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THE EMPIRICS OF THE SOLOW GROWTH MODEL: LONG-TERM EVIDENCE

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In this paper we reassess the standard Solow growth model, using a dynamic panel data approach. A new methodology is chosen to deal with this problem. First, unit root tests for individual country time series were run. Second, panel data unit root and cointegration tests were performed. Finally, the panel cointegration dynamics is estimated by (DOLS) method. The resulting evidence supports roughly one-third capital share in income, α.

JEL classification codes: O47, O50, O57, C33, and C52

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I. Introduction

The basic Solow growth model postulates stable equilibrium with a long-run constant income growth rate. The neoclassical assumption of analytical representation of the production function usually consists of constant returns to scale, Inada conditions, and diminishing returns on all inputs and some

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degree of substitution among them. Assuming a constant savings rate implies that every country always follows a path along an iso-savings curve. Exogenous rates of population growth and technology were useful simplifications at the time Solow wrote the original paper.

Despite its originality, there are some limitations in the Solow model. First, it is built on the assumption of a closed economy. That is, the convergence hypothesis supposes a group of countries having no type of interrelation. However, this difficulty can be circumvented if we argue, as Solow did, that every model has some untrue assumptions but may succeed if the final results are not sensitive to the simplifications used. In addition to the model proposed by Solow, there have been some attempts at constructing a growth model for an open economy, for example Barro, Mankiw, and Sala-I-Martin (1995).

The second limitation of the Solow model is that the implicit share of income that comes from capital (obtained from the estimates of the model) does not match the national accounting information. An attempt to eliminate this problem, done by Lucas (1988), involves enlarging the concept of capital in order to include that which is physical and human (the latter of which consists of education and, sometimes, health). The third limitation is that, the estimated convergence rate is too low even though attempts to modify the Solow model have impacts on this rate; e.g., Diamond model and open economy versions of the Ramsey-Cass-Koopmans model both have larger rates of convergence. And finally, the equilibrium rates of growth of the relevant variables depend on the rate of technological progress, an exogenous factor and furthermore, the individuals in the Solow model (and in some of its successors) have no motivation to invent new goods.

Notwithstanding the shortcomings mentioned, the Solow model became the cornerstone of economic literature focusing on the behavior of income growth across countries. Moreover, the conditional convergence of income among countries implies a negative correlation between the initial level of the real *per capita* GDP and the subsequent rates of growth of the same variable. Indeed, this result arises from the assumption of diminishing returns on each input, ensuring that a less capital-intensive country tends to have higher rates of return and, consequently, higher GDP growth rates. Detailed assessment of the convergence hypothesis and, particularly its validity across different estimation techniques are found, *inter alia*, in Barro and Sala-I-Martin (1995), Bernard and Durlauf (1996), Barro (1997) and Durlauf (2003).

The Solow neoclassical growth model was exhaustively tested in Mankiw, Romer, and Weil (1992). They postulated that the Solow neoclassical model fits the data better, once an additional variable - human capital - is introduced, which improves considerably the original ability to explain income disparities across countries. Investigating the limitations listed above, this paper uses another route: new econometric techniques, that select a group of countries with time series that present the same stochastic properties in order to make reliable estimates of physical capital share. This procedure provides a new empirical test of the Solow growth model, which yields new evidence on income disparity behavior across countries.

Recently, the profusion of remarkable advances in econometric methods has generated a new set of empirical tests for economic growth theories. In keeping with this trend, an important contribution was made by Islam (1995), whose paper reports estimates for the parameters of a neoclassical model in a panel data approach. In this case, the author admits on leveling effects for individual countries as heterogeneous fixed intercepts in a dynamic panel. Although the Mankiw, Romer, and Weil (1992) findings allow us to conclude that human capital performs an important role in the production function, Islam (1995) reaches an opposite conclusion, once a country's specific technological progress is introduced into the model.

Lee, Pesaran, and Smith (1997) presented an individual *random effect* version of the model, developed by Islam (1995), introducing heterogeneities in intercepts and in slopes of the production function in a heterogeneous dynamic panel data approach. These authors have concluded that the parameter homogeneity hypothesis can definitely be rejected. Indeed, they point out that different growth rates render the notion of convergence economically meaningless, because knowledge of the convergence rate provides no insights into the evolution of the cross-country output variance over time.

