On the optimality of bank capital requirement policy in a macroeconomic framework
ON THE OPTIMALITY OF BANK CAPITAL REQUIREMENT POLICY IN A MACROECONOMIC FRAMEWORK

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An increasingly widespread “macro-prudential” view holds that bank capital requirements should be loosened during recessions and tightened during expansions to avoid excessive credit and output swings. We present a dynamic general equilibrium framework that accounts for the effects of capital requirement policies on the saving decisions of households, and, through this channel, on bank loans and output. We evaluate optimal capital requirement policy in the presence of loan write-offs (loan supply) and productivity (loan demand) shocks. We show that capital requirements should be reduced in response to unanticipated loan write-offs. We also show that capital requirements should be tightened in anticipation of future declines in productivity, and loosened at the onset of recessions. We conclude that macro-prudential capital requirement policies can be optimal from a welfare standpoint, but they can also generate output and credit booms through general equilibrium effects.

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I. Introduction

The recent global financial crisis has revealed the limits of standing macroeconomic policy frameworks that rely on monetary and fiscal policies alone. These policies moderated business cycles and kept inflation low and stable, but they were unable to prevent a massive build-up of systemic financial risk and, ultimately, a deep and prolonged global recession (Blanchard, Dell’Ariccia, and Mauro 2010).

To address the shortcomings of current macroeconomic frameworks, an increasingly widespread “macro-prudential” view holds that prudential financial regulations should be used as policy instruments to limit the inherent procyclicality of financial systems (Borio, Furfine, and Lowe 2001; Borio 2003; Kashyap and Stein 2004). Specifically, prudential regulations should be loosened during recessions and tightened during expansions to limit systemic financial risk and dampen credit and output swings.

One argument that supports the counter-cyclical use of bank capital requirements goes as follows. During recessions, loan defaults cause bank capital write-offs that, in turn, force banks to raise new capital or withdraw maturing loans and accumulate cash assets in order to satisfy the required risk-weighted asset ratio. As raising new capital is typically difficult in bad times, banks tend to satisfy the requirement through loan supply reductions, which amplify the credit crunches and the recessions. These amplification effects can be avoided by lowering the capital requirements at the beginning of recessions. Similarly, during upswing phases of business cycles, credit booms and pro-cyclicality could be contained by tightening bank capital requirements.1

Though appealing, this argument overlooks the fact that the banking system’s lending capacity is determined, to a large extent, by the households’ willingness to provide savings in the form of bank deposits and equity holdings. The literature is missing a general equilibrium framework that accounts for the effects of capital requirement policies on the consumption-saving decisions of households and, through this channel, on credit and output. In this paper, we provide such

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1 Other regulatory and non-regulatory sources of pro-cyclicality in financial systems include those related to risk measurement, risk management techniques, individuals’ behavioral biases, etc. See Borio, Furfine, and Lowe (2001).
a framework and address the following question: how should bank capital requirements be set in different phases of the business cycle? In particular, should bank capital requirements be tightened during expansions and loosened during recessions, as the growing “macro-prudential” literature suggests?

This paper shows that a policy that tightens bank capital requirements in anticipation of future recessions and loosens such requirements at the onset of recessions is optimal. It also shows, however, that this policy can generate lending and output booms prior to recessions. Such a policy would not serve the macro-prudential purpose of leaning against the expansionary phase of the business cycle, as conventional views hold. The reason is that households and the economy must boost savings during the economic expansion in order to build up bank capital buffers. Higher savings channeled to the banking system, in turn, generate credit and output booms.

More specifically, we develop a dynamic general equilibrium model of a closed economy\(^2\) with a financial system in which (non-pecuniary) externalities motivate the need for minimum capital requirements. In this sense, this paper is related to Lorenzoni (2008), which models pecuniary externalities arising from borrowing constraints, and shows how prudential regulations can curb inefficient credit booms. Rather than modeling pecuniary externalities in detail, as Lorenzoni (2008) does, we model a deposit insurance system that transmitted cross-bank externalities through premiums that depend on aggregate leverage in the banking system (Acharya, Santos, and Yorulmazer 2010). Our aim, however, is to understand whether and how bank capital requirements should change over time; the way in which we rationalize the need for capital requirements at a given point in time plays only a secondary role. Instead, a key feature of our model is that households make dynamic decisions but all other agents, including banks and firms, make only static decisions. This implies that households’ consumption-saving and portfolio choices are the key drivers of dynamic changes in bank capital requirements. In order to present the mechanism linking household saving, bank capital requirements, and credit growth in a transparent manner, we abstract from

\(^2\) In a closed economy, the power of domestic savings to affect the banking system’s lending capacity is particularly evident. However, the results obtained in this paper would also apply to open economies subject to imperfect capital mobility, in which domestic and foreign savings function as imperfect substitutes.
the asset-shifting effect—whereby banks satisfy capital requirements by cutting back on loans and shifting toward cash assets—and banks’ dynamic decisions, including whether to hold capital buffers on top of the required minimum.\(^3\)

In its technical approach but not in its objective, this paper is closely related to Edwards and Végh (1997) and Díaz-Gimenez et al. (1992), which develop models that include meaningful roles for banks but do not fully “explain” the existence of banks. Given our framework, we derive the main result of the paper by considering cyclical changes in output driven by anticipated and temporary reductions in productivity. We show that the bank capital requirement should be increased in anticipation of future temporary reductions in productivity and output, and reduced at the onset of recessions, when the declines in productivity and output materialize. This policy is optimal from a welfare standpoint but generates lending and output booms prior to recessions.

