Online Appendix

to

PRICE-LEVEL TARGETING VERSUS INFLATION TARGETING IN A NEW KEYNESIAN MODEL WITH INFLATION PERSISTENCE

LUISA F. ACUÑA-ROA
Universidad de los Andes

JULIÁN A. PARRA-POLANIA[**]
Banco de República

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This appendix provides further technical details for derivation of some results in the paper "Price-level Targeting versus Inflation Targeting in a New Keynesian Model with Inflation Persistence". Appendix A provides details for the ITC and the PLT problems. Appendix B provides details for the IT' and the PLT' problems.

[**] Julián A. Parra-Polania (corresponding autor): Banco de la República (Central Bank of Colombia), Cra 7 No. 14-78, Piso 11, Bogotá, Colombia; jparrapo@banrep.gov.co. Luisa F. Acuña-Roa: Universidad de los Andes-Colombia (Andes University-Colombia), Cra. 1 Nº 18A-12, Bogotá, Colombia; lf.acuna10@uniandes.edu.co. All opinions and possible errors are the sole responsibility of the authors.

A. Derivations for the ITC and the PLT problems

A.1. Expression for V_t^{ITC}

For the ITC case we have:

$$z_t^{ITC} = \frac{\lambda}{\kappa}(y_{t-1}^{ITC} - y_t^{ITC}), \tag{A1}$$

$$y_t^{ITC} = \eta\left(y_{t-1}^{ITC} - \frac{\kappa}{\lambda}\varepsilon_t\right), \tag{A2}$$

(Equations (7) and (8) in the article).

Using these equations we can show that:

$$E_t(z_{t+i}^{ITC})^2 = \left(\frac{\lambda(1-\eta)}{\kappa}\right)^2 E_t(y_{t+i-1}^{ITC})^2 + \eta^2\sigma^2, \tag{A3}$$

$$E_t(y_{t+i}^{ITC})^2 = \eta^2 E_t(y_{t+i-1}^{ITC})^2 + \left(\frac{\eta\kappa}{\lambda}\right)^2 \sigma^2. \tag{A4}$$

Using $V_t^{ITC} = (z_t^{ITC})^2 + \lambda(y_t^{ITC})^2 + \beta\sum_{i=0}^{\infty}\beta^i E_t[(z_{t+i+1}^{ITC})^2 + \lambda(y_{t+i+1}^{ITC})^2]$ and the above equations we obtain:

$$V_t^{ITC} = (z_t^{ITC})^2 + \lambda(y_t^{ITC})^2 + \tag{A5}$$

$$\beta\sum_{i=0}^{\infty}\beta^i\left[\left(\left(\frac{\lambda(1-\eta)}{\kappa}\right)^2 + \lambda\eta^2\right)E_t(y_{t+i}^{ITC})^2 + \left(\frac{(\eta\kappa)^2}{\lambda} + \eta^2\right)\sigma^2\right].$$

Finally, from (A4) we know that

$$E_t(y_{t+i}^{ITC})^2 = \eta^{2i}(y_t^{ITC})^2 + \left(\frac{\eta\kappa}{\lambda}\right)^2 \frac{1-\eta^{2i}}{1-\eta^2}\sigma^2,$$

then by substituting this equation into (A5) and after some algebra we obtain

$$(z_t^{ITC})^2 + \lambda(y_t^{ITC})^2 + \frac{[(1-2\eta)\beta\lambda + \kappa^2 + \lambda]\eta^2}{(1-\beta)(1-\beta\eta^2)\lambda}\beta\sigma^2 + \frac{[(1-2\eta)\lambda + (\kappa^2+\lambda)\eta^2]\lambda}{(1-\beta\eta^2)\kappa^2}\beta(y_t^{ITC})^2, \tag{A6}$$

(Equation (9) in the article).

