

Valuing real options: frequently made errors

Pablo Fernández
IESE Business School
Camino del Cerro del Aguila 3
28023 Madrid, Spain
Telephone 34-91-357 08 09
e-mail: fernandezpa@iese.edu

In this paper, we analyze frequently made errors when valuing real options. The best way of doing it is through examples. We start by analyzing Damodaran proposal to value the option to expand the business of Home Depot. Some of the errors and problems of this and other approaches are:

- Assuming that the option is replicable and using Black and Scholes' formula.
- The estimation of the option's volatility is arbitrary and has a decisive effect on the option's value.
- As there is no riskless arbitrage, the value of the option to expand basically depends on expectations about future cash flows. However, Damodaran assumes that this parameter does not influence the option's value (he does not use it) because he assumes that the option is replicable.
- It is not appropriate to discount the expected value of the cash flows at the risk-free rate (as is done implicitly when Black and Scholes' formula is used) because the uncertainty of costs and sales in the exercise date may be greater or less than that estimated today.
- Damodaran's valuation assumes that we know exactly the exercise price.
- Believe that options' value increases when interest rates increase.
- "Play" with volatility.
- Valuing contracts as real options when they are not.

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The formulas used to value financial options are based on riskless arbitrage (the possibility of forming a portfolio that provides exactly the same return as the financial option) and are very accurate. However, we will see that very rarely does it make sense to use these formulas directly to value real options because real options are hardly ever replicable. However, we can modify the formulas to take non-replicability into account (see Appendix 2).

Some problems we encounter when valuing real options are:

1. difficulty in communicating the valuation due to its higher technical complexity than the present value
2. difficulty in defining the necessary parameters for valuing real options
3. difficulty in defining and quantifying the volatility of the sources of uncertainty
4. difficulty in calibrating the option's exclusiveness
5. difficulty in valuing the options adequately. In any case, their valuation is much less accurate than the valuation of financial options.

1. Real options

It is not possible to value correctly a firm or a project that provides some type of future flexibility –**real options** – using the traditional techniques for discounting future flows (NPV or IRR). There are many types of real options: options to exploit mining or oil concessions, options to defer investments, options to expand businesses, options to abandon businesses, options to change the use of certain assets...

A real option exists in an investment project when there are future possibilities for action when the solution of a current uncertainty is known. Oil concessions are a typical example. The oil well will be operated or not depending on the future price of oil. Designing a new product is also a real option: a firm has the option of expanding its production facilities or canceling distribution, depending on the market's future growth. Investments in research and development can also be analyzed using options theory¹.

Corporate policy strategists and professors have repeatedly reproached finance –and financial analysts– for their lack of tools for valuing investment projects' **strategic implications**. Before using

¹ See, for example, Grenadier, S. and A. Weiss (1997), "Investment in technological innovations: An option pricing approach", *Journal of Financial Economics* 44, pp. 397-416.

options theory, most new investments were made on the basis solely of qualitative corporate policy criteria. The numbers –if any– were crunched afterwards so that they could give the result that the strategist wanted to back his decision. Options theory seems to enable projects' strategic opportunities to be valued: by combining quantitative analysis of the options with qualitative and strategic analysis of the corporate policy, it is possible to make more correct and more rational decisions about the firm's future.

In this paper, we will study a few simple examples that will enable us to readily see that not considering the options contained in a project may lead us to undervalue it and, in general, turn down projects that we should undertake². We will also analyze a number of real options that are present in many investment projects: the option to expand the project, the option to defer the investment, and the option to use the investment for alternative purposes.

One classification of real options is the following:

REAL OPTIONS		
Contractual options	Growth or learning options	Flexibility options
Oil concessions	Expand	Defer the investment
Mining concessions	Research and development	Downsize the project
Franchises	Acquisitions	Alternative uses
	New businesses	Renegotiations of contracts
	New customers	Outsourcing
	Internet venture	Abandon
	Greater efficiency in increasing entry barriers	Modification of products

People also talk about compound options, which are those that provide new options when they are exercised. Rainbow options is the term used to describe those that have more than one source of uncertainty, for example, an oil concession in which the uncertainty arises from the price of oil, an uncertain quantity of barrels, and uncertain extraction costs³.

For example, some of Amazon's real options when it was only a company that sold books were⁴:

- New business options. zShops (a marketplace), AmazonAuctions (an auction market) and its new businesses: Drugstore.com (beauty and health products), Ashford.com (jewelry and gift items), Della.com (weddings and gifts), Pets.com (pets) and Greenlight.com (automobiles). Several of these options were exercised by acquisition. Between April 1998 and April 1999, Amazon made 28 acquisitions.
- Expansion options. Amazon entered the European market in 1999.

² Similarly, if the projects we are considering contain options that may be exercised by third parties (the future flexibility plays against us), non-consideration of the options contained by the projects will lead us to invest in projects that we should turn down.

³ For a compilation of the different types of real options, see the books published by Trigeorgis (1996), and Amram & Kulatilaka (1999), both with the same title: *Real Options*.

⁴ See Collura and Applegate's case study (2000) entitled "Amazon.com: Exploiting the Value of Digital Business Infrastructure"

- Growth options through new customers. Amazon started to sell music, videos and DVDs in 1998; software, toys, electronic products and home products in 1999; kitchenware and gardening products in 2000.
- Efficiency improvement options to increase the entry barriers. In 1999, Amazon invested more than \$300 million to improve its technological infrastructure. It patented the procedure called “1-Click”. Free greeting service. Verification of e-mail order.

2. Frequently made errors when valuing real options

The best way to analyze frequently made errors when valuing real options is through an example.

