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STATE DEPENDENCE OF AGGREGATED RISK AVERSION: EVIDENCE FOR THE GERMAN STOCK MARKET

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We propose a dynamic generalization of the Capital Asset Pricing Model (CAPM) that allows for a time-varying market price of risk (MPR) reflecting both cross market dependence and future investment opportunities. The realized volatility approach is employed to determine market risk. The advocated state space model takes autoregressive dynamics of the MPR and predetermined state variables into account. For the case of the DAX, the major German stock index, the empirical analysis strongly underpins time variation of risk compensation. The MPR is conditioned upon the EURIBOR, a national and an international term spread, returns of the Dow-Jones-Industrial-Average-Index (DOW), and a dummy variable hinting at excess activity of noise traders. Moreover, we document forecasting results based on a short horizon trading strategy. The proposed model is characterized by strong market timing ability.

JEL classification codes: C53, C12, C22, G12
Key words: risk-return trade-off, stock return predictability, noise trading, realized volatility, Kalman-filter

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I. Introduction

The risk-return relation raises controversy in the empirical financial literature. Guided by the implications of the intertemporal CAPM of Merton (1973) financial economists agree that risk taking should yield an expected gain. While some empirical studies confirm a significantly positive risk-return trade-off (Goyal and Santa-Clara 2003; Ghysels et al. 2005; Lundblad 2007), numerous empirical contributions, however, fail to confirm this conjecture (among others, French et al. 1987; Campbell and Hentschel 1992; Bali et al. 2005). Most surprisingly, Campbell (1987) and Nelson (1991) document a negative relation.[1]

An eventually negative market price of risk (MPR) supports the perspective that there are market conditions where even risk-averse investors regard stronger market fluctuations as a chance. Theoretical models introduced by Abel (1988) or Backus and Gregory (1993) show that a transitorily negative risk-return relation could be consistent with an investors' dynamic decision process. Based on a finite-state Markov chain, Backus and Gregory (1993) formulate a dynamic asset-pricing framework and show by means of a numerical example that the relation between risk premia and conditional variances might be negative even under risk aversion. Under capital mobility, the international landscape of risk, financing costs and (future) investment opportunities are likely to impact on the domestic MPR. Depending on the state of the economy, variance measures have to be either interpreted as risk or chance. Thus, the MPR is most likely state dependent and, thus, time-varying.

Time variation of the MPR is captured by Chou et al. (1992) who formalize it to follow a random walk. Hafner and Herwartz (1998) model the MPR conditional on lagged squared GARCH innovations. Employing data from 1836 to 2003 Lundblad (2007) analyzes the risk-return trade-off from a historical perspective. He argues that the mixed evidence with respect to the sign of the MPR might be caused by small sample inference or state dependence. Financial markets and the economy are characterized by structural changes over these two centuries. At business cycle frequencies Campbell and Cochrane (1999) conclude from a

[1] Among others, Glosten et al. (1993) provide evidence for an either positive or negative risk-return relation depending on the modeling approach. For a detailed overview see Ghysels et al. (2005).

theoretical model with habit formation that the MPR moves countercyclically. Empirically, Lundblad (2007) finds a procyclical MPR while results in Yu and Yuan (2011) suggest that the MPR does not respond to business cycle information.

Beside fundamental effects, a supposedly time-varying MPR might be induced by beliefs of heterogeneous investors. In various theoretical approaches (e.g., De Long et al. 1990; Lux 1997) stylized facts or puzzles of asset returns have been addressed by modeling so-called noise trader effects. As a particular result of this literature, the belief that rational investors set off the influence of noise traders by means of suitable trading strategies has been weakened. On the empirical side, some contributions uncover noise trader impacts on the time structure of asset returns or their volatility. Investigating the influence of feedback traders, Bohl and Reitz (2006) conclude for German stock market data that in high volatility periods feedback traders cause negative return autocorrelations. Verma and Verma (2007) analyze excess US stock returns and sentiments of both institutional and individual investors by means of a vector autoregressive exponential GARCH model. They diagnose a significant impact of investors' decision errors (noise trader effects) on stock market volatility.

In light of the foregoing considerations, it is natural to follow De Long et al. (1990) who point to the coincidence of both fundamental and noise trader implied risk. Noise trader implied volatility is less predictable than its fundamental counterpart. An increasing fraction of the former should command a higher risk compensation demanded by rational investors. At the aggregate level, this effect, however, could be counteracted by a decreasing fraction of rational investors in the market. Thus, the net market compensation for risk taking might decrease in response to more prevalent noise trading. Empirically, Verma and Soydemir (2009) measure investor sentiments from survey data. Analyzing the relation between sentiments and the MPR in a VAR framework they conclude that strong sentiments cause a negative reaction of the MPR. Moreover, Yu and Yuan (2011) emphasize in a two regime model that a generally positive MPR could be offset by noise trader activity. In summary, however, evidence on the influence of noise traders and of the composition of market participants on the risk-return relation is scarce.[2]

[2] For a detailed literature review see Verma and Soydemir (2009).

Apart from the underlying economic considerations and the potential of structural change, the divergent empirical evidence for the risk-return relationship might also reflect competing approaches to quantify the latent market risk (Ghysels et al. 2005). GARCH-in-mean models have been widely applied to implement dynamic generalizations of the CAPM. For instance, Chou et al. (1992) apply a GARCH-in-mean model to study the monthly risk-return relation for US stock market data. In contrast, Bali and Peng (2006) and Guo (2006) underpin the merits of realized volatility measures (see, e.g., Barndorff-Nielsen and Shephard 2002) for the determination of the risk-return relation.

