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EXTREMAL DEPENDENCE IN EUROPEAN CAPITAL MARKETS

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For a sample of six countries with dirty/free float regimes over 1999-2002—the United States, Japan, the Czech Republic, Poland, Switzerland, and the United Kingdom, we investigate whether paired currencies exhibit a pattern of asymptotic dependence on the euro. That is, whether an extremely large appreciation or depreciation in the nominal exchange rate of one country might transmit to the euro, and vice versa. In addition, we investigate whether stock markets of European countries, outside the Euro zone, have exhibited extreme-value dependence on their exchange rates against the euro. In general, after controlling for volatility clustering and inertia in returns, we do not find evidence of extreme-value dependence either between paired exchange rates or between paired stock indices and exchanges rates. However, for asymptotic independent paired returns, we find that tail dependency of exchange rates is stronger under large appreciations than under large depreciations. In addition, we find a weak association between large currency depreciations and declining stock prices.

JEL classification codes: C22, G10
Key words: extreme-value dependence, DVEC models.

I. Introduction

In the past few years, transmission of stock market returns and volatility in international financial markets (i.e., spillovers) has become an active research area in finance (e.g., Eun and Shim 1989; Lin, Engle and Ito 1994; Karolyi 1995; Hamao, Masulis, and Ng 2000; Ng 2000; Worthington and Higgs 2004). For instance, Karolyi

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(1995) focuses on the short-run dynamics of returns and volatility for stocks traded on the New York and Toronto stock exchanges. Ng (2000) in turn looks at spillovers from Japan and the United States to six countries of the Pacific Basin. Recent work in this area has resorted to wavelet analysis to quantify correlations between high- and low-frequency components of the data (e.g., Ramsey 2002; Gençay, Whitcher, and Selçuk 2005).

Another strand of the literature has focused on testing the existence of contagion in international financial markets. In a recent article, Forbes and Rigobon (2002) define contagion as a significant increase in cross-market linkages after one country, or a group of countries, experiences a shock. Otherwise, if co-movement does not increase significantly, but a high level of correlation persists in all periods, they call it interdependence. The authors conclude that there was no contagion during the Asian crisis, the 1994 Mexican devaluation, and the 1987 U.S. stock market crash. Earlier studies on examining changes in correlation around crisis are Calvo and Reinhart (1996), Frankel and Schmukler (1996), and Bailey, Chan, and Chung (2000). Additional references and a complete discussion on financial contagion can be found in a recent survey article by Karolyi (2003).

One limitation of using correlation coefficients to quantify market co-movements is that they give the same weight to extreme observations. Therefore, they are not an accurate measure of dependence if extreme observations present different patterns of dependence from the rest of the sample. Based on this fact, recent studies in finance have emphasized the importance of rare events, and have resorted to extreme value theory (e.g., McNeil and Frey 2000; Longin 2000; Longin and Solnik 2001). The literature in this area distinguishes between two types of extreme-value dependence: asymptotic dependence and asymptotic independence. Both forms of dependence allow dependence between relatively large values of each variable, but the largest values from each variable can take place jointly only when the variables are asymptotically dependent (see, for example, Coles, Heffernan and Twan 1999).

Early studies on extreme value theory primarily focused on asymptotic dependence. However, if two series are asymptotically independent, such an approach will overestimate extreme-value dependence and, thus, financial risk. This issue is discussed in two recent articles by Poon, Rockinger, and Tawn (2003, 2004). Poon et al. conclude that extreme-value dependence is usually stronger in bearish (left tails) than in bullish markets (right tails), and that some of this dependency can be explained by correlated conditional volatilities.

In the first place, this study tests for the existence of extreme-value dependence in exchange rate markets. As we know, paired currencies usually exhibit dependence due to differentials in short-interest rates, inflation rates, domestic money supply, industrial production, and/or cumulative trade balances. However, it is not obvious whether such dependency should also carry over to severe appreciations/ depreciations. Specifically, we focus on six countries with dirty/free float regimes over 1999-2002—the United States, Japan, the Czech Republic, Poland, Switzerland, and the United Kingdom, and investigate whether paired currencies exhibit a pattern of asymptotic dependence on the euro, since its adoption.[1]

Secondly, we investigate whether stock markets of European countries, outside the Euro zone, have exhibited extreme-value dependence on their exchange rates against the euro. In particular, the finance literature has documented the interrelation between stocks returns and exchange rates by extending the one-country capital asset pricing model (CAPM) to an international version (IAPM), in which currency risk must be priced (see Megginson 1997 for a thorough discussion and citations). To our knowledge, nobody has earlier pursued similar research.

