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AGGREGATION ISSUES IN THE ESTIMATION OF LINEAR PROGRAMMING PRODUCTIVITY MEASURES

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This paper demonstrates the sensitivity of the linear programming approach in the estimation of productivity measures in the primal framework. Specifically, the sensitivity to the number of constraints (level of dis-aggregation) and imposition of returns to scale constraints is evaluated. Further, the shadow or dual values are recovered from the linear program and compared to the market prices used in the ideal Fisher index approach. Empirical application to U.S. state-level time series data from 1960-2004 reveal productivity change decreases with increases in the number of constraints. Divergence in productivity measures is observed due to the choice of method imposed, various levels of commodity/input aggregation, and technology assumptions. Due to the piecewise linear approximation of the nonparametric programming approach, the shadow share-weights are skewed leading to the difference in the productivity measures due to aggregation.

JEL classification codes: O3, C6, Q1
Key words: aggregation, share-weights, single and multiple output and input, Malmquist productivity index, Malmquist total factor productivity index

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I. Introduction

The linear programming (LP) approach has gained popularity since the early 1990s due to its ability to impose little a priori functional form, handle multiple outputs-inputs without the need of price data, and accommodate weak and strong disposability assumptions. However, the LP approach, due to its piecewise linear approximation of the technology or theoretical frontier, is conditioned by the number of decision making units (DMU) and the number of constraints (in our case the level of input and output aggregation) in the model. The sensitivity of LP efficiency measures due to output and input aggregation has been established (Thomas and Tauer 1994; Tauer and Hanchar 1995; and Shaik 2007) and referred to as the "curse of dimensionality" problem (see, e.g., Thanassoulis et al. 2008: 320). The "curse of dimensionality" problem associated with an increase in the number of constraints (or level of disaggregation), leads to an increase or decrease in the number of reference points resulting in a decrease or increase in the efficiency and productivity measures. These aggregation issues have been addressed in the literature (Blackorby and Russell 1999; Färe and Zelenyuk 2003; and Simar and Zelenyuk 2003) with the use of dual input, output prices. However, explaining the aggregation issue in the primal framework without the explicit or implicit use of dual or shadow price is challenging.

This paper addresses the "curse of dimensionality" issue by demonstrating that the problem may be due to the shadow or dual values recovered from the constraints of the LP approach. The dual values of the LP constraints should reflect technology and economic behavior of individual DMUs (or states in this case). Theoretically (Caves, Christensen and Diewert 1982a and 1982b), the computation of productivity measures involves the use of market prices in the case of the ideal Fisher index approach, marginal product in the case of the parametric approach, and shadow or dual values in the case of LP approach. We also demonstrate the shadow or dual values recovered from the LP constraints depend on how the return to scale constraint is imposed in the estimation of the LP productivity measures. The input-based Malmquist productivity index (IMP) or output-based Malmquist productivity index (OMP) impose constant returns to scale (CRS) or variable returns to scale (VRS) restrictions simultaneously in the input and output constraints (see Färe et al. 1994; Färe et al. 1998; and Grifell-Tatje and Lovell 1995). In contrast, the Malmquist total factor productivity (MTFP) index model (see Bjurek 1996) imposes constant returns to scale independently in input and output constraints. Other advantages of the MTFP index (a Hicks-Moorsteen type index) over the standard Malmquist productivity index is that it always has a TFP interpretation, and that under weak assumptions of VRS and strong disposability of inputs and outputs, it is not unbounded. One can see the TFP discussion in Grifell-Tatjé and Lovell (1995) and in Bjurek (1996), and the issues of infeasibilities and unboundedness in Bjurek (1996).

Specifically, this research demonstrates the sensitivity of the LP approach by comparing the estimated productivity measures and the shadow or dual values (relative to the market prices of the ideal Fisher index approach) of the constraints of the LP model estimated at various levels of aggregation.[1] The following section presents the time-series linear programming OMP, IMP and MTFP index methods. In the Section III, a brief description of the U.S. state-level time series data from 1960-2004 is presented. Empirical application and the results along with the performance of methods are presented in Section IV followed by conclusions.

II. Linear programming approach

For the nonparametric programming approach, technology that transforms input vector $x_t = (x_{1t}, x_{2t}, \ldots, x_{it})$ into output vector $y_t = (y_{1t}, y_{2t}, \ldots, y_{jt})$ for each state $k = 1, 2, \ldots, K(48)$ over time $t = 1(1960), 2, \ldots T(2004)$ can be represented by the output set:

$$P\left(x_t^k\right) = \left\{ y_t^k : x_t^k \text{ can produce } y_t^k \right\}, \tag{1}$$

or input set:

$$L\left(y_t^k\right) = \left\{ x_t^k : y_t^k \text{ is produced by } x_t^k \right\}, \tag{2}$$

and follows the properties of strong disposability of outputs and inputs, and constant returns to scale (CRS) or variable returns to scale (VRS) as in Färe et al. (1994), Färe et al. (1998) and Grifell-Tatje and Lovell (1995).