In this paper we adopt a flexible state-space model that allows for a time varying MPR. Moreover, market risk is quantified with negligible estimation error by means of realized volatility estimates. As a particular merit of the proposed model, we allow several potential determinants to impact jointly on the MPR while the related literature has mainly focussed on single determinants of the risk return relationship (Lundblad 2007; Yu and Yuan 2011). The model nests four alternative representations. The random walk model as advocated in Chou et al. (1992), a more structural model in line with Hafner and Herwartz (1998), a first order autoregressive representation and a traditional formulation with a constant MPR. Moreover, the MPR is conditioned upon measurable predetermined economic states and an indicator of (excess) noise trader activity. To preview some results, we find strong evidence in favor of time variation of the MPR for daily excess returns of the DAX, the major German stock index, over the period from March 1998 to June 2007. In the most preferred specification we find a significant impact of an international term spread. The domestic term spread approximating expected future investment opportunities exerts an insignificantly negative impact on time varying risk compensation. Furthermore, a short term (riskless) interest rate and log-returns of the Dow-Jones-Industrial-Average-Index (DOW) are found to govern the MPR. A constructed measure of noise traders' prevalence is found to diminish significantly the risk-return relationship. Exploiting the time dependence of the MPR, the advocated state space model is characterized by a strong ability of both ex-post and ex-ante market timing.

In Section II the empirical model and the measurement of the latent volatility is outlined. Section III provides a brief description of the data and a discussion of processes that potentially determine the risk return relationship. In Section IV a best performing model specification is selected based on log-likelihood statistics and diagnostics of market timing ability. In Section V we analyze state dependence

of the MPR in light of this particular model. Section VI summarizes the main findings and concludes.

II. Time variation of the risk-return relationship

Our empirical model follows De Long et al. (1990) in regarding market risk as an aggregate of fundamental and noise trading implied risk. Moreover, in light of the mixed evidence on the sign of risk compensation we investigate the scope of predetermined macroeconomic state variables in explaining the MPR. Doing so we refer to these state variables as indicators of present and future investment opportunities and determinants of cross market performance (Backus and Gregory 1993).

At the implementation side of the model we notice that the MPR is latent and, accordingly, employ a flexible state-space representation. Moreover, market risk is determined by means of the nonparametric but consistent realized volatility estimator. Exploiting ultra high frequency quotes to determine market risk, we expect respective approximation errors to be (almost) negligible. Next, we introduce the state-space model and motivate the state dependence of the MPR in more detail.

A. Model design

Let y_t denote the market return in excess of the expected riskless interest rate. The state-space model is given by

$$y_t = \alpha + \lambda_t \sigma_{t|t-1} + \varepsilon_t, \varepsilon_t | \Omega_{\mathrm{t}-1} \sim N(0, h_t)\,, \tag{1}$$

$$h_t = \gamma_0 + \big(\gamma_1 + \gamma_2 \mathrm{I}(\varepsilon_{t-1} < 0)\big)\varepsilon_{t-1}^2 + \gamma_3 h_{t-1}\,, \tag{2}$$

$$\lambda_t = \beta_0 + \rho\lambda_{t-1} + \boldsymbol{x}_{t-1}^{'}\boldsymbol{\beta} + \upsilon_t, |\rho| < 1,\ \upsilon_t \backsim N(0, g), t = 1, \dots, T. \tag{3}$$

In the measurement equation (1) the excess return is determined by the conditional market volatility (Merton 1973).[3] We quantify this latent market risk by means of realized volatility estimates. For quasi maximum likelihood (QML) estimation, disturbances ε_t conditioned on information available at period $t-1$, Ω_{t-1}, are presumed normally distributed with mean zero and variance h_t. This variance process follows by assumption a threshold GARCH(1,1) (TGARCH(1,1)) specification as formalized in (2), with I(·) denoting an indicator function.

While commonly applied GARCH-in-mean approaches offer at best an unbiased approximation of market risk, the consistency of the realized volatility estimator implies that daily risk becomes 'observable' in the limit. Moreover, it is noteworthy that realized volatility estimates are essentially nonparametric and, opposite to GARCH counterparts, do not suffer from time variation of second order return heterogeneity. We use realized standard deviations, i.e., $\sigma_t = \sqrt{\sum_{m=1}^{M} r_{m,t}^2}$, where $r_{m,t}$ are intra-daily log returns, measured at a frequency of 15 sec.[4] For this particularly high frequency microstructure issues (Bandi and Russell 2006) can be discarded at the index level, while the precision of risk estimates increases with the intradaily frequency. Given the strong persistence of realized volatility estimates (Andersen et al. 2001) we follow Bali and Peng (2006) and employ the lagged realized volatility σ_{t-1} to approximate the expected risk $\sigma_{t|t-1}$ in (1).[5] Moreover, from an econometric perspective, the use of the lagged instead of the contemporaneous realized volatility in (1) avoids endogeneity issues.[6]

The TGARCH specification in (2) is employed since Glosten et al. (1993) and Lundblad (2007) emphasize the importance of asymmetric effects of positive and negative return innovations to the conditional volatility process h_t. While it has become a common practice to diagnose so-called leverage effects by means of TGARCH models, it is noteworthy that consistent realized variance estimates also capture leveraged responses to lagged stock returns (Andersen et al. 2001).

[3] It is noteworthy that the return representation in (1) could be interpreted as a discrete time approximation of the continuous semimartingale model (Back 1991) comprising time varying drift parameters (of sufficiently small order).

[4] Intraday returns are provided by the Deutsche Kapitalmarktdatenbank, Karlsruhe.

[5] For a more detailed justification of this approximation the reader may consult Bali and Peng (2006).

[6] Note that y_t almost equals the sum of intraday returns while the realized variance is the sum of squared intraday returns.

B. Time variation of the MPR

For the discovery of time varying determinants of risk compensation the models' state equation (3) is of core importance. It nests a number of special cases. Conditional on Ω_{t-1}, it comprises a zero ($\beta_0 = \rho = g = 0, \boldsymbol{\beta} = 0$), and constant ($\beta_0 \neq 0, \rho = g = 0,\ \boldsymbol{\beta} = 0$) MPR, random walk dynamics ($\rho = 1, \beta_0 = 0,\ \beta = 0$), or stationary autoregressive patterns ($|\rho| < 1, \beta_0 \neq 0, \boldsymbol{\beta} = 0$). The most general specification is obtained by conditioning the MPR additionally on economic state variables $\boldsymbol{x}_{t-1}$ that are thought to indicate (expected) investment opportunities and the markets risk attitude. Section III will be more explicit on the composition of $\boldsymbol{x}_{t-1}$. It is worthwhile to mention that the model allows for an expected risk compensation of either sign ($\beta_0 + \rho\lambda_{t-1} + \boldsymbol{x}'_{t-1}\boldsymbol{\beta} \lesseqgtr 0$). The disturbance terms that v_t enter the state equation are presumed to be normally distributed with mean zero and variance g.

