Online Appendix

to

ON THE OPTIMALITY OF BANK CAPITAL REQUIREMENT POLICY IN A MACROECONOMIC FRAMEWORK

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Proofs of the Lemmas and Propositions are presented here. The time paths of selected variables in response to the optimal bank capital requirement policy are also presented.

A. Proofs

Lemma 1. *The differential equation system defined by (25) and (26) for a constant productivity path* $A_t = A$*, when linearized around the steady state* (h^*, l^*)*, exhibits saddle-path stability*.

Proof. The linearization of the differential equations (25) and (26) around the steady state (h^*, l^*) for $A_t = A$ yields the following Jacobian matrix J^*:

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$$J^* = \begin{bmatrix} (\frac{\partial \dot{h}_t}{\partial h_t})_{h^*,l^*} & (\frac{\partial \dot{h}_t}{\partial l_t})_{h^*,l^*} \\ (\frac{\partial \dot{l}_t}{\partial h_t})_{h^*,l^*} & (\frac{\partial \dot{l}_t}{\partial l_t})_{h^*,l^*} \end{bmatrix}$$, where its elements are given by

$$(\frac{\partial \dot{h}_t}{\partial h_t})_{h^*,l^*} = h^* \cdot (1-\alpha \cdot \xi_x) > 0, \quad (\frac{\partial \dot{h}_t}{\partial l_t})_{h^*,l^*} = \frac{h^* \cdot A \cdot \left[\tilde{f}_{ll} - \frac{\theta}{l^*} \cdot (\tilde{f}_l - \frac{\tilde{f}}{l^*}) \right]}{\theta + \alpha \cdot \Delta_x \cdot h^*} \gtrless 0,$$

$$(\frac{\partial \dot{l}_t}{\partial h_t})_{h^*,l^*} = -l^* \cdot [1-\alpha \cdot \xi_x] < 0, \quad (\frac{\partial \dot{l}_t}{\partial l_t})_{h^*,l^*} = A \cdot (\tilde{f}_l - \frac{\tilde{f}}{l^*}) < 0 \cdot$$

The last inequality follows from the strict concavity of the function $\tilde{f}$ (the marginal product of bank loans is lower than the average product) and the steady state conditions. These conditions also imply that the determinant of the Jacobian matrix J^* is negative:

$$Det(J^*) = \frac{h^* \cdot A \cdot (1-\alpha \cdot \xi_x)}{\theta + \alpha \cdot \Delta_x \cdot h^*} \cdot \left\{ (\tilde{f}_l - \frac{\tilde{f}}{l^*}) \cdot \alpha \cdot \Delta_x \cdot h^* + l^* \cdot \tilde{f}_{ll} \right\} < 0 \cdot$$

Let δ_1, δ_2 denote the eigenvalues of the matrix J^* that solve the quadratic equation $\delta^2 - tr(J^*) \cdot \delta + Det(J^*) = 0$, where $tr(J^*)$ denotes the trace of J^*. As $Det(J^*) < 0$, the eigenvalues are of opposite signs. Hence, the solution of the dynamic system exhibits saddle-path stability.

Lemma 2. *If the production function is Cobb-Douglas, the following results are obtained in the comparative statics analysis of the steady state* (h^*, l^*) *: (i) the stock of bank loans* l^* *is increasing in the level of* *productivity, that is,* $\frac{\partial l^*}{\partial A} > 0$ *; (ii) the consumption-loan ratio* h^* *and the capital-asset ratio* x^* *are invariant to changes in productivity, that is,* $\frac{\partial h^*}{\partial A} = 0$ and $\frac{\partial x^*}{\partial A} = 0$.

Proof. The steady state equations $\dot{h}_t = 0 = G(h^*, l^*, A)$ and $\dot{l}_t = 0 = H(h^*, l^*, A)$ can be used to express h^* and l^* implicitly as functions of A. To evaluate the change of the steady state point $h^*(A)$, $l^*(A)$ when A changes, we must analyze the following system:

$$\begin{pmatrix} G_h & G_l \\ H_h & H_l \end{pmatrix} \cdot \begin{pmatrix} \dfrac{\partial h^*}{\partial A} \\ \dfrac{\partial l^*}{\partial A} \end{pmatrix} = \begin{pmatrix} -G_A \\ -H_A \end{pmatrix},$$

where G_h, G_l, H_h, H_l, G_A and H_A are evaluated at the initial steady state and are given by

$$G_h = (\frac{\partial \dot{h}_t}{\partial h_t})_{h^*,l^*},\ G_l = (\frac{\partial \dot{h}_t}{\partial l_t})_{h^*,l^*},\ H_h = (\frac{\partial \dot{l}_t}{\partial h_t})_{h^*,l^*},\ H_l = (\frac{\partial \dot{l}_t}{\partial l_t})_{h^*,l^*},\ G_A = \frac{h^* \cdot (\tilde{f}_l - \theta \cdot \frac{\tilde{f}}{l^*})}{\theta + \alpha \cdot \Delta_x \cdot h^*} \gtrless 0,\ H_A = \tilde{f} > 0.$$