We also evaluate optimal bank capital requirement policy in the presence of unanticipated and exogenous loan write-offs (loan supply shocks). Higher loan write-offs can be seen as a way of modeling a rise in non-performing loans and loan-loss provisions, such as the one observed during the early stages of the recent global financial crisis—in sub-prime mortgage loans in the US banking system by the end of 2006. In this case, we show that the capital requirement should be reduced temporarily to facilitate the process of credit and output recovery. Our analysis implies that a policy of holding the capital requirement constant over time is suboptimal and causes deadweight welfare losses.\(^4\)

Other papers in the literature have also studied the effects of changing bank capital requirements over the business cycle. Kashyap and Stein (2004) studies the

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\(^3\) Based on evidence that banks hold capital buffers that exhibit cyclical patterns, one strand of the literature explores whether such buffers can offset the effects of changes in capital requirements (Heid 2007; Repullo and Suarez 2012; Estrella 2004). Heid (2007) explains why buffers amplify credit cycles—increasing in downturns and decreasing in expansions—under risk-insensitive regulation such as Basel I. But it argues that the behavior of buffers changes when the regulation becomes more risk sensitive, as in the shift toward Basel II: buffers dampen credit cycles, but only partially offsetting the impact of changes in capital requirements.

\(^4\) In our framework, the real sector of the economy exhibits neoclassical features—wages and prices are flexible and adjust to clear output and input markets at all times—and changes in output are driven by productivity shocks or loan write-offs. We do not study the optimality of bank capital requirement policy in a framework with Keynesian features, in which output fluctuations are driven by changes in aggregate demand in the presence of wage or price rigidities.
asset-shifting mechanism described above and shows that bank capital requirement policies associated with the macro-prudential view can maximize social welfare in a one-period, stochastic model. Our approach differs from theirs in that we evaluate bank capital requirement policy in a dynamic general equilibrium model that accounts for interactions between household saving decisions and bank capital requirements. More broadly, this paper is also related to the large literature that studies the connection between bank regulation and aggregate fluctuations, including among others Bernanke and Lown (1991), Blum and Hellwig (1995), Goodhart, Hoffmann, and Segoviano (2004), and Peek and Rosengren (1995). Those studies have set the background for macro-prudential analysis. Unlike this paper, however, they do not explicitly evaluate or advocate the use of bank capital requirements as counter-cyclical policy instruments. Finally, real business cycle theories interpret economic fluctuations as optimal responses of agents to uncertainty in the rate of technological change in neoclassical economies (Kydland and Prescott, 1982; and Prescott, 1986). In relation to those theories, this paper shows that in economies that also include a banking system, the optimal responses to technological shocks require variations in bank capital requirements over time.

We organize the rest of this paper as follows. In Section II, we present the model. In Section III, we evaluate optimal bank capital requirement policy in the presence of productivity (loan demand) shocks and loan write-offs (loan supply shocks). In Section IV, we conclude.

II. Model

Consider a closed economy populated by households, firms, banks, deposit insurers, and the government. Households own the banks, consume the single storable good, and supply labor, bank capital, and deposits. Firms produce the single good using labor and physical capital which is fully financed with bank loans. Banks receive deposits and raise capital from households, provide loans to firms, and purchase deposit insurance from the insurers. Insurers offer deposit

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5 Pennacchi (2005) points out that Kashyap and Stein’s analysis does not account for the deposit insurance losses associated with lower capital requirements. To avoid implicit deposit insurance subsidies, we include a self-financed and risk-based deposit insurance system in our framework.
insurance contracts to banks, collect premiums, and pay back the deposits of failed banks. Finally, the government imposes full deposit insurance and capital requirements on banks.

A key feature of the model is that households make dynamic decisions but all other agents make only static decisions. For this reason, households’ consumption-saving and portfolio choices are the key drivers of dynamic changes in optimal bank capital requirements predicted by the model.

A. Households

The lifetime utility of the representative household is given by

$$W = \sigma \int_0^T \left( \frac{c_t^{1-\theta} - 1}{1-\theta} \right) e^{-\theta t} dt,$$  

(1)

where $c_t$ denotes consumption of the single good and $\theta > 0$ is the subjective discount rate. Note that the instantaneous utility function exhibits constant relative risk aversion measured by $\theta > 0$ and the household’s intertemporal elasticity of substitution is given by $\sigma = \frac{1}{\theta}$.

The household supplies inelastically one unit of labor in competitive labor markets: $n_t^h = 1$ for all $t$. It also holds a portfolio of assets $b_t^h$, composed of bank equity (capital) $k_t^h$ and bank deposits $d_t^h$, and the latter are determined by a deposit-in-advance constraint. Thus,

$$b_t^h = k_t^h + d_t^h; \quad d_t^h = \alpha \cdot c_t,$$  

(2)

The household’s flow constraint is given by

$$\dot{b}_t^h = r_t^h b_t^h + w_t - (r_t - r_t^d) d_t^h - c_t + \Omega_t^h,$$  

(3)

where a dot over a variable indicates the time derivative of the variable, $r_t^h$ is the real rate of return on bank equity, $r_t^d$ is the real deposit interest rate, $w_t$ is the
real wage per unit of labor service, and $\Omega^h_i$ denotes dividends from the banks. Bank equity holdings are subject to idiosyncratic risks that can be fully diversified because they are independent and the number of banks is large. Specifically, the household optimally holds equal equity positions in all banks, and thus the rate of return on the total household’s equity, $r_i^l$, is riskless. The household is born at time $t = 0$ with an endowment of assets $b^h_0 > 0$.

