

2 Preferences and Institutions

In this chapter we outline—in a very simple way—some of the important tenets of social choice theory and spatial voting theory. We ask when a voting equilibrium exists under pure majority rule, how to cope with problems created by the nonexistence of such an equilibrium, and, more generally, how majority rule aggregates individual policy preferences.

The discussion is first cast in a general framework, which we then specialize into simple policy examples as we proceed. Thus in section 2.1, we formulate a general policy problem and the policy preferences of individual voters, setting the stage for the subsequent analysis.

Then in section 2.2 we discuss different restrictions on individual policy preferences sufficient to guarantee the existence of a Condorcet winner. We also show that this policy constitutes a majority-voting equilibrium under certain assumptions about voting behavior and the process of voting. Essentially, these assumptions confine disagreement among the voters to a single dimension, a condition that is possible only if policy itself is unidimensional, or if individual voter heterogeneity is unidimensional.

In section 2.3, we discuss the consequences when these restrictions on preferences are violated: voting cycles may occur, and incentives for agenda manipulation and strategic voting obtain. To guarantee existence of a political equilibrium, we therefore need to impose additional structure on the collective choice problem by restricting the institutions that govern how policy decisions are made. In this context, we illustrate the ideas behind models of probabilistic voting, structure-induced equilibrium, and agenda setting.

Finally, in section 2.4, we take stock of the chapter's main conclusions and discuss their implications for our main purpose: the modeling of equilibrium economic policymaking in a representative democracy.

2.1 A General Policy Problem

Many positive analyses of the politics of economic policy share the same general structure. Consider thus a set of citizens affected by some vector of policies \mathbf{q} . (Throughout the book we denote vectors with boldface letters, whereas scalars are denoted with italics.) This set of citizens can be small (as in a committee) or large (as in the electorate). For now we allow for a small set, although the plausible set is large (and often approximated by a continuum) in most policy applications. These “voters” are indexed by their individual attributes. Introducing a notational convention that will also be followed throughout the book, we use superscript i to denote variables specific to individuals of type i . Thus, let α^i (possibly a vector) denote the specific features of voter i , capturing his idiosyncratic preferences, endowments, risks, technological opportunities, or other socioeconomic attributes. The α^i 's are assumed to be distributed among the citizens according to some given distribution. Individuals have

utility functions defined over bundles of consumption c^i ; they could also have some other well-defined economic objectives, such as profit functions.

In his role as an *economic agent*, an individual chooses his consumption bundle so as to maximize his utility function $U(c^i, q, p; \alpha^i)$, subject to some (budget or time) constraints $H(c^i, p, q; \alpha^i) \geq 0$, where p is some vector of market-determined data (prices or quantities). We can thus define the *indirect utility* of individual i as

$$\tilde{W}(q, p; \alpha^i) = \text{Max}_c [U(c^i, q, p; \alpha^i) \mid H(c^i, p, q; \alpha^i) \geq 0]. \quad (2.1)$$

Any *policy maker* setting q must respect the market-determined value of p and some further constraints, such as a balanced government budget or a resource constraint, that an atomistic private agent can neglect. Let us summarize these various constraints by $G(q, p) \geq 0$. Typically, the constraints will be binding, in which case the implicit function theorem allows us to write $p = P(q)$; that is, the market outcomes depend on policy and parameters.

In his role as a *political agent*, individual i engages in voting, lobbying, or some other form of political activity. His *policy preferences* govern his actions in these activities. We obtain these policy preferences from his indirect utility function \tilde{W} , taking into account the equilibrium constraints as summarized by $P(q)$. Let us define these reduced-form policy preferences by

$$\tilde{W}(q, P(q); \alpha^i) \equiv W(q; \alpha^i) \quad \longrightarrow \quad \text{Reduced form policy pref}$$

Using this notation, we can define the *preferred policy*, or the bliss point, of voter i as

$$q(\alpha^i) = \underset{q}{\text{Argmax}} W(q; \alpha^i) \quad (2.2)$$

Because of differences in α^i , different individuals typically have conflicting policy preferences.