This article is organized as follows. Section II presents a theoretical background on extreme-value dependence. Specifically, Section II.A defines the concepts of asymptotic dependence and asymptotic independence, based on Poon, Rockinger, and Twan's (2003, 2004) work. Meanwhile, Section II.B focuses on technical aspects of implementing the statistical tests referred to in Section II.A. Section III, which presents our estimation results, is divided into two parts. Section III.A presents statistical tests of asymptotic dependence of paired exchange rates for the above-mentioned sample of countries. Section III.B in turn focuses on tail dependence of exchange rates and stock markets between European countries in and outside the Euro zone. Finally, Section IV presents our main conclusions.

II. Theoretical background on extreme-value dependence

A. Asymptotic dependence and asymptotic independence

Focusing exclusively on the probability distribution of the maximum or the minimum of a sample is inefficient if other data on extreme values are available. Therefore, an alternative approach consists of modeling the behavior of extreme

[1] The Euro zone is made up by Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain.

values above a high threshold ("Peaks over threshold" or POT). The excess distribution, above a threshold u, is given by the conditional probability distribution

$$F_u(y) = \Pr(X - u \le y \mid X > u) = \frac{F(y+u) - F(u)}{1 - F(u)}, \quad y > 0. \tag{1}$$

Under some regularity conditions, there exists a positive function $\beta(u)$, for a large enough u, such that (1) is well approximated by the generalized Pareto distribution (GPD):

$$H_{\zeta,\beta(u)}(y) = \begin{cases} 1 - \left(1 + \dfrac{\zeta y}{\beta(u)}\right)^{-1/\zeta} & \zeta \neq 0 \\ 1 - \exp\left(-\dfrac{y}{\beta(u)}\right) & \zeta = 0 \end{cases}, \tag{2}$$

where $\beta(u)>0$, and $y \geq 0$ when $\zeta \geq 0$, and $0 \leq y \leq -\beta(u)/\zeta$ when $\zeta<0$ (see, for example, Coles 2001, or Embrechts, Klüpperberg and Mikosch 1997). If $\zeta>0$, F is said to be in the Fréchet family and $H_{\zeta\beta(u)}$ is a Pareto distribution. In most applications of risk management, the data comes from a heavy-tailed distribution, so that $\zeta>0$.

Poon, Rockinger, and Tawn (2003, 2004) introduce a special case of threshold modeling connected with the generalized Pareto distribution, for the Fréchet family. For this particular case, the tail of a random variable Z above a (high) threshold u can be approximated as

$$1 - F(z) = \Pr(Z > z) \sim z^{-1/\eta} L(z), \quad \textit{for } z > \mathrm{u}, \tag{3}$$

where $L(z)$ is a slowly varying function of z,[2] and $\eta>0$. If $L(z)$ is treated as a constant for all $z>u$, such that $L(z)=c$, and under the assumption of n independent observations, the maximum-likelihood estimators of η and c are

$$\hat{\eta} = \frac{1}{n_u} \sum_{j=1}^{n_u} \log\left(\frac{z_{(j)}}{u}\right), \quad \hat{c} = \frac{n_u}{n} u^{1/\hat{\eta}}, \tag{4}$$

where $z_{(1)},\ldots, z_{(n_u)}$, are the n_u observations above the threshold u, and $\hat{\eta}$ is known as the Hill estimator.

[2] A function L on $(0, \infty)$ is slowly varying if $\lim_{z\to\infty} L(tz)/L(z)=1$ for $t>0$.

In order to study dependency of paired returns, Poon et al. suggest transforming the original variables to a common marginal distribution. If (X,Y) are bivariate returns with corresponding cumulative distribution functions F_X and F_Y, the following transformation turns them into unit Fréchet marginals (S, T):

$$S = -\frac{1}{\ln F_X(X)}, \qquad T = -\frac{1}{\ln F_Y(Y)} \quad S>0, T>0. \tag{5}$$

Under this transformation, $\Pr(S>s)=\Pr(T>s)\sim s^{-1}$. As both S and T are on a common scale, the events $\{S>s\}$ and $\{T>s\}$, for large values of s, correspond to equally extreme events for each one. Given that $\Pr(S>s)\to 0$ as $s\to\infty$, the focus of interest is the conditional probability $\Pr(T>s|S>s)$, for large s. If (S, T) are perfectly dependent, $\Pr(T>s|S>s)=1$. By contrast, if (S, T) are exactly independent, $\Pr(T>s|S>s)=\Pr(T>s)$, which tends to zero as $s\to\infty$. Poon et al. define the following measure of asymptotic dependence:

$$\chi = \lim_{s\to\infty} \Pr(T>s \mid S>s) \qquad 0 \le \chi \le 1. \tag{6}$$

In particular, two random variables are called asymptotically dependent if $\chi>0$, and asymptotically independent if $\chi=0$.