In a given year, t, the concept of the output set can be represented by the output distance function for k decision-making units as:

$$OD_t\left(x_t^k, y_t^k\right)^{-1} = \max \theta : \ \theta y_t^k \in P\left(x_t^k\right), \tag{3}$$

[1] Other relative issues, slack and disposability are important but beyond the scope of the paper. We also will not be dealing with non-marketable goods or assume weak disposability in estimating productivity measures.

Similarly, the concept of input set can be represented by input distance function for k decision making units as:

$$ID_t^k(y_t^k, x_t^k)^{-1} = \min \lambda: \ \lambda x_t^k \in L(y_t^k) \cdot \tag{4}$$

A. Time-series output and input-based Malmquist productivity indices (OMP and IMP)

Following Shaik (1998) and Shaik et al. (2002), in time-series observations on a single economic unit (such as North Dakota), an *IMP* in year t relative to the final year T can be represented as follows. Consider the multiple of year t output that is revealed to be possible relative to the set of all observations including year T, using the year t bundle of inputs. If outputs could be doubled (the multiple is 2), then the productivity at time t is the inverse of this multiple, or 0.5. This concept can be represented by an output or input distance function evaluated for any year t using reference production possibilities set T as:

$$OD\left(x_t, y_t\right)^{-1} = \max_{\theta, z} \theta, \text{ s.t. } \theta y_{j,t} \leq zY_j \text{ , } zX_i \leq x_{i,t}, z \geq 0, \tag{5a}$$

$$ID\left(y_t, x_t\right)^{-1} = \min_{\lambda, z} \lambda, \text{ s.t. } y_{j,t} \leq zY_j, \ \lambda x_{i,t} \geq zX_i, z \geq 0, \tag{5b}$$

where $Y_j = (y_j^1, y_j^1, \ldots\ldots\ldots, y_j^T)$ and $X = (x_i^1, x_i^2, \ldots\ldots\ldots, x_i^T)$, the intensity variables $z \geq 0 (z = 0)$ identify the CRS (VRS) boundaries of the reference set.

Hence, the *OMP* measure for a single economic unit, between two time-periods t and $t+1$, given technology, is defined as:

$$OMP_t^{t+1} = \frac{OD(x_{t+1}, y_{t+1})}{OD(x_t, y_t)}, \tag{6}$$

and the *IMP* measure for a single economic unit, between two time-periods t and $t+1$, given technology, is defined as:

$$IMP_t^{t+1} = \frac{ID(y_{t+1}, x_{t+1})}{ID(y_t, x_t)}. \tag{7}$$

B. Time-series Malmquist total factor productivity index

Following Bjurek (1996), the time-series Malmquist total factor productivity index (*MTFP*), an alternative to the time-series OMP or IMP index, is the ratio of Malmquist output index (*MO*) and Malmquist input index (*MI*). The *MO* index measures the scalar change in outputs assuming the inputs are constant over time. Here inputs are constant, meaning that input usage does not change. Hence this would reflect the computation of an ideal Fisher output quantity index. Similarly the *MI* index measures the scalar decrease in inputs assuming the outputs are constant over time. Here outputs are constant, meaning that output produced does not change. Hence this would reflect the computation of an ideal Fisher input quantity index.

This concept of *MO* and *MI* indices can be represented by modifying equation (5a) and equation (5b) evaluated for any year t for a single firm employing a reference production possibility set T:

$$OD\left(x_t\,(=constant)y_t\right)^{-1}=\max_{\theta,z}\theta,\ \text{s.t. }\theta y_{j,t}\le zY_j\ ,x_{i,t}\ge zX_i\,,z\ge 0,x=constant, \tag{8a}$$

$$ID\left(y_t\,(=constant)x_t\right)^{-1}=\min_{\lambda,z}\lambda,\ \text{s.t. }\lambda x_{i,t}\ge zX_i\,,y_{j,t}\le zY_j\ ,z\ge 0,y=constant, \tag{8b}$$

where $Y_j = (y_j^1, y_j^1, \ldots\ldots\ldots, y_j^T)$ and $X = (x_i^1, x_i^2, \ldots\ldots\ldots, x_i^T)$, the intensity variables $z \geq 0 (z = 0)$ identify the constant (variable) return to scale boundaries of the reference set.