Damodaran proposes valuing the option to expand the business of Home Depot⁵. Home Depot is considering the possibility of opening a store in France. The store’s cost will be 24 million euros and the present value of the expected cash flows is 20 million euros. Consequently, the project’s value will be –4 million euros and it would not be a good idea. However, Home Depot believes that by opening this store, it will have the option to open another larger store in the next 5 years. The cost of the hypothetical second store would be 40 million euros and the present value of the expected cash flows is 30 million euros, although there is a lot of uncertainty regarding this parameter. Home Depot estimates the volatility of the present value of the expected cash flows of the second store at 28.3%. Damodaran uses Black and Scholes’ formula to value the option of opening the second store. According to him, the option of opening the second store is a call with the following parameters:

Option of opening the second store = Call (S=30; K=40; r = 1.06; t = 5 years; $\sigma=28.3\%$) = 7.5 million euros

Consequently, according to Damodaran, Home Depot should open the store in France because the project’s present value plus the value of the option to expand is $-4 + 7.5 = 3.5$ million euros. Some of the errors and problems of this approach are:

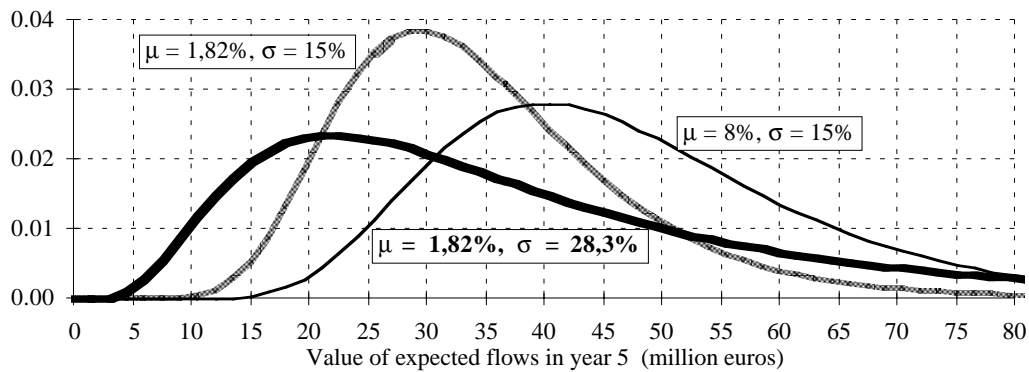
- **Assuming that the option is replicable.** This is why Black and Scholes’ formula is used in the valuation. It is fairly obvious that the option to open a second store is not replicable⁶.
- **The estimation of the option’s volatility is arbitrary and has a decisive effect on the option’s value.** Damodaran’s hypotheses regarding volatility (28.3%), present value of the expected cash flows (30 million), the option’s life (5 years) and the option’s replicability ($\mu = \ln(r) -$

⁵ See page 38 of Damodaran Aswath (2000), “The Promise of Real Options”, Journal of Applied Corporate Finance, volume 13, number 2.

⁶ To get around the non-replicability issue, Amram and Kulatilaka define real options as “the subset of strategic options in which the decision to exercise the option is basically determined by financial instruments or assets traded on markets”. The problem is that, according to this definition, only a few oil and mining concessions would be real options. See page 10 of Amram Martha,

$\sigma^2/2= 1.82\%$) are synthesized in the distribution of the expected cash flows in 5 years' time shown in Figure 1.⁷

Figure 1. Distribution of the expected cash flows in 5 years' time according to Damodaran



It is obvious that a volatility of 28.3% per year means assuming an enormous scatter of cash flows, which is tantamount to having no idea what these cash flows may be. One thing is that a greater uncertainty increases real options' value and another altogether that real options may have a high value (i.e. must undertake projects) because we have not the slightest idea of what may happen in the future. Figure 1 also shows the shape of two distributions with annual volatilities of 15%.

- As there is no riskless arbitrage, **the value of the option to expand basically depends on Home Depot's expectations about future cash flows.** However, Damodaran assumes that this parameter does not influence the option's value (he does not use it) because he assumes that the option is replicable.
- **It is not appropriate to discount the expected value of the cash flows at the risk-free rate** (as is done implicitly when Black and Scholes' formula is used). Although a real option will be exercised when a future uncertainty is settled (in this case, if the first store is a success), this does not mean that it is a risk-free project. The present value of the cash flows (30 million euro in the above example) is calculated using a rate that reflects the estimated risk today. Once the outcome of the first store is known, if it is a failure, the second store will not be opened; if it is a success, the second store will be opened, but the project of opening the second store will still have risks: the uncertainty of costs and sales in five years' time may be greater or less than that estimated today. Therefore, the cash flows must be discounted at a rate (r_K) that is greater than the risk-free rate.

and Nalin Kulatilaka (2000), "Strategy and Shareholder Value Creation: The Real Options Frontier", Journal of Applied Corporate Finance. Volume 13, Number 2, pp. 8-21.

⁷ Another way of expressing the scatter is that Damodaran assumes that the value of the expected cash flows in 5 years' time will lie between 22 and 79 with a probability of 66%; and between 12 and 149 with a probability of 95%.

Table 1 shows the value of the option to open the second store using formula 6 for a non-replicable option. The table shows that the value of the option to open the second store offsets the 4 million euros of negative value generated by opening the first store if:

1. with low volatilities, the firm has very good prospects regarding the second store’s cash flows (large μ)
2. The volatility is very high. In this case, even with extremely unfavorable expectations regarding future cash flows (negative μ), the option’s value is high. However, as we have already remarked, it is best to not take too much notice of these values. If we did, firms would have to establish themselves in those countries where they have most uncertainty (countries they do not know or countries whose future is completely unknown to them) because the option of expanding in the future would have a very high value.

Table 1. Value of Home Depot’s option to expand with different expectations for μ and volatility, $r_K = 1.09$; $S = 30$; $K = 40$; $t = 5$ years. (Million euros)
(The probability of exercising the option is given in Italics)