Coles, Heffernan and Tawn (1999) point out that two random variables, which are asymptotically independent (i.e., $\chi=0$), may show, however, different degrees of dependence for finite levels of s. Therefore, they propose the following measure of asymptotic independence:

$$\bar{\chi} = \lim_{s\to\infty} \frac{2\log(\Pr(S>s))}{\log(\Pr(S>s, T>s))} - 1, \quad -1 \le \bar{\chi} \le 1. \tag{7}$$

Values of $\bar{\chi}>0$, $\bar{\chi}=0$ and $\bar{\chi}<0$ are an approximate measure of positive dependence, exact independence, and negative dependence in the tails, respectively. In particular, $\bar{\chi}$ resembles a correlation coefficient, and it is identical to the Pearson correlation coefficient under normality.

Poon et al's tail-dependence test is based on the (χ, $\bar{\chi}$) pair, which makes it possible to characterize both the form and degree of extreme-value dependence. For asymptotically dependent variables, $\bar{\chi}=1$ and the degree of dependence is measured by $\chi>0$. For asymptotically independent variables, $\chi=0$ and the degree of dependence is measured by $\bar{\chi}$.

The above tail-dependence test rests on the fact that

$$\Pr(Z>z)=z^{-1/Z}L(z) \qquad \text{for } z>u, \tag{8}$$

for some high threshold u, where $Z=\min(S,T)$. Equation (8) shows that z is the tail index of the univariate random variable Z. Therefore, it can be computed by using the Hill estimator, constrained to the interval (0, 1]. Under the assumption of independent observations on Z, Poon et al. show that

$$\hat{\bar{\chi}} = 2\hat{\zeta} - 1 = \frac{2}{n_u}\left(\sum_{j=1}^{n_u}\log\left(\frac{z_{(j)}}{u}\right)\right) - 1, \quad Var(\hat{\bar{\chi}}) = \frac{(\hat{\bar{\chi}}+1)^2}{n_u}, \tag{9}$$

where $\hat{\bar{\chi}}$ is asymptotically normal.

The null hypothesis of asymptotic dependence (i.e., $\bar{\chi}=1$) is rejected if $\hat{\bar{\chi}} + 1.96\sqrt{Var(\hat{\bar{\chi}})} < 1$. In that case, we conclude that the two random variables are asymptotically independent (i.e., c =0), and the degree of dependency is measured by $\bar{\chi}$. Otherwise, if the null hypothesis cannot be rejected, c is estimated under the assumption that $\bar{\chi} = \xi = 1$, where $\hat{\chi} = \frac{un_u}{n}$, and $Var(\hat{\chi}) = \frac{un_u(n-n_u)}{n^3}$.

B. Threshold selection

In order to compute the Hill estimator of the tail index referred to above *(z)*, one has to choose an appropriate threshold (u). The simplest procedure is to plot the Hill estimator on u, and find such u for which it stabilizes (see, for instance, Tsay 2002, chapter 7). In practice, however, in some cases such graphical procedure may not shed much light on the optimal threshold to be selected. Consequently, formal methods to choose u have been devised. A discussion on different adaptive-threshold selection can be found in Matthys and Beirlant (2000).

The authors distinguish two approaches to estimating the optimal threshold u. One consists of constructing an estimator for the asymptotic mean-squared error (AMSE) of the Hill estimator, and choosing the threshold that minimizes it. This approach includes a bootstrap method (e.g., Danielson, de Haan, Peng, and de Vries 2001). The second approach directly derives an estimator for u, based on the representation of the AMSE of the Hill estimator. The exponential regression model-studied in detailed in Beirlant, Diercks, Goegebeur, and Matthys (1999), and further discussed in Matthys and Beirlant (2003)-falls into this class. Given that the exponential regression approach is both straightforward and computationally fast, it is our choice to find the optimal threshold.[3] We next describe the steps involved in this procedure.

Feuerverger and Hall (1999) and Beirlant et al (1999) derive an exponential regression model for the log-spacings of upper statistics

$$j(\log(X_{n-j+1,n}) - \log(X_{n-j,n})) \sim \left(\gamma + b_{n,k}\left(\frac{j}{k+1}\right)^{-\rho}\right) f_j \quad 1 \le j \le k, \tag{10}$$

where $X_{1,n} \le X_{2,n} \le \ldots \le X_{n,n}$, $b_{n,k} = b\left(\frac{n+1}{k+1}\right)$, $1 \le k \le n-1$, $(f_1, f_2, \ldots, f_k)$ is a vector of independent standard exponential random variables, and $\rho \le 0$ is a real constant.