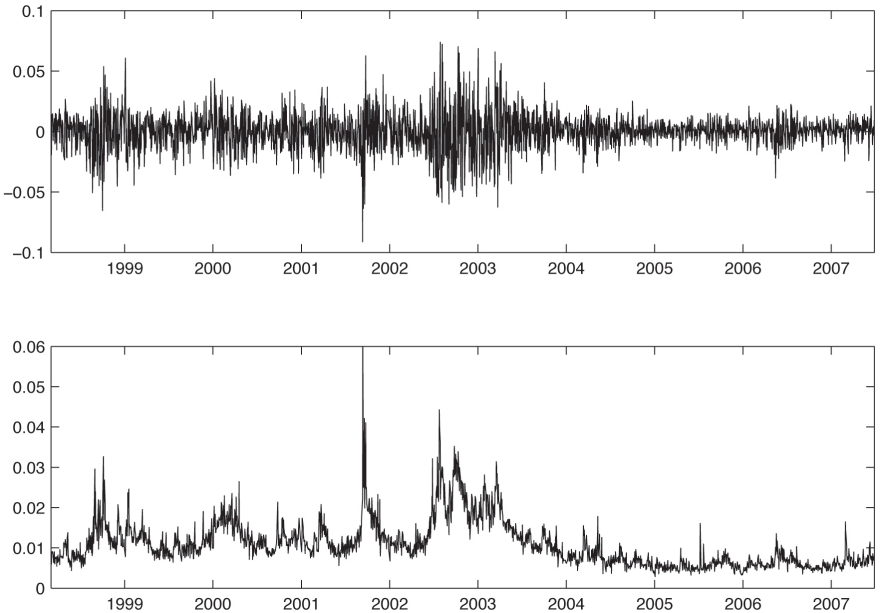
III. Data and state indicators

We investigate dynamic features of DAX returns over the period March 2, 1998 to June 29, 2007 covering 2365 daily observations.[7] Daily excess returns of the DAX are measured as $y_t = r_t - is_{t-1}^{EU}$, where r_t and is_{t-1}^{EU} denote the DAX log-return and the (lagged) EURIBOR rate (the FIBOR until December 31st 1998) with three month maturity, respectively.

Figure 1 shows the time path of DAX returns and realized standard deviations. Estimates of market risk reflect volatility clustering as a stylized second order moment feature of speculative price processes. The first order autocorrelation coefficient of σ_t is 0.89, which supports the use of σ_{t-1} as an approximation of the conditional market risk. The first half of the observation period is characterized by a relatively high volatility with strong peaks occurring in September 2001 and during 2002. Beginning in 2004 we observe a reduction of the unconditional return volatility.

[7] Owing to marked structural shifts of financial variables (term spreads, monetary policy rates) we exclude more recent data from the analysis. State dependence of the MPR during the current financial crisis is, however, an interesting direction for further research.

Figure 1. Excess returns on the DAX and realized volatility estimates



Descriptive statistics of the (log) realized volatility $(\ln(\sigma_t))$ and standardized DAX returns (r_t/σ_t) are given in Table 1. Similar to Andersen et al. (2001) we find that realized standard deviations are approximately log-normally distributed. The unconditional kurtosis and skewness of $\ln(\sigma_t)$ are 2.65 and 0.29, respectively.

Table 1. Descriptive statistics

	Mean	Volatility	Skewness	Kurtosis
σ_t	0.011	0.006	1.805	8.982
$\ln(\sigma_t)$	-4.650	0.448	0.286	2.650
r_t/σ_t	0.044	1.230	-0.029	2.876

Notes: Moments of the (log) realized standard deviations and standardized daily returns.

To analyze time variation of the MPR, subsequently, we introduce some observable indicators and a dummy variable hinting at excess presence of noise trading implied risk. While state dependence of the MPR has been investigated at the monthly frequency so far (Lundblad 2007; Yu and Yuan 2011), we focus on a daily frequency. As a potentially important determinant noise trader activity might explain movements of the MPR at higher frequencies (Tetlock 2007).

A. Observable state indicators

Backus and Gregory (1993) identify general and future economic conditions that impact on the MPR such that the effective direction of the risk-return relationship is state dependent. Apart from economic conditions one may conjecture that time variation of the MPR stems from changes in the risk attitudes of an representative investor, or, more realistic, from changes of the average market risk attitude (ARA). Note that the latter reflects the actual composition of active market participants which is likely unstable over time. As potential sources of ARA variation one may consider time dependence of long-run perceptions of cross market performance or of future prospects of excess earnings. Such dynamics are likely to impact on the composition of the rational, i.e., risk averse, market participants. The worse the expectation of the future development of the domestic stock market in comparison with international or fixed income opportunities, the higher is the likelihood of market exit for the most risk averse investors. As a consequence, ARA is likely to shrink and, thus, might command a reduction of the equilibrium risk premium.[8]

Regarding the state equation in (3) a number of observable variables are collected in $\boldsymbol{x}_{t-1}$ for conditioning the MPR. To approximate local and international investment opportunities and market prospects the set of empirical measures comprises stock market and fixed income time series characterizing the German (European) and US market. The financial literature underpins the role of US markets as a major force influencing international stock markets (Ramchand and Susmel 1998). Explicitly, the covariates in $\boldsymbol{x}_{t-1}$ comprise[9]

1. The log-returns of the DOW Jones Industrial Average index, $x_{t-1,1} = r_{t-1}^{DOW}$.
2. The ('riskless') EURIBOR rate, $x_{t-1,2} = is_{t-1}^{EU}$.
3. A domestic term spread, $x_{t-1,3} = il_{t-1}^{EU} - is_{t-1}^{EU}$, where il_{t-1}^{EU} is the rate of a German government bond with maturity of 10 years.