In this general setting, a positive analysis of economic policymaking amounts to specifying an institution and asking how it aggregates political actions, based on individual policy preferences, into equilibrium policies. Unfortunately, one of the most famous results in the social sciences demonstrates that the search for a general answer to this question is a futile exercise. Arrow's (1951) ingenious impossibility theorem shows that no general rule enables a democracy to consistently aggregate individual preferences into policy choices. One implication of this theorem of particular interest for our purposes is that majority rule—despite its apparent predominance in real-world politics—does not generate well-defined equilibrium policies, unless we restrict its applicability either to individual policy preferences of a specific form or to political institutions of a specific type. Exploration of such restrictions has been the subject matter of a great deal of subsequent research, in social

choice as well as in the spatial theory of voting.¹ In the next two sections, we point to some of the most celebrated results from this endeavor. Many of the general results are illustrated by specific policy examples.

2.2 Restricting Preferences

The aim of this section is to study preference aggregation by pure majority rule. We define pure majority rule by the following three assumptions:

- A1. *Direct democracy.* The citizens themselves make the policy choices.
- A2. *Sincere voting.* In every vote, each citizen votes for the alternative that gives him the highest utility according to his policy preferences (indirect utility function) $W(q; \alpha^i)$.
- A3. *Open agenda.* Citizens vote over pairs of policy alternatives, such that the winning policy in one round is posed against a new alternative in the next round and the set of alternatives includes all feasible policies.

We will certainly not maintain these assumptions in subsequent sections and chapters, when we go on to study policy choice in a representative democracy. They are useful, however, for explaining the logic of some basic theoretical results.

The Marquis de Condorcet, a French mathematician and philosopher, had pointed to the prospective problems of finding a stable outcome from majority rule already in the eighteenth century, by demonstrating how pairwise voting over policy alternatives may fail to produce a clear-cut winner. In other words, majority rule may not lead to a transitive binary relation between policy alternatives. Below, we give examples of this failure of majority rule to produce a clear winner, which is often referred to as the Condorcet paradox. In this section, however, we state and discuss sufficient conditions for existence of a well-defined majority winner.

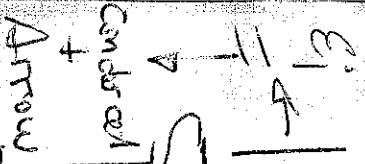
2.2.1 One-Dimensional Policy

Let us make the following definition:

DEFINITION 1. A **Condorcet winner** is a policy q^* that beats any other feasible policy in a pairwise vote.

Suppose now that the policy space is unidimensional, so that q is a scalar. In this case, a simple way to rule out the Condorcet paradox goes back, at least, to Black's seminal (1948)

¹ Innan 1987 includes a very nice survey of work in these areas and how they relate to Arrow's theorem. Mueller's (1989) textbook and the contributions in Mueller 1997 also provide useful perspectives on this vast body of work.



study of decisionmaking in a committee. Following Black, the policy preferences defined in (2.2) are said to be *single peaked* for voter i if his preference ordering for alternative policies is dictated by their relative distance from his bliss point, $q(\alpha^i)$: a policy closer to $q(\alpha^i)$ is preferred over more distant alternatives. Specifically:

DEFINITION 2. Policy preferences of voter i are **single peaked** if the following statement is true:

If $q'' \leq q' \leq q(\alpha^i)$ or, if $q'' \geq q' \geq q(\alpha^i)$, then
 $W(q''; \alpha^i) \leq W(q'; \alpha^i)$. (2.3)

We have a simple, but useful, first result:

PROPOSITION 1. If all voters have single-peaked policy preferences over a given ordering of policy alternatives, a Condorcet winner always exists and coincides with the median-ranked bliss point.

To prove this *median-voter theorem*, we can rely on a simple "separation argument." Fix the parameter vector at some value, order the individuals according to their bliss points $q(\alpha^i)$, and label the median-ranked bliss point by q^m .² Suppose that q^m is pitched against some other policy $q'' < q^m$. By (2.3), every individual whose bliss point satisfies $q^m \leq q(\alpha^i)$ prefers q^m to q'' , since it is closer to his bliss point. By A2, these individuals also vote for q^m . The coalition voting for supporting q^m thus constitutes a majority. Applying an analogous argument to $q'' > q^m$ we obtain the result that q^m is a Condorcet winner.

