.68	-1.15

Note: Money metric utility and net income are per adult equivalent. Social Welfare is the equally distributed equivalent level of money metric utility (or net income).

C. Welfare changes for subgroups

For different subgroups in the population, Table 5 compares average expected welfare and net income changes per adult equivalent for those above the tax-free threshold who are thus affected by the tax rate change. The net income and welfare changes are not necessarily equal even for individuals without a labour supply response. The existence of unchanged Marshallian labour supply does not necessarily imply the absence of an excess burden.

Furthermore, a welfare change larger or smaller than the net income change in absolute terms may be found at another labour supply point than the observed labour supply point, even if utility is still optimal at the original observed labour supply. In addition, differences between the two measures could arise for individuals without labour response because their partners in couple households may have changed their labour supply.

Couples are affected to the largest degree in terms of the proportion of households affected and the average net income and welfare changes, because they are on

Table 5. Average welfare and net income changes per adult equivalent

	Equivalent Variation (*EV*) > 0			*EV* = 0	Total	
	% of IU	Income change per adult equivalent ($/year)	*EV* per adult equivalent ($/year)	% of IU	Income change per adult equivalent ($/year)	*EV* per adult equivalent ($/year)
By income unit (IU) type						
Couples	80.5	-1,684	-1,411	19.5	-1,417	-1,187
Single men	70.1	-1,645	-1,385	29.9	-1,154	-971
Single women	54.1	-1,326	-1,189	45.9	-718	-644
Single parents	46.9	-1,189	-755	53.1	-532	-338
By labour force status						
Full time	97.7	-1,882	-1,559	2.3	-1,839	-1,523
Non-participant	24.5	-686	-661	75.5	-168	-162
Part-time	85.5	-865	-797	14.5	-740	-682
Unemployed	13.7	-641	-569	86.3	-88	-78
By labour supply response						
Working more	100.0	-2,058	-1,569	0.0	-2,058	-1,569
No change	52.5	-1,153	-1,145	47.5	-605	-601
Working less	100.0	-1,871	-1,725	0.0	-1,871	-1,725
Total	70.0	-1,627	-1,358	30.0	-1,233	-1,029

average at a higher income level than the other groups. Relatively fewer single parents are affected but, if so, their decrease in net income is relatively large. The difference between the net income change and equivalent variation is largest for single parents because it is the demographic group experiencing the largest labour supply responses.

As expected, those working full-time are more likely to be affected than the other groups and they have a larger decrease in net income and welfare if they are affected. The unemployed are least likely to be affected, followed by the non-participants including those who are retired and/or have other sources of income than from labour supply. The average income changes for non-participants and unemployed, if they are affected, are similar, but the corresponding welfare losses are clearly lower for the unemployed than for non-participants. Again, this shows that different conclusions may be reached regarding the group affected to the largest degree by a policy change depending on whether net income changes or welfare changes are measured.

V. Conclusions

This paper has examined the calculation of compensating and equivalent variations in the context of labour supply modelling, where highly nonlinear budget constraints are common. The standard method of computing welfare changes may not give appropriate values if the computation involves hours levels for which a linearised virtual budget constraint indicates a different net income compared with the exact nonlinear budget constraint. Special attention was given to the context of discrete hours models, where there is a random utility component, giving rise to a probability distribution over available hours levels as the predicted outcome of policy changes. Such discrete hours models have gained importance because they are being more widely adopted as a result of their substantial advantages in preference estimation.

The implementation of the method in the context of microsimulation, using econometrically estimated direct utility functions for particular demographic groups, was examined here. A method of producing the probability distribution of welfare changes for each individual was proposed. The method allows fully for the nonlinearity of the budget constraint facing each individual. Moreover, the method is fully consistent with the discrete choice framework, can be used in an expected hours of labour supply framework, but is also compatible with the use of 'calibration', ensuring that, for all individuals, their optimal labour supply before a hypothetical tax change is equal to the observed (discretised) labour supply reported in the dataset.

The special case of quadratic direct utility functions, which are widely used in labour supply modelling, was discussed and used in the empirical example.

To illustrate the use of the approach in microsimulation, a policy change involving an increase in non-zero marginal tax rates was simulated. An advantage of using welfare change measures is that they can take into account the value of leisure or home production time. This advantage is of particular importance in policy evaluations which allow for labour supply responses. Therefore, measured differences between evaluations using welfare measures and those obtained using only changes in net incomes were examined. The results showed that very different conclusions may be reached regarding individual comparisons, overall comparisons using social welfare functions and identification of those demographic groups affected to the largest degree by a policy change, depending on whether net income changes or welfare changes are measured. It was found that the marginal excess burden can take a wide range of values for individuals and subgroups in the population. Substantial marginal welfare costs associated with an increase in income tax rates were measured, in particular for single parents.

Given the increasing use of behavioural microsimulation models in tax and social security policy evaluations, the procedures outlined in this paper offer considerable scope for extending the range of analyses and measures generally used to judge the effects of proposed reforms. These new procedures allow the evaluation of any changes in leisure and home production time available to the income units in addition to the usual evaluation of changes in disposable income due to policy reforms. The differences in results compared with alternative approaches, reported in the empirical example, were substantial in individual cases, and there could be potentially more substantial differences overall in a different setting, such as the introduction of a different policy change. The proposed approach is relatively straightforward to implement, does not require indirect utility and expenditure functions, can be used with any distribution function of random utility from which values can be drawn, and it allows for the nonlinearity of the budget constraint while being fully consistent with the discrete choice framework.