The *MTFP* index for a single economic unit maintaining the index productivity notion is represented as:

$$MTFP \equiv \frac{MO}{MI} = \frac{OD\left(x_{t+1}(=constant),y_{t+1}\right)}{OD\left(x_t\,(=constant),y_t\right)} * \frac{ID\left(y_t\,(=constant),x_t\right)}{ID\left(y_{t+1}(=constant),x_{t+1}\right)}\ . \tag{9}$$

To illustrate the sensitivity of the nonparametric program approach to the level of commodity aggregation, we compare the share-weights recovered from the dual values implicit in the linear programming constraints. For comparison, the share-weights are recovered from the dual values (*dv*) of the output (input) constraints defined in equation (5a) of OD (equation 5b of ID) as well as from the output (input) constraints in equation (8a) of *MO* (equation 8b of *MI*) of *MTFP*.

The *dv* of the linear programming input equations (5b) and (8b) and output equations (5a) and (8a) constraints are normalized to one, and are equivalent to the share-weights. Following Shaik (1998) and Shaik et al. (2002), the nonparametric implicit output and input share-weights in terms of dv are represented as:

$$RS_j = \frac{dv_j}{\sum_j dv_j} \ , \tag{10}$$

and

$$CS_i = \frac{dv_i}{\sum_i dv_i} \ , \tag{11}$$

where RS_j and CS_i are the implicit output and input share-weights recovered from the linear programming constraint and dv are the dual values obtained from the output and input linear programming constraints.

III. U.S. agriculture data

The U.S. Department of Agriculture's Economic Research Service (ERS) constructs and publishes the state and aggregate production accounts for the farm sector.[2] The features of the state and national production accounts are consistent with the gross output model of production and are well documented in Ball et al. (1999). Output is defined as gross production leaving the farm, as opposed to real value added (quantity index, base 1960=100). All inputs are quantity index with 1960=100. Finally, quantity indexes are constructed as the weighted sum of the rate of growth of the components, where the weights are the respective value (output or input) shares. As such, the indexes measure the annual rates of change in the output or input aggregate.

The state-wise annual growth rate of the variables' employed in the estimation of productivity for the period 1960-2004 is presented in Table 1. The annual growth rate is defined as $[x_{2004}/x_{1960})^{1/n}-1]*100$, where x is input or output variable and n is the number of years in the time period. Within outputs, the average annual growth rate across all the states for crops is 1.464 followed by livestock with 0.942 and other farm revenue with 0.715. In the input category, capital (-0.339), land (-0. 881) and labor (-2.187) had a negative average annual growth rate across all the states compared to positive average annual growth rate of energy (0.444), material (0.7) and chemicals

[2] The data are available at the USDA/ERS website http://www.ers.usda.gov/data/agproductivity/.

Table 1. State-wise annual output and input growth rates[1], 1960-1996

State	Aggregate output	Crops	Livestock	Other farm revenue	Aggregate input	Land	Labor	Capital	Chemicals	Energy	Materials
AL	1.609	0.591	2.132	1.974	0.307	-1.358	-2.661	0.368	0.727	0.779	1.914
AZ	1.450	1.454	1.790	-0.490	-0.058	-2.397	-0.776	0.802	2.901	0.540	0.478
AR	2.726	2.270	3.563	0.889	0.806	-0.186	-2.399	0.885	4.695	0.608	1.966
CA	2.236	2.490	1.975	0.777	0.585	-0.645	-0.753	0.105	3.166	0.523	1.524
CO	1.701	1.284	1.990	1.907	0.612	-0.701	-1.619	-0.248	3.997	0.840	1.530
CT	0.367	0.787	-0.355	0.524	-1.764	-2.176	-2.654	-1.781	-0.870	-0.181	-0.811
DE	2.440	1.615	2.772	2.652	0.653	-0.805	-2.570	-0.342	-0.322	1.859	1.945
FL	2.049	2.516	1.544	-0.943	0.621	-0.912	-0.231	1.266	1.259	1.199	1.470
GA	2.151	1.735	2.458	1.383	0.258	-1.466	-2.303	0.030	1.979	0.411	1.529
ID	2.403	2.333	2.655	-0.160	0.407	-0.597	-1.400	0.274	4.074	1.940	1.248
IL	1.227	2.260	-1.729	1.289	-0.695	-0.247	-2.789	-0.528	4.005	-0.425	-0.774
IN	1.415	2.322	-0.122	-0.089	-0.822	-0.385	-2.891	-0.583	3.278	-0.526	-0.480
IA	1.327	2.302	0.261	0.292	-0.504	-0.122	-2.836	-0.375	4.950	0.458	-0.035
KS	1.705	1.416	2.013	2.169	0.665	-0.114	-1.694	-0.229	4.936	0.809	1.639
KY	1.423	1.136	1.323	2.489	-0.159	-0.392	-2.419	0.770	1.782	0.935	1.605
LA	1.603	1.967	0.908	0.108	-0.294	-0.737	-2.974	0.093	3.699	0.245	0.734
ME	0.028	-0.675	0.679	-1.021	-1.814	-1.849	-2.885	-1.437	-2.322	0.061	-1.267
MD	1.474	1.802	1.158	1.289	-0.322	-1.300	-2.365	-0.874	-0.132	0.578	0.887
MA	-0.423	0.463	-2.326	0.534	-2.627	-1.960	-4.197	-1.460	-1.741	-0.332	-1.588
MI	1.355	1.945	0.443	0.202	-1.004	-0.780	-2.694	-0.821	2.163	0.161	0.369