Under direct and sincere voting by individuals (i.e., A1-A2) and the additional assumption of an open-agenda process (i.e., A3), we have the following:

COROLLARY 1. q^m is the unique equilibrium policy (stable point) under pure majority rule, that is, under A1-A3.

The reason is simple: q^m beats any previous winner the first time it comes up and cannot be beaten in any subsequent vote.

From a general perspective, unidimensionality and single-peakedness are very strong assumptions. Unidimensionality of q severely restricts the available policy instruments, which may be implausible in many applications. If private agents make no economic choices, single-peakedness of W is fulfilled by the innocuous assumption that private utility is concave (in the single policy instrument). But interesting economic policy problems do

² Let us assume either that the set of voters describes a continuum, or that it is finite but odd in number. The median bliss point is then unambiguously defined. An even-numbered finite set of voters requires a slight technical variation in the present and the following arguments that keeps track of odd and even numbers.

single
peakedness
+ single crossing

q^m, q^m

include endogenous private choices and market outcomes. When private choices depend on policy, concavity of the primitive utility function U is no guarantee of a well-behaved indirect utility function W , as is well known from the analysis of optimal taxation. In particular, problems may arise when market outcomes, which themselves depend on policy instruments, enter individual policy preferences, say through externalities, indivisibilities, or government budget constraints. Our model represents this possibility formally by allowing $P(q)$ to enter as an argument of W .

There are, however, more general sufficient conditions. One such general condition is the single-crossing property formulated by Gans and Smart (1996). An essentially equivalent condition, order-restricted preferences, was formulated by Rothstein (1990). Both conditions impose restrictions on the character of voter heterogeneity rather than on the shape of individual preferences. Specifically, suppose that in addition to the policy variable q , the individual parameter α^i is also unidimensional with a domain on the interval \mathcal{V} . The interval \mathcal{V} thus denotes the set of voters. The Gans-Smart condition can be stated as

DEFINITION 3. The preferences of voters in \mathcal{V} satisfy the **single-crossing property** when the following statement is true:

$$\text{If } q > q' \text{ and } \alpha^{i'} > \alpha^i, \text{ or if } q < q' \text{ and } \alpha^{i'} < \alpha^i, \text{ then} \quad (2.4)$$

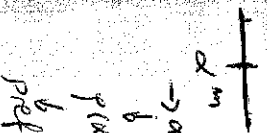
$$W(q; \alpha^i) \geq W(q'; \alpha^i) \Rightarrow W(q; \alpha^{i'}) \geq W(q'; \alpha^{i'})$$

In other words, single crossing enables us to project preferences over q on the set of voter types, \mathcal{V} . This condition is distinct from single-peakedness but has similar implications. Specifically

PROPOSITION 2. If the preferences of voters in \mathcal{V} satisfy the single-crossing property, a Condorcet winner always exists and coincides with the bliss point of the voter with the median value of α^i .

To prove this proposition, label the critical value of α^i as α^m . Then, by (2.4), every voter with $\alpha^i \geq \alpha^m$ prefers $q(\alpha^m)$ to any $q < q(\alpha^m)$. Similarly, everyone with $\alpha^i \leq \alpha^m$ prefers $q(\alpha^m)$ to any $q > q(\alpha^m)$. In other words, $q(\alpha^m)$ wins a pairwise vote against any conceivable alternative. Clearly, the argument is related to the separation argument applied above, in the case of single-peaked preferences where the voters were ranked according to their individually preferred policies. But here, the monotonicity of policy preferences instead allows us to rank voters according to their individual types.

It is easily verified whether the single-crossing condition is satisfied, since it is closely related to the familiar Spence-Mirrlees condition on marginal rates of substitution. Specifically, suppose that preferences are defined over a two-dimensional choice variable, but



policy is effectively one-dimensional because of a budget or resource constraint. Single crossing as defined above is then equivalent to the Spence-Mirrlees condition that marginal rates of substitutions can be ordered by individual type.

Evidently, single crossing, like single-peakedness, is capable of generating the existence of a political equilibrium under pure majority rule.