However, most classical econometric theory has been predicated on the assumption that observed data come from stationary processes. A preliminary glance at graphs of most economic time series or even at the historical track record of economic forecasting is enough to invalidate that assumption, since economies do evolve, grow, and change over time in both real and nominal terms.

Binder and Pesaran (1999) showed that a way exists to solve this question providing that a stochastic version of the Solow model is substituted for the original one. This requires explicitly treating technology and labor as stochastic processes with unit roots, which thus provide a methodological basis for using *random effects* in the equations estimated in the panel data approach. Once these settings are taking as given, Binder and Pesaran (1999) infer that the convergence parameter estimate is interpreted purely in terms of the dynamic random components measured in the panel data model, without any further information about convergence dynamics itself. Binder and Pesaran (1999) also conclude that the stochastic neoclassical growth model developed is not necessarily a contradiction, despite the existence of unit roots in the *per capita* output time series.

The purpose of this paper is threefold. First, a detailed time series analysis is carried out on a country-by-country basis in order to build the panel. The purpose of this procedure is to emphasize some evidence relating to unit roots and structural breaks in the data. Based on previous results, an alternative selection criterion is used for aggregating time series with the same features. Finally, the original Solow growth model results are validated by estimating the panel data model based on the procedure already described.

The remainder of the paper is organized as follows: the Solow growth model panel data and the status of the current research is discussed in section II. Section III considers data and sample selection issues. A country-by-country analysis and a data set re-sampling are carrying out in section IV. Section V contains the results for unit roots and cointegration tests for the entire panel. The final Solow growth model, which embodies an error-correction mechanism, is presented in section VI. In the next section, issues on the validity and interpretation of the convergence hypothesis are discussed. Concluding remarks are presented in section VIII.

II. Solow's Growth Model in a Dynamic Panel Version

This section presents the basics of the Solow model and discusses the corrections needed in order to build a dynamic panel version. The methodology is the same as that presented by Islam (1995) and restated in Lee, Pesaran, and Smith (1997).

Let us assume a Cobb-Douglas production function:

$$Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha} \tag{1}$$

where Y, K, L and A denote output, physical capital stock, labor force, and technology respectively. Capital stock change over time is given by the following equation:

$$\frac{\partial k / \partial t}{k} = \frac{sf(k)}{k} - (n + g + \delta) \tag{2}$$

where $k \equiv K/AL$ and $y \equiv Y/AL$, and s is the constant savings rate $s \in (0,1)$.

After taking logs on both sides of equation (1), the income *per capita* steady-state is:

$$\ln\left(\frac{Y(t)}{L(t)}\right) = \ln\big(A(0)\big) + gt + \frac{\alpha}{1-\alpha}\ln\big(s\big) - \frac{\alpha}{1-\alpha}\ln(n + g + \delta) \tag{3}$$

which is the equation obtained by Mankiw, Romer, and Weil (1992). These authors implicitly assume that the countries are already in their current steady state.

Apart from differences in the specific parameter values for each country, there is an additional term, $\ln\big(A(0)\big) + gt$ in equation (3), which deserves attention. Mankiw, Romer, and Weil (1992) assume g is the same for all countries, so gt is the deterministic trend and $\ln\big(A(0)\big) = a + \varepsilon$, where a is a constant and ε is the country-specific shock. However, the same cannot be said about *A(0)* since this term reflects the initial technological endowments of an individual economy.

This point is reinforced by Islam (1995) and Lee, Pesaran, and Smith (1997), who argue that this specification form generates loss of information on the technological parameter dynamics. The reason for this is that the panel data approach is the natural way to specify all shifts in the specific shock terms for a given country, ε. In order to proceed, we assume a law of motion for the behavior of the per capita incomes near the steady state. Let y^* be the equilibrium level for the output per effective worker, and $y(t)$, its actual value at time t. An approximation of y in the neighborhood of the steady state produces a differential equation that generates the convergence path. After some algebraic work on equation (3), we derive the same equation as Mankiw, Romer, and Weil (1992) by which to analyze the path of convergence across countries:

$$\ln(y_t) - \ln(y_{t-1}) = \left(1 - e^{-\lambda\Delta t}\right)\frac{\alpha}{1-\alpha}\ln(s) - \left(1 - e^{-\lambda\Delta t}\right)\frac{\alpha}{1-\alpha}\ln(n+g+\delta) - \left(1 - e^{-\lambda\Delta t}\right)\frac{\alpha}{1-\alpha}\ln(y_{t-1}) \quad (4)$$