Let $|M|$ denote the determinant associated with the matrix of partial derivatives of $G(.)$ and $H(.)$ with respect to h and l. It is straightforward to verify that $|M| < 0$ when evaluated at the steady state. Use Cramer's rule to obtain

$$\frac{\partial h^*}{\partial A} = \frac{-G_A \cdot H_l + G_l \cdot H_A}{|M|} = \frac{h^* \cdot A}{|M| \cdot \left(\theta + \alpha \cdot \Delta_x \cdot h^*\right)} \cdot \left\{ \tilde{f} \cdot \tilde{f}_{ll} - \tilde{f}_l \cdot (\tilde{f}_l - \frac{\tilde{f}}{l^*}) \right\} \gtrless 0,$$

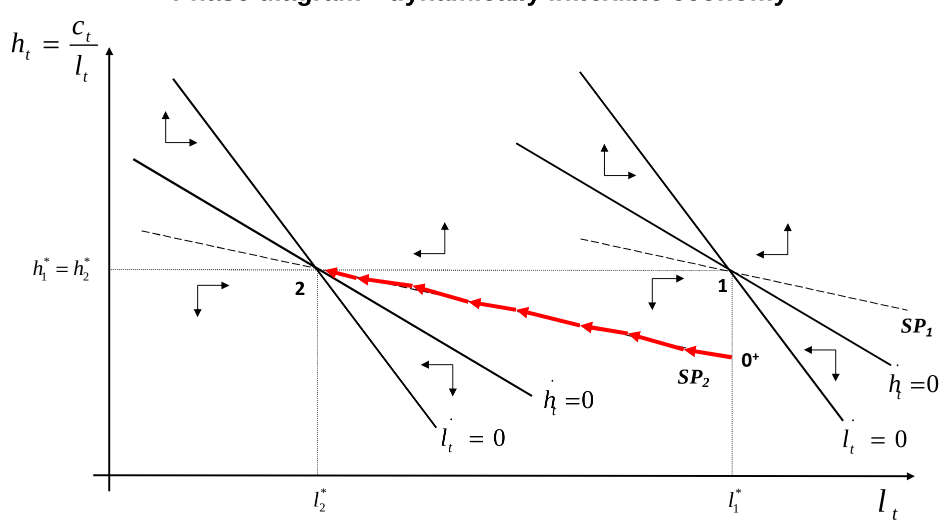
$$\frac{\partial l^*}{\partial A} = \frac{-G_h \cdot H_A + G_A \cdot H_h}{|M|} = \frac{-h^*}{|M|} \cdot \left\{ \tilde{f} \cdot (1-\theta) - \tilde{f} \cdot \alpha \cdot \xi_x + l^* \cdot \tilde{f}_l \right\} > 0.$$

Note that, if we consider the Cobb-Douglas production function $\tilde{f}(l,p) = l^{\gamma}\cdot(\frac{1}{p})^{1-\gamma}$, $0<\gamma<1$, it follows that

$\frac{\partial h^*}{\partial A} = 0$. As $x_t = 1-\alpha\cdot h_t$, it also follows that $\frac{\partial x^*}{\partial A} = 0$.

Figure A1 shows the phase diagram corresponding to an economy which is defined as "dynamically *inflexible*." Along a perfect foresight equilibrium path with constant productivity ($A_t = A_1$), and for any arbitrary initial level of bank loans l_0, the economy determines the initial value of h_t at the corresponding point on the saddle path (SP_1). Then, the economy travels over time along the saddle path until it converges to the steady state 1. Note that in this model h_t is a jumping variable, whereas l_t is predetermined.[1]

Figure A1. Unanticipated and permanent reduction in productivity
Phase diagram—dynamically inflexible economy



[1] We justify our modeling of l_t as a non-jumping variable as follows. Typically, banks hold liquid assets as well as long-maturity loans, which, to a large extent, cannot be liquidated or extended further immediately after the realization of shocks. Thus, we interpret that, at every instant, the stock of bank loans is predetermined. This interpretation, in turn, allows us to simplify our analysis by ignoring the liquid assets that banks typically hold.