Example 1. Redistributive distortionary taxation. Consider a simplified version of the well-known model of redistribution financed by distortionary taxation, formulated by Romer (1975), Roberts (1977), and Meltzer and Richard (1981). We provide a more extensive treatment and several extensions of this model in chapter 6. In our version, the i^{th} individual has quasi-linear preferences:

$$w^i = c^i + V(x^i),$$

where c^i and x^i denote individual consumption and leisure, respectively, and $V(\cdot)$ is a well-behaved concave utility function. The private budget constraint is

$$c^i \leq (1 - q)l^i + f,$$

where q denotes the income tax rate, l^i the individual labor supply, and f a lump sum transfer. The real wage is exogenous and normalized at unity. Individual productivity differs, such that individuals have different amounts of "effective time" available. That is, individuals are subject to the "time constraint"

$$1 - \alpha^i \geq x^i + l^i, \quad (2.5)$$

where α^i is individual productivity. We assume that α^i is distributed in the population with mean α and median α^m . It is easy to verify that optimal labor supply satisfies

$$l^i = L(q) - (\alpha^i - \alpha), \quad (2.6)$$

where $L(q) \equiv 1 - \alpha - V_x^{-1}(1 - q)$ is decreasing in q by the concavity of $V(\cdot)$.³ A higher tax rate thus distorts the labor-leisure choice and induces the consumer to work less. Furthermore, more-productive consumers work more at every tax rate. Let l denote average labor supply. Since the average of α^i is α , we have $l = L(q)$. The government budget constraint can therefore be written:

$$f \leq ql \equiv qL(q).$$

Let q be the policy variable with f determined as a residual. By straightforward substitution

3. Maximize the utility of individual i 's subject to the budget and time constraints. The first-order condition implies $1 - q = V_x(1 - \alpha^i - l^i)$, where a subscript denotes a derivative. Take the inverse of $V_x(\cdot)$ and simplify to get the expression for l^i in the text. Note that $L_q = 1/V_{xx}(x^i) < 0$.

into the individual utility function, we can express the policy preferences of individual i as

$$W^i(q; \alpha^i) \equiv L(q) + V(1 - L(q) - \alpha) - (1 - q)(\alpha^i - \alpha). \quad (2.7)$$

It is easy to see that the indirect utility function in (2.7) fulfills the single-crossing condition (2.4). Suppose that the average labor supply $L(q)$ were convex enough to imply non-single-peaked preferences $W^i(q; \alpha^i)$. Then monotonicity of individual labor supply in α^i is still sufficient to guarantee the existence of a well-defined Condorcet winner, namely the tax rate preferred by the voter with median productivity and income.

Problem 2 of this chapter gives an example of a policy problem with similar properties.

2.2.2 Multidimensional Policy, Unidimensional Conflict

What if the policy is instead multidimensional? From a general point of view, the existence problem gets significantly worse (the following section explains why in more detail). Nevertheless, we can still find monotonicity restrictions on the policy preferences that guarantee the existence of a majority winner. The single-crossing property defined above can be generalized to multidimensional policies, but in that case it may be harder to verify whether it is satisfied. A simple sufficient condition can be found, however, that again ensures the existence of a Condorcet winner. This condition is less general than single crossing, but it relies on a very similar idea: voter heterogeneity is limited in that voters' preferences for a multidimensional policy can be projected on a unidimensional space in which different voters can be ordered by their type. Following Grandmont (1978), we label this condition "intermediate preferences" (even though Grandmont's definition is slightly more general).

Specifically, let \mathbf{q} be a vector of policies and $W(\mathbf{q}; \alpha^i)$ be the policy preferences of a voter of type α^i . As before, the parameter α^i is unidimensional, with a domain on the interval \mathcal{V} . Thus, although policy is multidimensional, voters differ only in one dimension, namely with regard to the parameter α^i . Then we can state:

DEFINITION 4. Voters in the set \mathcal{V} have **intermediate preferences**, if their indirect utility function $W(\mathbf{q}; \alpha^i)$ can be written as

$$W(\mathbf{q}; \alpha^i) = J(\mathbf{q}) + K(\alpha^i)H(\mathbf{q};), \quad (2.8)$$

where $K(\alpha^i)$ is monotonic in α^i , for any $H(\mathbf{q})$ and $J(\mathbf{q})$ common to all voters.