Table 1. (continued) State-wise annual output and input growth rates[1], 1960-1996

State	Aggregate output	Crops	Livestock	Other farm revenue	Aggregate input	Land	Labor	Capital	Chemicals	Energy	Materials
MN	1.454	2.263	0.426	0.584	-0.370	-0.213	-2.632	-0.246	4.032	0.967	0.595
MS	1.718	1.151	2.089	1.641	-0.229	-1.082	-3.923	-0.009	2.122	0.484	1.947
MO	1.131	1.918	0.114	0.780	-0.456	-0.225	-1.986	-0.026	3.312	-0.180	0.041
MT	1.265	1.499	0.413	1.998	-0.088	-0.215	-1.038	-0.041	4.044	0.403	0.011
NE	2.242	2.500	1.864	2.765	0.654	-0.051	-1.854	-0.028	4.699	0.865	1.761
NV	1.751	2.692	1.148	0.270	0.528	-1.471	-0.426	0.618	3.476	2.250	1.283
NH	-0.205	1.027	-1.326	0.211	-2.138	-2.242	-3.041	-1.690	-1.862	0.049	-1.603
NJ	-0.163	0.518	-1.584	0.012	-1.777	-1.286	-2.245	-1.344	-0.737	-0.664	-1.345
NM	2.220	1.307	2.911	0.009	0.787	-0.092	-0.871	0.307	2.042	1.097	1.984
NY	0.279	0.392	0.172	-0.558	-1.165	-1.336	-2.558	-1.036	-0.761	-0.375	-0.177
NC	1.913	0.516	3.755	1.947	0.093	-1.276	-3.557	-0.147	1.392	0.336	3.061
ND	1.839	2.297	-0.103	0.921	-0.033	-0.125	-1.489	-0.426	5.678	0.313	0.194
OH	1.074	1.677	0.023	-0.318	-1.036	-0.421	-2.959	-0.830	1.761	-0.015	0.161
OK	1.113	0.771	1.934	-1.564	0.542	-0.180	-1.174	0.124	3.531	0.583	1.818
OR	2.171	2.967	0.788	0.397	-0.372	-0.770	-1.223	-0.568	2.510	0.974	0.268
PA	1.284	1.349	1.179	0.729	-0.492	-0.943	-1.748	-0.397	-0.026	0.342	0.639
RI	-0.388	0.611	-2.632	1.893	-2.774	-1.930	-4.136	-2.005	-1.921	-0.875	-1.931
SC	1.065	0.245	2.461	0.233	-0.516	-1.411	-3.224	-0.406	1.492	-0.150	1.466
SD	1.635	2.551	0.500	1.002	0.143	-0.057	-1.618	-0.548	7.991	0.579	0.615
TN	0.779	1.303	-0.046	0.861	-0.324	-0.675	-2.264	0.279	1.731	0.061	1.176

Table 1. (continued) State-wise annual output and input growth rates[1], 1960-1996

State	Aggregate output	Crops	Livestock	Other farm revenue	Aggregate input	Land	Labor	Capital	Chemicals	Energy	Materials
TX	1.582	1.257	1.851	1.665	0.457	-0.301	-1.431	0.201	3.586	-0.189	1.678
UT	1.452	1.487	1.401	0.614	-0.074	-1.127	-1.616	0.060	0.935	1.089	1.017
VT	0.239	-0.161	0.393	-0.530	-1.338	-1.993	-2.776	-1.125	-1.595	0.473	-0.242
VA	1.113	0.687	1.458	0.808	-0.386	-0.945	-2.794	-0.488	0.677	0.227	1.590
WA	2.402	2.746	1.739	1.127	0.688	-0.721	-0.970	-0.371	2.427	0.775	1.670
WV	0.288	0.281	0.191	0.418	-0.967	-1.201	-2.324	-1.117	-1.599	0.180	0.598
WI	0.703	1.296	0.252	0.454	-0.853	-0.666	-2.857	-0.668	3.181	0.669	0.264
WY	0.893	1.116	0.735	0.193	0.248	-0.209	-1.131	-0.267	2.340	0.573	1.195
Average[2]	1.315	1.464	0.942	0.715	-0.342	-0.881	-2.187	-0.339	2.014	0.444	0.700

Notes: [1]Annual growth rate is defined as $\left[(x_{2004}/x_{1960})^{1/n}-1\right]*100$, where x is the input or output variable and n is the number of years in the time period. [2]A simple average across states.