Proposition 1. *If the economy is dynamically inflexible, the capital requirement should be increased temporarily in response to an unanticipated and permanent reduction in productivity.*

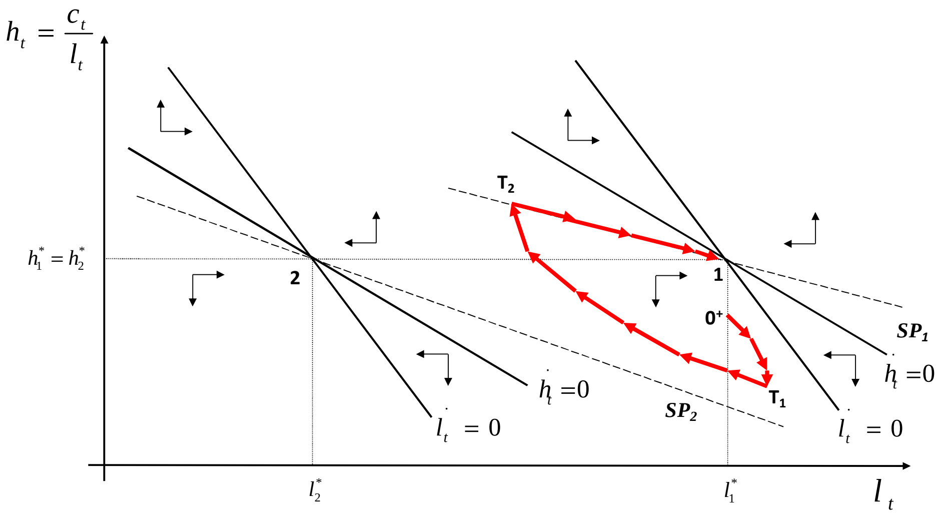
Proposition 1 can be proved as follows. Suppose that the economy is initially at steady state 1 in Figure A1. Consider an unanticipated and permanent reduction in productivity A_t at $t = 0$, from the initial level A_1 to the new level A_2 $(A_2 < A_1)$. Steady state 2 corresponds to the new permanent value of productivity. Immediately after the shock, at time $t = 0^+$, the consumption-loan ratio h_t jumps down to intersect the saddle path SP_2. Thereafter, the economy travels along the saddle path SP_2 until it converges to steady state 2. During this transitional dynamics, the consumption-loan ratio h_t increases while the stock of bank loans l_t decreases.

Proposition 2. *In a dynamically inflexible economy, the optimal capital requirement policy in the presence of anticipated and temporary reductions in productivity is characterized as follows: (i) the capital requirement should be increased in anticipation of future temporary reductions in productivity; (ii) the capital requirement should be reduced at the onset of recessions. Under this policy, the economy exhibits lending and output booms prior to recessions.*

Proposition 2 can be proved as follows. Consider an anticipated and temporary reduction in productivity A_t in an economy which is initially at steady state 1 in Figure A2. All agents learn at time $t = 0$ that at a future date $t = T_1$ the productivity parameter A_t will decrease temporarily from A_1 to A_2 $(A_2 < A_1)$ and then return to the initial level A_1 at time $t = T_2$. Steady states 1 and 2 correspond to productivity levels A_1 and A_2, respectively. On impact, the economy jumps down to a point such as 0+, and then follows the indicated trajectory.[2]

[2] Driven by the corresponding laws of motion, the economy evolves over time so as to converge at time $t = T_1$ on a dynamic trajectory that is located above the saddle path SP_2. During the low productivity period between times $t = T_1$ and $t = T_2$, the economy travels along this unique trajectory that converges to the saddle path associated with steady state 1 (SP_1) exactly at time $t = T_2$. Note that as changes in productivity have been fully anticipated, the consumption-loan ratio cannot jump at times $t = T_1$ or $t = T_2$.

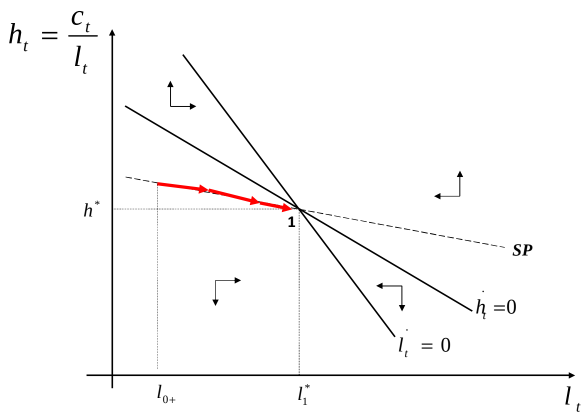
Figure A2. Anticipated and temporary reduction in productivity
Phase diagram—dynamically inflexible economy