If the threshold u is fixed at the $(k+1)^{th}$ largest observation, the Hill estimator can be rewritten as

$$H_{n,k} = \frac{1}{k}\sum_{j=1}^{k} j(\log(X_{n-j+1,n}) - \log(X_{n-j,n})). \tag{11}$$

The Hill estimator so expressed is the maximum-likelihood estimator of γ in the reduced model

$$j(\log(X_{n-j+1,n}) - \log(X_{n-j,n})) \sim \gamma\, f_j, \qquad 1 \le j \le k.$$

From the above, the AMSE of the Hill estimator is given by

$$AMSE\ H_{k,n} = \left(\frac{b_{n,k}}{1-\rho}\right)^2 + \frac{\gamma^2}{k}. \tag{12}$$

Therefore, the optimal threshold k_n^{opt} is defined as the one that minimizes (12):

$$k_n^{opt} \equiv \arg\min_k (AMSE\ H_{k,n}) = \arg\min_k \left(\left(\frac{b_{n,k}}{1-\rho}\right)^2 + \frac{\gamma^2}{k}\right). \tag{13}$$

[3] Matthys and Berlaint (2000) carry out simulation exercises under different distributional assumptions to compare the adaptive-threshold selection methods they discuss. They find that the exponential-regression method performs quite well, and that it even outperforms the bootstrap method for moderate sample sizes (e.g. 500).

The algorithm for the exponential regression goes as follows: (i) In expression (10), fix ρ at $\rho_0 = -1$ and calculate least-squares estimates $\hat{\gamma}_k$ and $\hat{b}_{n,k}$ for each $k \in \{3,\ldots, n\}$; (ii) Determine $\overline{AMSE\ H_{k,n}} = \left(\frac{\hat{b}_{n,k}}{1-\hat{\rho}_k}\right)^2 + \frac{\hat{\gamma}_k^2}{k}$ for $k \in \{3,\ldots, n\}$, with $\hat{\rho}_k \equiv \rho_0$;[4] (iii) Determine $\hat{k}_n^{opt} = \arg\min_{3\le k\le n}(\overline{AMSE\ H_{k,n}})$ and estimate γ by $H_{\hat{k}_n^{opt}}$.

The first step of the algorithm boils down to running a linear regression of $j(\log(X_{n-j+1,n}) - \log(X_{n-j,n}))$ on a constant term and $\frac{j\,(n+1)}{(k+1)^2}$, for each $k \in \{3,\ldots,n\}$.

III. Estimation results

A. Tail dependence of selected nominal exchange markets and the euro

This section studies whether the currencies of European economies outside the Euro zone, which are characterized by dirty/free float regimes, and the Japanese Yen and U.S. dollar exhibit extreme-value dependence on euro.[5] The sample is comprised by the United States, Japan, the Czech Republic, Poland, Switzerland, and the United Kingdom. The data series were all obtained from the web site of the Bank of Canada, and are measured at a daily frequency. The sample period spans from January 1999 to December 2002. Exchange rates are expressed in Canadian dollars per unit of local currency,[6] and returns are continuously compounded. In this case, a negative (positive) return is associated with depreciation (appreciation) of the local currency against the Canadian dollar.

Table 1 shows some summary statistics of the data. Daily returns are zero, on average, and all return series reject the assumption of normality, according to the Jarque-Bera test. For all countries, autocorrelation coefficients are not in general statistically significant, which suggests that daily returns are close to white noise.

[4] Matthys and Beirlant (2000) point out that for many distributions the exponential-regression method works better, in MSE-sense, if the nuisance parameter ρ is fixed at some value ρ_0 rather than estimated.

[5] A dirty float is a type of floating exchange rate that is not completely free because Central Banks interfere occasionally to alter the rate from its free-market level.

[6] This is the way nominal exchange rates are reported at the web site of the Bank of Canada. The choice of a numeraire is, however, arbitrary.