(2.014). The productivity computed based on the average annual growth rate of output (1.315) and input (-0.342) leads to average annual productivity growth rate of 2.136.

IV. Empirical application and results

To illustrate the sensitivity of the LP to the level of aggregation, equation (5a), the output based Malmquist productivity measures OMP, and equations (8a) and (8b), the Malmquist total factor productivity measures MTFP, are estimated for various levels of commodity and input aggregations using state-level data from 1960-2004. First, productivity measures estimated by alternative models are compared to the ideal Fisher index productivity measure. Second, the shadow or dual values of the LP constraints for disaggregate Malmquist productivity index and the Malmquist total factor productivity index are compared to the market prices used in the Fisher index.

The state-wise annual productivity growth rate estimated for the period 1960-2004 using OMP and MTFP index time series models for various levels of aggregation are presented in Table 2.[3] Specifically, two levels of dis-aggregation were considered: (1) single output and single input (SOSI) model with an aggregate input and aggregate output; and (2) multiple output and multiple input (MOMI) model with 6 inputs and 3 outputs.[4]

For aggregate or SOSI technology, the OMP estimated an annual growth rate of 2.136 for CRS (1.55 for VRS) that is identical (different) to the ideal Fisher index measure. Since the SOI or aggregate technology is immune to the divergences in productivity, measures such as share-weights are not used in the estimation process of the LP model. In contrast, the MTFP index for the SOSI model estimated an annual productivity growth rate of 1.9127 for CRS and VRS technology. Even though the MTFP index has a TFP interpretation, the productivity measures estimated under CRS or VRS assumptions are expected to be identical given equation (9) was estimated under constant input (output) for MO (MI). This is different from the ideal Fisher index productivity measure of 2.136 (Table 2). The annual productivity measures estimated by the ideal Fisher productivity index, the OMP index, and the MTFP index for the SOSI models are graphically presented for a single state, North Dakota, in Figure 1.

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[3] The detailed annual productivity measures computed can be obtained from the authors.

[4] Results from other levels of disaggregation, (1) single output and multiple input (SOMI) model with an aggregate output and 6 inputs; and (2) multiple output and single input (MOSI) model with an aggregate input and three outputs, are available from the authors.

Table 2. State-wise annual productivity growth rates, 1960-2004

State	Ideal Fisher index	Variable returns to scale				Constant returns to scale			
		OMP		MTFP		OMP		MTFP	
		MOMI	SOSI	MOMI	SOSI	MOMI	SOSI	MOMI	SOSI
AL	1.7868	1	1.1381	1.4965	2.3537	1	1.7868	1.4965	2.3537
AZ	1.9621	1	1.9372	1.2557	1.8624	1	1.9621	1.2557	1.8624
AR	2.3373	1	0.998	2.3118	4.8143	1	2.3373	2.3118	4.8143
CA	2.0802	1	1.3163	1.9897	3.5166	1	2.0802	1.9897	3.5166
CO	1.6236	1	0.9347	1.8902	2.8109	1	1.6236	1.8902	2.8109
CT	2.6272	1.1872	1.3061	0.8493	0.5294	1.3217	2.6272	0.8493	0.5294
DE	2.2077	1	2.0862	2.1225	3.9668	1.1245	2.2077	2.1225	3.9668
FL	1.8851	1	1.0069	1.1868	3.2912	1	1.8851	1.1868	3.2912
GA	2.3204	1	2.2309	2.003	2.9272	1	2.3204	2.003	2.9272
ID	2.4248	1	1	1.4285	3.4952	1	2.4248	1.4285	3.4952
IL	2.3695	1	1.7314	1.0229	1.2651	1	2.3695	1.0229	1.2651
IN	2.729	1	1.8821	0.9988	1.2981	1	2.729	0.9988	1.2981
IA	2.2719	1	1.8096	1.1598	1.4413	1	2.2719	1.1598	1.4413
KS	1.5875	1	0.9695	2.0278	2.8837	1	1.5875	2.0278	2.8837
KY	2.0293	1	1.8887	1.8937	1.7579	1.0663	2.0293	1.8937	1.7579
LA	2.3355	1	2.2156	1.2627	1.7922	1	2.3355	1.2627	1.7922
ME	2.3075	1	1.2168	0.6983	0.4444	1.1174	2.3075	0.6983	0.4444
MD	2.2332	1	1.9317	1.8314	1.6708	1.2131	2.2332	1.8314	1.6708
MA	2.738	1	1	0.7334	0.2493	1	2.738	0.7334	0.2493