		μ											
		-20.0%	-10.0%	-5.0%	0.0%	1.82%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%
σ	1.0%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	1.7	3.1	4.6	6.2
		<i>0.0%</i>	<i>0.0%</i>	<i>0.0%</i>	<i>0.0%</i>	<i>0.0%</i>	<i>0.0%</i>	<i>4.6%</i>	<i>70.9%</i>	<i>99.7%</i>	<i>100%</i>	<i>100%</i>	<i>100%</i>
	5.0%	0.0	0.0	0.0	0.0	0.0	0.4	0.8	1.4	2.3	3.5	4.9	6.4
		<i>0.0%</i>	<i>0.0%</i>	<i>0.0%</i>	<i>0.5%</i>	<i>3.9%</i>	<i>21.6%</i>	<i>36.8%</i>	<i>54.4%</i>	<i>71.1%</i>	<i>84.2%</i>	<i>92.7%</i>	<i>97.1%</i>
	10.0%	0.0	0.0	0.0	0.3	0.7	1.5	2.1	2.9	3.8	4.8	6.1	7.4
		<i>0.0%</i>	<i>0.0%</i>	<i>0.8%</i>	<i>9.9%</i>	<i>19.0%</i>	<i>34.7%</i>	<i>43.3%</i>	<i>52.2%</i>	<i>61.0%</i>	<i>69.2%</i>	<i>76.6%</i>	<i>82.9%</i>
	15.0%	0.0	0.0	0.2	1.1	1.8	3.0	3.7	4.6	5.5	6.6	7.8	9.2
		<i>0.0%</i>	<i>0.9%</i>	<i>5.4%</i>	<i>19.6%</i>	<i>27.9%</i>	<i>39.7%</i>	<i>45.5%</i>	<i>51.5%</i>	<i>57.4%</i>	<i>63.1%</i>	<i>68.6%</i>	<i>73.7%</i>
	20.0%	0.0	0.2	0.8	2.3	3.3	4.8	5.6	6.6	7.6	8.8	10.1	11.5
		<i>0.2%</i>	<i>3.9%</i>	<i>11.5%</i>	<i>26.0%</i>	<i>33.0%</i>	<i>42.2%</i>	<i>46.6%</i>	<i>51.1%</i>	<i>55.5%</i>	<i>59.9%</i>	<i>64.2%</i>	<i>68.3%</i>
	25.0%	0.1	0.7	1.7	3.9	5.1	6.9	7.9	8.9	10.1	11.3	12.7	14.2
		<i>1.1%</i>	<i>7.9%</i>	<i>16.8%</i>	<i>30.3%</i>	<i>36.2%</i>	<i>43.8%</i>	<i>47.3%</i>	<i>50.9%</i>	<i>54.4%</i>	<i>58.0%</i>	<i>61.4%</i>	<i>64.8%</i>
28.3%	0.2	1.1	2.5	5.2	6.5	8.5	9.5	10.7	11.9	13.3	14.7	16.3	
	<i>2.1%</i>	<i>10.7%</i>	<i>19.8%</i>	<i>32.5%</i>	<i>37.8%</i>	<i>44.5%</i>	<i>47.6%</i>	<i>50.8%</i>	<i>53.9%</i>	<i>57.0%</i>	<i>60.1%</i>	<i>63.1%</i>	
30.0%	0.2	1.4	3.0	5.9	7.3	9.4	10.5	11.7	13.0	14.4	15.9	17.5	
	<i>2.7%</i>	<i>12.0%</i>	<i>21.1%</i>	<i>33.4%</i>	<i>38.5%</i>	<i>44.8%</i>	<i>47.8%</i>	<i>50.7%</i>	<i>53.7%</i>	<i>56.6%</i>	<i>59.6%</i>	<i>62.4%</i>	
35.0%	0.6	2.5	4.7	8.2	10.0	12.4	13.6	15.0	16.5	18.0	19.7	21.5	
	<i>5.0%</i>	<i>15.7%</i>	<i>24.6%</i>	<i>35.7%</i>	<i>40.1%</i>	<i>45.5%</i>	<i>48.1%</i>	<i>50.6%</i>	<i>53.2%</i>	<i>55.7%</i>	<i>58.2%</i>	<i>60.7%</i>	
40.0%	1.2	4.0	6.8	11.1	13.2	16.0	17.4	19.0	20.6	22.4	24.3	26.3	
	<i>7.5%</i>	<i>18.9%</i>	<i>27.4%</i>	<i>37.4%</i>	<i>41.3%</i>	<i>46.1%</i>	<i>48.3%</i>	<i>50.5%</i>	<i>52.8%</i>	<i>55.0%</i>	<i>57.2%</i>	<i>59.4%</i>	
55.0%	4.9	11.4	16.8	24.3	27.7	32.2	34.5	36.9	39.5	42.2	45.1	48.2	
	<i>14.8%</i>	<i>26.1%</i>	<i>33.1%</i>	<i>40.8%</i>	<i>43.6%</i>	<i>47.2%</i>	<i>48.8%</i>	<i>50.4%</i>	<i>52.0%</i>	<i>53.6%</i>	<i>55.3%</i>	<i>56.9%</i>	

- Damodaran’s valuation assumes that we know exactly the cost of opening the second store and that it will be 40 million euros. Obviously, there is uncertainty as to how much it will cost to open a store in five years’ time. The formula (6) used in Table 4 assumes that the risk of the opening cost is equal to the risk of the cash flows generated by opening the store, which is not entirely correct. Normally, the cash flows generated by opening the store will have a greater risk than the opening cost and should be discounted at a higher rate.

Other errors.

- **Believe that options' value increases when interest rates increase.** For example, Keith and Michaels⁸ say, “an increase in interest rates increases the option's value, in spite of its negative effect on the net present value, because it reduces the present value of the exercise price”. This is wrong because the negative effect of increased interest rates on the present value of the expected cash flows (as on the value of shares) is always greater than the positive effect of the reduction of the present value of the exercise price.

- **“Play” with volatility.** The best way to explain what we mean when we say “play” is with an example. To value an oil concession where we have uncertainty regarding the number of barrels, Damodaran⁹ proposes calculating the volatility (σ) in the following manner:

$\sigma^2 = \sigma_p^2 + \sigma_q^2 + \sigma_{pq}$, where σ_p is the volatility of the oil price, σ_q is the volatility of the quantity of barrels of oil, and σ_{pq} is the covariance between price and quantity. Apart from the difficulties in estimating the parameters σ_q and σ_{pq} , it is obvious that, with this method, we will assign a higher value to the option by assigning it a high volatility. The more sources of uncertainty there are, the greater will be the volatility.