The household’s problem is to choose the paths of consumption and asset holdings $\{c_t, k^h_t, d^h_t\}$ to maximize its utility (1) subject to constraints (2), (3), and the no-Ponzi condition $\lim_{t \to \infty} b^h_t e^{-\beta t} \geq 0$, taking as given the paths of the rates of return, wages, and dividends $\{r^l_t, r^d_t, w_t, \Omega^h_t\}$, and the endowment $b^h_0$. The current-value Hamiltonian is given by

$$H \equiv \left( \frac{c_t^{-\theta} - 1}{1 - \theta} \right) + \lambda_t \cdot \{r_t b^h_t + w_t - (r_t - r^d_t) d^h_t - c_t + \Omega^h_t\}, \tag{4}$$

where $\lambda_t$ is the costate variable. The first-order conditions with respect to the control $c_t$ and the law of motion for the costate variable are given by

$$c_t^{-\theta} = \lambda_t \cdot [1 + \alpha \cdot (r_t - r^d_t)], \tag{5}$$

$$\dot{\lambda}_t = \lambda_t \cdot (\beta - r_t). \tag{6}$$

The optimization conditions also include (3) and the transversality condition $\lim_{t \to \infty} \lambda_t e^{-\beta t} = 0$. According to (5), the household equates the marginal utility of consumption to the marginal value of wealth at every instant $t$. The price of current consumption reflects the cost of holding deposits: the deposit-in-advance requirement $\alpha$ multiplied by the equity-deposit spread $r_t - r^d_t$.\footnote{\lambda_t$ can be interpreted as the marginal value (measured in utility terms) of the household’s wealth at time $t$. Also, the equity-deposit spread $r_t - r^d_t$ is, in equilibrium, positive. Although both bank equity and deposits allow the household to store value, only the latter provides liquidity services and thus yields a lower return.}
B. Firms

Firms are indexed by \( i \), produce output \( y_{it} \) by employing bank loans \( l_{it} \) and labor \( n_{it} \), and are subject to idiosyncratic productivity shocks \( A_{it} \). The production function is given by

\[
y_{it} = A_{it} \cdot f(l_{it}, n_{it}) ,
\]

(7)

where \( f(.) \) is strictly increasing and concave in both arguments. Productivity shocks \( A_{it} \) are represented by two states: the high-productivity state, \( A_{it} = A_t = \frac{A_i}{p} \), and the low-productivity state, \( A_{it} = 0 \), which occur with probabilities \( p \) and \( 1 - p \), respectively. Thus, the expected productivity of any firm \( i \) is \( E(A_{it}) = A_t \). Firms are uniformly distributed in the interval \([0,1]\), and, by the “law of large numbers,” the fraction of firms with high productivity is (ex-post) \( p \).

Each firm \( i \) receives a loan from bank \( i \) in the amount \( l_{it} \), and the loan return is contingent on the productivity state of the firm. For simplicity, we assume that bank lending is specialized: each bank \( i \) lends to a single firm \( i \), while firm \( i \) only borrows from bank \( i \).\(^7\) Thus, we can interpret that firms act as bank agents, and free entry of firms ensures that loan returns are maximized. The firm chooses the optimal amount of labor \( n_{it} \), conditional on the realization of the productivity shock \( A_{it} \), taking as given the loan \( l_{it} \) and the market wage rate \( w_t \).

In the high-productivity state, the return on bank \( i \)'s loan is \( A_t \cdot f(l_{it}, n_{it}) - w_t n_{it} \), and the firm's first-order condition is given by \( w_t = A_t \cdot f_n(l_{it}, n_{it}) \), which implicitly defines its demand for labor as \( n_{it}^* = n_t(A_t, l_{it}, w_t) \). In the low-productivity state, firm \( i \)'s labor demand and the return on bank \( i \)'s loan are 0. Thus, the labor demand of firm \( i \) and the loan return of bank \( i \), \( 1 + r_{it}^l \), are given by\(^8\)

\(^7\)Our view is that this specialization arises from banks’ expertise in monitoring certain industries or activities and transaction costs of diversification. This assumption allows us to introduce a meaningful deposit insurance scheme, as banks with zero revenue realizations are unable to pay back deposits.

\(^8\)In Section III, the business cycle will affect a firm’s (expected) productivity \( A_t \) but not its loan default probability \( p \); in contrast, other studies model cyclical variations in loan default probabilities (Heid 2007; Repullo and Suarez 2012; Estrella 2004). Both approaches, however, imply that banks’ expected loan returns vary with the business cycle.
Note that the indirect return per unit of bank $i$’s loan $1 + r^l_i$ in the high-productivity state is obtained by plugging the optimization condition $w_i = \overline{A}_i \cdot f_n(l_i, n^*)$ and $n^*(.)$ into the firm’s objective function, and then applying Euler’s theorem.

C. Banks

Bank $i$ holds a portfolio of loans $l_i$, capital $k_i$, and deposits $d_i$; its balance sheet satisfies

$$l_i = k_i + d_i.$$  \hspace{1cm} (9)

Bank $i$’s loan and equity returns, $1 + r^l_i$ and $1 + r^e_i$, are state contingent, whereas its deposit return $1 + r^d_i$ is market determined and riskless, as all deposits are fully insured. Bank $i$ enters into a fairly priced, full-deposit insurance contract with the insurer. According to the contract, the bank pays the insurer a premium per unit of loan $\tau_i = \tau(k_i, l_i, r^d_i, p)$ in the high-revenue state, and the insurer assumes the deposit liabilities of the bank in the zero-revenue state. The premium $\tau(.) \cdot l_i$ is decreasing in bank $i$’s capital and increasing in bank $i$’s assets, that is, $\frac{\partial \tau(.)}{\partial k_i} < 0$, $\frac{\partial \tau(.)}{\partial l_i} > 0$. Let $\Omega_n^k$ denote bank $i$’s state contingent profits, which are paid as dividends to households. Bank $i$’s expected profit function, $E(\Omega_n^k)$, is given by

$$E(\Omega_n^k) = p \cdot \overline{A}_i \cdot f_n(l_i, n^*) \cdot l_i - E(1 + r^e_i) \cdot k_i - p \cdot (1 + r^d_i) \cdot d_i - p \cdot \tau(.) \cdot l_i. \hspace{1cm} (10)$$