Appendix

A. The MITTS model

The microsimulation is based on a sample of representative Australian households in the 2000/2001 Survey of Income and Housing Cost. This is a survey of the

Australian population at the time of the policy change of interest. Detailed information is available on each household and on the individuals in the households. This allows the social security payments received and income tax paid for each individual and household to be computed according to the tax and social security rules at any point in time or according to a hypothetical set of rules. Using the weights provided by the Australian Bureau of Statistics, the sample can be weighted to obtain population amounts.

The MITTS microsimulation model is based on estimated parameters for a structural labour supply model. A more detailed general description of the behavioural microsimulation modelling approach used in this analysis can be found in Creedy and Kalb (2005b) and specific information on MITTS can be found in Creedy et al. (2002, 2004). MITTS calculates net incomes for each household at all predetermined discrete labour supply points based on the wage rates of individuals (either observed in the data or imputed, using the estimated wage, other income, and some individual and household characteristics. The preference function and wage function estimates are reported in Kalb (2002) and Kalb and Scutella (2002) respectively. The net incomes can be calculated imposing different tax and transfer systems, allowing hypothetical and real policy changes to be analysed. Together with the net incomes at all labour supply points, the estimated parameters from the structural labour supply model are key inputs in the behavioural component of the microsimulation model. As mentioned above, the behavioural labour supply responses presented in this paper are based on quadratic utility functions with preference parameters which are allowed to vary with an individual's characteristics. The approach follows the discrete choice approach taken by Van Soest (1995) and Blundell et al. (2000).

The behavioural simulation begins by recording the discrete hours level for each individual that is closest to the observed hours level. Labour supply is kept constant for some groups who are expected to differ in their responses (that is, be less responsive) compared to the average working-age individual. These groups are the self-employed, those on disability payments, full-time students and people over 65 years of age. Then, given the parameter estimates of the utility function (which vary according to a range of demographic characteristics), a set of random draws is taken from the conditional distribution of the random utility term (specified as an extreme value distribution) for those whose labour supply is allowed to change. This conditional distribution takes the discretised optimal hours level observed before the reform into account; see Bourguignon, Fournier and Gurgand (1998). That is, the error terms drawn from this distribution all place the individual at the observed labour supply before the reform. The sets of conditional error terms are then used to determine the optimal hours level after the policy change. A total of 100 such sets of "successful draws" are produced. Conditional on this set of random draws, a probability distribution over the set of discrete hours for each individual under the new tax and transfer structure is generated. Thus the same random utility components are used before and after the reform. However, the tax and transfer changes affect net household incomes, and as a result changes the deterministic utility levels and potentially the optimal level of labour supply. The expected labour supply after the reform is calculated as the average outcome across all draws of the error terms. For further details, see Creedy and Kalb (2005b) for a detailed description of the estimation, specification and simulation in behavioural microsimulation modelling.

B. No-solution cases

Section III.B mentions the possible violation of the condition of increasing utility with income before reaching the desired utility level when using quadratic utility functions. As a result of this violation, there may be no solution for the *EV* (or *CV*) at some of the discrete labour supply points. An indication of the extent to which the required condition is violated is given in Table A1, for the policy reform examined here of a five percentage point increase in all positive income tax rates. For single individuals, 11 discrete labour supply points are used and 100 sets of error terms are drawn to produce the probability distribution of post-reform labour supply. This means that for each individual, 1100 equations involving the quadratic utility function need to be solved. For each couple family, labour supply choices of the two partners are simultaneously determined. Couples can choose from 66 hours combinations, made up of 6 labour supply points for partnered men and 11 points for partnered women; hence 6600 equations have to be solved for each couple. The results from running this policy simulation show that there are relatively few points where there is no feasible solution. As shown in Table A1, they concern about 3.6% of the total number of equations that need to be solved and 30.5% of all income units are affected for at least one of their equations.

Table A1 shows that there are no no-solution cases for single men and very few for couple families, but in 13.79% of the equations there is no solution for single females and 99.2% of all single women are affected. That is, on average 152 out of the 1100 equations have no solution for single women. Single parents follow closely: in 10.07% of the equations there is no solution and 93.47% of all single parents are affected.

Table A1. Distribution and frequency of no-solution cases by demographic groups (in %)

Income unit (IU) type	IU with at least one equation without solution	Equations without solution (for IU with at least one equation without a solution)	Equations without solution (for all equations and IU)
Couples	7.09	2.21	0.16
Single men	0.00	0.00	0.00
Single women	99.19	13.90	13.79
Single parents	93.47	10.77	10.07
All	30.53	11.86	3.62

The equations for which no solution can be found occur infrequently at labour supply points below 20 hours of work per week and are most prevalent at the 45 and 50 hours points. In the vast majority of these cases, the optimal utility level is at an observed labour supply lower than the relevant hours for which no solution can be found. Since the lowest compensation possible is required, these points at which no solution can be found due to low preferences for income are not relevant, because they do not result in the lowest compensation across all labour supply points.

The more frequent occurrence of no-solution points for single women and single parents can be explained by comparing parameter estimates in the utility function with those for other groups. Single women tend to have a lower preference for income relative to leisure time. This preference for income also tends to decrease more steeply with hours of work compared with other groups. For single parents, the preference for income is decreasing with age up to about age 45 and overall is relatively low as well. As a result, it is more difficult to compensate single women and older single parents at higher labour supply levels.

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