Proposition 3. *In a dynamically inflexible economy, the capital requirement should be reduced temporarily in response to an unanticipated reduction in loan supply (loan write-offs).*

Proposition 3 can be proved as follows. The economy is initially at the steady state shown in Figure A3. At $t=0$, an unanticipated and negative loan supply shock that reflects loan write-offs reduces the household's assets and the stock of loans from l_1^* to l_{0+}. The consumption-loan ratio h_t jumps up to intersect the corresponding saddle path SP. During the transitional dynamics, the economy travels along the saddle path, converging toward the steady state in the long-run.

Figure A3. Unanticipated loan write-offs
Phase diagram—dynamically inflexible economy



B. Optimal bank capital requirement policy: time paths of selected variables

Figure 1 (continued). Unanticipated and permanent reduction in productivity
Time paths of selected variables—dynamically inflexible economy

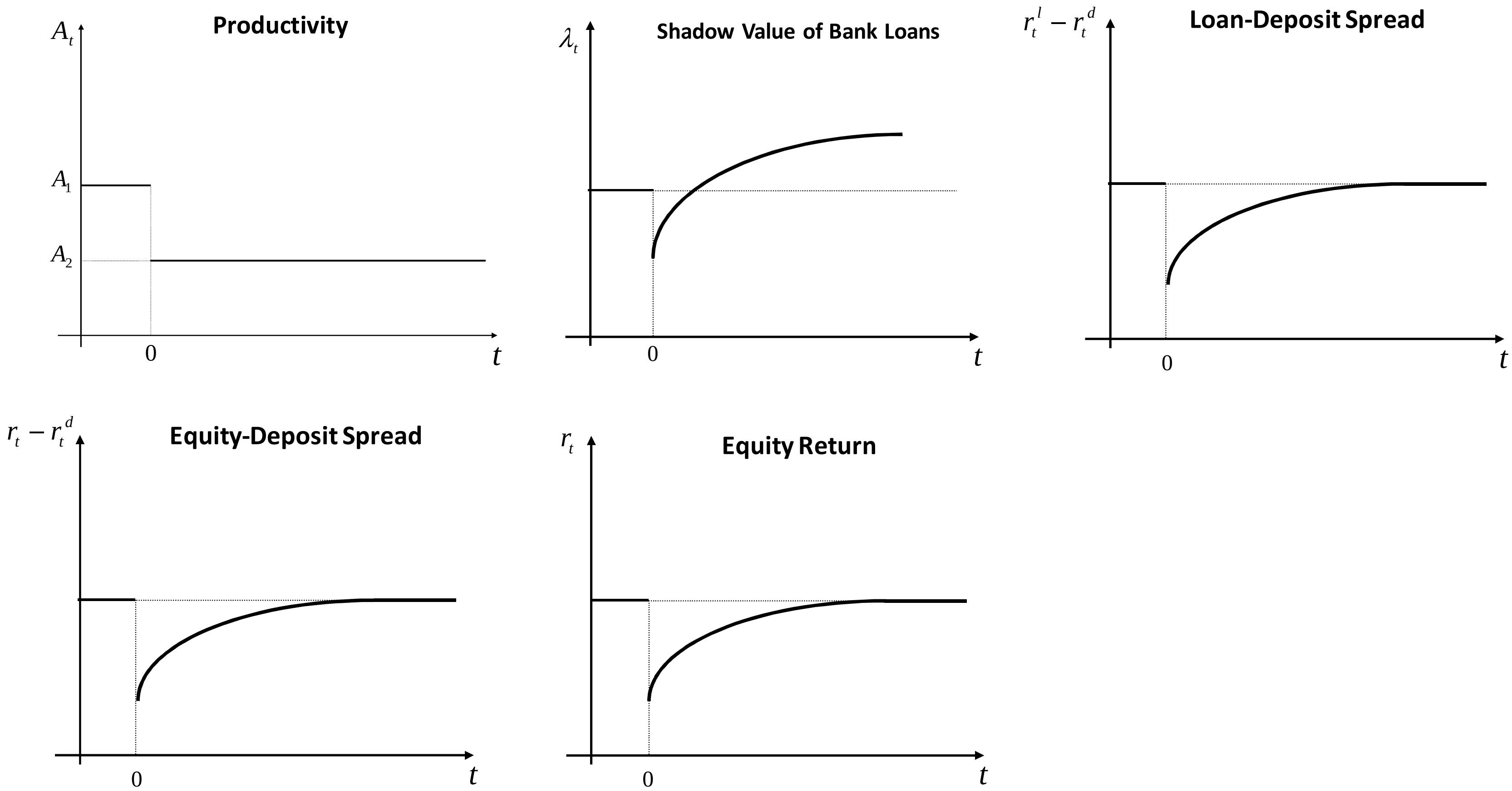


Figure 2 (continued). Anticipated and temporary reduction in productivity
Time paths of selected variables—dynamically inflexible economy