Bank $i$’s problem is to choose $k_i$, $l_i$, and the equity returns $1 + r^e_i$ so as to maximize its expected profits (10), subject to its balance sheet constraint (9) and
the equity-holder participation constraint, \( E(1 + r_{it}) = 1 + r_t \), taking as given the rates of return \( r_t \), \( r_{it} \) and the wage rate \( w_t \). Households can diversify away the specific risk of holding bank \( i \)'s capital, and the participation constraint ensures that the expected return of bank \( i \)'s capital \( E(1 + r_{it}) \) is equal to the market-wide return, \( 1 + r_t \). The first-order conditions of bank \( i \)'s problem are given by

\[
\bar{A}_i \cdot [f_i + f_n \cdot n_i^*] - w_t \cdot n_i^* = 1 + r_i^d + \frac{\partial \tau(\cdot) \cdot l_{it}}{\partial l_{it}}, \tag{11}
\]

\[
1 + r_t = p \cdot [1 + r_i^d - \frac{\partial \tau(\cdot) \cdot l_{it}}{\partial k_{it}}]. \tag{12}
\]

Equation (11) is bank \( i \)'s first-order condition with respect to \( l_{it} \). The bank equates the expected marginal benefit and the expected marginal cost of financing new loans with deposits (the amount of bank capital remains constant). The expected marginal benefit is given by the increased production of firm \( i \) in the high-productivity state. In such a state, additional lending boosts production directly, \( \bar{A}_i \cdot f_i \), and indirectly, by increasing the productivity of labor, \( \bar{A}_i \cdot f_n \cdot n_i^* \). The latter benefit is not fully internalized by the bank because firm \( i \) pays a larger wage bill \( (w_t \cdot n_i^*) \). The expected marginal cost is the sum of the deposit return, \( 1 + r_i^d \), and the increase in the deposit insurance premium paid in the high-productivity state, \( \frac{\partial \tau(\cdot) \cdot l_{it}}{\partial l_{it}} \).

Equation (12) is the bank’s first-order condition with respect to \( k_{it} \). The bank equates the expected marginal benefit and the expected marginal cost of substituting deposits for capital to finance its loans (the amount of loans remains constant). The expected marginal benefit is the sum of the deposit return, \( p \cdot (1 + r_i^d) \), and the reduction in the deposit insurance premium associated with a higher capital-asset ratio, \( -p \cdot \frac{\partial \tau(\cdot) \cdot l_{it}}{\partial k_{it}} \). The expected marginal cost is the expected return on equity, \( E(1 + r_{it}) \).

Conditions (11) and (12) determine bank \( i \)'s demands for deposits \( d_{it} \) and equity \( k_{it} \), its loan \( l_{it} \), and its capital-asset ratio \( x_{it} = \frac{k_{it}}{l_{it}} \) as functions of \( r_t \), \( r_i^d \), \( w_t \), \( A_i \), and \( p \):
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Free entry ensures that banks earn zero expected profits. As bank $i$'s profit is zero in the low-revenue state, it must also be zero in the high-revenue state. Its equity return $1 + r_i$ is given by

$$
1 + r_i = \begin{cases} 
\overline{A}_i \cdot f_i(\cdot, \overline{A}_i, \overline{r}_i, \cdot, w_i) - (1 + r_i^d) \cdot d_r(\cdot) - \tau[x_i^*(\cdot), r_i^d, p] \cdot l_r(\cdot) & \text{if } A_i = A_i \not= 0 \\
0 & \text{if } A_i = 0 
\end{cases}.
$$

(D) Deposit insurers

The representative deposit insurer collects fair insurance premiums from banks with positive revenue realizations and pays the deposits of banks with zero revenue realizations. It also incurs operational costs $C(d, l)$ when bank $i$ fails, where $C(\cdot)$ is linearly homogeneous and strictly increasing and convex in the banking system’s aggregate deposits $d_r$ and loans $l_r$. $C(\cdot)$ represents the costs of liquidating loans of failed banks—including fire sale costs associated with widespread bankruptcies—as well as the administrative costs of dealing with depositors. The insurer’s zero-expected-profit condition is given by

$$
p \cdot \tau_i \cdot l_i - (1 - p) \cdot [d_i \cdot (1 + r_i^d) + C(d, l)] = 0,
$$

where the first term, $p \cdot \tau_i \cdot l_i$, is the expected revenue, $(1 - p) \cdot d_i \cdot (1 + r_i^d)$ is the expected payout to depositors, and $(1 - p) \cdot C(d, l)$ is the expected operational cost. The insurer sells contracts to a large number of banks: actual revenues and costs equal expected ones—actual profits are zero.

Note the presence of cost externalities in the insurance industry, whereby bank $i$'s insurance premium depends not only on its own expected losses but also on...
those of other banks. From the properties of the function $C(.)$ mentioned above, we can write $C(d_t, l_t) = l_t \cdot c(x_t)$, where $c(x_t)$ satisfies $c'(x_t) < 0$, $c'(x_t) > 0$. The insurer perceives the costs $C(.)$ as being "fixed" or independent of the individual bank’s balance sheet, but these costs increase with leverage in the system and affect the level of the bank’s premium.

In the broader context of the model, the cost function $C(.)$ introduces a benefit of financing banks with equity capital: reducing real costs of deposit insurance provision. This advantage of equity capital plays against the liquidity-provision advantage of deposits and both jointly determine banks’ capital structure. Also, the cost externalities introduce a role for government regulation. In the absence of regulation, deposit insurance is underpriced (in the margin) and hence banks are undercapitalized.

To further understand the role of the cost function $C(.)$, note also that if insurance provision were assumed to be costless, that is, $C(d_t, l_t) = 0$, all deposit risk could be fully diversified at no cost. In this case, banks would not use capital to reduce deposit insurance premiums and could not offer households a positive equity-deposit spread; in equilibrium, banks would set $d_u = l_u$. Also, if deposit insurance provision were not subject to cost externalities, that is, $C(d_t, l_t) = C(d_u, l_u)$, then the government would not need to impose capital requirements because banks would choose to hold socially optimal capital-asset ratios.