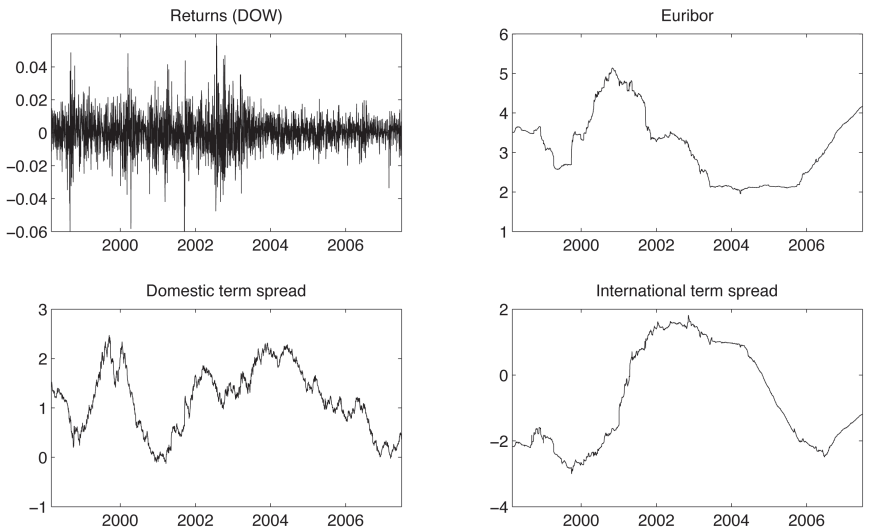
[8] Taking a historic perspective, Lundblad (2007) analyses economic conditions as potential long and medium run triggers of the MPR. He finds a recession indicator, a measure of the overall size of the equity market, external trade and government spending to govern the MPR during the 19th and 20th century. Our interest is, however, more on medium to short run fluctuations of the MPR which can be explained by changes of the ARA. Consequently, we employ state indicators that are comparable to those used in Yu and Yuan (2011).

[9] Throughout, time series with daily frequency are drawn from DATASTREAM.

4. An international term spread, is $x_{t-1,4} = is_{t-1}^{EU} - is_{t-1}^{US}$, where is_{t-1}^{US} is the US interbank rate with maturity of three month.

The financial processes $x_{t-1,j}$, for j=1,...,4, are displayed in Figure 2. In the light of the time series properties of these processes they can be classified as showing patterns of stronger or weaker persistence. While the former might signal medium term economic prospects, the latter are more likely to invoke transitory dynamics of the MPR.

Figure 2. Time paths of daily financial and monetary processes



B. Noise trader implied risk

Apart from observable state indicators, transitory dynamics that stem from the effective market participation of (less risk averse or even risk neutral or risk loving) noise traders can be seen as potential sources of ARA variation. A priori one could expect both a positive or a negative effect of noise trader participation on the MPR. The first direction refers to the conjecture that rational investors demand a higher risk premium if they become aware of the increasing participation of noise traders in the market. The second direction reflects that the MPR is a weighted average price over all market activities. The average price of risk should decrease if more traders act without consideration of risk.

Since noise trader activity is not observable, we construct a dummy variable which is supposed to identify periods of higher noise trader participation in the stock market. A similar approach is followed by Yu and Yuan (2011) who construct a monthly dummy variable based on the annual composite sentiment index of Baker and Wurgler (2006).[10] However, we are interested in market conditions at a daily frequency and, therefore, we do not rely on the composite sentiment index. The proposed noise trading indicator builds upon the presumption that in periods where fundamental risk is mainly responsible for the state of market risk, noise traders have likely reduced their activity. To be specific, we evaluate the following sequence of rolling regressions over time windows that comprise Δ trading days:

$$\ln \sigma_\tau = c + \ln \boldsymbol{z}_{\tau-1} \boldsymbol{\phi} + \zeta_\tau, \tau = t - \Delta, \dots, t. \tag{4}$$

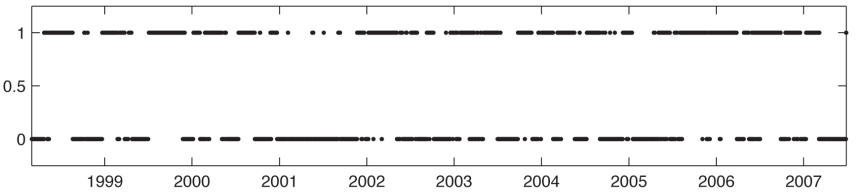
In explaining (log) market risk, $\boldsymbol{z}_{\tau-1}$ comprises absolute changes of the fixed income rates, i.e., $is_{\tau-1}^{EU}, il_{\tau-1}^{EU}, is_{\tau-1}^{US}$, absolute returns of the DOW Jones, $|r_{\tau-1}^{DOW}|$, and absolute changes of the EURO/US-Dollar log exchange rate.

From the regression model in (4) we take the degree of explanation and expect this quantity to be relatively small if the contribution of noise traders to market risk is high. After implementing the auxiliary regressions over rolling windows of size $\Delta = 60$ we evaluate the empirical distribution of the degrees of explanation $\{R_t^2\}_{t=\Delta+1}^T$.[11] The median of this distribution is taken to separate days of relatively weak and strong market participation of noise traders. Thus, high contributions of noise trader risk are supposed for trading days for which $NT_t = I\left(R_t^2 < \text{median}\ (\{R_t^2\}_{t=\Delta+1}^T)\right) = 1$. Accordingly, $NT_t = 0$ indicates trading days when market risk was mostly characterized by fundamental factors. For entering the state equation in (3) we set $x_{t-1,5} = NT_{t-1}$. The binary process is displayed in Figure 3.

[10] The composite sentiment index of Baker and Wurgler (2006) is given by the first principal component of six variables which are likely related to investors sentiments: closed-end fund discount, NYSE share turnover, number of IPOs, average first-day return of IPOs, equity share in new issues and dividend premium.