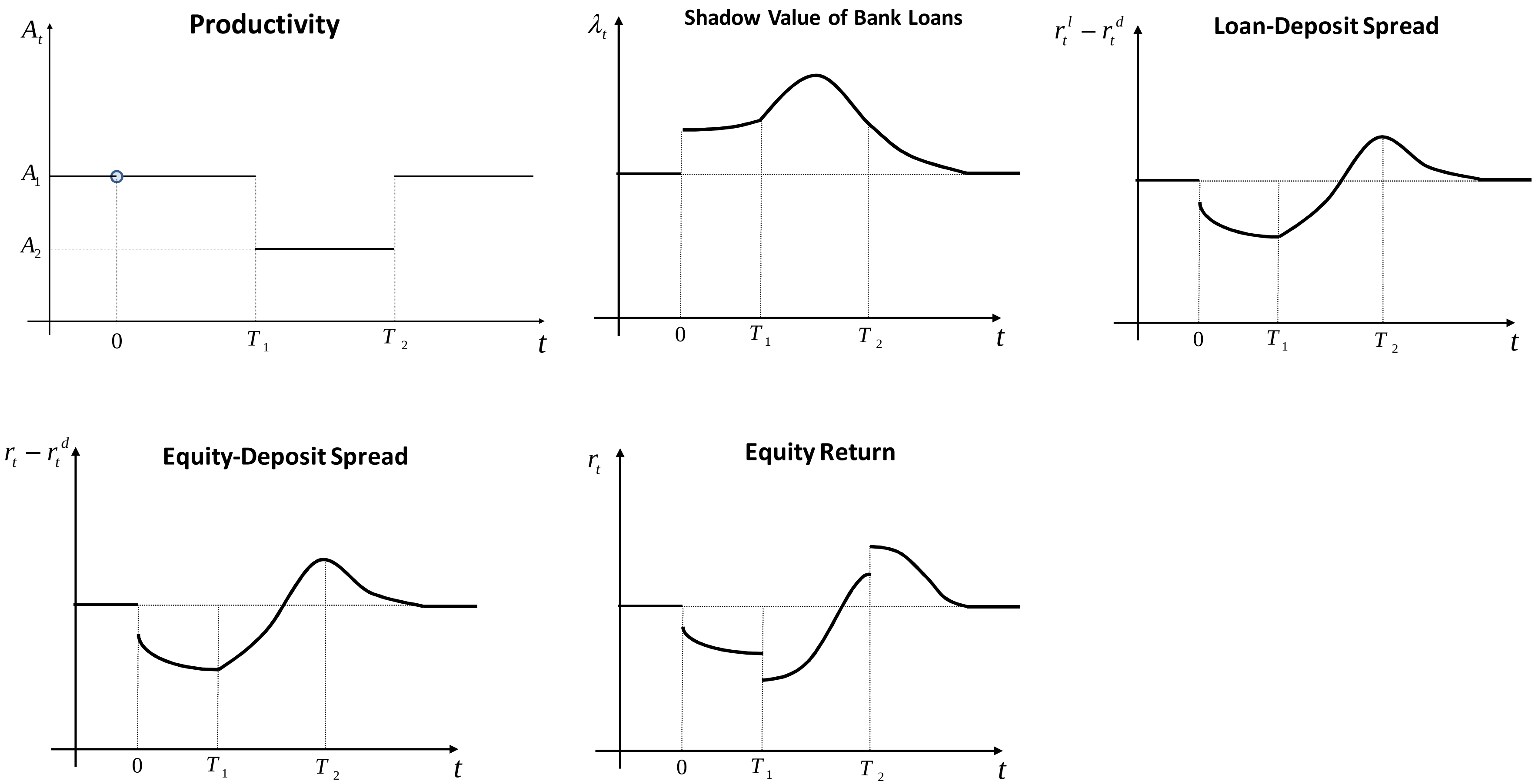


Figure 3 (continued). (Unanticipated) loan write-offs
Time paths of selected variables—dynamically inflexible economy