Clearly, it is easily verified whether the voters' preferences satisfy this condition. If so, we have a very useful result. Let $\mathbf{q}(\alpha^m)$ be the policy preferred by the median value of α^i in the set \mathcal{V} . Then once more:

At the same time all the cycling and instability existing models suggested, however, did not seem to be a feature of democratic decision making in practice. A number of new ideas that were starting to make their way into political theory in the years around 1980, however, sowed seeds of a more optimistic view. These ideas appeared in the context of the *probabilistic voting* model associated with Hinich (1977), Coughlin and Nitzan (1981), and Ledyard (1981, 1984),⁵ the *structure-induced equilibrium* model associated with Shepsle (1979) and Shepsle and Weingast (1981),⁶ and the *agenda-setter* model associated with Romer and Rosenthal (1978, 1979).

These ideas stem from a common premise: policy choices are not made by the citizens themselves under direct democracy, but are delegated to elected representatives. Assumption A1 is thus relaxed, appealing to the organization of decision making in real-world democracies. The details of how policy is chosen impose additional structure on the political process that, in turn, can give rise to a well-defined equilibrium. Each of these approaches identifies a specific aspect of political institutions as a crucial determinant of policy. Probabilistic voting is a theory of electoral competition in which politicians offer policy platforms to the voters and specific assumptions are made about the voters' behavior. Structure-induced equilibrium and the agenda-setter model apply to collective decisions in smaller groups of political representatives, like a committee or a legislature, in which representatives have well-defined policy preferences and the institution imposes a particular procedure for decision making. Thus in these models the policy decision is the outcome of a game with a well-defined extensive form. In the following, we briefly introduce the main ideas behind each of these approaches.

Probabilistic Voting The traditional starting point for analyzing electoral competition was the classical theory of Hotelling (1929) and Downs (1957). By the 1970s, formal results had verified the main insight of these authors. Suppose, as did Hotelling and Downs, that elections involve two identical politicians (or parties). The politicians are opportunistic in the sense of being purely office-motivated: they strive to maximize their vote share or, alternatively, the probability of winning. Moreover, these politicians can make binding commitments to policy platforms in the course of the electoral campaign. The outcome is a Condorcet winner, if such a policy exists. In the next chapter, we formally derive this result in the context of a specific policy example. But the intuition is very simple and closely related to the separation arguments in section 2.2. Faced with only two policy alternatives (the two electoral platforms), all citizens vote sincerely. Thus a policy platform coinciding with a Condorcet winner always captures at least half of the vote when it is up against

5. The probabilistic voting model in its later form was preceded by Hinich, Ledyard, and Ordeshook 1972.

6. The ideas behind structure-induced equilibrium had earlier appeared in Kramer 1972.

any other platform. Consequently, the situation in which both candidates select the policy preferred by the pivotal voter is the only one where no candidate (party) can discontinuously increase his probability of winning (its vote share). As the candidates are office-motivated, this is, by definition, a Nash equilibrium.

With multidimensional policy conflict, on the other hand, Downsian electoral competition games generally do not have any equilibria. This is a direct consequence of the cycling problems discussed above. If no policy dominates any other policy, one candidate can always find another policy that is preferable for a majority of the voters, given any policy platform proposed by the other candidate. The objective functions of the office-seeking candidates thus become highly discontinuous throughout the policy space.

Probabilistic voting models essentially smooth out these objectives by introducing uncertainty—from the candidates' viewpoint—about the mapping from policy to aggregate voting behavior. The argument comes in several guises, with different degrees of micropolitical foundations. Individual voters may abstain from voting if the proposed policies are too far away from their ideal points or if not too much is at stake. Or the candidates may perceive the probability that a particular voter (or group of voters) votes for a particular candidate as a continuous function, rather than a step function, of the distance between the two platforms. In either case, the expected number of votes becomes a smooth function of the policy platform, which guarantees existence of a Nash equilibrium under some regularity conditions on the underlying utility and distribution functions.

To illustrate these points more precisely, consider a policy choice like the one discussed in example 3. Thus two types of public consumption, q_1 and q_2 , are financed by general taxation. As before, q_1 and q_2 are the policy variables with taxes residually determined. There are I voter types, $i = 1, \dots, I$ (with I a large number), who evaluate policy according to preferences $W(\mathbf{q}; \alpha^i)$. But now these preferences do not fulfill the monotonicity property imposed in example 3, meaning that an open-agenda process over pairs of policy vectors could give rise to cycling. Two parties, A and B , simultaneously announce their policy platforms ahead of the election, \mathbf{q}_A and \mathbf{q}_B respectively. The party winning the election implements his promised policy, and parties maximize the probability of winning.