- **Valuing contracts as real options when they are not.** For example, the contract held by Áurea, a firm that manages freeway concessions, by virtue of which Dragados will offer Áurea the concession management contracts for all the freeways it is contracted to construct for the next 15 years (this initial term can be extended by mutual agreement between the two parties). The price at which Dragados will offer each concession to Áurea will be 95% of the value determined (at the time of the offer, at the end of the concession) by an independent valuer who is acceptable to both Dragados and Áurea. Áurea has the option of buying (at that time) each concession's equity for 95% of the value determined (at the time of the offer) by the independent valuer¹⁰. If Áurea exercises the option, it will buy the equity from Dragados and take on the freeway's debt. It is obvious that this contract is composed of a series of real options, one call per concession. However, each of the calls comprising the contract is an *in-the-money* call¹¹.

In this case, the price of the underlying asset is the value determined by the valuer (V), and the exercise price is 95% of this value (0.95 V). Consequently, there is no uncertainty (from a purely economic viewpoint) regarding the future exercise of the options: all of the options will be exercised because they enable a concession having a value V to be bought for 0.95 V.

⁸ See page 14 of Leslie, K.J. and M.P. Michaels (1997), “The Real Power of Real Options”, The McKinsey Quarterly, Number 3, pp. 5-22.

⁹ Damodaran Aswath (1999), “The Promise and Peril of Real Options”, Working Paper, Stern School of Business.

¹⁰ The valuations made by independent valuers (that are acceptable to both Dragados and Áurea) of each concession are very accurate, according to the opinion of Valora managers.

¹¹ An *in-the-money* call is an option whose exercise price is less than the underlying asset's price.

This option is similar to a call on a GM share whose exercise price will be 95% of GM's share price at the time of exercise. What is the value of this call? It is 5% of GM's share price today, irrespective of the exercise date and the volatility.

The value of the contract held by Áurea is, therefore, the present value of 5% of the value of the equity of the concessions that Dragados will offer Áurea during the next 15 years¹².

3. Methods for valuing real options

Real options can be valued using the following methods:

- If they are replicable, using Black and Scholes' formula, the formulas developed for valuing exotic options¹³, by simulation, the binomial formula, or by solving the differential equations characterizing the options.
- If they are not replicable, by any of the above methods but taking into account the non-replicability. For example, it is not possible to apply Black and Scholes' formula but rather the modified formula, explained in section 7.

As an exercise, I propose that the reader think how he should value my Argentinean friend's livestock company:

Dear Pablo: The reason for this message is to briefly consult you about the use of real options. It so happens that I am valuing a livestock company that owns a number of farms in the province of Salta. One of the farms (whose value without options I have already calculated) is located between two towns. The towns have grown and a residential estate has been built very close. There is a distinct possibility of an urban development project in the future and, if this should occur, the land could be worth 8 times more than what it is worth as a livestock farm. The point is that as time goes by, the likelihood that this will happen will increase. Could you give me your opinion?

Another exercise. I propose that the reader identify the errors made in the valuation of Yahoo given in Table 2. A renowned international consulting firm using what it called "an innovative valuation model" performed the valuation¹⁴.

The value of the shares (93.355 billion) is the sum of the present value of the cash flows (52.946 billion) and the value of the real options (40.409 billion). The present value of the cash flows is obtained by discounting the free cash flow forecast at a rate of 13.3%. The options' value is calculated using Black and Scholes' formula with the parameters given in Table 2.

¹² One could consider more years by assigning a probability to the renewal of the contract when the 15-year period has expired.

¹³ The interested reader can see Fernández, P. (1996), "Derivados exóticos" and "Valoración de opciones por simulación", IESE research documents n. 308 and 309.

¹⁴ The consulting firm also stated that "the advantage of this methodology lies in the fact that it enables absolute valuations to be obtained for Internet companies, avoiding the invariably dangerous valuations of firms operating in this sector".

Table 2. Valuation of Yahoo performed by a renowned international consulting firm

1. Value of future flows							
(million dollars)	1999	2000	2001	2002	2003	2004	Terminal value
Sales	589	1,078	1,890	3,034	4,165	5,640	
EBIT	188	399	756	1,365	1,999	2,876	
Free Cash Flow	103	216	445	842	1,255	1,832	104,777
Net present value of free cash flows	52,346						
+ net cash	600						
Equity value	52,946						

Risk-free rate: 6.3%. Market Risk Premium: 4%. Beta of Yahoo: 1.74. WACC = 13.3%. Long term growth (free cash flow): 8.25%

2. Value of options (million dollars)			
Electronic commerce		Advertising revenues	
Present value of sales	37,684	Present value of sales	79,531
time to exercise (years)	5	time to exercise (years)	5
Exercise price	37,684	Exercise price	79,531
Volatility	88.4%	Volatility	85.9%
1 + annual interest rate	1.133	1 + annual interest rate	1.133
Value of option (sales)	29,017	Value of option (sales)	60,445
Net margin	45.17%	Net margin	45.17%
Value of option (flows)	13,107	Value of option (flows)	27,303

3. Value of Yahoo shares (million dollars)	
Present value of flows	52,946
Value of option on electronic commerce	13,107
Value of option on advertising revenues	27,303
Value of Yahoo shares	93,355

Some questions to help the reader identify errors:

- According to the cash flow forecasts, how big will Yahoo be in 2010, in 2020 and in 2050?
- Is it correct to say that the company’s value is the present value of the expected cash flows plus the options on those same flows?
- Does it make sense to use the WACC to calculate the options’ value?
- What is the sense of the 5-year term used to calculate the options’ value?
- What do you think about the hypothesis that the options’ underlying asset is the present value of sales?
- Is it correct to use Black and Scholes’ formula to value the options?
- What do you think about the volatilities used to value the options?

Finally, one piece of information. Yahoo’s equity market value on 23 April 2001 was 10.16 billion dollars.

4. Applying options theory in a firm

If the real options cannot be replicated, using financial option formulas is completely inappropriate for valuing real options, as all the formulas are based on the existence of a replicate portfolio. The logic of options theory is based on arbitrage: as it is possible to form a replicate portfolio

that will have exactly the same return as the option we are trying to value, (in order to avoid arbitrage) the option must have the same value as the replicate portfolio. If it is not possible to form the replicate portfolio, this reasoning loses its entire basis.

