Online Appendix

to

On central bank independence and political cycles

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1 Online appendix: solving the model

1.1 Households

Consumers' problem is solved in two stages: first minimize the cost of buying a certain level of composite good C_t and then maximize their utility function subject to their budget constraint. Minimization implies the solution of the following problem:

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj$$
$$s.t \quad : \quad C_t \leq [\int_0^1 c_{jt}^{\frac{\zeta-1}{\zeta}} dj]^{\frac{\zeta}{\zeta-1}}$$

Let ψ_t be the Lagrange multiplier associated to the constraint, the FOC with respect to c_{jt} is:

$$p_{jt} = \psi_t [\int_0^1 c_{jt}^{\frac{\zeta-1}{\zeta}} dj]^{\frac{\zeta}{\zeta-1}-1} c_{jt}^{-\frac{1}{\zeta}}$$

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But $[\int_0^1 c_{jt}^{\frac{\zeta-1}{\zeta}} dj]^{\frac{1}{\zeta-1}} = C_t^{\frac{1}{\zeta}}$, so the former condition can be rewritten as:

$$\begin{aligned} p_{jt} &= \psi_t C_t^{\frac{1}{\zeta}} c_{jt}^{-\frac{1}{\zeta}} \\ c_{jt} &= C_t \left(\frac{p_{jt}}{\psi_t}\right)^{-\zeta} \end{aligned}$$

Using the definition of the composite good:

$$\begin{aligned} C_t &= \int_0^1 c_{jt}^{\frac{\zeta-1}{\zeta}} dj]^{\frac{\zeta}{\zeta-1}} \\ C_t &= [\int_0^1 C_t^{\frac{\zeta-1}{\zeta}} \left(\frac{p_{jt}}{\psi_t}\right)^{1-\zeta} dj]^{\frac{\zeta}{\zeta-1}} \Rightarrow \\ \psi_t^{-\zeta} &= [\int_0^1 p_{jt}^{1-\zeta} dj]^{\frac{\zeta}{\zeta-1}} \\ \psi_t &= [\int_0^1 p_{jt}^{1-\zeta} dj]^{\frac{1}{1-\zeta}} \equiv P_t \end{aligned}$$

Substituting the definition of the Lagrange multiplier in the FOC obtained earlier, the demand for individual good j is:

$$c_{jt} = C_t \left(\frac{p_{jt}}{P_t}\right)^{-\zeta}$$

The second step will give us the Euler equation and the labor supply:

$$\max_{C_t, N_t, B_t} E_t \sum_{i=0}^{\infty} \beta^i [\frac{(\alpha G_{t+i})^{1-\sigma}}{1-\sigma} + \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \frac{\int_0^1 N_{jt+i}^{1+\eta} dj}{1+\eta} + \theta^s A_{t+i}]$$

$$s.t \quad : \quad C_t + \frac{B_t}{P_t} = \frac{w_t \int_0^1 N_{jt}\ dj}{P_t} + (1+i_{t-1})\frac{B_{t-1}}{P_t} + \Pi_t$$

Let ι_t be the corresponding Lagrange multiplier; then FOC with respect to $C_t; C_{t+1}; N_{jt}$ and B_t are, respectively:

$$\begin{aligned} C_t^{-\sigma} &= \iota_t \\ \beta E_t C_{t+1}^{-\sigma} &= \iota_{t+1} \\ N_{jt}^{\eta} &= \iota_t \frac{w_t}{P_t} \\ \frac{\iota_t}{P_t} &= (1+i_t)\frac{\iota_{t+1}}{P_{t+1}} \end{aligned}$$

Using the fourth FOC and combining the first three, I obtain the Euler equation and labor supply already mentioned:

$$\begin{aligned}
\frac{C_t^{-\sigma}}{\beta E_t C_{t+1}^{-\sigma}} &= (1+i_t)\,\frac{P_t}{P_{t+1}} \Rightarrow \\
C_t^{-\sigma} &= \beta\,(1+i_t)\,E_t\frac{P_t}{P_{t+1}}C_{t+1}^{-\sigma} \qquad \textit{and} \\
N_{jt}^{\eta} &= C_t^{-\sigma}\frac{w_t}{P_t}
\end{aligned}$$