Table 1. Descriptive statistics of returns on nominal exchange rates

	Czech Republic	Euro	Japan	Poland	Switzerland	U.K.	United States
# observations	1,002	1,002	1,002	1,002	1,002	1,002	1,002
Average	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Median	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Interquartile range	0.010	0.009	0.009	0.008	0.009	0.007	0.009
Kurtosis	3.706	4.130	3.980	5.264	4.459	3.682	3.980
Skewness	0.171	0.291	0.294	–0.280	0.290	0.202	0.294
r_1	0.024	0.001	–0.014	0.082	–0.010	-0.004	–0.014
p-value	0.453	0.966	0.652	0.009	0.758	0.900	0.652
r_{13}	0.005	–0.011	–0.005	0.042	–0.039	–0.040	–0.005
p -value	0.863	0.720	0.870	0.183	0.213	0.203	0.870
r_{26}	0.017	0.016	–0.016	–0.006	0.001	0.007	–0.016
p-value	0.591	0.604	0.602	0.850	0.979	0.831	0.602
p-value JB test	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: (1) The sample period is 1999-2002. The data are measured at a daily frequency, and the numeraire is the Canadian dollar. (2) r_j represents the autocorrelation coefficient of order j. (3) JB test stands for the Jarque-Bera test to detect departures from normality. (4) Data source: The Bank of Canada.

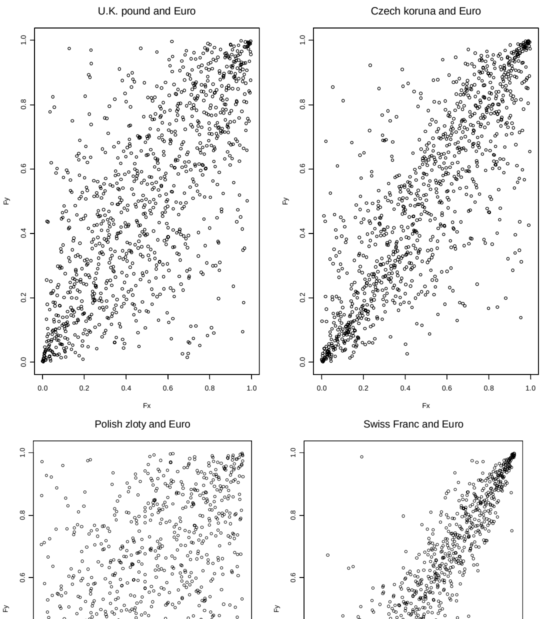
Pair-wise correlation in exchange rates usually reflects differentials in short-interest rates, inflation rates, domestic money supply, industrial production, and/ or cumulative trade balances. The question we would like to answer is whether such correlation translates into extreme-value dependence. That is, whether either a large appreciation or depreciation in the nominal exchange rate of one country may transmit to another.

The presence of extreme-value dependence can be informally assessed by the following graphical procedure. Let us consider left-tail dependence, and let (X_t,Y_t), t=2,…,T, be independent observations of negative returns, with unknown distribution F.[7] The random variables $u_t = F_X(X_t)$ and $v_t = F_Y(Y_t)$ are both distributed as uniform, where F_X and F_Y are the marginal distribution functions. We next examine the degree of association between large values of u_t and v_t (see, for example, Coles, Heffernan, and Tawn 1999). Since F_X and F_Y are unknown, estimates are obtained from the empirical distribution functions.

[7] As a convention, in the literature of extreme value theory returns are negated. So, large negative returns become large positive returns.

Figure 1 depicts left-tail dependence on the euro of the European economies, from outside the Euro zone, contained in our sample.[8] In this case, the Czech koruna/euro and Swiss franc/euro pairs are the ones that exhibit the strongest patterns of dependency under high depreciations (i.e., there is strong correlation between paired observations located at the upper-right corner of each graph).

Figure 1. Left-tail dependency of paired exchange rates



Notes: (1) The data are daily and span from January 1999 to December 2002. The source is Morgan Stanley. (2) (X_t,Y_t) represents a pair of negative returns, t=2,…,T. The random variables $u_t = F_X(X_t)$ and $v_t = F_Y(Y_t)$ are both distributed as uniform, where F_X and F_Y are the marginal distribution functions. An informal procedure to detect extreme-value dependence consists of examining if large values of u_t and v_t are positively associated (that is, if there is a strong association between paired observations at the upper-right corner of each graph).

[8] When looking at left-tail dependence, what we do is to concentrate on large depreciations of paired currencies against the numeraire (Canadian dollar).

Next, we formally test for tail dependence using the statistical tools presented in Sections II.A and II.B. We first test for asymptotic dependence in the raw data. Next, given that dependency may be partially due to volatility persistence and inertia in returns, we filter the data for both conditional heteroscedasticity and serial correlation. For such purpose, we use a diagonal VEC model or DVEC(1, 1):

$$r_t = c + \beta r_{t-1} + \varepsilon_t \quad t=2, \ldots, T, \tag{14}$$

where $\boldsymbol{r}_t$ is a k x 1 vector of returns, **c** is a k x 1 vector of constant terms, $\boldsymbol{r}_{t-1}$ is a k x 1 vector containing the first lag of $\boldsymbol{r}_t$, β is a k x 1 vector, and ε_t is a k x 1 white-noise vector with zero mean. The matrix variance-covariance of ε_t is given in this case by