Table 2. (continued) State-wise annual productivity growth rates, 1960-2004

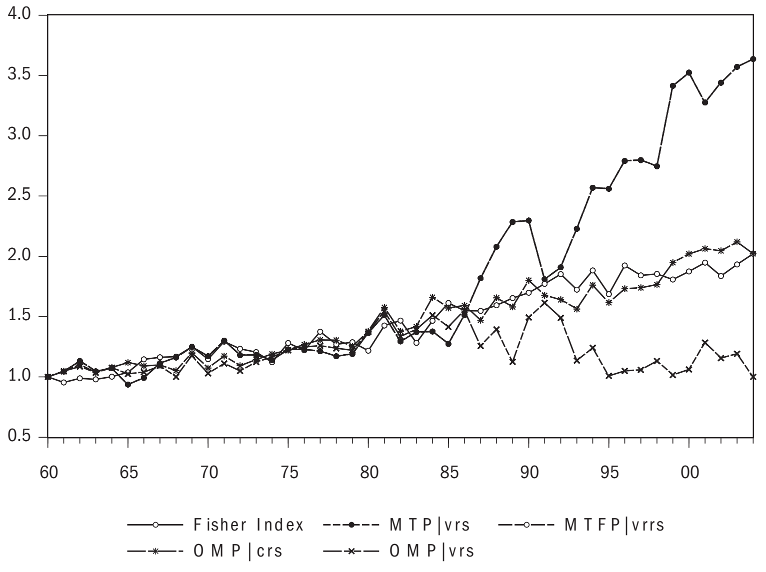
State	Ideal Fisher index	Variable returns to scale				Constant returns to scale			
		OMP		MTFP		OMP		MTFP	
		MOMI	SOSI	MOMI	SOSI	MOMI	SOSI	MOMI	SOSI
MI	2.8855	1	1.8328	1.1789	1.1641	1	2.8855	1.1789	1.1641
MN	2.2626	1	1.9598	1.213	1.621	1	2.2626	1.213	1.621
MS	2.3861	1	2.3255	2.1031	1.9411	1	2.3861	2.1031	1.9411
MO	2.0374	1	1.6585	1.144	1.3501	1	2.0374	1.144	1.3501
MT	1.8317	1	1.8003	1.2619	1.6926	1	1.8317	1.2619	1.6926
NE	2.0222	1	1	2.326	3.6364	1	2.0222	2.326	3.6364
NV	1.7232	1	1.1265	1.6116	2.7678	1	1.7232	1.6116	2.7678
NH	2.4116	1	1.0173	0.7369	0.3447	1.0718	2.4116	0.7369	0.3447
NJ	2.0827	1	1	0.6685	0.4147	1.0017	2.0827	0.6685	0.4147
NM	1.8879	1	1.1706	1.2963	3.8233	1	1.8879	1.2963	3.8233
NY	1.9205	1	1.1879	0.8425	0.6692	1	1.9205	0.8425	0.6692
NC	2.2504	1	2.3082	1.3335	2.446	1	2.2504	1.3335	2.446
ND	2.3041	1	2.2704	1.1058	2.2371	1	2.3041	1.1058	2.2371
OH	2.5839	1	1.617	1.0099	1.012	1	2.5839	1.0099	1.012
OK	1.2903	1	1.386	1.0389	2.0984	1	1.2903	1.0389	2.0984
OR	3.1089	1	2.6286	1.3746	2.2226	1	3.1089	1.3746	2.2226
PA	2.2172	1	1.8181	1.4517	1.4219	1.1105	2.2172	1.4517	1.4219
RI	2.9781	1	1.0666	0.518	0.2367	1.0261	2.9781	0.518	0.2367
SC	2.0334	1	1.611	1.1664	1.2763	1	2.0334	1.1664	1.2763