E. Government

The cost externalities in the insurance industry imply that government intervention aimed at forcing banks to internalize the external effects of their decisions can improve upon the decentralized, free market equilibrium. Equation (15) implies

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9 Acharya, Santos, and Yorulmazer (2010) describe the fire-sale costs incurred by deposit insurers during systemic financial crises and argues that both individual and systemic risks should determine the insurance premiums paid by banks. Consistent with this idea, our framework assumes that cross-bank externalities are transmitted through the deposit insurance system. An additional source of externalities is that deposit insurers employ industry-specific factors that are in high demand at times of systemic stress (such as bank auditors). In contrast to our approach, the banking literature justifies the imposition of bank capital requirements on the basis of externalities transmitted through the payments system (see Berger, Herring, and Szego 1995).
that bank $i$’s marginal insurance costs, which do not include the external effects, are given by

$$\frac{\partial \tau_i(l_i)}{\partial l_i} = \left(\frac{1-p}{p}\right) \cdot (1+r_i^d), \quad \frac{\partial \tau_i(l_i)}{\partial k_i} = -\left(\frac{1-p}{p}\right) \cdot (1+r_i^d). \tag{16}$$

The inclusion of the external effects yields the following marginal insurance costs:

$$\frac{\partial \tau_i(l_i)}{\partial l_i} = \left(\frac{1-p}{p}\right) \cdot [1+r_i^d + c(x_i) - c'(x_i) \cdot x_i], \quad \frac{\partial \tau_i(l_i)}{\partial k_i} = -\left(\frac{1-p}{p}\right). \tag{17}$$

\[1+r_i^d - c'(x_i)].

Note that the exclusion of the external effects understates both the marginal costs of increasing loans and the marginal benefits of increasing capital. Hence, the banking equilibrium without government intervention implies lower than socially optimal capital-asset ratios.

The first-best equilibrium could be attained through a system of taxes and lump-sum transfers. Alternatively, the government could directly regulate the pricing of insurance contracts, so that banks’ marginal insurance costs include their systemic contributions. Finally, the government could impose capital requirements on individual banks. To do so, the government solves bank $i$’s optimization conditions (11) and (12) using the marginal insurance cost functions which include the external effects (17). Note that when the external effects are internalized, the insurance premium per unit of loan $\tau_i = \tau(x_i, r_i^d, p) = -\left(\frac{1-p}{p}\right) \cdot [(1-x_i) \cdot (1+r_i^d) + c(x_i)]$ is a decreasing and convex function of bank $i$’s capital-asset ratio:

$$\frac{\partial \tau_i}{\partial x_i} = -\left(\frac{1-p}{p}\right) \cdot [1+r_i^d - c'(x_i)] < 0, \quad \frac{\partial^2 \tau_i}{\partial x_i^2} = \left(\frac{1-p}{p}\right) \cdot c''(x_i) > 0.$$

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10 At this point we assume that all banks are equal, which is the case in the equilibrium, see Subsection F.
11 The government could also choose to operate the deposit insurance system directly. As noted above, in the absence of government intervention, operational costs are considered as fixed by individual insurers. Hence, competitive insurers price contracts without including (marginal) systemic effects.
Henceforth, we assume that the government always uses bank capital requirements to eliminate the cross-bank externalities. We refer to the resulting capital-asset ratio, $\chi_i$, as the minimum “capital requirement” that must be imposed on bank $i$, which is always binding due to the static nature of the banking problem.

F. Equilibrium conditions

Let $d_i^h = \int_0^1 d_i^* \cdot di$ be the aggregate demands for deposits and equity from the banking system. Let $l = \int_0^1 l^* \cdot di$ be the aggregate stock of loans, and $n^f = \int_0^1 n^* \cdot di$ be the aggregate demand for labor in the economy. An equilibrium in this economy satisfies the following market-clearing conditions:

$$d_i^h = d_i^h = d_i^*, \quad k_i^h = k_i^* = k_i,$$

(18)

$$n_i^h = n_i^f = n_i = 1.$$  

(19)

Note that (2), (9), (13), and (18) imply that, in equilibrium, the aggregate stock of household’s assets $b_i^h$ is equal to the aggregate stock of bank loans $l_i$. Hence, $\lambda_i^h$ is the economy’s “shadow value” of loans, that is, the marginal value (in household’s utility terms) of the aggregate stock of loans at time $t$.

The equilibrium must also satisfy the condition that the household allocates bank capital evenly across banks to fully eliminate bank capital risk, that is, $k_i^h = k_i^*$. It also follows from (13) and (15) that all banks pay the same insurance premium per unit of loan, $\tau_i = \tau_i^*$. Condition (19) implies that the labor employed in a high-productivity firm is $n^*(\lambda_i, l_i, w_i, \lambda_i^h) = \frac{1}{p}$, which implicitly defines the wage rate in terms of $l_i, A_i$, and $p$. It also implies $n_i^h(\lambda_i, l_i, w_i) = 0$.

The banking system’s profit, $\Omega_i^h$, is certain by the law of large numbers and equal to the expected profit of each bank $i$. To obtain it, plug the insurer’s zero-expected-
optimal bank capital requirement policy in a macroeconomic framework

profit condition (15) and the equilibrium conditions into bank \( i \)'s expected profit function (10), and integrate over all banks:

\[
\Omega_i = E(\Omega^b_i) = A \cdot f(l, \frac{1}{p}) - w - (1 + r_t) \cdot k - (1 + r^d_t) \cdot d - (1 - p) \cdot c(x_t) \cdot l_t.
\] (20)

The aggregate flow constraint is obtained from the flow constraint of the household, (3), the aggregate bank profit function, (20), and the equilibrium conditions (18) and (19):

\[
\mathbf{b} = l = A \cdot \bar{f}(l, p) - c - \xi(x, p) \cdot l,
\] (21)

where \( \bar{f}(l, p) = f(l, \frac{1}{p}) \), \( \xi(x, p) = 1 + (1 - p) \cdot c(x) \), and the latter function \( \xi(.) \) is decreasing and convex in \( x \); \( \xi_x = (1 - p) \cdot c'(x) < 0 \), \( \xi_{xx} = (1 - p) \cdot c''(x) > 0 \). According to (21), the household’s consumption and the operational cost of the deposit insurance industry are subtracted from output to obtain the economy’s instantaneous saving flow.12