1.2 Firms

To obtain the $HNKPC$, note that forward-looking firms, also have a two-step solution to set their optimal price; they first minimize costs from where the real marginal cost is obtained and then choose the price p_{jt} that maximizes the present value of expected utilities. The first part of the problem implies solving:

$$\begin{aligned}
&\min_{N_{jt}} \frac{w_t}{P_t}N_{jt} \\
s.t \quad : \quad & y_{jt} = Z_t N_{jt}
\end{aligned}$$

Let ϕ_t be the Lagrange multiplier, then, the FOC is:

$$\phi_t = \frac{\frac{w_t}{P_t}}{Z_t}$$

So, ϕ_t is the real marginal cost. On the other hand, each firm must face the demand for its differentiated product, then total demand of product j $\left(c_{jt}^T\right)$ is given by individual demand from households and government:

$$\begin{aligned}
c_{jt}^T &= c_{jt} + g_{jt} \\
&= C_t\left(\frac{p_{jt}}{P_t}\right)^{-\zeta} + G_t\left(\frac{p_{jt}}{P_t}\right)^{-\zeta} \\
&= \left(\frac{p_{jt}}{P_t}\right)^{-\zeta}(C_t + G_t) \\
&= \left(\frac{p_{jt}}{P_t}\right)^{-\zeta} C_t^T
\end{aligned}$$

Imposing market clear condition for goods, it must be true that:

$$\begin{aligned}
y_{jt} &= c_{jt}^T \\
&= \left(\frac{p_{jt}}{P_t}\right)^{-\zeta} C_t^T
\end{aligned}$$

Then the pricing decision for the forward-looking firm can be expressed as:

$$\max_{p_{jt}^{fo}} E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i}[(\frac{p_{jt}^{fo}}{P_{t+i}})^{1-\zeta}(1-\tau_{t+i}) - \phi_{t+i}\left(\frac{p_{jt}^{fo}}{P_{t+i}}\right)^{-\zeta}]C_{t+i}^T$$

The FOC with respect to the optimal price is:

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i}[(1-\zeta)\,(\frac{p_{jt}^{fo}}{P_{t+i}})^{-\zeta}(1-\tau_{t+i})\frac{1}{P_{t+i}}+\zeta\phi_{t+i}\left(\frac{p_{jt}^{fo}}{P_{t+i}}\right)^{-\zeta-1}\frac{1}{P_{t+i}}]C_{t+i}^{T}=0$$

As all firms face the same problem, they will all set the same optimal price p_t^{fo} so that:

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i}[(1-\zeta)\,(\frac{p_t^{fo}}{P_{t+i}})(1-\tau_{t+i})+\zeta\phi_{t+i}]C_{t+i}^{T}P_{t+i}^{\zeta}=0$$

The stochastic discount factor is $\Delta_{i,t+i}=\beta^i(C_{t+i}/C_t)^{-\sigma}$, so p_t^{fo} must satisfy:

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{-\sigma}\,(C_{t+i}+G_{t+i})\,[(1-\zeta)\,\frac{p_t^{fo}}{P_t}\frac{P_t}{P_{t+i}}(1-\tau_{t+i})+\zeta\phi_{t+i}]P_{t+i}^{\zeta}=0$$

Then solving for $\frac{p_t^{fo}}{P_t}$:

$$\frac{p_t^{fo}}{P_t}=\frac{\zeta}{\zeta-1}\frac{E_t\sum_{i=0}^{\infty}\omega^i\beta^iC_{t+i}^{-\sigma}(C_{t+i}+G_{t+i})\phi_{t+i}(\frac{P_{t+i}}{P_t})^{\zeta}}{E_t\sum_{i=0}^{\infty}\omega^i\beta^iC_{t+i}^{-\sigma}(C_{t+i}+G_{t+i})(1-\tau_{t+i})(\frac{P_{t+i}}{P_t})^{\zeta-1}}$$