[11] To initialize the regression we employ pre-sample data such that the noise trader measure starts at March 2, 1998.

Figure 3. Time path of noise trader dummy



Notes: A value of one indicates stronger prevalence of noise trading implied risk (NT_t = 1).

Apparently, excess noise trading is clustering in the sense that the measure identifies a couple of time spans where risk is mostly determined fundamentally. For instance, for the majority of trading days in 2001 we find that fundamental factors are most relevant for stock market risks, while in the second half of 1999 noisy components turn out to dominate market risk.

IV. Empirical analysis

The Kalman-Filter is employed to estimate the parameters of the state-space model in (1) to (3), $\boldsymbol{\psi} = (\alpha_0, \rho, \beta_0, \beta', g, \boldsymbol{\gamma}')'$, $\boldsymbol{\gamma} = (\gamma_0, \gamma_1, \gamma_2, \gamma_3)'$, by means of (quasi) maximum likelihood techniques. It is noteworthy that the state space model allows estimating all parameters in $\boldsymbol{\psi}$ simultaneously, and, thus, it should offer efficiency gains in comparison with sequential approaches (Whitelaw 1994; Ludvigson and Ng 2007).

For the purpose of model selection, four alternative models nested in the general state space representation are discussed. The constant MPR specification is compared with models characterized by dynamic MPRs. We rely on both, common model diagnostics and statistics that describe market timing ability. Market timing ability is assessed alternatively by means of in-sample diagnostics and rolling window model implementations that offer an ex-ante perspective. Conditional on the most preferable model, the dynamic features of the MPR will be discussed in Section V.

A. Model selection

Before considering the estimation results in detail it has to be noted that the TGARCH parameter estimates of (2) are almost invariant across alternative specifications of the conditional mean model. This underpins the coherence of the

considered model specifications. For instance, employing the most general state representation to formalize the conditional mean yields the following estimated TGARCH variance equation (t-ratios below coefficient estimates):

$$h_t = \underbrace{6.77E-7}_{(2.25)} + \left(\underbrace{4.92E-4}_{(0.06)} + \underbrace{0.07}_{(4.42)} I_{\varepsilon_{t-1}<0} \right) \varepsilon_{t-1}^2 + \underbrace{0.927}_{(66.77)} h_{t-1}. \tag{5}$$

Emphasizing the findings of Glosten et al. (1993) it turns out that at conventional significance levels only squared negative lagged returns impact on the variance process.[12]

Further estimation and diagnostic results are documented in Table 2. The model formalizing a constant MPR achieves a log-likelihood of 7052.06. Introducing time-variation of the MPR, the random walk formulation indicates an insignificant decline of the log-likelihood statistic. In contrast, the purely autoregressive specification and the model conditioning on the explanatory content of predetermined exogenous variables (unrestricted model) show strong significance according to the respective LR statistics (19.33 and 58.20).[13] Moreover, the comparison of the purely autoregressive representation with the fully conditional model underpins joint significance of the predetermined state variables. Thus, based on log-likelihood statistics we conclude that the model conditioning simultaneously on stationary autoregressive patterns of the MPR, macroeconomic processes and an indicator of risk emergence fits the data most appropriately. Next we evaluate rival specifications of the proposed state space model in terms of market timing ability.

[12] It is noteworthy that the unconditional features of DAX returns standardized by the nonparametric realized volatility estimates ($r_t \,/\, \sigma_t$) in Table 1 come closer to the Gaussian distribution in comparison with estimated TGARCH innovations ($\hat{\varepsilon}_t \,/\, \sqrt{\hat{h}_t}$). Corresponding moments of the latter are: mean -0.004, variance 0.999, skewness -0.209, kurtosis 3.571 (see also Andersen et al. 2001).

[13] The Jarque-Bera statistic hints at a violation of the conditional normality assumption. However, the (quasi) log-likelihood improvement offered by the fully conditional model appears sufficiently strong such that the LR statistics are likely significant under misspecification of the log likelihood function.

Table 2. Parameter estimates

	All	AR	Constant	RW
Conditional mean equation (1)				
$\hat{\alpha}$	−5.37E-4 *(−0.90)*	2.32E-04 *(−0.50)*	2.44E-05 *(0.04)*	5.75E-05 *(0.24)*
$\hat{\lambda}$	-	-	0.014 *(0.23)*	-
MPR determinants equation (3)				
Intercept	0.438 *(1.62)*	-0.023 *(-0.40)*	-	-
AR	-0.078 *(-1.72)*	-0.019 *(-0.33)*		1.000
r_{t-1}^{DOW}	7.330 *(4.02)*	-	-	-
is_{t-1}^{EU}	-0.096 *(-1.88)*	-	-	-
$il_{t-1}^{EU} - is_{t-1}^{EU}$	-0.028 *(-0.44)*	-	-	-
$is_{t-1}^{EU} - is_{t-1}^{US}$	-0.040 *(-1.95)*	-	-	-
NT_{t-1}	-0.121 (-2.26)	-	-	-
g	0.794 *(14.00)*	0.780 *(11.00)*	-	5.46E-07 *(4.6E-4)*
GI	2.080 *(0.00)*	0.814 *(0.02)*	−0.472 *(-)*	−0.472 *(-)*
GO	0.921 *(0.02)*	-0.442 *(0.88)*	−0.334 *(0.70)*	0.506 *(0.09)*
log-likelihood	7081.16	7071.39	7052.06	7051.85
LR	58.20 *(14.10)*	19.33 *(5.99)*	-	-
JB	898.87	937.35	930.17	932.58

Notes: Parameters estimated for model specifications with constant MPR, time-varying MPR which follows a stationary autoregressive process or a random walk and the MPR conditioned on state variables (t-ratios are displayed in parentheses). Measures of ex-ante and ex-post market timing ability (GI and GO, resampling p-values in parentheses) are reported. Since for both the constant MPR and random walk model insample estimates are positive throughout, we did not imitate the scenarios of random signalling for these models. Model diagnostics are log-likelihood, LR (5% critical values in parentheses), and Jarque-Bera statistics.