Table 2. (continued) State-wise annual productivity growth rates, 1960-2004

State	Ideal Fisher index	Variable returns to scale				Constant returns to scale			
		OMP		MTFP		OMP		MTFP	
		MOMI	SOSI	MOMI	SOSI	MOMI	SOSI	MOMI	SOSI
SD	1.9455	1	1.9257	1.3662	2.213	1	1.9455	1.3662	2.213
TN	1.6407	1	1.4179	1.1151	1.2254	1	1.6407	1.1151	1.2254
TX	1.6506	1	1.1169	1.758	2.4871	1	1.6506	1.758	2.4871
UT	1.9774	1	1.9343	1.8496	1.8503	1.1203	1.9774	1.8496	1.8503
VT	2.0407	1	1.195	0.9054	0.6072	1.0007	2.0407	0.9054	0.6072
VA	1.9583	1	1.6456	1.3608	1.3828	1	1.9583	1.3608	1.3828
WA	2.1368	1	1.8953	2.0309	3.9611	1	2.1368	2.0309	3.9611
WV	1.7625	1	1.1415	0.9345	0.7349	1.0112	1.7625	0.9345	0.7349
WI	2.0152	1	1.3978	1.1569	0.9323	1	2.0152	1.1569	0.9323
WY	1.3343	1	1.3428	1.3154	1.6677	0.9861	1.3343	1.3154	1.6677
Average	2.1362	1.0074	1.55	1.3612	1.9127	1.0244	2.1362	1.3612	1.9127

Notes: OMP is the output-based Malmquist productivity index, MTFP is the Malmquist total factor productivity index, CRS is constant returns to scale, VRS is variable returns to scale, MOMI is multiple output and input and SOSI is single output and input.

Figure 1. North Dakota: annual TFP estimated by Fisher, OMP and MTFP indices for Single Output Single Input (SOSI), 1960-2004

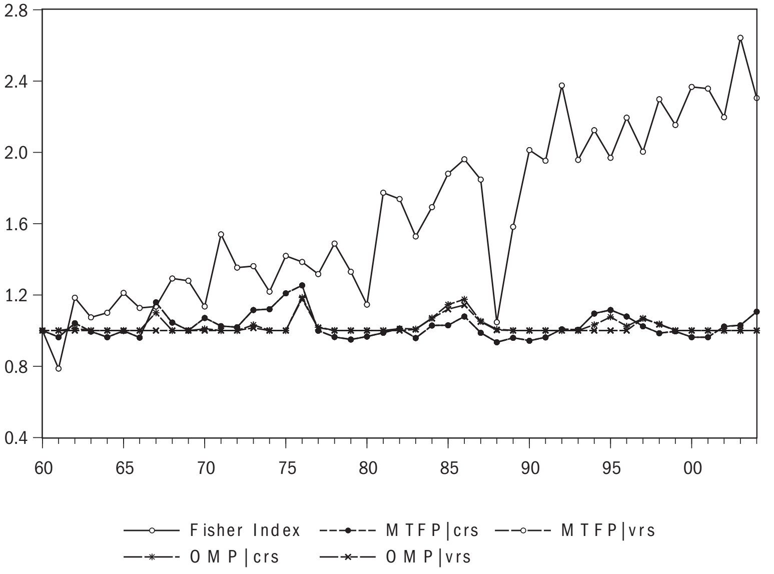


Notes: OMP is the output Malmquist productivity index, MTFP the Malmquist total factor productivity index, crs (vrs) the constant (variable) returns to scale.

Results for disaggregate or multiple output and multiple input (MOMI) model with 6 inputs and 3 outputs are also presented in Table 2. The OMP index estimated an annual productivity growth rate of 1.0244 for CRS (1.0074 for VRS), while the MTFP index estimated an annual growth rate of 1.3612 for CRS and VRS technology. These annual productivity growth rates for the MOMI models were different from ideal Fisher index measure. Further, the estimated annual productivity growth rate from the MOMI model is different from the SOSI model. Figure 2 presents the annual productivity measures estimated by the ideal Fisher productivity index, the OMP index, and the MTFP index for the MOMI models for a single state, North Dakota.

This difference in the annual productivity growth rates due to "curse of dimensionality" problem is consistent with the efficiency (Hanchar and Tauer 1995; and Tauer and Thomas 1994) measures. In a productivity framework it is obvious that the "curse of dimensionality" problem leads to decreased productivity growth measures and the results in Table 2 support the argument. In addition, results also show the sensitivity of the use of CRS and VRS technology due to the composition of the theoretical frontier (or envelope).

Figure 2. North Dakota: annual TFP estimated by Fisher, OMP and MTFP indices for Multiple Output Multiple Input (MOMI), 1960-2004



Notes: OMP is the output Malmquist productivity index, MTFP the Malmquist total factor productivity index, crs (vrs) the constant (variable) returns to scale.

