Online Appendix

to

REGIONAL INFLATION DYNAMICS AND INFLATION TARGETING. THE CASE OF PERU

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Following theoretical results in Phillips (1986), Park and Phillips (1988) and Sims et al. (1990), this Appendix shows that both the aggregation and long-run homogeneity hypotheses can be tested using standard results from classical regression theory. In particular, it is shown that the statistics involved are asymptotically Gaussian even when the data are not stationary. This is the starting point for Wald coefficient tests that are asymptotically χ^2.

A Aggregation hypothesis

The VAR(p) model

$$\boldsymbol{y}_t = \boldsymbol{A}_1 \boldsymbol{y}_{t-1} + \boldsymbol{A}_2 \boldsymbol{y}_{t-2} + \cdots + \boldsymbol{A}_p \boldsymbol{y}_{t-p} + \boldsymbol{\varepsilon}_t \tag{17}$$

can be rewritten as

$$\boldsymbol{y}_t = \boldsymbol{C}_1 \Delta \boldsymbol{y}_{t-1} + \boldsymbol{C}_2 \Delta \boldsymbol{y}_{t-2} + \cdots + \boldsymbol{C}_{p-1} \Delta \boldsymbol{y}_{t-p+1} + \boldsymbol{H} \boldsymbol{y}_{t-1} + \boldsymbol{\varepsilon}_t \quad , \tag{18}$$

where $\boldsymbol{H} = \boldsymbol{A}_1 + \boldsymbol{A}_2 + \cdots + \boldsymbol{A}_p$, $\boldsymbol{C}_1 = \boldsymbol{A}_1 - \boldsymbol{H}$ and $\boldsymbol{C}_r = \boldsymbol{C}_{r-1} + \boldsymbol{A}_r$ for $r = 2, \ldots, p-1$.

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If $\boldsymbol{y}_t$ is stationary, then the least squares estimators of the $\boldsymbol{C}$ matrices and of $\boldsymbol{H}$ will be asymptotically Gaussian under standard conditions. On the other hand, following Sims et al. (1990) and Hamilton (1994, section 18.2), when $\boldsymbol{y}_t$ is nonstationary, the least squares estimators of the $\boldsymbol{C}$ matrices in (18) are consistent and remain asymptotically Gaussian, whereas the corresponding estimator of $\boldsymbol{H}$ becomes superconsistent and have a nonstandard distribution characterized by functionals of Weiner processes. More formally, it is found that for $r = 1, 2, \dots, p - 1$

$$\sqrt{T}\mathrm{vec}\left(\widehat{\boldsymbol{C}}_r - \boldsymbol{C}_r\right) \xrightarrow{d} N(\boldsymbol{0}, \boldsymbol{V}_r) \quad \text{whereas} \quad \sqrt{T}\mathrm{vec}\left(\widehat{\boldsymbol{H}} - \boldsymbol{H}\right) = o_p(1)\,, \tag{19}$$

where $\mathrm{vec}(\boldsymbol{M})$ is the vector obtained by stacking the columns of matrix $\boldsymbol{M}$, and $\boldsymbol{V}_r$ denotes a positive definite covariance matrix.

The results in (19) are significant since they imply that hypothesis tests involving linear combinations of the $\boldsymbol{C}$ matrices and $\boldsymbol{H}$ are dominated asymptotically by the coefficients with the slower rate of convergence, namely those in the $\boldsymbol{C}$ matrices. In particular, tests involving linear combinations of the $\boldsymbol{A}$ matrices in (17) other than the sum $\boldsymbol{H}$ have the usual limiting distributions. A leading application of this result is that lag length tests on any variable or any set of variables can be conducted with standard *F*-tests regardless of whether the variable in question is stationary (Sims et al. 1990).

Recall that, for a given $r = 1, 2, \dots, p$, the aggregation restrictions have the form $a_{ij}(r) - (w_{ij}/w_{ik})\, a_{ik}(r)$ for $j \neq k$ and $k \neq i$. These restrictions can be expressed more compactly as $\boldsymbol{R}_r\mathrm{vec}(\boldsymbol{A}_r) = \boldsymbol{0}$, where $\boldsymbol{R}_r$ is a $p(n-2) \times n^2$ matrix whose typical row contains a 1, the ratio $-w_{ij}/w_{ik}$ and $n^2 - 2$ zeroes. In terms of the coefficientes of the reparameterized model (18), the aggregation restrictions are (consider $\boldsymbol{C}_p = \boldsymbol{0}$)

$$\begin{aligned} &\boldsymbol{R}_r\mathrm{vec}(\boldsymbol{A}_r) \equiv \boldsymbol{R}_r\mathrm{vec}(\boldsymbol{C}_r - \boldsymbol{C}_{r-1}) = \boldsymbol{0} \quad \text{for } r = 2, \dots, p \;, \\ &\boldsymbol{R}_1\mathrm{vec}(\boldsymbol{A}_1) \equiv \boldsymbol{R}_1\mathrm{vec}(\boldsymbol{C}_1 + \boldsymbol{H}) = \boldsymbol{0} \end{aligned} \tag{20}$$