Islam (1995) demonstrated that a correlation between *A(0)* and all included independent variables of the model is observable. Once this model is taken, Islam (1995) derives the growth regressions as in equation (5):

$$\ln(y_{i,t}) = e^{-\lambda\tau}\ln(y_{i,t-1}) + \left(1 - e^{-\lambda\tau}\right)\frac{\alpha}{1-\alpha}\sum\ln(s_{i,t}) + \left(1 - e^{-\lambda\tau}\right)\frac{\alpha}{1-\alpha}\sum\ln(n+g_{i,t}+\delta) + \left(1 - e^{-\lambda\tau}\right)\frac{\alpha}{1-\alpha}\ln(A(0)) + \eta_t + v_{i,t} \quad (5)$$

where, η_t is the time trend of technological change and $v_{i,t}$ is the transitory error term, with expected value equal to zero, that varies across countries and across time.

After applying a panel data estimation method to equation (5), interesting results come up immediately. Though the cross-sectional results of Mankiw, Romer, and Weil (1992) produce an average 2 percent annual rate of convergence, the estimates obtained in a panel data framework are more volatile. This observation is supported by Islam (1995), who allows for heterogeneities only in the intercept terms and finds annual convergence rates ranging from 3.8 to 9.1 percent. Alternatively, Lee, Pesaran, and Smith (1997) find annual rates of convergence of approximately 30 percent. Furthermore, Caselli, Esquivel, and Lefort (1996) suggest an annual convergence rate of 10 percent, after conditioning out the individual heterogeneity and by introducing instrumental variables to consider the dynamic endogeneity problem. Nerlove (1996), by contrast, finds annual convergence rate estimates that are even lower then those generated by cross-section regressions. He also argues that his findings are due to the finite sample bias of the estimator adopted in the empirical tests of the neoclassical growth model.

Choosing a panel data approach incurs both advantages and disadvantages.[1] A main disadvantage is that the nature of a panel structure as well as the procedure of decomposition of the constant term into two additive terms and a time-specific component does not necessarily seem theoretically natural in many cases. However, one significant advantage comes from solving the problem of interpreting standard cross-section regressions. In particular, the dynamic equation typically displays correlation between lagged dependent variables and unobservable residual, for example, the Solow residual. Therefore, the resulting regression bias depends on the number of observations and disappears only when that figure approaches infinity. This point is one of the most important issues treated in this paper, mainly because it complicates in interpreting convergence regression findings in terms of poor countries, narrowing the gap between themselves.

Most research on this topic admits time spans in estimating the panels as opposed to use of an entire time series (a recent exception is Ferreira, Issler, and Pessôa (2000)). In fact, this could conceal important problems such as unit roots and structural breaks. Moreover, as long as a first-order-integrated stochastic process *I(1)* is detected for a set of time series, the possibility of a panel data error-correction representation cannot be discarded.

Certainly this methodology leads to another puzzle, as stressed by Islam (1995); i.e. the larger the time span, short-term disturbances may loom larger. However, this paper deals with the time dimension of the panel in a more precise form. All the procedures (such as specifying the right stochastic processes underlying time series behavior) are used in order to guarantee the absence of bias in the estimated parameters, which the most efficient strategy in handling this problem, is to start with an individual investigation for each time series included in the time dimension of the panel, before estimating the panel parameters.

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[1] Durlauf and Quah (1999) provide a good source for supporting these arguments.

III. Data Set and Samples

Following the traditional approach in dealing with growth empirics, as in Mankiw, Romer, and Weil (1992); Nerlove (1996); and Barro (1997), among others, we use data on real national accounts, compiled by Heston and Summers (1991) and known as Penn World Tables Mark 5.6. This includes time series based on real income, real government and private investment spending, and population growth for 1959-1989. The countries included in our sample are described in Table 1.

Note that countries included are only the ones which compose the *First-Order Integrated Sample* (IS), as described in next section.