Let π_P^i be the probability perceived by the candidates that voter i votes for party P , $P = A, B$, and suppose that these probabilities refer to independent events for different voters. Then the expected vote share of party P is

$$\pi_P = \frac{1}{I} \sum_{i=1}^I \pi_P^i \quad (2.9)$$

Under Downsian electoral competition with two identical parties, π_P^i jumps discontinuously from 0 to 1 as voter i always votes with certainty for the party that promises the better policy.

Because of these discontinuous jumps, a Nash equilibrium in the electoral competition game may fail to exist. One way or the other, probabilistic voting models instead assume that $\pi_A^i = F^i(W(\mathbf{q}_A; \alpha^i), W(\mathbf{q}_B; \alpha^i))$, where $F^i(\cdot)$ is a smooth and continuous function, increasing in the first argument and decreasing in the second. This smoothness implies that a small unilateral deviation by one party does not lead to jumps in its expected vote share and thus gives rise to well-defined equilibria.

An interesting special case, which we use extensively throughout the book, restricts these probabilities to take the form $\pi_A^i = F^i(W(\mathbf{q}_A; \alpha^i) - W(\mathbf{q}_B; \alpha^i))$, where $F^i(\cdot)$ is a continuous and well-behaved cumulative distribution function (c.d.f.), associated with a probability distribution. Furthermore, suppose that parties maximize their expected vote share.⁷ In this case, party A sets \mathbf{q}_A to maximize:

$$\pi_A = \frac{1}{I} \sum_{i=1}^I F^i(W(\mathbf{q}_A; \alpha^i) - W(\mathbf{q}_B; \alpha^i)). \quad (2.10)$$

Clearly, party B faces a symmetric problem, and in a Nash equilibrium with simultaneous policy announcements both candidates announce the same equilibrium policies: $\mathbf{q}_A = \mathbf{q}_B$. Moreover, the first-order conditions for a maximum of (2.10), evaluated at the equilibrium policy \mathbf{q}_A , and taking \mathbf{q}_B as given, can be written as

$$\begin{aligned} \sum_{i=1}^I f^i(0) W_{q_{1A}}(\mathbf{q}_A; \alpha^i) &= 0 \\ \sum_{i=1}^I f^i(0) W_{q_{2A}}(\mathbf{q}_A; \alpha^i) &= 0. \end{aligned}$$

In these expressions, $f^i(0)$ denotes the density corresponding to the c.d.f. $F^i(\cdot)$, evaluated at 0 (namely in equilibrium). Thus the equilibrium under this form of electoral competition implements the maximum of a particular weighted social welfare function, where voter i receives weight $f^i(0)$. Voters with higher $f^i(0)$ weigh more heavily, because in a neighborhood of the equilibrium they are more likely to reward policy favors with their vote. That is, more "responsive" voters, who have a higher density $f^i(0)$, receive a better treatment under electoral competition. Clearly, if all voters are equally responsive (if they all have the same value of $f^i(0)$), this form of electoral competition implements the utilitarian optimum.

7. When discussing probabilistic voting in subsequent chapters, we provide some microfoundations for the individual probabilities π_p^i . We also assume that parties maximize the probability of winning, that is, the probability that the vote share exceeds $\frac{1}{2}$, rather than maximize the vote share *per se*. In most cases the equilibrium is not sensitive to these slightly different assumptions, which are generally dictated by the models from which π_p^i is derived.

Alternative assumptions about how π_p^i depends on voters' welfare yield different specific results, though with a similar flavor

The probabilistic voting model has become a useful tool for posing positive and normative questions in voting theory and applications. We rely on versions of this model in different chapters of the book and return to it already in chapter 3.