A.2. Solution for the PLT problem

For PLT, we can state the problem of the central bank as

$$Min_{y_t}\, L_t^{PLT} + \beta E_t V_{t+1}^{PLT}, \tag{A7}$$

(Equation (13) in the article). To obtain a solution we postulate functional forms[1] for the law of motion of w_t and the value function V_{t+1}^{PLT}:

$$w_t = \phi(w_{t-1} + \varepsilon_t), \tag{A8}$$

[1] We use equation (A5), with only one coefficient ϕ, just for the sake of simplicity. Instead, we could start by postulating an equation with potentially different coefficients for w_{t-1} and ε_t and then we could show that the solution for both coefficients is equal (i.e. just one coefficient is required).

$$V_{t+1}^{PLT} = \delta_0 + \delta_w w_t^2, \tag{A9}$$

these functional forms are later verified (and their coefficients ϕ, δ_0, δ_w determined) when finding a final solution to the model.

Remember the Phillips curve can be written as

$$w_t - w_{t-1} = \beta(E_t w_{t+1} - w_t) + \kappa y_t + \varepsilon_t, \tag{A10}$$

(Equation (11) in the article).

Solving (A10) for w_t yields

$$w_t = \frac{w_{t-1} + \beta E_t w_{t+1} + \kappa y_t + \varepsilon_t}{1+\beta}, \tag{A11}$$

and equation (A8) implies that

$$E_t w_{t+1} = \phi^2(w_{t-1} + \varepsilon_t). \tag{A12}$$

Using (A11) and (A12) and solving (A7) we can obtain the following expression:

$$y_t^{PLT} = -\frac{(1+\beta\delta_w)(1+\beta\phi^2)\kappa}{(1+\beta\delta_w)\kappa^2 + (1+\beta)^2\mu}(w_{t-1} + \varepsilon_t), \tag{A13}$$

and substituting this result and (A12) into (A11):

$$w_t = \frac{(1+\beta)(1+\beta\phi^2)\mu}{(1+\beta\delta_w)\kappa^2 + (1+\beta)^2\mu}(w_{t-1} + \varepsilon_t). \tag{A14}$$

We can use equations (A9), (A11)-(A14) and the fact that $E_t[w_{t-1}\varepsilon_t] = 0$ and $E_t[\varepsilon_t^2] = \sigma^2$ to find an expression for the value function V_t^{PLT}:

$$V_t^{PLT} = \beta\delta_0 + \frac{(1+\beta\delta_w)(1+\beta\phi^2)^2\mu}{(1+\beta\delta_w)\kappa^2 + (1+\beta)^2\mu}(w_{t-1}^2 + \sigma^2). \tag{A15}$$

Comparing equations (A14) and (A15) with (A8) and (A9), we verify the functional forms postulated above and determine the value of the coefficients by solving the following system:

$$\phi = \frac{(1+\beta)(1+\beta\phi^2)\mu}{(1+\beta\delta_w)\kappa^2 + (1+\beta)^2\mu}, \tag{A16}$$

$$\delta_w = \frac{(1+\beta\delta_w)(1+\beta\phi^2)^2\mu}{(1+\beta\delta_w)\kappa^2 + (1+\beta)^2\mu}, \tag{A17}$$

$$\delta_0 = \beta\delta_0 + \delta_w\sigma^2. \tag{A18}$$

A.3. Expression for $V_t^{IT|PLT}$

From equations (A13) and (A14) we can show that:

$$E_t(z_{t+i}^{PLT})^2 = (1-\varphi_1)^2 E_t w_{t+i-1}^2 + \varphi_1^2\sigma^2, \tag{A19}$$

$$E_t(y_{t+i}^{PLT})^2 = \varphi_2^2(E_t w_{t+i-1}^2 + \sigma^2), \tag{A20}$$

where $\varphi_1 \equiv \frac{(1+\beta)(1+\beta\phi^2)\mu}{(1+\beta\delta_w)\kappa^2+(1+\beta)^2\mu}$ and $\varphi_2 \equiv \frac{(1+\beta\delta_w)(1+\beta\phi^2)\kappa}{(1+\beta\delta_w)\kappa^2+(1+\beta)^2\mu}$.