Table 1. Countries Included in Analysis

First order stationary sample		
Algeria	Guinea-Bissau	Papua New Guinea
Argentina	Guyana	Korea, Rep. of
Austria	Italy	Romania
Bangladesh	Jamaica	Russia
Belgium	Jordan	South Africa
Benin	Kenya	Sri Lanka
Bolivia	Madagascar	Swaziland
Botswana	Mali	Thailand
Burkina Faso	Mauritania	The Gambia
Canada	Mauritius	Togo
Central African, Rep. of	Morocco	Trinidad and Tobago
Chad	Myanmar	Uganda
Cyprus	Namibia	United States
Denmark	New Zealand	Yugoslavia
Domican Republic	Nicaragua	Zaire
Fiji	Nigeria	Zambia
Ghana	Pakistan	Zimbabwe
Greece	Panama	

An important issue should be considered here. Since the present goal is to validate the standard Solow's model, we should consider doing the same thing with its augmented version (Mankiw, Romer, and Weil, 1992) in the same way. The ideal procedure would be to introduce some measures of human capital, then testing for unit root and a possible cointegration relationship with other variables in the model. This could be done by using the Barro-Lee education attainment dataset (Barro and Lee, 2001).

However, the data compiled by these authors consist in a panel of observations based on five-year spans. Since the data covers 1960-1995, we have only 8 data points, which preclude any test for unit roots, even if corrected for small samples.

IV. Time Series Preliminary Analysis

In fact, accurate country-by-country time series analyzes has been carried out, the first of which was performed by Nelson and Plosser (1982). Building on these results, in this paper, we investigate the dynamic structure of the three time series cited above for each of the 123 countries in the data set.

The first step involves running augmented Dickey and Fuller (1979) and Phillips and Perron (1988) unit root tests. In addition, for the time series, we suspect there is a structural break and therefore, Lee and Strazicich (2001) unit roots tests are performed. Based on these procedures, 30 countries are dropped from the original data set because they fail to match the integration degree of the time series tested[2] and the final and definite sample covers the remaining 93 countries. The empirical results obtained in this paper permit us to state the following:[3]

First Fact: For 20 countries out of 93, an *I(2)* stochastic process was adjusted. The real per capita income fits into a first-order integrated stochastic process, *I(1)*, for 73 countries, which accounts for 80 percent of the sample. Furthermore, for 20 countries out of these 73, where the *per capita* income

[2] In fact, these countries' time series follow a mixture of the *I(1)*, *I(2)*, and *I(0)* processes.

[3] The tables containing individual country results are available upon request.

is *I(1)*, the null hypothesis of a single endogenous structural break is not rejected.[4]

Second Fact: The *per capita* physical capital time series, obtained using a proxy variable defined as the sum of private and public investment spending, is a first-order integrated stochastic process *I(1)*, with no exceptions for countries in the sample.

Third Fact: The growth rates of population across countries are characterized by first-order integrated stochastic processes *I(1)*. The calculated values for ADF and PP tests do not allow us to reject the null hypothesis for 101 countries in the data set. The remaining 22 time series are all stationary. Furthermore, the same 53 countries that follow an *I(1)* stochastic process for real per capita income are, in fact, contained in this sample of 101 countries.

One of the major problems related to recent empirical works on growth models concerns the adoption of an *ad hoc* procedure for choosing samples for analysis. Therefore, though choosing a specific procedure that classifies countries into oil producers, industrialized and developed nations and others, seems to be consistent, nothing can be said about its reliability.

Once this argument is accepted as reasonable, we select the groups based on stochastic characteristics of the processes that drive the entire variables behavior set. On a country-by-country basis, it is clearly possible to aggregate countries in an entirely new binary fashion: countries where the real *per capita* income growth rates are represented by a stationary process, which can include structural breaks, and countries where these time series are integrated. This procedure results in three samples: the *First-Order Integrated Sample* (IS), the *Second-Order Integrated Sample* (IIS), and the *Stationary Sample* (SS).[5] In this paper, only the first sample is admitted; research on the remaining samples has already been carried out by the present authors.

Additionally, empirical reasoning appropriate to support this alternative methodology. Once the conclusion that the right frame for a growth model is

[4] Tables available upon request.

[5] The stationary sample contains a structural break time series subset.

based on cross-sections and time-series dimensions of a panel, the next step involves performing a panel data unit root test. Then first the existence of a cointegration relationship among the variables that constitute the model has to be tested for. If this relationship is supported by the data, the estimation of an error-correction mechanism is the final step.