In the following paragraphs, we have included a number of considerations on the practical application of options theory to the analysis of investment projects.

1. High interest rates mean high discount rates, which reduces the present value of future flows. Obviously, this should decrease the value of the option to undertake a project. However, high discount rates also reduce the present value of the option's exercise price. This compensatory effect helps sustain the option's value when interest rates increase, which may give certain types of project -particularly growth options- an enormous value that should be taken into account when analyzing investments.

2. Kester¹⁵ suggests one feature of options that should be considered: to what extent the holder of an option has an exclusive right to exercise it. Unlike share options, there are two types of growth option: exclusive and shared. The former are the more valuable because they give their holder the exclusive right to exercise them. These options derive from patents, unique knowledge of the market held by the firm or technology that its competitors cannot imitate.

Shared growth options are less valuable. They represent "collective" opportunities held by the industry, such as, for example, the possibility of entering a market that is not protected by high entry barriers or of building a new factory to supply a particular geographical segment of the market. Cost reduction projects are normally shared options, because, as a general rule, they can also be undertaken by competitors.

3. Kester also suggests that when analyzing investment projects, firms should classify the projects in accordance with the options they include. The classification using the traditional criteria of replacement, cost reduction, capacity increase, and new product introduction is not very useful. A more appropriate classification would be to distinguish between projects whose future benefits are mainly generated through cash flows (simple options) and those whose future benefits include subsequent investment options (compound options). Simple growth options -such as routine cost reductions and maintenance and replacement projects- only create value through the cash flows generated by the underlying assets.

Compound growth options -such as research and development projects, a major expansion in an existing market, entry in a new market, and acquisitions (of new businesses or firms)- lead to new investment opportunities and affect the value of the existing growth options. The compound options' complexity, their role in giving shape to the firm's strategy and, even, their impact on the organization's survival require a deeper analysis. The firm must view these projects as part of a larger group of projects or as a series of investment decisions that follow a time continuum. In the light of the firm's strategy, its executives must ask themselves whether a particular option will provide suitable investment opportunities in the appropriate markets, within a suitable time frame, that are matched to their firm's needs.

¹⁵ See Kester, W. Carl, (1984) "Today's Options for Tomorrow's Growth", Harvard Business Review, March-April, pp. 153-160.

4. The firm must separate the projects that require an immediate decision on the entire project from those in which there is decision flexibility in the future. Finally, the firm must ask itself if it can realize all the option's benefits or whether they will also be available for other competitors.

5. When examining investment opportunities from the option valuation viewpoint, managers will find it easier to recognize that: a) the conventional NPV may undervalue certain projects by eliminating the value of the options already existing in the project; b) projects with a negative NPV can be accepted if the value of the option associated with future flexibility exceeds the NPV of the project's expected cash flows; and c) the extent of the undervaluation and the degree to which managers can justifiably invest more than what the conventional rules regarding the NPV would indicate can be quantified using options theory¹⁶.

6. The options' framework indicates that the value of the management's future flexibility is greater in more uncertain environments. This value is greatest in periods with high interest rates and availability of the investment opportunities during extended periods. Consequently, contradicting generally held opinion, greater uncertainty, high interest rates, and more distant investment horizons (when part of the investment can be deferred) are not necessarily harmful for an investment opportunity's value. Although these variables reduce a project's static NPV, they can also increase the value of the project's options (value of management flexibility) to a level that may counteract the previous negative effect.

7. A real option will only be valuable if it provides a sustainable competitive advantage. This competitive advantage basically depends on the nature of the competitors (normally, if competition is fierce and the competitors are strong, the advantage's sustainability will be less) and on the nature of the competitive advantage (if it is a scarce resource, for example, scarce buildable land, the advantage's sustainability will be greater).

¹⁶ For a good study on the application of real options to mining companies, see Moel and Tufano (2000), "When are real options exercised? An empirical study of mine closings". Working Paper. Harvard Business School.

Appendix 1

Black and Scholes' formula for valuing financial options

The value of a call on a share, with an exercise price K and which can be exercised at the time t, is the present value of its price at the time t, i.e. $\text{MAX}(S_t - K, 0)$, where S_t is the share's price at the time t.

Consequently:

$$\text{Call} = \text{NPV} [\text{MAX}(S_t - K, 0)] = \text{NPV} [S_t / S_t > K] P[S_t > K] - \text{NPV} [K / S_t > K] P[S_t > K]$$

The first term of the subtraction is the present value of the share's price (provided that it is greater than K) multiplied by the probability that the share's price will be greater than K. The second term of the subtraction is the present value of the exercise price ($K r^{-t}$) multiplied by the probability that the share's price will be greater than K.

It can be shown (see Appendix 1) that if the price of the asset with a risk S follows a course

$$S_t = S_0 e^{(\mu t + \sigma \epsilon \sqrt{t})}$$

and we assume that¹⁷ $\mu = \ln(r) - \sigma^2 / 2$, then:

$$\text{NPV} [S_t / S_t > K] P[S_t > K] = S N(x)$$

$$\text{NPV} [K / S_t > K] = r^{-t} E [K / S_t > K] = K r^{-t}$$

$$P[S_t > K] = N(x - \sigma \sqrt{t}), \text{ where }^{18} x = [\ln(S / Kr^{-t}) / (\sigma \sqrt{t})] + \sigma \sqrt{t} / 2.$$

Consequently, Black and Scholes' formula is:

$$\text{Call} = S N(x) - K r^{-t} N(x - \sigma \sqrt{t}),$$

$$\text{where } x = [\ln(S / Kr^{-t}) / (\sigma \sqrt{t})] + \sigma \sqrt{t} / 2.$$

$N(x - \sigma \sqrt{t})$ is the probability that the option will be exercised, i.e., $P[S_t > K]$.