Using $V_t^{IT|PLT} = (z_t^{PLT})^2 + \lambda(y_t^{PLT})^2 + \beta\sum_{i=0}^{\infty}\beta^i E_t[(z_{t+i+1}^{PLT})^2 + \lambda(y_{t+i+1}^{PLT})^2]$ and the above equations we obtain:

$$V_t^{IT|PLT} = (z_t^{PLT})^2 + \lambda(y_t^{PLT})^2 + \tag{A21}$$

$$\beta\sum_{i=0}^{\infty}\beta^i[(1-\varphi_1)^2 + \lambda\varphi_2^2]E_t w_{t+i-1}^2 + (\varphi_1^2 + \lambda\varphi_2^2)\sigma^2.$$

Finally, from (A14) we can see that

$$E_t w_{t+i}^2 = \varphi_1^{2i} w_t^2 + \varphi_1^2 \frac{1-\varphi_1^{2i}}{1-\varphi_1^2}\sigma^2,$$

then by substituting this equation into (A21) and after some algebra we obtain

$$(z_t^{PLT})^2 + \lambda(y_t^{PLT})^2 + \frac{(1+\beta(1-2\phi))(1+\beta\phi^2)^2\phi^2\mu^2+\lambda\kappa^2\delta_w^2}{(1-\beta)(1-\beta\phi^2)(1+\beta\phi^2)^2\mu^2}\beta\sigma^2 + \tag{A22}$$

$$\frac{\left((1-\phi)^2(1+\beta\phi^2)^2\mu^2+\lambda\kappa^2\delta_w^2\right)\kappa^2\delta_w^2}{(1-\beta\phi^2)(1+\beta\phi^2)^4\phi^2\mu^4}\beta(y_t^{PLT})^2,$$

(Equation (26) in the article).

B. Derivations for the IT' and the PLT' problems

B.1. Solution for IT'

For the IT' case, the central bank minimises $L_t^{IT'} = \pi_t^2 + \mu y_t^2$ subject to

$$\pi_t - \gamma\pi_{t-1} = \beta\,(E_t\pi_{t+1} - \gamma\pi_t\,) + \kappa y_t + \varepsilon_t. \quad \text{(B1)}$$

To solve this problem we postulate

$$\pi_t^{IT'} = \psi_1^{IT'}\pi_{t-1}^{IT'} + \psi_2^{IT'}\varepsilon_t,$$

for inflation, and $V_{t+1}^{IT'} = \delta_0^{IT'} + \delta_\pi^{IT'}\pi_t^2$ for the value function. Then following a similar procedure to that described in Appendix A.2. we find that the relevant coefficients in the postulated forms can be determined using the following system:

$$\vartheta_1^{IT'} \equiv \frac{1}{1+\beta\left(\gamma-\psi_1^{IT'}\right)}, \quad \text{(B2)} \qquad \vartheta_2^{IT'} \equiv \kappa\left(\vartheta_1^{IT'}\right)^2\left(1+\beta\gamma\delta_\pi^{IT'}\right), \quad \text{(B3)}$$

$$\psi_1^{IT'} = \vartheta_1^{IT'}\left(\gamma + \kappa\psi_3^{IT'}\right), \quad \text{(B4)} \qquad \psi_2^{IT'} = \psi_1^{IT'}/\,\gamma, \quad \text{(B5)}$$

$$\psi_3^{IT'} = -\frac{\gamma\vartheta_2^{IT'}}{k\vartheta_2^{IT'}+\gamma\mu}, \quad \text{(B6)} \qquad \psi_4^{IT'} = \psi_3^{IT'}/\,\gamma, \quad \text{(B7)}$$

$$\delta_\pi^{IT'} = \left(\psi_1^{IT'}\right)^2\left(1+\beta\delta_\pi^{IT'}\right) + \mu\left(\psi_3^{IT'}\right)^2, \quad \text{(B8)}$$

where the first two equations correspond to definitions we have used to collect some parameters.