The optimal price set by each forward and backward-looking firm must be linearized around its steady state in order to obtain the $HNKPC$, where inflation is defined as $\pi_t \equiv \ln\left(\frac{P_t}{P_{t-1}}\right)$. First define Q_t as the relative price a firm sets when it has to adjust prices, so $Q_t \equiv \frac{p_t^*}{P_t}$; note that in steady state, when all firms can adjust prices it must be true that, $Q_{SS}=1$; and also note that $\widehat{q}_t=\ln\left(\frac{Q_t}{Q_{SS}}\right)$. Second, the general price level P_t is a weighted average between the price set by adjusting firms and the general price level the period before; it is also necesary the equation for the optimal price set by adjusting firms (forward and backward-looking) and finally, the FOC for forward-looking firms. So the relevant equations are:

$$Q_t \equiv \frac{p_t^*}{P_t};Q_t^{fo}\equiv\frac{p_t^{fo}}{P_t};Q_t^{ba}\equiv\frac{p_t^{ba}}{P_t} \tag{B.1}$$

$$P_t^{1-\zeta}=(1-\omega)(p_t^*)^{1-\zeta}+\omega P_{t-1}^{1-\zeta} \tag{B.2}$$

$$p_t^*=(1-\Theta)\,p_t^{fo}+\Theta p_t^{ba} \tag{B.3}$$

$$p_t^{ba}=p_{t-1}^* \tag{B.4}$$

$$\frac{p_t^{fo}}{P_t}=\frac{\zeta}{\zeta-1}\frac{E_t\sum_{i=0}^{\infty}\omega^i\beta^iC_{t+i}^{-\sigma}(C_{t+i}+G_{t+i})\phi_{t+i}(\frac{P_{t+i}}{P_t})^{\zeta}}{E_t\sum_{i=0}^{\infty}\omega^i\beta^iC_{t+i}^{-\sigma}(C_{t+i}+G_{t+i})(1-\tau_{t+i})(\frac{P_{t+i}}{P_t})^{\zeta-1}} \tag{B.5}$$

Expressing $(B.2)$ in percentage deviations around the steady state, we can see that:

$$\begin{aligned} 1 &= (1-\omega)(\frac{p_t^*}{P_t})^{1-\zeta} + \omega\left(\frac{P_{t-1}}{P_t}\right)^{1-\zeta} \Rightarrow \\ 0 &= (1-\omega)\widehat{q}_t - \omega\pi_t \\ \widehat{q}_t &= \frac{\omega}{1-\omega}\pi_t \end{aligned}$$

From $(B.4)$, I can express the price set by backward-looking firms as:

$$\begin{aligned} \frac{p_t^{ba}}{P_t} &= \frac{p_{t-1}^*}{P_{t-1}}\frac{P_{t-1}}{P_t} \Rightarrow \\ Q_t^{ba} &= Q_{t-1}\frac{P_{t-1}}{P_t} \Rightarrow \\ \widehat{q}_t^{ba} &= \widehat{q}_{t-1} - \pi_t \end{aligned}$$

I use this in $(B.3)$, previously expressed in percentage deviations around the steady state:

$$\begin{aligned} Q_t &= (1-\Theta)\,Q_t^{fo} + \Theta Q_t^{ba} \Rightarrow \\ \widehat{q}_t &= (1-\Theta)\,\widehat{q}_t^{fo} + \Theta\widehat{q}_t^{ba} \Rightarrow \\ \widehat{q}_t &= (1-\Theta)\,\widehat{q}_t^{fo} + \Theta\widehat{q}_{t-1} - \Theta\pi_t \end{aligned}$$

Let $\mu \equiv \frac{\zeta}{\zeta-1}$, $(B.5)$ can be expressed as:

$$\begin{aligned} & Q_t\left[E_t\sum_{i=0}^{\infty}\omega^i\beta^i C_{t+i}^{-\sigma}(C_{t+i}+G_{t+i})(1-\tau_{t+i})(\frac{P_{t+i}}{P_t})^{\zeta-1}\right] \\ = \quad & \mu E_t\sum_{i=0}^{\infty}\omega^i\beta^i C_{t+i}^{-\sigma}(C_{t+i}+G_{t+i})\phi_{t+i}(\frac{P_{t+i}}{P_t})^{\zeta} \end{aligned}$$

So that the LHS (which I will denote by Θ) can be approximated by:

$$\begin{aligned} \Theta \approx \; & \Theta\,(SS) + \left.\frac{\partial\Theta}{\partial Q_{t+i}}\right|_{SS}\left(Q_{t+i}^{fo}-1\right) + \left.\frac{\partial\Theta}{\partial C_{t+i}}\right|_{SS}(C_{t+i}-C) + \left.\frac{\partial\Theta}{\partial G_{t+i}}\right|_{SS}(G_{t+i}-G) \\ & + \left.\frac{\partial\Theta}{\partial\tau_{t+i}}\right|_{SS}(\tau_{t+i}-\tau) + \left.\frac{\partial\Theta}{\partial\frac{P_{t+i}}{P_t}}\right|_{SS}\left(\frac{P_{t+i}}{P_t}-1\right) \end{aligned}$$