It is important to remember that this formula assumes that the option can be replicated and, therefore:

1. Considers that $\mu = \ln(r) - \sigma^2 / 2$
2. Calculates the present value using the risk-free rate.

Table A1.1. Value of the call and analysis of how the call's value is affected by changes in its parameters (million euros)

Share price	Exercise price	Risk-free rate	Volatility	Time to exercise	Dividends	CALL
180	200	5%	30%	1 year	0	17.24
200						28.35
	180					25.51
		6%				17.91
			33%			19.39
				1.1 year		18.63
					20	9.05

¹⁷ This can only be assumed if the option is replicable. This requirement is based on the fact that when a financial instrument can be valued by arbitrage (it can be replicated from other existing instruments), the price ratios move within a risk-free probability range. In this probability range, the expected value of a share's price (and whose price today is S euros) is equal to the expected value of investing those euros at the risk-free rate:

$$E(S_t) = S e^{(\mu + \sigma^2/2)t} = S r^t$$

¹⁸ It is important to realize that $P[S_t > K] = N(x - \sigma \sqrt{t})$ only if $\mu = \ln(r) - \sigma^2/2$. This condition is imposed by the fact that the option can be replicated.

Applying Black and Scholes' formula to the call on 10,000,000 shares, where the price of each share is 18 euros, the exercise price is 20 euros per share, the volatility is 30%, the time is 1 year, and the interest rate is 5% (like the oil option), we obtain:

$$x = -0.038568. N(x) = 0.4846. N(x - \sigma\sqrt{t}) = 0.3675. S N(x) = 87.23 \text{ million euros.}$$

$$K r^{-t} N(x - \sigma\sqrt{t}) = 69.99 \text{ million euros}$$

And consequently, the call's value is:

$$\text{Call} = 17.24 \text{ million euros} = 87.23 - 69.99.$$

Table A1.1 shows a sensitivity analysis of this call's value.

A derivation of Black and Scholes' formula¹⁹. Valuation of a call

In this Appendix, we will demonstrate Black and Scholes' formula for valuing a call on a share (therefore, a replicable option) using the simplest procedure. We assume that the share's return follows a normal process and that its price follows a course such that:

$$[1] \quad S_t = S e^{(\mu t + \sigma \varepsilon \sqrt{t})}$$

The share's expected value is given by the equation:

$$[2] \quad E(S_t) = S e^{(\mu + \sigma^2/2)t}$$

Where: μ = return expected by the investor per unit of time. $\mu t = E[\ln(S_t/S)]$

σ = annual volatility of the share in percent

ε = normal random variable of zero mean and variance equal to unity

By definition, the call's value now ($t=0$) must be the net present value of the future cash flows generated by it. We know the cash flow that the option's holder will receive on the exercise date, i.e., the maximum of the values $(S_t - K)$ and 0: $\text{Max}(S_t - K, 0)$. Consequently:

$$C = \text{NPV} [\text{Max}(S_t - K, 0)] = \text{NPV} [(S_t - K) / S_t > K] P[S_t > K] + \text{NPV} [0] P[S_t > K] =$$

$$[3] \quad = \text{NPV} [S_t / S_t > K] P[S_t > K] - \text{NPV} [K / S_t > K] P[S_t > K]$$

Before calculating equation [3], we should make clear one important point. If two investors were to calculate the option's NPV using different expectations about the share's future value (with different μ), they would obtain different results. However, if the two investors agree on their volatility expectations, they must also agree on the option's price because the option can be replicated with shares and bonds. Consequently -and this is a general rule for valuing financial instruments that can be constructed from other (replicable instruments)- it is not possible to calculate the NPV using the investor's return expectations. Instead, a fixed return expectation must be used, so that all investors use the same expectation even though individually they may have different expectations.

When a financial instrument can be valued by riskless arbitrage -it can be replicated from other existing instruments-, the price ratios move between a risk-free probability range. In this range, the expected value of a share's price (and whose price today is S euros) is equal to the value expected from investing those euros at the risk-free rate:

$$[4] \quad E(S_t) = S e^{(\mu + \sigma^2/2)t} = S r^t$$

because, where $r = 1 + \text{risk-free rate}$:

$$[5] \quad \mu = \ln(r) - \sigma^2/2$$

* Calculation of NPV $[K / S_t > K] P[S_t > K]$

The present value of K , if $S_t > K$, will be equal to its expected value discounted at the rate r . This value is K , which is a data we know. Thus:

$$[6] \quad \text{NPV} [K / S_t > K] = r^{-t} E [K / S_t > K] = K r^{-t}$$

To calculate the **probability** that the option will be exercised, i.e., the probability that the share's value will be greater than the exercise price on the exercise date, we will take into account equation [1]. Thus:

¹⁹ The formula was published for the first time in F. Black and M. Scholes: "The Pricing of Options and Corporate Liabilities", *The Journal of Political Economy*, May-June 1973, pp. 637-654.

$$P[S_t > K] = P[S e^{(\mu t + \sigma \varepsilon \sqrt{t})} > K] = P[\mu t + \sigma \varepsilon \sqrt{t} > \ln(K/S)] = P[\varepsilon > -\frac{\ln(S/K) + \mu t}{\sigma \sqrt{t}}]$$

ε is a normal random variable of zero mean and variance equal to unity. In a normal distribution, the following equation is met: $P[\varepsilon > -H] = P[\varepsilon < H]$

$$\text{Consequently: } P[S_t > K] = P[\varepsilon < \frac{\ln(S/K) + \mu t}{\sigma \sqrt{t}}]$$

Considering [5] and substituting in the above equation, we obtain:

$$P[S_t > K] = P[\varepsilon < \frac{\ln(S/K) + \ln(r^t) - t\sigma^2/2}{\sigma \sqrt{t}}]$$

As ε is a normal distribution (0,1), the following equation is met:

$$P[S_t > K] = N\left[\frac{\ln(Sr^t/K) - t\sigma^2/2}{\sigma \sqrt{t}}\right]. \quad \text{Defining } x \text{ as: } x = \frac{\ln(Sr^t/K) - t\sigma^2/2}{\sigma \sqrt{t}}$$

we obtain the expression²⁰: $P[S_t > K] = N(x - \sigma \sqrt{t})$. Taking into account equation [6]:

$$[7] \quad \text{NPV}[K/S_t > K] P[S_t > K] = K r^{-t} N(x - \sigma \sqrt{t})$$

Calculation of NPV [S_t / S_t > K] P[S_t > K]