$$\boldsymbol{\Sigma}_t = \mathbf{A}_0 + \mathbf{A}_1 \otimes (\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}') + \mathbf{B} \otimes \boldsymbol{\Sigma}_{t-1} \quad t=2, ..., T, \tag{15}$$

where $\boldsymbol{A}_0$, $\boldsymbol{A}_1$, $\boldsymbol{B}$, Σ_t and $\varepsilon_{t-1}\varepsilon_{t-1}'$ are k x k matrices, for t=2, …,T, and $\otimes$ denotes the Hadamar product (e.g., Bollerslev, Engle, and Wooldridge 1988; Zivot and Wang 2003, chapter 13). In order to obtain the elements of Σ_t, only the lower part of the system in (15) is considered. All estimation was carried out with the statistical package S-Plus 6.1.

Test results of asymptotic dependence are reported in Table 2, Panels (a) and (b). As mentioned earlier, $\bar{\chi}$ and χ are measures of extreme-value independence and dependence, respectively. For the raw data (Panel (a)), almost all paired returns strongly reject the existence of asymptotic dependence (i.e., $\bar{\chi} = 1$) in both tails. The only exceptions are the Swiss franc/euro pair, for which we cannot reject the null hypothesis in either tail at any conventional significance level; and, the Czech koruna/euro and UK pound/euro pair, for which we cannot reject left-tail dependence at the 1-percent level, and right-tail dependence at any conventional significance level. In these cases, asymptotic dependence is measured by χ. As stated in equation (6), χ gives the probability of joint occurrence of more extreme events. For instance, there is about a 74-percent chance that a large depreciation (appreciation) in the euro will translate into a large depreciation (appreciation) of the Swiss franc.

For those cases in which we reject that $\bar{\chi} = 1$, we conclude that paired exchange rates are asymptotically independent, and $\bar{\chi}$ gives a measure of asymptotic independence. In particular, for the raw data we see that for most cases the Pearson correlation is smaller than $\bar{\chi}$. This indicates that the former can be a poor measure of tail dependence. For instance, for the U.K. pound/euro pair the Pearson correlation considerably underestimates right-tail dependence (i.e., large joint appreciations of the U.K. pound and the euro against the Canadian dollar), while it is an acceptable measure of left-tail dependence. For all cases, $\overline{\chi}$ is statistically different from zero.

Panel (b) of Table 2 reports tests of tail dependence for the filtered data. Our results show that the existence of asymptotic dependence in the raw data is primarily due to correlated volatilities, and to a lesser extent to serial correlation. (As reported in Table 1, the latter is relatively small). Indeed, in most cases $\overline{\chi}$ noticeably decreases for the filtered data (e.g., left- and right-tail dependence of the Swiss franc/euro pair), but it remains statistically significant.

An interesting feature is that some asymptotically independent pairs exhibit more tail dependency¾ measured by $\overline{\chi}$ ¾ in the right than in the left tail for both the raw and filtered data (e.g., U.K. pound/euro and Japanese Yen/euro). In other words, currencies tend to exhibit more dependency under large appreciations than under large depreciations.

B. Extreme-value dependence between European stock and exchange rate markets

The finance literature has stressed the relationship between stock returns and exchange rate fluctuations. Indeed, several authors have worked on the international version of the capital asset pricing model (IAPM), which states that the excess return on an individual asset is a linear function of the excess return on the world market portfolio and the excess return on the exchange rate of the domestic currency against one or more foreign currencies (see, for instance, De Santis and Gérard 1998). Other authors have focused on co-movements of exchange rates and stock prices under severe economic slowdowns. For instance, Hashimoto and Ito (2004) analyze the behavior of exchange rates and stock prices in Hong Kong, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan, and Thailand over the Asian crisis of 1997-1999. They conclude, among others, that the Indonesian, Korean and Thai currency depreciations and the Hong Kong stock market decline affected other currencies and stock markets in the region during the crisis.