(a) $\Theta\,(SS) = \frac{C^{\,-\sigma}(C+G)(1-\tau)}{1-\omega\beta}$

(b) $\left.\frac{\partial\Theta}{\partial Q_{t+i}}\right|_{SS}(Q_{t+i}-1) = \frac{C^{\,-\sigma}(C+G)(1-\tau)}{1-\omega\beta}\widehat{q}_t^{fo}$

(c) $\left.\frac{\partial\Theta}{\partial C_{t+i}}\right|_{SS}(C_{t+i}-C) = (1-\tau)\sum_i\omega^i\beta^i\left[(1-\sigma)\,C^{1-\sigma} - \sigma C^{-\sigma}G\right]E_t\widehat{c}_{t+i}$

(d) $\left.\frac{\partial\Theta}{\partial G_{t+i}}\right|_{SS}(G_{t+i}-G) = (1-\tau)\sum_i \omega^i\beta^i C^{-\sigma} G E_t\widehat{g}_{t+i}$

(e) $\left.\frac{\partial\Theta}{\partial \tau_{t+i}}\right|_{SS}(\tau_{t+i}-\tau) = -\tau C^{\,-\sigma}(C+G)\sum_i \omega^i\beta^i E_t\widehat{\tau}_{t+i}$

(f) $\left.\frac{\partial\Theta}{\partial \frac{P_{t+i}}{P_t}}\right|_{SS}\left(\frac{P_{t+i}}{P_t}-1\right) = (1-\tau)\,C^{\,-\sigma}(C+G)\sum_i \omega^i\beta^i\,(\zeta-1)\,(E_t\widehat{p}_{t+i}-\widehat{p}_t)$

On the other hand, denote the RHS by Φ, so that it can be approximated by:

$$\begin{aligned}\Phi \quad\approx\quad & \Phi(SS) + \left.\frac{\partial\Phi}{\partial C_{t+i}}\right|_{SS}(C_{t+i}-C) + \left.\frac{\partial\Phi}{\partial G_{t+i}}\right|_{SS}(G_{t+i}-G) \\ & + \left.\frac{\partial\Phi}{\partial \phi_{t+i}}\right|_{SS}\left(\phi_{t+i}-\phi\right) + + \left.\frac{\partial\Phi}{\partial \frac{P_{t+i}}{P_t}}\right|_{SS}\left(\frac{P_{t+i}}{P_t}-1\right)\end{aligned}$$

(a) $\Phi(SS) = \frac{C^{\,-\sigma}(C+G)\mu\phi}{1-\omega\beta}$

(b) $\left.\frac{\partial\Phi}{\partial C_{t+i}}\right|_{SS}(C_{t+i}-C) = \mu\phi\sum_i \omega^i\beta^i\left[(1-\sigma)\,C^{1-\sigma}-\sigma C^{-\sigma}G\right]E_t\widehat{c}_{t+i}$

(c) $\left.\frac{\partial\Phi}{\partial G_{t+i}}\right|_{SS}(G_{t+i}-G) = \mu\phi\sum_i \omega^i\beta^i C^{-\sigma} G E_t\widehat{g}_{t+i}$

(d) $\left.\frac{\partial\Phi}{\partial \phi_{t+i}}\right|_{SS}\left(\phi_{t+i}-\phi\right) = \mu\phi C^{\,-\sigma}(C+G)\sum_i \omega^i\beta^i E_t\widehat{\phi}_{t+i}$

(e) $\left.\frac{\partial\Phi}{\partial \frac{P_{t+i}}{P_t}}\right|_{SS}\left(\frac{P_{t+i}}{P_t}-1\right) = \mu\phi C^{\,-\sigma}(C+G)\sum_i \omega^i\beta^i\zeta\,(E_t\widehat{p}_{t+i}-\widehat{p}_t)$

Equating both sides and canceling terms:

$$\widehat{p}_t^{fo}+\widehat{q}_t^{fo} = (1-\omega\beta)\left[\frac{\tau}{1-\tau}\sum_i \omega^i\beta^i E_t\widehat{\tau}_{t+i} + \sum_i \omega^i\beta^i\left(E_t\widehat{\phi}_{t+i}+E_t\widehat{p}_{t+i}\right)\right]$$