V. Panel Data Unit Root and Cointegration Tests

The panel data unit root test calculated is based on an original approach incorporated in recent econometric literature. For the sake of this test, the null hypothesis refers to non-stationary behavior of the time series, connection admitting the possibility that the error terms are serially correlated with different serial correlation coefficients in cross-sectional units. The test is calculated as an averaged ADF t-statistic, as presented in Im, Pesaran, and Shin (2002):

$$y_{i,t} = \alpha_i + \beta_i t + \rho y_{i,t-1} + v_{i,t} \tag{6}$$

The calculated values of the Im, Pesaran, and Shin panel data unit root test are reported in Table 2.

Table 2. Panel Data Unit Root Test Estimates - IPS

Variables	IPS $\bar{t}$-test	IPS $\overline{LM}$-test
Income growth	-30.6558	37.4643
Per capita income	-1.8544	2.1694
Saving rates	-1.0288	1.9242
Population growth	0.0869	0.9186

The null hypothesis of one unit root is rejected only in the case of the real income growth variable. For the other four variables, the calculated unit root test estimates are not cause for rejecting the null hypothesis. Moreover, the test is calculated in a panel representation that accounts for a both constant term and a time-trend component as in equation (6).

Considering a recent paper by Strauss and Yigit (2003), in which the authors show that the Im, Pesaran, and Shin panel data unit root test is potentially biased, we take into account another test for panel unit roots.[6] This test, developed by Hadri and Larsson (2000) has two main advantages: first, it considers stationarity as a null hypothesis, so it can also be used for a confirmatory analysis together with other tests; second, it corrects for heterogeneous error variances across cross sections and serial correlation-over-time dimension, the main problem with which the IPS test cannot deal. Results are presented in Table 3.

Table 3. Panel Data Stationarity Test Estimates - Hadri & Larson

Variables	Z_τ	P-value
Income growth	-1.132	0.8712
Per capita income	65.888	0.0000
Saving rates	42.726	0.0000
Population growth	57.222	0.0000

Again, the null hypotheses of stationarity of all panels under individual heterocedasticity and time series correlation are rejected, excepting for income growth, indicating that all countries have their growth variables guided by a unit root process.

The cointegration tests performed in this paper resemble those in Kao (1999) and Pedroni (1992, 1999) with the null hypothesis regarding estimated equation as not cointegrated. Four types of cointegration tests are, therefore performed. First, the Dickey-Fuller t-based test (Kao $DF\text{-}\rho$) is calculated. Second, an augmented Dickey-Fuller t-based test (Kao $DF\text{-}t\rho$) is also calculated. Finally, a panel t-parametric statistic (Pedroni ρ_{NT}), calculated on the basis of pooling along the within-dimension, and a group t-parametric statistic resulting from (Pedroni $t\text{-}\rho_{NT}$), which relies on a pooling along the between-dimension is calculated. The final estimates for all tests are in Table 4.

[6] We acknowledge an anonymous referee for pointing out this potential pitfall.

Table 4. Cointegration Test Estimates for the Solow Model

Test type	Statistic	Probability
Kao: D-F_{ρ}	-700.489	0.0000
Kao: D-$F_{t\rho}$	-409.482	0.0000
Pedroni_{ρ}	-13.538	0.0000
$\text{Pedroni}_{t\rho}$	-17.630	0.0000

Once the estimated results for all tests proved to be significant compared to the cut-off significance values, the null hypothesis of no cointegration was rejected. Therefore, the next step involves the estimation of an error-correction model for the Solow growth model, which is the main topic of discussion in the next section.

VI. The Solow Model in an Error-Correction Presentation

The estimated error-correction model[7] is based on a reparameterization of an autoregressive distributed lag model represented by ARDL (p, q). If the time series observations can be stacked for each group in the panel, the ECM can be written as follows:

$$\Delta \mathbf{y}_i = \phi \mathbf{y}_{i,-1} + \beta_i \mathbf{X}_i + \sum_{j=1}^{p-1} \lambda_{i,j}^{*} \Delta \mathbf{y}_{i,-j} + \sum_{j=1}^{p-1} \delta_{i,j}^{*} \Delta \mathbf{X}_{i,-j} + \mathbf{D}\gamma_i + \varepsilon_i \tag{7}$$

$\forall i, i = 1,\dots,T$ where $\mathbf{y}_i = (y_{i,1},\dots,y_{i,T})^t$ is a T *x* 1 vector of observations on real *per capita* income for the i^{th} group of the panel; $\mathbf{X}_i = (x_{i,1},\dots,x_{i,T})^t$ is a T *x* k matrix of observations on the independent variables of the model, which vary across groups and across time, i.e., population growth rates and per capita physical capital accumulation rates, and $\mathbf{D} = (d_1,\dots,d_T)^t$ is a matrix of dimension T *x* S that includes the observations on time-invariant independent variables as intercepts and time-trend variables.