Motivated by this literature, we would like to find out whether stock and exchange rate markets exhibit a pattern of extreme-value dependence. Our previous results show evidence, at least in the raw data, of extreme-value dependence of the currencies of the U.K., the Czech Republic, and Switzerland on the euro. Therefore, in this section we analyze whether a sharp appreciation (depreciation) of one of

Table 2. Tail dependency of paired nominal exchange rates: Selected economies and the euro

Paired return	r	Left tail							Right tail						
		k^*	$\bar{\chi}$	s.e	t-test $\bar{\chi}=1$	p-value	χ	s.e	k^*	$\bar{\chi}$	s.e	t-test $\bar{\chi}=1$	p-value	χ	s.e
(a) Raw data															
Yen/euro	0.344	170	0.392	0.101	-6.014	0.000	—	—	167	0.534	0.119	-3.924	0.000	—	—
USD/euro	0.344	168	0.484	0.114	-4.507	0.000	—	—	183	0.468	0.109	-4.902	0.000	—	—
U.K. pound/euro	0.717	155	0.713	0.138	–2.087	0.018	0.527	0.039	195	0.947	0.139	–0.382	0.351	0.541	0.035
Czech koruna/euro	0.815	199	0.762	0.125	–1.909	0.028	0.620	0.039	159	0.842	0.146	–1.083	0.139	0.661	0.048
Polish zloty/euro	0.431	154	0.549	0.125	–3.612	0.000	—	—	186	0.498	0.110	–4.568	0.000	—	—
Swiss franc/euro	0.947	199	0.962	0.139	–0.273	0.392	0.736	0.047	98	0.858	0.188	–0.757	0.224	0.789	0.076
(b) Filtered data															
Yen/euro		184	0.181	0.087	–9.405	0.000	—	—	171	0.233	0.094	-8.139	0.000	—	—
USD/euro		155	0.428	0.115	-4.981	0.000	—	—	153	0.248	0.101	-7.445	0.000	—	—
U.K. pound/euro		117	0.070	0.099	–9.400	0.000	—	—	119	0.422	0.130	–4.435	0.000	—	—
Czech koruna/euro		199	0.149	0.081	–10.452	0.000	—	—	160	0.262	0.100	–7.402	0.000	—	—
Polish zloty/euro		199	0.326	0.094	–7.164	0.000	—	—	53	–0.054	0.130	–8.112	0.000	—	—
Swiss franc/euro		155	0.366	0.110	–5.781	0.000	—	—	195	0.290	0.092	–7.693	0.000	—	—

Notes: (1) The sample period for Panels (a) and (b) is 1999-2002. The data are measured at a daily frequency, and were obtained from the Bank of Canada. (2) r is the Pearson correlation coefficient over the whole sample period. (3) k* represents the optimal threshold obtained by the exponential-regression procedure. (4) $\bar{\chi}$ is computed based on tail-index estimation of Fréchet transformed margins of daily co-exceedances of return pairs, Z=min(S,T). Asymptotic dependence cannot be rejected if $\bar{\chi}=1$. In that case, the degree of dependence is measured by c>0.

Table 3. Tail dependency between paired exchange rates and stock markets in Europe

Paired return	r	Left tail k^*	$\overline{\chi}$	s.e	t-test $\overline{\chi}=1$	p-value	Right tail k^*	$\overline{\chi}$	s.e	t-test $\overline{\chi}=1$	p-value
(a) Raw data											
Swiss stock index/ Swiss franc	0.221	193	0.442	0.104	–5.378	0.000	183	0.313	0.097	–7.086	0.000
Czech stock index/ Czech koruna	–0.043	130	0.150	0.085	–10.287	0.000	158	0.102	0.088	–10.244	0.000
U.K. stock index/ U.K. pound	–0.059	155	0.261	0.101	–7.304	0.000	94	–0.073	0.096	–11.218	0.000
(b) Filtered data											
Swiss stock index /Swiss franc		197	0.369	0.098	–6.470	0.000	199	0.272	0.091	–8.074	0.000
Czech stock index/ Czech koruna		199	0.240	0.088	–8.641	0.000	77	–0.004	0.101	–9.975	0.000
U.K. stock index/ U.K. pound		162	0.023	0.080	–12.149	0.000	199	0.148	0.085	–10.010	0.000

Notes: (1) The data are daily and span from November 1999 to October 2004. They were obtained from Morgan Stanley. The percent variation of each exchange rate is expressed as the appreciation rate of the domestic currency against the euro. Stock indices are measured in local currency. (2) r is the Pearson correlation coefficient over the whole sample period. (3) k* represents the optimal threshold obtained by the exponential-regression procedure. (4) $\overline{\chi}$ is computed based on tail-index estimation of Fréchet transformed margins of daily co-exceedances of return pairs, Z=min(S,T). Asymptotic dependence cannot be rejected if $\overline{\chi}=1$. In that case, the degree of dependence is measured by c>0.