Solving the system for each particular set of parameter values we can approximate $V_t^{IT|IT'} \equiv \sum_{i=0}^{\infty}\beta^i E_t\left[\left(\pi_{t+i}^{IT'} - \gamma\pi_{t+i-1}^{IT'}\right)^2 + \lambda\left(y_{t+i}^{IT'}\right)^2\right]$ by calculating this sum for a large number of periods (5000) and taking into account that

$$E_t\left(\pi_{t+i}^{IT'}\right)^2 = \left(\psi_1^{IT'}\right)^2 E_t\left(\pi_{t+i-1}^{IT'}\right)^2 + \left(\psi_2^{IT'}\right)^2\sigma^2, \quad \text{(B9)}$$

$$E_t\left(y_{t+i}^{IT'}\right)^2 = \left(\psi_3^{IT'}\right)^2 E_t\left(\pi_{t+i-1}^{IT'}\right)^2 + \left(\psi_4^{IT'}\right)^2\sigma^2, \quad \text{(B10)}$$

$$E_t\left(\pi_{t+i}^{IT'}\pi_{t+i-1}^{IT'}\right) = \psi_1^{IT'} E_t\left(\pi_{t+i-1}^{IT'}\right)^2. \quad \text{(B11)}$$

B.2. Solution for PLT'

For the PLT' case, the central bank minimises $L_t^{PLT'} = p_t^2 + \mu y_t^2$ subject to the Phillips curve which can be rewritten as

$$p_t - p_{t-1} - \gamma(p_{t-1} - p_{t-2}) = \beta\,[E_t(p_{t+1} - p_t) - \gamma(p_t - p_{t-1})] + \kappa y_t + \varepsilon_t. \quad \text{(B12)}$$

To solve this problem we postulate functional forms

$$p_t^{PLT'} = \psi_1^{PLT'}p_{t-1}^{PLT'} + \psi_2^{PLT'}p_{t-2}^{PLT'} + \psi_3^{PLT'}\varepsilon_t,$$

for the price level, and $V_{t+1}^{PLT'} = \delta_{t-1}^{PLT'} + \delta_1^{PLT'} p_t p_{t-1} + \delta_2^{PLT'} p_t^2$ for the value function. Then following a similar procedure to that described in Appendix A.2. we find that the relevant coefficients in the postulated forms can be determined using the following system:

$$\vartheta_1^{PLT'} \equiv \frac{1}{1+\beta\left(1+\gamma-\psi_1^{PLT'}\right)}, \tag{B13}$$

$$\vartheta_2^{PLT'} \equiv \kappa\gamma\left(\vartheta_1^{PLT'}\right)^2\left(1+\beta\gamma\delta_2^{PLT'}\right), \tag{B14}$$

$$\vartheta_3^{PLT'} \equiv 1+\gamma+\beta\left(\psi_2^{PLT'}+\gamma\right), \tag{B15}$$

$$\psi_1^{PLT'} = \frac{\gamma\vartheta_1^{PLT'}\left(\mu\vartheta_3^{PLT'} - \frac{\beta\kappa^2\vartheta_1^{PLT'}\delta_1^{PLT'}}{2}\right)}{k\vartheta_2^{PLT'}+\gamma\mu}, \tag{B16}$$

$$\psi_2^{PLT'} = -\frac{\gamma^2\mu\vartheta_1^{PLT'}}{k\vartheta_2^{PLT'}+\gamma\mu}, \tag{B17}$$