[7] ECM, henceforth.

Assuming that disturbances are identical and independently distributed across countries and over time, and that the roots of the ARDL model are outside the unit circle, it is possible to ensure that there exists a long-run relationship between $y_{i,T}$ and $x_{i,T}$, defined by the following equation:

$$y_{i,t} = -\left(\frac{\beta_i'}{\phi_i}\right)x_{i,t} + \eta_{i,t} \tag{8}$$

where the error term, $\eta_{i,t}$, is a stationary process. Clearly, we conclude that the order of integration of the variable $y_{i,t}$ is, at most, equal to the order of integration of the regressors.

In order to write equation (7) in a more compact and intuitive manner, we set the long-run coefficients on $\mathbf{X}_{i,t}$ as $\theta_i = \beta_i / \phi_i$ to be the same across groups, namely $\theta_i = \theta$, which results the ECM expression:

$$\Delta\mathbf{y}_i = \phi_i\xi_i(\theta) + \mathbf{W}_i\kappa_i + \varepsilon_i \tag{9}$$

where $\xi_i(\theta) = \mathbf{y}_{i,-1} - \theta\mathbf{X}_i$ (10)

is the error-correction component of the entire ECM representation.

The introduction of dynamic panel data methodology affects the traditional analysis of growth model. Due to the inner structure of this model, this admits the effects of variables in levels and lags, on the estimation step. Relying on a theoretical statistical viewpoint, the procedure is optimal, since all parameters of the model are estimated by maximizing a likelihood function. Another difference concerns the interpretation of short and long-run estimated coefficients. Basically, once an error-correction model is admitted, one is dealing with actual values, though estimating only observed long-run parameters.

Finally, the common procedure in estimating an unrestricted and a restricted form of the basic empirical specification, in an error-correction model allows for different interpretations. This happens because cointegration vector estimation is sensitive to a linear combination of the variables.

The estimated results of a dynamic *fixed-effects* panel model in an error-correction form are presented in Table 5.

Table 5. Dynamic Panel Estimates for the Growth Model

Unrestricted regression	
Variables	Long-run coefficients
ln (s)	0.4926
	(0.1435) *
ln (n + g + δ)	-1.2787
	(0.3565) *
$\phi\theta$	0.4926
ϕ	-0.0742
Model adjustment statistics	
AIC	1,832.1600
SC	1,678.3900
LR stat. for long-run parameters	224.0441
p-value	0.0000
Restricted regression	
Variables	Long-run coefficients
ln (s) - ln (n + g + δ)	0.5144
	(0.0420) *
Implied α	0.3396
$\phi\theta$	0.5144
ϕ	-0.0696
Model adjustment statistics	
AIC	1,902.4500
SC	1,753.0000
LR stat. for long-run parameters	135.0147
p-value	0.0000

Notes: The dependent variable is ln(y). Numbers in parenthesis refer to standard deviations and the signal * indicates significance at 5% levels. Sample size: 1,484.

Based on these, it is possible to state the following: first, the coefficient of savings and population growth rates shows the theoretically predicted signs, but not at the same level of magnitude. This finding apparently contradicts the hypothesis of constant returns to scale, indicating the prevalence of decreasing returns, since the magnitude of the effective depreciation variable is, in the modulus, twice the magnitude of the savings rate. However, if this is true when the restricted equation is estimated, the correct physical capital share cannot be found.

Now, taking into account the restricted equation, the estimated parameter provides an implicit capital share whose value is exactly one third. Additionally, the sign of this coefficient was found to be positive. Thus, our results support the standard view of $\alpha = 1/3$, once the implied α is calculated.

In contrast to the widespread claim that the Solow model explains cross-country variability in labor productivity largely by appealing to variations in technologies, the two readily observable variables on which the Solow model focuses account, in fact, for most of the variations in *per capita* income.

Concerning the countries' time series which are proved to be represented by an *I(2)* integrated stochastic process, an additional a concluding remark should be added. The possible absence of diminishing returns to capital, a key property upon which the endogenous growth theory relies, is an assumption that many authors have provided a basis for in the recent literature. This is the case, for example, of Lucas (1988), Romer (1990), and Rebelo (1991). Based on the analysis in this paper, it remains an open question whether this sort of endogenous growth models are sufficient to explain the dynamic behavior of economies whose income growth stochastic process embodies an acceleration property.