Table 4. Tail dependency in European stocks markets

Paired return	r	Left tail							Right tail						
		k*	$\overline{\chi}$	s.e	t-test $\overline{\chi}=1$	p-value	χ	s.e	k*	$\overline{\chi}$	s.e	t-test $\overline{\chi}=1$	p-value	χ	s.e
(a) Raw data															
Switzerland/Euro zone	0.806	144	0.961	0.163	–0.241	0.405	0.598	0.047	199	0.946	0.138	–0.391	0.348	0.584	0.038
Czech Rep./Euro zone	0.378	161	0.420	0.112	–5.181	0.000	—	—	25	–0.227	0.155	–7.939	0.000	—	—
U.K /Euro zone	0.849	191	0.998	0.145	–0.016	0.494	0.629	0.042	194	0.923	0.138	–0.556	0.289	0.628	0.042
(b) Filtered data															
Switzerland/Euro zone		125	0.076	0.096	–9.599	0.000	—	—	199	0.194	0.085	–9.520	0.000	—	—
Czech Rep./Euro zone		198	0.311	0.093	–7.392	0.000	—	—	192	0.161	0.084	–10.004	0.000	—	—
U.K./Euro zone		196	0.158	0.083	–10.176	0.000	—	—	199	0.212	0.086	–9.174	0.000	—	—

Notes: (1) The Euro index includes Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain. (2) The data are daily and span from November 1999 to October 2004. They were obtained from Morgan Stanley. Stock indices are in local currency. (3) r is the Pearson correlation coefficient over the whole sample period. (4) k* represents the optimal threshold obtained by the exponential-regression procedure. (5) $\overline{\chi}$ is computed based on tail-index estimation of Fréchet transformed margins of daily co-exceedances of return pairs, Z=min(S,T). Asymptotic dependence cannot be rejected if $\overline{\chi}=1$. In that case, the degree of dependence is measured by c>0.

these countries' currency against the euro might be associated with a sharp increase (decline) of its stock prices.[9]

A formal test for tail dependence between a local currency appreciation (depreciation) and a bullish (bearish) domestic stock market is in Table 3, for the raw and filtered data. The domestic stock market return is in local currency. The data are daily and were obtained from Morgan Stanley's web site. The sample period spans from November 1999 to October 2004. We again use a DVEC model as a filter. Our testing results do not show evidence of asymptotic dependence for any country under consideration. In addition, $\bar{\chi}$ suggests that the degree of left- and right-tail dependency is relatively weak between the domestic stock market and the parity of the local currency against the euro. For instance, for the Czech Republic, $\bar{\chi}$ is statistically insignificant for the right tail (i.e., joint occurrence of a large positive return on the stock market index and a large appreciation of the Czech koruna against the euro)

We next investigate whether European stock markets exhibit asymptotic dependence. The evidence reported in Table 4 suggests that extreme returns on one country's stock market are associated to a greater extent with extreme returns in other stock markets than with sharp fluctuations of its own currency against the euro. In particular, for the raw data, we find asymptotic dependence in both tails between Switzerland and the Euro zone, and between the U.K. and the Euro zone.

Although, after controlling for conditional volatility and serial correlation in returns, asymptotic dependence disappears, we still find a positive and statistically significant association —measured by $\bar{\chi}$ ¾ in both tails for the Czech Republic/ Euro zone and U.K./Euro zone pairs, and in the right tail for the Switzerland/Euro zone pair. In other words, for asymptotically independent paired stock returns, a poor (good) performance of the Euro zone stock markets will translate into bearish (bullish) stock markets of the European countries outside the Euro zone.

IV. Conclusions

Extreme value theory studies the stochastic behavior of a process at unusually large or small levels. This study tests for the existence of extreme-value dependence in exchange rate markets. Specifically, we focus on six countries with dirty/free

[9] The intuition is as follows. If one country's currency sharply depreciates, authorities are likely to increase interest rates to offset capital outflows. In turn, higher interest rates put a downward pressure on the domestic stock market. See Hashimoto and Ito (2004) for further discussion.

float regimes over 1999-2002—the United States, Japan, the Czech Republic, Poland, Switzerland, and the United Kingdom, and investigate whether paired currencies exhibit a pattern of asymptotic dependence on the euro, since its adoption. In order to quantify tail dependence, we resort to statistical techniques used in recent finance applications of extreme value theory. In addition, we investigate whether the stock markets of the European countries in our sample, which are outside the Euro zone, have exhibited extreme-value dependence on their exchange rates against the euro.

Our findings show, in general, no evidence of asymptotic dependence between paired exchange rates, after controlling for correlated volatilities and serial correlation in returns. For finite return levels, the degree of dependence is usually stronger under large appreciations than under large depreciations. As to co-movements of stocks markets between Euro countries and countries outside the Euro zone, in general we find a positive association between bullish (bearish) markets, but not asymptotically. Moreover, our results show that tail dependence between European stocks markets of countries outside the Euro zone and their parities against the euro is relatively weak.

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