G. Solution

Plug (17), \( n^s(\bar{A}, l, w) = 0 \), and the equilibrium conditions into bank \( i \)'s first-order conditions (11) and (12) to write them as follows:

\[
E(1 + r^d_{i}) = A \cdot \bar{f}(l, p) = r_{i}^d + \xi(x, p) - x \cdot \xi_x(x, p),
\] (22)

---

12 Note that the insurance premium embeds two components: one corresponds to transfers that are received by depositors (households) in failed banks, and the other corresponds to real operational costs. Only the latter are social costs and thus are reflected in the aggregate flow constraint (21).
For given rates of return, \( r_t \) and \( r^d_t \) equation (23) determines the optimal financing structure of the banking system \( x_t \), and then, equation (22) determines the size of the banking industry \( l_t \). Note that the latter is limited by the decreasing marginal productivity of loans in the economy.

Also, the loan-deposit and equity-deposit spreads, \( r^l_t - r^d_t \) and \( r_t - r^d_t \) are decreasing in the capital-asset ratio \( x_t \), where \( r^l_t = E(r^l_0) \); from (11) and (12), the derivatives with respect to \( x_t \) are the following:

\[
\frac{\partial [r^l_t - r^d_t]}{\partial x_t} = -x_t \cdot \xi_{xx}(x_t, p) < 0, \quad \frac{\partial [r_t - r^d_t]}{\partial x_t} = -\xi_{xx}(x_t, p) < 0.
\]

This implies that the demand for capital is downward sloping and an increase (decrease) in \( x_t \) is associated with narrower (wider) spreads.

From equations (5), (6), and (23), we can express the household’s Euler equation in terms of the time variation in the equity-deposit spread and the capital-asset ratio, as follows:

\[
\dot{\theta} \cdot \frac{c_t}{c_t} = r_t - \beta \cdot \frac{\alpha}{[1 + \alpha \cdot (r_t - r^d_t)]} \cdot \frac{\partial (r_t - r^d_t)}{\partial t} = r_t - \beta + \frac{\alpha \cdot \xi_{xx}}{[1 + \alpha \cdot (r_t - r^d_t)]} \cdot x_t.
\]

This expression makes clear how a change in the capital requirement exerts an independent influence on consumption growth—beyond the effect of changes in \( r_t \). As the requirement increases over time, the equity-deposit spread goes down, reducing the effective cost of consumption—the household’s (opportunity) cost of holding the bank deposits needed to finance consumption. These price changes motivate the household to increase consumption growth. Combining equations (21), (24), and \( x_t = 1 - \alpha \cdot \frac{c_t}{l_t} \), we obtain the equations that characterize the equilibrium behavior of this economy for any initial aggregate stock of assets \( b^h_0 = l_0 \). The equations can be expressed in terms of \( h_t = \frac{c_t}{l_t}, l_t, \) and \( A_t \), as follows:
Consider a constant path of productivity $A_t = A$. Now, (25) and (26) form a system of differential equations in $h_t$ and $l_t$. Let $(h^*, l^*)$ denote the steady state values of $h_t$ and $l_t$ obtained by setting $h_t = l_t = 0$ in (25) and (26). Useful properties of the economy's equilibrium are summarized in Lemmas 1 and 2. All proofs are shown in the Appendix available online.

**Lemma 1.** The differential equation system defined by (25) and (26) for a constant productivity path $A_t = A$, when linearized around the steady state $(h^*, l^*)$, exhibits saddle-path stability.

Figure A1 in the online Appendix shows the phase diagram corresponding to an economy which is defined below as “dynamically inflexible.” The economy determines the initial value of $h_t$ at the corresponding point on the saddle path $(SP)$. In this model $h_t$ is a jumping variable, whereas $l_t$ is predetermined.\(^{13}\)

**Lemma 2.** If the production function is Cobb-Douglas, the following results are obtained in the comparative statics analysis of the steady state $(h^*, l^*)$: (i) the stock

\[ \frac{\alpha \cdot \xi(x, p)}{1 - \alpha \cdot \xi_x(x, p)} \]

where $\Delta_x = \frac{\alpha \cdot \xi(x, p)}{1 - \alpha \cdot \xi_x(x, p)}$.

\[ (1 - \theta) \cdot \xi(x, p) - \alpha \cdot h_t \cdot \xi_x(x, p) \right\}, \]

\[ l_t = \left[ A_t \cdot \frac{\tilde{f}(l, p)}{l_t} - h_t - \xi(x, p) \right], \quad (26) \]

\[ [\theta + \alpha \cdot \Delta_x \cdot h_t] \cdot h_t = h_t \left\{ \theta \cdot h_t - \beta + A_t \cdot \left[ \tilde{f}_t(l, p) - \theta \cdot \frac{\tilde{f}(l, p)}{l_t} \right] \right\}, \]

\[ (1 - \theta) \cdot \xi(x, p) - \alpha \cdot h_t \cdot \xi_x(x, p) \right\}, \]

\[ l_t = \left[ A_t \cdot \frac{\tilde{f}(l, p)}{l_t} - h_t - \xi(x, p) \right], \quad (26) \]

13 We justify our modeling of $l_t$ as a non-jumping variable as follows. Typically, banks hold liquid assets as well as long-maturity loans, which, to a large extent, cannot be liquidated or extended further immediately after the realization of shocks. Thus, we interpret that, at every instant, the stock of bank loans is predetermined. This interpretation, in turn, allows us to simplify our analysis by ignoring the liquid assets that banks typically hold.
of bank loans \( l^* \) is increasing in the level of productivity, that is, \( \frac{\partial l^*}{\partial A} > 0 \); (ii) the consumption-loan ratio \( h^* \) and the capital-asset ratio \( x^* \) are invariant to changes in productivity, that is, \( \frac{\partial h^*}{\partial A} = 0 \) and \( \frac{\partial x^*}{\partial A} = 0 \).