Therefore, using the results in (19), we have that under the null hypotheses in (20) (consider $\widehat{\boldsymbol{C}}_p = \boldsymbol{0}$),

$$\begin{aligned} &\sqrt{T}\boldsymbol{R}_r\mathrm{vec}\left(\widehat{\boldsymbol{C}}_r - \widehat{\boldsymbol{C}}_{r-1}\right) \xrightarrow{d} N(0, \boldsymbol{\Omega}_r) \quad \text{for } r = 2, \dots, p \;, \\ &\sqrt{T}\boldsymbol{R}_1\mathrm{vec}\left(\widehat{\boldsymbol{C}}_1 + \widehat{\boldsymbol{H}}\right) = \sqrt{T}\boldsymbol{R}_1\mathrm{vec}\left(\widehat{\boldsymbol{C}}_1\right) + o_p(1) \xrightarrow{d} N(0, \boldsymbol{\Omega}_1)\,. \end{aligned} \tag{21}$$

where $\boldsymbol{\Omega}_r$ denote positive definite covariance matrices for $r = 1, 2, \dots, p$. It follows that, regardless of the stationarity properties of the data, the statistics involved in testing aggregation hypothesis are asymptotically Gaussian, and thus the corresponding Wald tests have the usual asymptotic χ^2 distribution.

B. Long-run homogeneity

Consider the stylized model, equation (8) with $p = 1$,

$$\Delta y_t = -\gamma\ (y_{t-1} - x_{t-1}) + \theta x_{t-1} + \ \varepsilon_t \quad , \tag{22}$$

where $\Delta x_t = v_t$ and ε_t are stationary processes with zero mean and unconditional variances σ_v^2 and σ_ε^2, respectively. It is assumed that v_t and ε_t are uncorrelated at all leads and lags, which essentially implies that equation (22) is correctly specified. The arguments below are still valid for augmented equations that include lags of Δx_t and Δy_t such that the uncorrelatedness between v_t and ε_t is guaranteed.

The purpose is to test $H_0 :\ \theta = 0$ in (22). For concreteness let $z_{t-1} = y_{t-1} - x_{t-1}$, which is stationary under H_0. It is not difficult to verify that, because $z_t \sim I(0)$, and $y_t \sim\ I(1)$,

$$\frac{1}{T}\frac{\sum_t x_{t-1}\, z_{t-1}\ \sum_t z_{t-1}\, \varepsilon_t}{\sum_t (z_{t-1})^2} = \frac{1}{T}\frac{O_p(T)O_p(\sqrt{T})}{O_p(T)} = o_p(1) \quad \text{and} \quad \frac{1}{T^2}\frac{(\sum_t x_{t-1}\, z_{t-1})^2}{\sum_t (z_{t-1})^2} = \frac{1}{T^2}\frac{O_p(T^2)}{O_p(T)} = o_p(1) \tag{23}$$

On the other hand, let $W_\varepsilon(.)$ and $W_v(.)$ be two standard Weiner processes on $C(0,1)$, associated to the standardized series $\varepsilon_t/\sigma_\varepsilon$ and v_t/σ_v, respectively. These processes are uncorrelated and hence, due to the Gaussianity of the increments, independent. A standard result for integrated processes (cf Hamilton, 1994, section 17.5) is that

$$\frac{1}{T^2}\sum_t (x_{t-1})^2 \xrightarrow{d} \sigma_v^2 \int_0^1 W_v\,(r)^2 dr \equiv\ \mathfrak{D}_2 \ , \tag{24}$$

whereas Phillips (1986: 327) and Park and Phillips (1988, Lemma 5.1) show that

$$\frac{1}{T}\sum_t x_{t-1}\, \varepsilon_t \xrightarrow{d} \sigma_v\ \sigma_\varepsilon \int_0^1 W_v\,(r) dW_\varepsilon(r) = N(0, \sigma_\varepsilon^2 \mathfrak{D}_2) \equiv\ \mathfrak{D}_1 \ . \tag{25}$$

The above limiting distribution $\mathfrak{D}_1$ is mixed Gaussian. This is to be understood as a normal distribution with variance proportional to $\mathfrak{D}_2$ which is itself a random drawing from the space of positive scalars, in this case quadratic functionals of a Weiner process.

Finally, let s^2 denote the usual maximum likelihood estimator of the regression error variance, which is consistent for σ_ε^2. Upon gathering the results in (23), (24) and (25), the *t* statistic for testing $H_0 :\ \theta = 0$ satisfies, under this null hypothesis,

$$\tau = \frac{\sum_t x_{t-1}\,\varepsilon_t - \sum_t x_{t-1}\,z_{t-1}\sum_t z_{t-1}\,\varepsilon_t / \sum_t (z_{t-1})^2}{\sqrt{s^2\sum_t (x_{t-1})^2 - s^2(\sum_t x_{t-1}\,z_{t-1})^2 \,/\, \sum_t (z_{t-1})^2}} = \frac{\sum_t x_{t-1}\,\varepsilon_t / T}{\sqrt{s^2\sum_t (x_{t-1})^2 / T^2}} + o_p(1) \xrightarrow{d} \frac{\mathfrak{D}_1}{\sqrt{\sigma_\varepsilon^2\,\mathfrak{D}_2}} \equiv N(0,1) \;, \quad (26)$$

which follows from the continuous mapping and Cramér theorems. Therefore, the limiting distribution of τ is standard Gaussian, even though the least squares estimator of θ itself is not asymptotically Gaussian (the limiting distribution of $T\hat{\theta}$ is $\mathfrak{D}_1/\mathfrak{D}_2$). Similar arguments show that the asymptotic χ^2 statistics for tests for general restrictions on the coefficients θ across equations are also generated in the standard way.

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