The present value of S_t is equal to its expected value discounted at the rate r :

$$\begin{aligned} \text{NPV}[S_t / S_t > K] P[S_t > K] &= r^{-t} E[S_t / S_t > K] P[S_t > K] = E[S_t / S_t > K] P[S_t > K] = \\ &= \int_{-x + \sigma \sqrt{t}}^{\infty} S e^{(\mu t + \sigma \varepsilon \sqrt{t})} \frac{e^{-\varepsilon^2/2}}{\sqrt{2\pi}} d\varepsilon = S e^{\mu t} \int_{-x + \sigma \sqrt{t}}^{\infty} \frac{e^{(\sigma \varepsilon \sqrt{t} - \varepsilon^2/2)}}{\sqrt{2\pi}} d\varepsilon = S e^{(\mu + \sigma^2/2)t} \int_{-x + \sigma \sqrt{t}}^{\infty} \frac{e^{-\left(\frac{\sigma \sqrt{t} - \varepsilon}{2}\right)^2}}{\sqrt{2\pi}} d\varepsilon = \end{aligned}$$

To solve this integral, we change variables: $v = \sigma \sqrt{t} - \varepsilon$; $dv = -d\varepsilon$

Then: for $S_t = K$; $\varepsilon = -x + \sigma \sqrt{t}$; $v = x$. For $S_t = \infty$; $\varepsilon = \infty$; $v = -\infty$

With these results: $E[S_t / S_t > K] P[S_t > K] = S e^{(\mu + \sigma^2/2)t} N(x)$

On the other hand, taking into account [4], the following is met: $e^{(\mu + \sigma^2/2)t} = r^t$

Therefore: $E[S_t / S_t > K] P[S_t > K] = S r^t N(x)$. Consequently:

$$[8] \quad \text{NPV}[S_t / S_t > K] P[S_t > K] = r^{-t} E[S_t / S_t > K] P[S_t > K] = S N(x)$$

Substituting [7] and [8] in [3], we obtain Black and Scholes' formula for a call:

$$\text{Call} = S N(x) - K r^{-t} N(x - \sigma \sqrt{t}) \quad \text{where } x = \frac{\ln(Sr^t/K) - t\sigma^2/2}{\sigma \sqrt{t}}$$

$N(x)$ is an integral that has no explicit solution. However, most statistics books contain tables with the cumulative probability function of a normal distribution and many spreadsheets already contain the function $N(x)$.

Factors that determine a financial option's value

Let us briefly recall the definitions of *call* and *put*. A call is a contract that gives its holder (the buyer) the right (not the obligation) to buy a certain number of shares, at a predetermined price, at any time before a certain date (American option) or only on that date (European option). The buyer has the alternative of exercising or not his right, while the seller must sell at the buyer's order.

A put is a contract that gives its holder (the seller) the right (not the obligation) to sell a certain number of shares, at a predetermined price, at any time before a certain date (American option) or only on that date (European option).

²⁰ It is important to realize that $P[S_t > K] = N(x - \sigma \sqrt{t})$ only if $\mu = \ln r - \sigma^2/2$. This condition is imposed by the fact that the option can be replicated with shares and bonds.

The six basic variables that affect the option's price are:

- The share price to which the option is referenced (S)
- The option's exercise price (K)
- The share's volatility
- The risk-free interest rate
- The dividends that the share will receive before the exercise date
- The time remaining until the last exercise date

The share price to which the option is referenced (S). A call's value increases with the share price, while the put's value decreases. In the case of a European option, this is obvious. At the time of exercising the option, the call's holder may opt for paying the exercise price (K) and receiving a share with a value S: his gain is (S-K), so it is to his interest that S be high.

At the time of exercising the option, the holder of a put realizes a gain (K-S) as he receives K in exchange for a share: the lower the share's price, the higher his gain will be.

The option's exercise price (K). An increase in the exercise price (K) decreases the value of a call and increases the value of a put. When exercising a call, its holder gains (S-K). Thus, it is to his interest that the payment to be made be small. The situation is the opposite for the holder of a put. If he exercises the option, he will gain (K-S). The exercise price is the amount that he will receive, so it is to his interest that it be high.

The share's volatility. Both if the option is a call or a put, its value will be greater the higher the volatility forecast for the future of the share to which it is referenced. This is because the holder of an option benefits from the oscillations in the share's price in a certain direction (upwards if the option is a call and downwards if it is a put), while he is protected against movements in the opposite direction.

The risk-free interest rate. The interest rate affects an option's value because the net present value of the option's exercise price depends on the interest rates. Thus, a call's value increases with higher interest rates, because the exercise price's NPV decreases as the discount rate, that is, the interest rate increases. In the case of a put, the situation is the opposite: its value decreases when the interest rate increases.

The dividends that the share will receive before the exercise date. Dividends affect the option because when a share pays a dividend, the share's market price is adjusted to reflect the dividend paid (it decreases). Thus, the holder of a call will prefer that the share not pay dividends or that it pay the lowest dividend possible. The holder of a put will prefer that the share pay the highest dividend possible because this will mean that the share's price on the exercise date will be lower.

The time remaining until the last exercise date. The time to exercise affects the option's value through three variables mentioned previously:

Volatility: the longer the time to the exercise date, the greater is the possibility that the share's price will increase or decrease.

Exercise price: the longer the time to the exercise date, the lower is the exercise price's NPV.

Dividends: the longer the time to the exercise date, the higher are the dividends that the firm will pay.

However, not all these variables affect the option's value in the same way. The total effect will depend on the sum of each of these three variables' partial effects. Generally speaking, in the case of American options, the value of both calls and puts increases the longer the time to the exercise date. In the case of European options, each case must be studied individually.

Replication of the call

Let us assume that the price of the shares can follow two different courses, as shown in Figure A1.1. A (bullish) course reaches a price in one year's time of 254.66 million euros and the (bearish) course reaches 139 million euros. An intuitive approach to the valuation would conclude that the investor with bullish expectations would be prepared to pay more for the option than the investor with bearish expectations. However, this reasoning is incorrect. Both will agree (if the volatility expected by both is 30%) in valuing the option at 17.24 million euros. The reason for this is that buying today 87.23 million euros of shares and borrowing 69.99 million euros (net outlay: 17.24 million euros) in one year's time will have the same position as buying the option, whatever the future course followed by the share's price.