According to Lemma 2, a more productive economy has higher steady state bank credit. More important, however, is the fact that in this model the optimal capital requirement remains unchanged in the presence of productivity-driven growth. Hence, it is straightforward to extend the application of this model to growing economies: the capital requirement would remain constant when the economy grows along its trend path and change during business cycles.

**Definition 1.** The economy is dynamically inflexible if and only if
\[
\frac{1}{\sigma} = \theta > \frac{\tilde{h} \cdot l^*}{(\tilde{f}_l - \tilde{f}_w)}
\]
is satisfied.

In a dynamically inflexible economy, household’s intertemporal elasticity of substitution in consumption is low and the production function is sufficiently concave. For such an economy, household’s saving is relatively unresponsive and the cost of reallocating resources over time using production is high because the marginal productivity of capital (loans) decreases rapidly. In contrast, a dynamically flexible economy could reallocate resources over time at lower cost.\(^{14}\)

In the particular case of a Cobb-Douglas production function
\[
\tilde{f}(l, p) = F \cdot \left( \frac{1}{p} \right)^{\gamma}, \quad 0 < \gamma < 1,
\]
the condition for dynamic inflexibility boils down to
\[
\frac{1}{\sigma} = \theta > \gamma.
\]
This condition is almost certainly satisfied in real world economies because most (but not all) empirical estimates of households’ intertemporal elasticity of substitution in consumption \( \sigma \) obtain values less than 1.\(^{15}\)

\(^{14}\) Note that Lemmas 1 and 2 hold for both dynamically flexible and dynamically inflexible economies.\(^{15}\) Most studies estimate values of \( \sigma \) significantly less than 1; these include Hall (1988), Dynan (1993), and Attanasio and Browning (1995). A few studies estimate values of \( \sigma \) greater than 1; these include Mulligan (2002), Gruber (2006), and Vissing-Jorgensen and Attanasio (2003).
III. Optimal bank capital requirement policy

In this section, we evaluate optimal bank capital requirement policy and the dynamic response of the economy in the presence of productivity (loan demand) shocks and loan write-offs (loan supply shocks). In subsection A, we consider unanticipated and permanent reductions in productivity. This exercise serves to illustrate how the model works and explain the transmission of shocks and capital requirement policies. The main purpose of this paper, however, is to evaluate cyclical changes in bank capital requirements and specifically whether they should be increased in anticipation of future recessions. For this reason, in subsection B we consider anticipated and temporary reductions in productivity and present the main results of this paper. Finally, in subsection C we evaluate bank capital requirement policy in the presence of loan write-offs.

A. Unanticipated and permanent reductions in productivity

The following proposition characterizes the optimal bank capital requirement policy in the presence of unanticipated and permanent reductions in productivity.

Proposition 1. If the economy is dynamically inflexible, the capital requirement should be increased temporarily in response to an unanticipated and permanent reduction in productivity.

See proof in the online Appendix. Figure 1 shows the time paths of selected variables, which are derived from Figure A1 and the relations established in section II. The capital requirement $\chi_t$ jumps up on impact (at $t = 0$) and decreases over

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16 The literature has documented the existence of unanticipated and sudden changes in productivity often associated with business cycles, and the more gradual and predictable changes in productivity associated with technological revolutions. Harberger (1998) refers to these alternative ways of achieving cost reductions as the “yeast versus mushroom” issue. He shows that total factor productivity (TFP) improvements tend to be somewhat unpredictable across sectors and for the aggregate economy. He shows that a “mushroom” process best describes the historical evolution of TFP growth in the US—where unanticipated improvements popped up in a fashion that was not easy to predict. More predictable “yeast” processes, however, also exist and are associated with technological revolutions. For an example, see Jorgenson et al. (2007) who document the contribution of information technology to the observed growth in US productivity.
time, returning to its steady state level in the long-run. Output $y_t$ jumps down on impact due to the discrete fall in productivity, and then decreases smoothly over time as the stock of bank loans declines toward its (lower) long-run level. Similarly, consumption $c_t$ and deposits $d_t$ jump down on impact and decrease smoothly toward lower steady state levels. The time paths of other variables are shown in the online Appendix.

Figure 1. Unanticipated and permanent reduction in productivity
Time paths of selected variables—dynamically inflexible economy

Intuitively, the economy’s response at $t = 0$ is as follows. The decline in productivity reduces the economy’s marginal productivity of capital, the wage rate, and all rates of return (on lending, deposit, and equity). Households’ adjust to lower permanent income and lower returns on savings by reducing current consumption and dissaving. Note that because this economy is dynamically inflexible, the permanent income effect associated with the shock (which lowers consumption)
dominates the intertemporal consumption substitution effect caused by the lower deposit and equity rates (which lowers saving and increases consumption).\textsuperscript{17}

The supply of deposits declines in tandem with consumption. As aggregate bank loans cannot change on impact, the discrete fall in deposits implies an equal increase in bank equity, from which it follows that the optimal capital-asset ratio increases.\textsuperscript{18} Banks boost capital funding and reduce deposit funding, and the equity-deposit spread narrows.

In sum, in this exercise the capital requirement must increase to accommodate the household’s optimal consumption-saving response to the permanent productivity shock. It plays this role in an environment in which banks’ capital structures are flexible and aggregate credit cannot be immediately adjusted. Such a policy response amplifies the output decline, but it is optimal from a welfare standpoint: it balances out the economy’s need to run down its capital stock and bank credit with the households’ distaste for consumption fluctuations.

\textbf{B. Anticipated and temporary reductions in productivity}

In this subsection we study optimal bank capital requirement policy in the presence of anticipated and temporary reductions in productivity. Proposition 2 summarizes the main result.