B. Market timing ability

Measurement

As a particular means of model evaluation we consider in-sample and out-of-sample timing ability of the proposed state space model. To assess the economic performance of forecasting schemes, predictions are translated into trading signals. Throughout, positive (negative) excess return forecasts serve as a buy (sell) signal, with feasible short selling. The cumulated in-sample return of a dynamic trading scheme reads as

$$GI^{(\bullet)} = \sum_{t=\omega+1}^{T} \hat{I}_t^{(\bullet)} y_t \,, \tag{6}$$

where $\hat{I}_t^{(\bullet)} = \mathrm{I}\big(\hat{y}_t^{(\bullet)} > 0\big) - \mathrm{I}(\hat{y}_t^{(\bullet)} < 0)$ is a binary estimate ($\hat{I}_t^{(\bullet)} \in \{-1, +1\}$) that depends on a particular MPR specification indicated by '•'. As explained next, the time index ω separates estimation and forecasting samples in the out-of-sample analysis. For a better comparability of in- and out-of-sample market timing ability, in-sample diagnostics are also conditioned upon observations $y_t, t \geq \omega + 1$.

The total return in (6) is an in-sample estimate that fully exploits the available time series information in Ω_T. Alternatively, we employ rolling estimations to quantify ex-ante market timing ability. Ex-ante total returns are

$$GO^{(\bullet)} = \sum_{t=\omega+1}^{T} \hat{I}_{t|t-1}^{(\bullet)} y_t \,, \tag{7}$$

where trading signals are extracted from one step ahead forecasts conditioned upon sample information Ω_{t-1}, i.e., $\hat{I}_{t|t-1}^{(\bullet)} = \mathrm{I}\big(\hat{y}_{t|t-1}^{(\bullet)} > 0\big) - \mathrm{I}\big(\hat{y}_{t|t-1}^{(\bullet)} < 0\big)$.

To evaluate if a particular model success, $GI^{(\bullet)}$ or $GO^{(\bullet)}$, is systematic rather than random, we implement a resampling design that contrasts the outcome of random signalling against the model based strategy. Notice that a random strategy would unsystematically attach buy or sell signals to particular (out-of-sample) excess returns obtaining

$$GI^{(\bullet,*)} = \sum_{t=\omega+1}^{T} I_t^{(\bullet,*)} y_t \quad \text{or} \quad GO^{(\bullet,*)} = \sum_{t=\omega+1}^{T} I_{t|t-1}^{(\bullet,*)} y_t \,,$$

where $I_t^{(\bullet,*)}$ and $I_{t|t-1}^{(\bullet,*)}$ are drawn with replacement from the distribution of model implied sequences ('runs') of positive and negative trading signals.[14] To imitate random model performance under the failure of market timing, we draw 10000 realizations of artificial excess returns $GI^{(\bullet,*)}$ or $GO^{(\bullet,*)}$. If a particular model specification fails in market timing, one would expect the simulated distribution of $GI^{(\bullet,*)}$ ($GO^{(\bullet,*)}$) to have high mass in the neighborhood of the actual sample statistic $GI^{(\bullet)}$ ($GO^{(\bullet)}$). Therefore we regard small 'p-values', $\text{Prob}\left[GI^{(\bullet,*)} > GI^{(\bullet)}\right]$ $\left(\text{Prob}\left[GO^{(\bullet,*)} > GO^{(\bullet)}\right]\right)$, as an indication of in-sample (out-of-sample) market timing ability.

Evidence

In-sample and out-of-sample measures of market timing ability are documented in the lower panel of Table 2. To initialize the analysis of market timing ability we choose $\omega = 1000$. This corresponds to the period from March 2, 1998 to February 14, 2002. The size of rolling time windows is also 1000.

In-sample results confirm the strong dominance of dynamic model specifications conditioning the MPR on predetermined state variables. Similar to the constant MPR model, the extraction of trading signals from the random walk specification obtains a negative net return over the sample horizon. Extracting trading signals from the most general model obtains a cumulated return $GI^{(\bullet)} = 2.08$ which differs significantly from earnings obtained with random signaling $\left(\text{Prob}\left[GI^{(\bullet,*)} > GI^{(\bullet)}\right] = 0.0018\right)$. Contrasting the purely autoregressive and most general model confirms the significant informational content of the predetermined exogenous variables.

Out-of-sample results further underpin the scope of conditioning the MPR on predetermined state variables. The models which are based on the constant

[14] As an alternative to drawing (runs of) trading signals with replacement we also use sequences $I_t^{(\bullet,*)}$ and $I_{t|t-1}^{(\bullet,*)}$ drawn without replacement. Inferential results from sampling without replacement are qualitatively equivalent to those obtained from sampling with replacement.

and autoregressive MPR formulation yield negative out-of-sample cumulative returns $GO^{(\bullet)} = -0.44$ and $GO^{(\bullet)} = -0.33$, respectively. In contrast, conditioning on predetermined variables yields an out-of-sample excess return of $GO^{(\bullet)} = 0.92$ which exceeds significantly the outcome of random signaling ($\text{Prob}[GO^{(\bullet,*)} > GO^{(\bullet)}] = 0.02$).

V. Determinants of the market price of risk

Having identified the fully conditional model as most accurate to describe excess DAX returns from both a statistical and economic perspective, we now interpret this particular model and its implied MPR path. To address which economic conditions govern the premium per unit of market risk, we presume that investors differ in their trading behavior. Given the high dimension of investment decisions and, moreover, legal or self regulations, investment strategies are likely heterogeneous across agents. For instance, some financial traders consider stock or fixed income markets globally while others focus their strategy at domestic opportunities. Moreover, investment strategies could be classified according to their more technical or fundamental aptitude. Hence, we first regard the MPR as a market feature influenced by the prospects of less risky fixed income markets. The second is to interpret it as a reflection of both international competition across financial markets and future investment opportunities. Furthermore, the MPR is conditioned on the prevalence of noise trading implied risk.