Structure-Induced Equilibrium The next model of collective choice, structure-induced equilibrium, disregards elections. Instead it analyzes the decisions by a group of representatives, with given policy preferences, which is in charge of making policy decisions in a committee or a legislature. The political institution prescribes some procedure for reaching a consensus. Specifically, consider a situation in which the decision can be split in different stages, each stage being under the jurisdiction of a specific committee or being the outcome of a separate vote. The specific assumptions are most easily illustrated in the context of a concrete example. Reconsider therefore example 3, in which policy choice consists of two types of public consumption, q_1 and q_2 , financed by general taxation. Policy choice is delegated to a legislature with three members i —we can interpret these as three parties, representing three groups of citizens—who evaluate policy according to preferences $W(q; \alpha^i)$. These preferences do not, however, fulfill the monotonicity property imposed in example 3, so that an open-agenda process over pairs of policy vectors gives rise to cycling. Figure 2.2 illustrates examples of such preferences: the legislators' most preferred policies are given by the points $q(\alpha^i)$, $i = 1, 2, 3$, with surrounding elliptic indifference contours.

Imagine that the decisions on each publicly provided good are made in an open-agenda process in which legislators vote separately and sequentially over each dimension. First decisions are made over, say, q_1 , then over q_2 for a given q_1 . All alternatives are compared pairwise in each dimension separately. This approximates the legislative practice of letting two different committees handle the two types of public consumption and only allowing the legislature to consider amendments under the jurisdiction of one committee at a time. A crucial assumption is that all legislators vote sincerely.

Consider the last stage, in which a vote is taken over q_2 for a given q_1 . As figure 2.2 is drawn, voter 1 is the median voter. He is constrained to pick a point along the vertical line corresponding to the value of q_1 that was selected at the first stage. He thus selects the tangency point between the vertical line and his indifference curve. As we vary q_1 , we thus trace out voter 1's "reaction function," the locus of points where the indifference curves of voter 1 have vertical slope. Consider now the first stage, at which a vote over q_1 is taken. Here, as figure 2.2 is drawn, the median voter is voter 3, who realizes that the final equilibrium will be a point on voter 1's reaction function. Voter 3 can choose which point by selecting a value of q_1 . His best choice is obviously a point where voter 1's

2.4 Discussion

What does the discussion in this chapter suggest for our main task, namely to build positive models of policymaking in representative democracies? Quite a few things, we believe. For one, we learn from the classical results in social choice and voting theory that we cannot hope for a general model of universal applicability. Thus, we have to judge on a case-by-case basis how to deal with specific policy applications.

We also get some guidance on how to make those judgments. One lesson is that what type of policy we want to investigate is important. For some applications, it may be a reasonable assumption that the essence of political disagreement runs in a single dimension. Such applications may include tax and spending policy with long-run consequences for the size of the public sector or aggregate demand management with short-run consequences for inflation and unemployment. In both cases, the major conflict is likely to be highly correlated with income in a single left-to-right dimension. If we are willing to make this simplification, we can generate policy preferences implying the existence of a Condorcet winner, using assumptions no more restrictive than those made in other applied work in macroeconomics, public finance, or contract theory. If, in addition, we are willing to assume that the policies are determined in Downsian electoral competition, we have a particularly simple equilibrium to analyze: the median-voter optimum.

But many, perhaps most, policies in public finance, regulation, or trade policy do not fall into this convenient class of general-interest politics. They are inherently multidimensional, because their benefits are targeted to specific, well-defined groups. In those instances of special-interest politics, we have seen that there are several ways of making predictions about the policy outcome. But we have also seen that precise predictions necessitate precise assumptions about the institutions in the policy process. Empirical observation, rather than analytical ease or theoretical principle, must then guide the modeling.

Specifically, analytical convenience makes it tempting to overuse the median-voter solution. Suppose we doubt that Downsian electoral competition successfully captures the political process governing the selection of a specific policy. Then a Condorcet winner may not be the right candidate for equilibrium policy, even if policy preferences are sufficiently well behaved to ensure its existence. The next three chapters will make this point more precisely. These chapters use a common policy example to illustrate different possibilities of modeling political equilibria in a representative democracy. Here the elected policymakers' decisions interject an important filter between voters' policy preferences and equilibrium policy. Therefore, we explore the policy outcomes under different assumptions about the objectives of these elected politicians, contrasting opportunistic and partisan objectives.