Setting up a simple endogenous growth model, such as the ***AK*** model,[8] we can easily observe the absence of diminishing returns to capital. An economy described by this model can display positive long-run per capita growth without any technological progress, which is coherent with the presence of an embodied acceleration component.

[8] Like Barro and Sala-I-Martin (1995), we also think that the first economist to use a production function of the ***AK*** type was Von Neumann (1937).

Many authors, including Barro and Sala-I-Martin (1995) and those cited in their references therein, mention that one way to think about the absence of diminishing returns to capital in the ***AK*** production function is to consider a broad concept of capital encompassing both physical and human components. Unfortunately, as pointed out in section III, there is no appropriate data for carrying out an econometric analysis based on the approach presented here. Thus, such new other ideas as learning-by-doing, discussed by Arrow (1962) and Romer (1990), and purposeful activity, such as R&D expenditures, as in Romer (1987), and Aghion and Howitt (1998), should be considered. Finally, concerning convergence, unlike the neoclassical model, the ***AK*** formulation does not predict absolute or conditional convergence, which constitutes a substantial failing of the model, because conditional convergence appears to be an empirical regularity. This is certainly a matter for further research.

VII. Interpreting Convergence

Interpreting income convergence hypotheses by panel data estimation is a controversial issue, because it centers on the interpretation of estimated speed of convergence and, consequently, its validity.

In Bernard and Durlauf (1996), the authors argue that cross-sectional convergence tests, as performed by Mankiw, Romer, and Weil (1992) and others are based on the fact that data are in *transition towards a limiting distribution* and, therefore, the convergence hypothesis must be interpreted as a catching-up. The same reasoning should be applied to a panel data approach. Furthermore, the authors arrogate that time series tests assume that data sets are generated by economies *near their limiting distributions*, and convergence must be interpreted to mean that initial conditions have no effect on the expected value of output differences across countries. Consequently, a given approach is appropriate depending upon whether one regards the data as better fitting by *transition* or *steady state dynamics*.

In this paper, the estimation of an ECM provides us with a framework to interpret convergence by either type of dynamics without violating Proposition 6 in Bernard and Durlauf (1996). First, concerning cross-sectional and panel data tests, we found that the expected value of growth income across countries is negative while the difference between initial incomes is positive. At the same time, the existence of an error correction term, ϕ, implies that in the long run, the system is *I(0)*, so that absence of unit roots is consistent with the convergence hypothesis under the time series structure of the model.

In this fashion, the long-run behavior of income across countries furnishes us with a proxy for the speed of convergence, i.e. the *error correction term*. In our model, this term assumes the value of 0.0742. In other words, economies, on the average, will converge at a 7.42 % rate, a more reasonable result than the usual 2 % rate.

VIII. Concluding Remarks

This paper rests on empirical evidence obtained to support the original Solow growth model, which in fact happens, since the implied capital share on output is approximately the same as that predicted by given national accounting data. Furthermore, this finding on the share of capital output allows us to arrive at larger and less restrictive conditional coefficient of the speed of convergence.

The dynamic *fixed-effects* Solow growth model provides a tight theoretical framework within which to interpret the stochastic process behind income growth. Once the nature of stochastic processes is taken into account, an important issue arises when estimating the long-run behavior of income growth that is not significantly different from the predictions of the Solow model, including the evidence for diminishing returns to scale. Since panel data the cointegration technique assigns a fixed effect to allow for country-specific heterogeneities, conditional convergence does not lose its meaning.

In order to apply a panel data cointegration technique, a panel unit root test is calculated the result obtained does not allow us to reject the null hypothesis. Therefore, an error correction model representation is estimated following the usual time series procedure. Though an immediate comparison to a single time series error correction representation is not direct, it is reasonable to assume the estimated coefficient for ϕ is equivalent of the error correction term, i.e., the speed of convergence.

Admittedly, our procedure is not complete. This is so because the selection criterion suggested in this paper does not apply to when all countries are included in the sample. However, this is a solvable problem, since recent developments in econometric techniques deal with this problem.

Moreover, a new branch of research is open on the empirics of the Solow growth model, mainly for those countries manifesting the existence of structural breaks and multiple unit roots in the stochastic processes generating their income paths.

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