A. Earning prospects and cross market allocation

Slope estimates in Table 2 show that the MPR decreases significantly at the 10% level if the EURIBOR increases. Naturally, this variable indicates not only the refinancing opportunities of banks but also of companies. Increasing costs of refinancing reduce the number of attractive investment opportunities and, subsequently, the earning prospects of companies. The short-term EURIBOR directly reflects the competition of stock and fixed income markets. Thus, an ascent of short-term interest rates encourages fixed income investments. It is likely that the most risk averse agents leave the stock market and reinvest their capital for the benefit of fixed income markets. In consequence, ARA is likely to shrink. Thus, for given market risk, the demanded compensation decreases as a result of a diminished MPR.

The effect of the domestic term spread has an unexpected sign. The term structure mirrors the trade-off between current and future opportunities of refinancing and its increase might lead to an advance in current profitable investments. Moreover, it is commonly associated with an expected acceleration of real economic activity (Estrella and Hardouvelis 1991). Strengthened earning prospects improve the relative attractiveness of stock markets. Thus, relatively more risk averse investors will participate in the stock market and the ARA is likely to increase which contradicts the negatively estimated impact on the MPR. However, the estimated effect is statistically insignificant at common significance levels which is in line with findings in Yu and Yuan (2011).

With 10% significance the MPR is negatively affected by the lagged international term spread $x_{t-1,4} = is_{t-1}^{EU} - is_{t-1}^{US}$. This measure reflects two distinct aspects. First, it covers non-synchronous changes of German and US market conditions of refinancing. In this perspective, a change in the international term spread hints at distinct economic prospects and refinancing costs in both markets (Bali and Peng 2006). For instance, an increase in the European base rate worsens the investment opportunities of European corporations while credit costs of American corporations remain (in absolute terms) unchanged. Consequently, market participants downgrade their expectations of European (German) market returns. Under these conditions the most risk averse agents might leave the German market for the benefit of foreign investments. As a result, ARA and thereby the MPR face a reduction. Second, owing to the uncovered interest parity the international term spread comprises expected exchange rate adjustments and a compensation for currency risk (Frankel 1992).[15] De Santis and Gerard (1998) analyze the role of currency risk for asset pricing in an extended international CAPM framework. Based on the intertemporal CAPM, however, our focus is on the risk compensation paid at a domestic market. Therefore no strict theoretical considerations on the impact of currency risk can be made within this (limited) context. However, one may argue that with increasing term spreads an uprise of general investment risks or currency risks is perceived on stock markets. From this perspective again most risk averse agents may withdraw risky investments for

[15] We thank an anonymous referee for hinting at this important view at the international term spread.

the benefit of less risky assets. Thereby, ARA will shrink at the domestic market invoking a reduction of the demanded risk compensation.

Lagged returns at US stock markets have a significantly positive influence on the MPR. Periods of, on average, positive DOW returns might be interpreted as an early indication of global economic acceleration and, thus, could attract more risk averse investors into the stock market. Then, with an acceleration of ARA, the compensation per unit of risk will also increase.

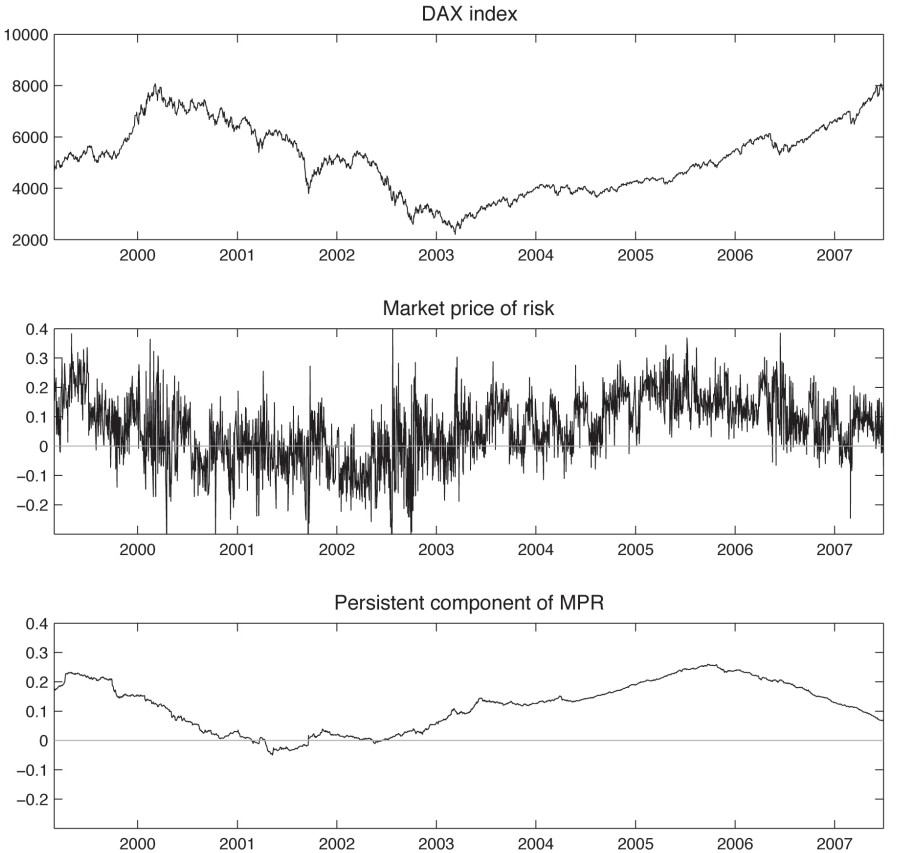
B. Noise trader activity

The proposed measure of noise trader activity exerts a significantly negative impact on the MPR. Thus, the effect that the average price of risk decreases if more (less risk averse) noise traders participate actively in the market, dominates the effect that rational investors claim for a compensation of increased risk due to noise trader participation. This evidence is in line with the results of Yu and Yuan (2011) and Verma and Soydemir (2009). Thus, noise trader activity impacts on the MPR not only at a monthly but also at the daily frequency. Moreover, these results underpin the argument of De Long et al. (1990) that noise trader participation is priced and that rational investors cannot offset their influence by means of arbitrage.