Finally, the importance of the policymaking institutions suggests an interesting topic of research. Different policymaking institutions aggregate conflicting policy preferences in

different ways. Looking for systematic associations between policy outcomes and political institutions, then, constitutes a largely unexplored research program in comparative politics. We return to that theme later in the book, especially in part 3.

2.5 Notes on the Literature

A general overview of many theoretical results discussed in this chapter can be found in the survey by Inman (1987), which offers a nice perspective on central results in social choice from the perspective of political economy. Enelow and Hinich 1984 discusses the advances made in spatial voting theory, as does the more recent survey by Ordeshook (1997). Much of the material in the chapter is also covered in textbook form by Ordeshook (1986) and Mueller (1989).

References to the classical contributions discussed in the chapter are given in the text. For the specific contributions and approaches used as building blocks of analysis in future chapters, we give more detailed references in context.

2.6 Problems

1. Existence and nonexistence of a Condorcet winner under simple majority rule

Consider three voters indexed by $i \in \{1, 2, 3\}$, each characterized by an intrinsic parameter α^i , where $\alpha^1 < \alpha^2 < \alpha^3$. Agent i derives a utility $W(q_j; \alpha^i)$ over policy q_j . Three possible policies $q_j \in \{q_1, q_2, q_3\}$ can be implemented. A policy is selected by simple majority rule.

a. The preferences of agent $i \in \{1, 2, 3\}$ are such that

$$W(q_1; \alpha^1) > W(q_3; \alpha^1) > W(q_2; \alpha^1)$$

$$W(q_2; \alpha^2) > W(q_1; \alpha^2) > W(q_3; \alpha^2)$$

$$W(q_3; \alpha^3) > W(q_2; \alpha^3) > W(q_1; \alpha^3)$$

Moreover, the agenda is open and agents vote sincerely. Prove that no Condorcet winner exists under majority rule. Discuss.

b. Suppose that agents have the same preferences as in (a) but agent 1 is the agenda setter. He selects two rounds in which all agents vote sincerely. What is the optimal agenda from the perspective of agent 1? Suppose now that agent 1 sets the agenda and agents 2 and 3 vote sincerely. Can agent 3 improve his welfare by voting strategically? Discuss.

c. Suppose that the agents have the following preferences:

$$W(q_1; \alpha^1) > W(q_2; \alpha^1) > W(q_3; \alpha^1)$$

$$W(q_2; \alpha^2) > W(q_1; \alpha^2) > W(q_3; \alpha^2)$$

$$W(q_3; \alpha^3) > W(q_2; \alpha^3) > W(q_1; \alpha^3),$$

with $q_1 < q_2 < q_3$. Is there a Condorcet winner? Explain

d. Suppose that the preferences of agent 2 are such that

$$W(q_2; \alpha^2) > W(q_1; \alpha^2) > W(q_3; \alpha^2),$$

with $q_1 < q_2 < q_3$. Construct the preferences (ordering) of agents 1 and 3 so that they verify the single-crossing property. Then show that the median voter is a Condorcet winner.

2. Simple majority rule and unidimensional public consumption

Consider a society inhabited by a continuum of citizens and normalize the size of the population to 1. Suppose that the preferences of agent i over a publicly provided good y and a privately provided good c^i are

$$w^i = c^i + \alpha^i V(y),$$

where $V(\cdot)$ is a concave, well-behaved function and α^i is an intrinsic parameter of agent i distributed according to $F(\cdot)$ with mean α . Assume, in addition, that all individuals have initial resources in private good $e^i = 1$ for all i . Suppose also that one unit of private good is required to produce one unit of public good. Last, suppose that to finance the production of the public good, the government raises a tax q on each individual so that agent i 's budget constraint is $c^i \leq 1 - q$.

a. What is the (utilitarian) social optimum in this economy?

b. Compute each individual's policy preferences. What is the preferred policy $q(\alpha^i)$ of agent i ?

c. Under majority rule, what is the selected policy? Compare this to the social optimum. When does the social optimum coincide with the equilibrium policy?

d. Suppose now that each agent's preferences are given by

$$w^i = c^i + (\alpha^i - \hat{\alpha})^2 V(y),$$

where $\hat{\alpha}$ is a given value of α^i . Again, compute the social optimum as well as the policy preferences of individuals. Do we reach the same conclusions as in question (c)?