I bring forward this expression, take the expected value in t and then replace it again in the previous equation so that $\widehat{p}_t^{fo}$ and $\widehat{q}_t^{fo}$ will be expressed only in terms of $\widehat{p}_{t+1}^{fo}$ and $\widehat{q}_{t+1}^{fo}$:

$$\widehat{q}_t^{fo} = (1-\omega\beta)\left(\frac{\tau}{1-\tau}\widehat{\tau}_t+\widehat{\phi}_t\right)+\omega\beta E_t\widehat{q}_{t+1}^{fo}+\omega\beta E_t\pi_{t+1}$$

It is also possible to express $E_t\widehat{q}_{t+1}^{fo}$ in terms of $E_t\widehat{q}_{t+1}$; $\widehat{q}_t$ and $E_t\pi_{t+1}$. Finally, recall that

$$\begin{aligned}\widehat{q}_t &= \frac{\omega}{1-\omega}\pi_t \Rightarrow \\ E_t\widehat{q}_{t+1} &= \frac{\omega}{1-\omega}E_t\pi_{t+1}\end{aligned}$$

Substituting these two equations in the resulting one from the previous steps, I obtain an expression for the inflation rate:

$$\pi_t = \Lambda_1 \widehat{\tau}_t + \Lambda_2 E_t \pi_{t+1} + \Lambda_3 \pi_{t-1} + \Lambda_4 \widehat{\phi}_t$$

where

$$\begin{aligned}
\Lambda_0 &\equiv \left(\frac{\omega}{1-\omega} + \frac{\omega\beta\Theta}{1-\Theta} + \Theta \right)^{-1} \\
\Lambda_1 &\equiv \Lambda_0 \left(1-\Theta\right)\left(1-\omega\beta\right) \frac{\tau}{1-\tau} \\
\Lambda_2 &\equiv \Lambda_0 \frac{\beta}{\left(1-\Theta\right)} \left[\Theta + \frac{\omega}{\left(1-\omega\right)} \right] \\
\Lambda_3 &\equiv \Lambda_0 \frac{\omega\Theta}{1-\omega} \\
\Lambda_4 &\equiv \Lambda_0 \left(1-\Theta\right)\left(1-\omega\beta\right)
\end{aligned}$$

1.3 Optimal fiscal policy under central bank independence

Optimal fiscal policy under central bank independence is obtained from the incumbent's maximization problem:

$$\begin{aligned}
& \max_{G_t} E_t \frac{\left[\left(1-\alpha\right) G_t\right]^{1-\sigma}}{1-\sigma} + \beta\Gamma E_t \frac{\left[\left(1-\alpha\right) G_{t+1}\right]^{1-\sigma}}{1-\sigma} \\
st &: \\
E_t \pi_{t+1} &= \frac{\lambda\theta_P}{\Lambda_4 \left(\theta_P \eta + \sigma\right)} \left(\frac{\Lambda_2}{\beta} x_t - E_t x_{t+1} + \beta \Lambda_3 E_t x_{t+2} \right) \\
\widehat{x}_t &= \theta_P \widehat{c}_t + \theta_G \widehat{g}_t - \widehat{y}_t^f \\
\Gamma &= 1 - F\left[-\Gamma_G \left(E_t \widehat{g}_{t+1}^a - E_t \widehat{g}_{t+1}^b \right) + \Gamma_\pi \left(E_t \pi_{t+1}^a - E_t \pi_{t+1}^b \right) - \Gamma_\mu E_t \mu_t^a \right] \\
and \;\; G_t &= \tau_t Y_t + \varepsilon_t + \rho_t \;\; holds \;\; \forall t
\end{aligned}$$

The FOC is then:

$$E_t G_t^{-\sigma} = \frac{f\left(\chi\right) \Gamma_\pi \lambda \theta_P \Lambda_2 \theta_G}{\Lambda_4 \left(\theta_P \eta + \sigma\right) G \left(1-\sigma\right)} E_t G_{t+1}^{1-\sigma} \tag{1}$$

where $\chi \equiv -G \left(E_t \widehat{\tau}_{t+1}^a - E_t \widehat{\tau}_{t+1}^b \right) + \Gamma_\pi' \left(E_t \pi_{t+1}^a - E_t \pi_{t+1}^b \right).$