C. The DAX market price of risk

After the discussion of potential MPR determinants, we turn the attention to the time path of the model implied MPR for the German stock market. The filtered MPR for the most preferable model is displayed in the middle panel of Figure 4. The average MPR $(\bar{\lambda})$ is 0.0606. Noting an unconditional standard deviation of $4.82E-5$, $\bar{\lambda}$ is positive at conventional significance levels. The positiveness of the unconditional MPR confirms former results, for instance, Goyal and Santa-Clara (2003), Ghysels et al. (2005) and Lundblad (2007). While the maximum MPR is 0.4694 (August 8, 1999) the minimum MPR is negative (-0.5865, September 18, 2001). The model implied MPR is time-varying and negative for 639 out of 2365 trading days. These results are not in line with an a-priori presumed uniformly positive relation between price uncertainty and returns. However, transitory dynamics of the MPR may hide a generally positive dynamic evolution of the risk return relationship. To address this aspect, we next uncover a smooth component of the MPR and analyze it in more detail.

Figure 4. The DAX and the market price of risk



Notes: The upper panel displays DAX price quotes. The middle panel presents the filtered time varying MPR as implied by the fully conditional model with parameter estimates provided in the first column of Table 2. The long run behavior of the MPR as implied by the EURIBOR and the national and international term spreads is displayed in the lower panel.

To make the impact of explanatory variables on the expected MPR explicit, it is convenient to develop λ_t in (3) recursively and to take the conditional expectation. Thereby we obtain,

$$\mathrm{E}(\lambda_t|\Omega_{t-1}) = \kappa_t + \sum_{j=0}^{t-1} \rho^j \left(\boldsymbol{x}'_{t-j-1}\boldsymbol{\beta}\right) + \sum_{j=1}^{t-1} \rho^j v_{t-j} + \rho^t \lambda_0, \tag{8}$$

where $\kappa_t = \beta_0(1-\rho^t)\,/\,(1-\rho)$. $\mathrm{E}(\lambda_t|\Omega_{t-1})$ is decomposed into a level component κ_t, as well as impacts of historic explanatory processes $\left(\sum_{j=0}^{t-1}\rho^j(\boldsymbol{x}'_{t-j-1}\boldsymbol{\beta})\right)$, lagged residuals $\left(\sum_{j=1}^{t-1}\rho^j v_{t-j}\right)$, and of the initial condition $(\rho^t\lambda_0)$. Provided that $|\rho|<1$, the latter two components vanish with increasing horizon *t*. In the same time, more persistent variables in $\boldsymbol{x}_{t-j-1}$ govern the MPR in the longer or medium term. The persistence of the MPR depends on the autoregressive parameter ρ, and, as can be noted from Figure 2, particularly on both term spreads and the EURIBOR. In light of these considerations we approximate the more persistent component of the MPR as

$$\tilde{\lambda}_t = \kappa_t + \sum_{j=0}^{t-1}\rho^j\left(\tilde{\boldsymbol{x}}'_{t-j-1}\tilde{\boldsymbol{\beta}}\right), \tag{9}$$

where $\tilde{\boldsymbol{x}}_{t-j-1}$ comprises the persistent financial processes mentioned before and $\tilde{\boldsymbol{\beta}}$ the corresponding parameters. To quantify $\tilde{\lambda}_t$ empirically, we employ the estimated coefficients of the model specification documented in the first column of Table 2 and the first 250 observations as initialization period. Estimation results are provided in the lower panel of Figure 4. As it turns out, the persistent component of the MPR is positive for most periods. Only for 154 out of 2115 trading days in the mid of 2001 it becomes slightly negative. Its minimal value is -0.0498.

While the DAX displayed in the upper panel of Figure 4 is typically characterized by procyclical movements, it is hard to diagnose such patterns for the MPR. Thus, we neither confirm the empirical evidence of Lundblad (2007) in favour of a procyclical MPR nor the theoretically suggested countercyclical movements (Campbell and Cochrane 1999).

VI. Conclusions

We formalize a dynamic model that allows time dependence of the risk-return trade-off for the major German stock index (DAX) over the time period March, 1998 to June, 2007. The model is set out in state-space form making use of realized volatility measures to quantify the latent market risk consistently. The risk-return relation is simultaneously conditioned upon predetermined economic state variables, covering the risk free rate, a national and an international term spread, log-returns of US stock markets and a constructed measure hinting

at excess activity of noise traders. Relying on common model diagnostics and market timing ability, predetermined state characteristics are strongly significant to explain time variation of the risk-return relationship.

Unconditionally the estimated MPR is (significantly) positive and, thus, in line with the general view that risk taking should pay in terms of excess revenue. This is further confirmed by evidence for a mostly positive persistent component of the MPR. Reflecting the market composition of risk aversion on the one hand and (future) international investment opportunities on the other hand, also time clusters of negative risk-return relationships are diagnosed. Depending on the time series features of the state variables impacting on the MPR, transitory (US stock returns, likelihood of noise trading) and persistent characteristics (domestic fixed income rates, international term spread) of the risk-return relationship are uncovered.

In this work we concentrate on data up to June 2007 and neglect developments during the current crisis. Many financial processes — in particular monetary policy rates and term spreads — reflect quantitative easing and, thus, a marked change in the local and global landscapes of risk. How macroeconomic conditions, monetary policy stances and noise trader activity impact jointly on the MPR during the current crisis is an important question for future research. Moreover, rational and irrational investors likely respond in a distinct fashion to economic states. At the same time rational investors are heterogeneous with regard to their degree of risk aversion. Therefore, further analysis could address the determination of market implied dynamics of investors' risk attitudes across financial markets (not only stock markets) by means of stylized features of financial market processes (for instance, leptokurtosis, correlation structures and higher order dependencies). For this purpose, a tempting direction of future research appears to be to integrate information processed in derivative markets (Doran and Ronn 2008) into the state-space framework adopted in this work.

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