$$\psi_3^{PLT'} = -\psi_2^{PLT'}/\,\gamma, \tag{B18}$$

$$\psi_4^{PLT'} = -\frac{\vartheta_2^{PLT'}\vartheta_3^{PLT'} + \frac{\beta\kappa^2\gamma\vartheta_1^{PLT'}\delta_1^{PLT'}}{2}}{k\vartheta_2^{PLT'}+\gamma\mu}, \tag{B19}$$

$$\psi_5^{PLT'} = \frac{\gamma\vartheta_2^{PLT'}}{k\vartheta_2^{PLT'}+\gamma\mu}, \tag{B20}$$

$$\psi_6^{PLT'} = -\psi_5^{PLT'}/\,\gamma, \tag{B21}$$

$$\delta_1^{PLT'} = \left(\psi_1^{PLT'}\right)^2\left(1+\beta\delta_2^{PLT'}\right) + \beta\psi_1^{PLT'}\delta_1^{PLT'} + \mu\left(\psi_4^{PLT'}\right)^2, \tag{B22}$$

$$\delta_2^{PLT'} = 2\psi_1^{PLT'}\psi_2^{PLT'}\left(1+\beta\delta_2^{PLT'}\right) + \beta\psi_2^{PLT'}\delta_2^{PLT'} - \frac{2\gamma\mu\psi_4^{PLT'}}{k\vartheta_2^{PLT'}+\gamma\mu}, \tag{B23}$$

where the first three equations correspond to definitions we have used to collect some parameters.

Solving the system for each particular set of parameter values we can approximate $V_t^{IT|PLT'} \equiv \sum_{i=0}^{\infty}\beta^i E_t\left[\left(\pi_{t+i}^{PLT'} - \gamma\pi_{t+i-1}^{PLT'}\right)^2 + \lambda\left(y_{t+i}^{PLT'}\right)^2\right]$ by calculating this sum for a large number of periods (5000) and taking into account that $\pi_{t+i}^{PLT'} \equiv p_{t+i}^{PLT'} - p_{t+i-1}^{PLT'}$ and

$$E_t\left(p_{t+i}^{PLT'}\right)^2 = \left(\psi_1^{PLT'}\right)^2 E_t\left(p_{t+i-1}^{PLT'}\right)^2 + 2\psi_1^{PLT'} + \psi_2^{PLT'} E_t\left(p_{t+i-1}^{PLT'}p_{t+i-2}^{PLT'}\right)\left(\psi_2^{PLT'}\right)^2 E_t\left(p_{t+i-2}^{PLT'}\right)^2 + \left(\psi_3^{PLT'}\right)^2\sigma^2, \tag{B24}$$

$$E_t\left(y_{t+i}^{PLT'}\right)^2 = \left(\psi_4^{PLT'}\right)^2 E_t\left(p_{t+i-1}^{PLT'}\right)^2 + 2\psi_4^{PLT'}\psi_5^{PLT'} E_t\left(p_{t+i-1}^{PLT'}p_{t+i-2}^{PLT'}\right) + \left(\psi_5^{PLT'}\right)^2 E_t\left(p_{t+i-2}^{PLT'}\right)^2 + \left(\psi_6^{PLT'}\right)^2\sigma^2, \tag{B25}$$

$$E_t\left(p_{t+i}^{PLT'} p_{t+i-1}^{PLT'}\right) = \psi_1^{PLT'} E_t\left(p_{t+i-1}^{PLT'}\right)^2 + \psi_2^{PLT'} E_t\left(p_{t+i-1}^{PLT'} p_{t+i-2}^{PLT'}\right), \tag{B26}$$

$$E_t\left(p_{t+i}^{PLT'} p_{t+i-2}^{PLT'}\right) = \psi_1^{PLT'} E_t\left(p_{t+i-1}^{PLT'} p_{t+i-2}^{PLT'}\right) + \psi_2^{PLT'} E_t\left(p_{t+i-2}^{PLT'}\right)^2. \tag{B27}$$