Next, we identify the "curse of dimensionality" problem in reference to the shadow or dual values of the LP constraints. We also demonstrate the weights or shadow prices recovered depend on how the CRS or VRS constraint is imposed in the estimation of the OMP and MTFP indexes. To accomplish this objective, we compare the endogenous share-weights recovered from the dual values of the linear programming constraints of the OMP and MTFP programming method for various levels of commodity and input aggregation. Also, we compare the endogenous share-weights recovered from the programming approach to the exogenous share-weights of the ideal Fisher index approach from 1960-2004.The average input and output shares of the ideal Fisher index approach, the OMP programming approach, and the MTFP programming approach for the disaggregate model are presented in Table 3.[5] Results in Table 3 indicate that the average shadow shares of the OMP and MTFP programming approach are different from the exogenously observed

[5] The annual shadow or dual prices recovered from the linear program approach can be obtained from the authors.

market shares of the ideal Fisher index approach.[6] For example, in the ideal Fisher index approach, the average land, labor, capital, chemicals, energy and materials share are 9%, 27%, 13%, 6%, 4% and 41%, respectively. Compared to the ideal Fisher index approach, the average shadow or dual values input shares computed for OMP programming approach with VRS technology are 30%, 16%, 20%, 9%, 12% and 13% respectively for land, labor, capital, chemicals, energy and materials. Similar average shadow or dual values input shares with CRS technology are 15%, 24%, 9%, 15%, 12% and 26% respectively for land, labor, capital, chemicals, energy and materials. This is different from the shares used in the ideal Fisher index approach and recovered from the LP approach with VRS technology.

Similarly, the average shadow or dual values output shares computed from OMP programming approach with VRS (CRS) are 28%, 50% and 23% (28%, 54% and 18%) respectively for crops, livestock and other farm revenue. However, they are different from the output shares used in the ideal Fisher index. In the ideal Fisher index approach, crop and livestock had a share of 49% and 46%, respectively, with the remaining attributed to other farm revenue. In contrast, the output and input shares recovered by the MTFP programming approach under CRS and VRS technology were identical. These shares were different from the shares used in the ideal Fisher index approach and recovered from the OMP programming approach with VRS and CRS technology.

One of the main reasons for the difference in the productivity measures across models is the use of share-weights to form the technology or theoretical frontier (envelope). Unlike the ideal Fisher index approach, the average share-weights or shadow prices used in the programming approach are driven by the number of input and output constraints used in the estimation. For example, with a 6 input-3 output disaggregation model, the OMP or MTFP linear programming approach allocates maximum share-weight on a single input with a huge positive rate-of-change, resulting in a very low productivity measure. Alternatively, if the OMP or MTFP linear programming approach allocates maximum share-weight on a single input with a lowest rate-of-change, then the productivity measures would be very high.

[6] Due to the piecewise linear approximation of the programming approach for some inputs or outputs, the shares approximated from the linear programming constraints might attach zero or 100 percent weight. The shares present in the Table 3 are averaged across the whole time period.

Table 3. U.S average market shares and shares estimated from disaggregate output and input model, 1960-2004

Model	Output shares			Input shares					
	Crops	Livestock	Other farm revenue	Land	Labor	Capital	Chemicals	Energy	Materials
Fisher Index	49%	46%	4%	9%	27%	13%	6%	4%	41%
Variable returns to scale:									
OMP	28%	50%	23%	30%	16%	20%	9%	12%	13%
MTFP	35%	51%	14%	49%	9%	17%	5%	10%	10%
Constant returns to scale:									
OMP	28%	54%	18%	15%	24%	9%	15%	12%	26%
MTFP	35%	51%	14%	49%	8%	17%	5%	10%	10%

Notes: OMP is the output-based Malmquist productivity index and MTFP is the Malmquist total factor productivity index.

V. Conclusions

This paper examines the sensitivity of nonparametric programming productivity measures to the choice of commodity/input aggregation and imposition of CRS/VRS technology compared to the traditional ideal Fisher index approach using U.S. state-level data from 1960-2004. The importance of share-weights in explaining the sensitivity of the nonparametric productivity measures is illustrated by comparing the implicit shadow shares recovered from the dual values of the linear programming constraints in the OMP and MTFP programming methods to the observed shares of the ideal Fisher index.

The analyses at the U.S. state level indicates productivity measures estimated from the OMP programming approach with CRS technology is identical to the ideal Fisher index productivity measures for aggregate (single output and single input) technology. Divergence in productivity measures is observed not only due to choice of method –OMP and MTFP methods and various levels of commodity and input aggregation, but also between CRS and VRS technology. Due to the piecewise linear approximation of the nonparametric programming approach, the shadow share-weights are skewed leading to the difference in the productivity measures across methods, models and various levels of commodity aggregation.

The importance of the results reported in this paper will depend upon the researcher's objectives and the availability of data. If prices are available utilizing the price information (as share-weights) in the computation of productivity measures, either by the index and or linear programming approach will provide similar productivity measures. However, for the unpriced, non-market goods, like environmental pollution, the unavailability of price information would motivate researchers to apply the programming approach to estimate the productivity measures as well as to recover the shadow prices.

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