\textbf{Proposition 2.} \textit{In a dynamically inflexible economy, the optimal capital requirement policy in the presence of anticipated and temporary reductions in productivity is characterized as follows: (i) the capital requirement should be increased in anticipation of future temporary reductions in productivity; (ii) the capital requirement should be reduced at the onset of recessions. Under this policy, the economy exhibits lending and output booms prior to recessions.}

\textsuperscript{17} In contrast, in a dynamically flexible economy, households would tolerate larger consumption fluctuations over time. They would increase consumption in response to lower permanent income and returns (the intertemporal consumption substitution effect would dominate the income effect). The optimal capital requirement would fall on impact, and the economy would run down its capital stock at a faster pace. Note, however, that a positive consumption response to a negative income shock is empirically implausible.

\textsuperscript{18} The aggregate stock of bank loans cannot change on impact because, in equilibrium, it is equal to the aggregate stock of household’s assets (wealth), which is a predetermined, non-jumping variable (footnote 13).
See proof in the online Appendix. Figure 2 shows the time paths of selected variables. The capital requirement $x_t$ jumps up on impact (at $t = 0$) and increases over time in anticipation of the future decline in productivity, reaching its maximum level when the recession starts at time $t = T_1$. During the low productivity period, between times $t = T_1$ and $t = T_2$, the capital requirement falls gradually, reaching a minimum level at the end of the recession at time $t = T_2$. Thereafter, the capital requirement converges from below to its steady state value. The time paths of other variables are shown in the online Appendix.

Figure 2. Anticipated and temporary reduction in productivity
Time paths of selected variables—dynamically inflexible economy

Note that the economy exhibits a lending and output boom prior to the recession. Households adjust to lower future income by reducing consumption $c_t$ and deposits $d_t$ at time $t = 0$—when income has not yet fallen. This forward-looking increase in household saving fuels credit and output booms between times $t = 0$ and $t = T_1$ as the economy builds up bank capital buffers. Output jumps down
when the discrete fall in productivity hits the economy at time $t = T_1$. The stock of loans declines gradually between times $t = T_1$ and $t = T_2$, but the capital requirement policy limits the severity of the credit crunch. Once the low productivity period is over, output and the stock of bank loans increase gradually toward their steady state levels.

In sum, these results show the optimality of a cyclically sensitive bank capital requirement policy—one in which the capital requirement is tightened during expansions and loosened during recessions. In contrast to conventional views, however, the previous analysis suggests that this policy will not help mitigate output and credit booms that precede recessions and financial crisis episodes. The insight that we obtain from our general equilibrium analysis is that in order to build counter-cyclical bank capital buffers, the economy needs to increase savings in normal times (economic expansions) which in turn originate (or exacerbate) output and credit booms.19

C. Unanticipated reductions in loan supply

In this subsection we evaluate optimal bank capital requirement policy in the presence of loan supply shocks. The main result is described in the following proposition.

**Proposition 3.** In a dynamically inflexible economy, the capital requirement should be reduced temporarily in response to an unanticipated reduction in loan supply (loan write-offs).

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19 The case of an unanticipated and temporary reduction in productivity can also be analyzed with the help of Figure A2 in the online Appendix. In response to the shock, the economy jumps down vertically at time $0^+$ and the capital requirement rises upwards in one step. In contrast to the anticipated case, however, in the unanticipated case the economy starts moving in a north-west direction immediately: the stock of loans and the capital requirement decline over time until the end of the low-productivity period (indicated to occur at time $T_2$ in Figure A2). Visually, the dynamic behavior of the capital requirement can be illustrated with the help of Figure 2 if we ignore the time elapsed between $t = 0$ and $t = T_1$ (by collapsing both dates into a single point in the time line): after the shock, the dynamics of $x$ will be similar to the one shown in the figure after time $t = T_1$. 

See proof in the online Appendix. Figure 3 shows the time paths of selected variables. The capital requirement $x_t$ jumps down on impact and increases over time, whereas the amount of bank loans $l_t$ increases gradually after the initial shock as the economy returns to the steady state. Output $y_t$ jumps down on impact due to the discrete fall in loans and then rises monotonically toward its long-run level as the stock of loans is restored. Consumption $c_t$ and deposits $d_t$ jump down on impact and then increase gradually during the transition to the steady state. The time paths of other variables are shown in the online Appendix.

Figure 3. Unanticipated loan write-offs
Time paths of selected variables—dynamically inflexible economy

Intuitively, the adjustment process is as follows. *On impact*, as loans and output fall, the marginal productivity of loans and all the rates of return increase. The spike in returns and the lower wage income induce households to reduce consumption and deposits. The equity-deposit spread $r_t - r^d_t$ widens, and this in turn induces banks to finance their loans with less equity relative to deposits,
thereby reducing the capital-asset ratio. During the transition, the stock of loans is gradually rebuilt, output and the capital requirement rise while interest rates decline. Finally, consumption and deposits increase while the economy returns to the steady state.

IV. Conclusion

This paper addresses a fundamental economic and policy question. How should regulators set bank capital requirements in different phases of the business cycle? In particular, should such requirements be tightened during expansions and loosened during recessions, as a growing “macro-prudential” literature suggests?

These questions are addressed in a macroeconomic framework that has many advantages. On the one hand, the model’s microeconomic structure allows for meaningful deposit insurance and capital requirements. On the other hand, the model’s aggregate structure allows for an analytical solution and resembles a standard Ramsey neoclassical growth model—with the added twist that the physical capital stock is built with bank loans, which are in turn financed with households’ deposits and equity holdings.

This paper’s main contributions are twofold. First, it shows that a policy that tightens bank capital requirements in anticipation of future recessions and loosens such requirements at the onset of recessions is optimal in a dynamic macroeconomic framework. The general equilibrium approach used in this paper stresses the accommodating role of bank capital requirement policy to changes in household’s consumption-saving decisions triggered by exogenous macroeconomic shocks. Second, it shows that under the optimal policy, the economy exhibits lending and output booms prior to recessions. In contrast to conventional views, this result suggests that the policy cannot be used to lean against the expansionary phase of the business cycle. The reason is that the economy needs to boost savings during the economic expansion in order to build the counter-cyclical bank capital buffers. Higher savings, in turn, exacerbate credit and output booms.
References