Figure A1.1. Two possible courses for the price of 10,000,000 shares during the next year. The price today is 180 million euros. The price in one year's time according to the bullish course will be 254.6 million euros and 139 million euros according to the bearish course.

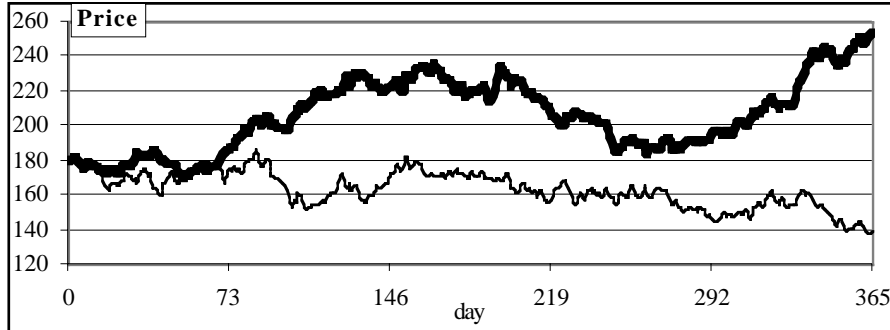


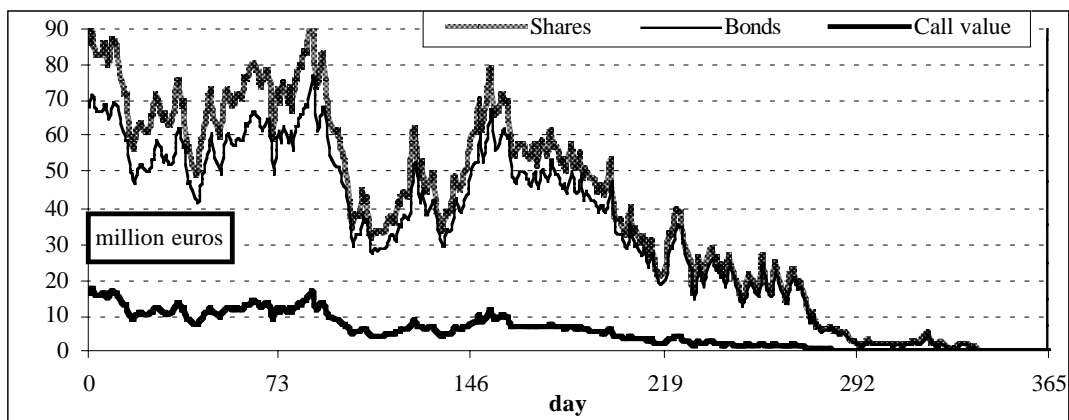
Figure A1.2 shows the option's replicate if the share price follows the bearish course. Initially, (day 0) 87.231 billion euros in shares (4,846,100 shares) must be bought and 69.994 billion euros must be borrowed.

During the following year, this portfolio must be altered as indicated by Black and Scholes' formula, which is calculated every day. On day 1, the share's price was 18.05 euros. Calculating the value of the call on day 1 gives 17.44 million euros (88.07 - 70.63). This means that the portfolio on day 1 must have 88.07 million euros invested in shares (if the share's price is 18.05 euros, it must have 4,879,200 shares). As 4,846,100 shares were held on day 0, 33,100 shares must be bought on day 1, which means an outlay of 0.6 million euros. However, this share purchase is financed entirely with debt. On day 1, the total loan will be the loan of day 0 plus one day's interest plus the new loan to buy the 331 shares:

$$69.994 \times 1.05^{1/365} + 0.6 = 70.6 \text{ million euros}$$

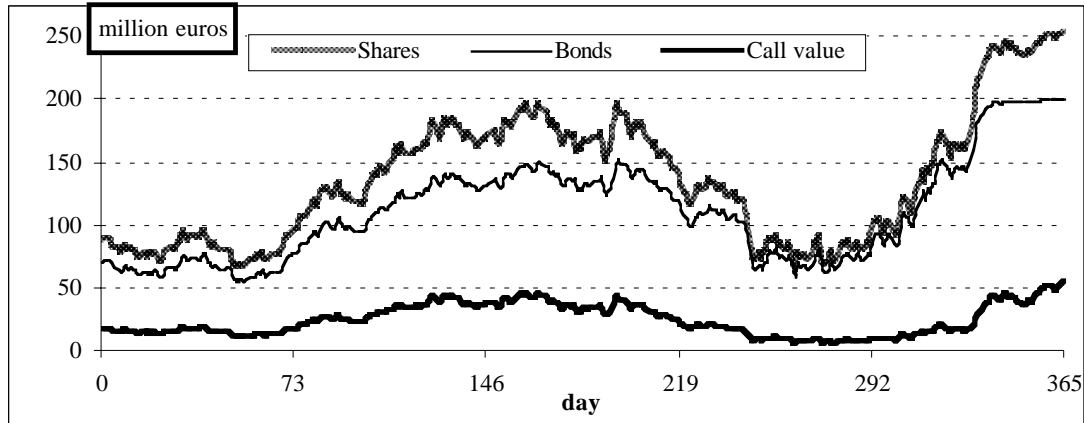
By varying the replicate portfolio in this manner throughout the year (if the share price rises, shares are bought with borrowed money; if the share price falls, shares are sold and part of the loan is repaid), Figure 2 shows how the composition of the option's replicate portfolio will vary. On any given day of the year, the call's value is identical to that of the replicate portfolio. At the end (day 364), the option is not worth anything because the share's final price is 13.50 euros. The replicate portfolio on day 365 is not worth anything either because it has neither shares nor debt.

Figure A1.2. Replicate of the call if the share follows the bearish course. In one year's time, both the option and the replicate portfolio will be worth zero (there will be neither shares nor debt).



In a similar manner, Figure A1.3 shows the option’s replicate portfolio if the share follows a bullish course. On day 365 the option is worth 54.66 million euros, as is also the replicate portfolio, which will have 254.66 million euros in shares and 200 million euros in debt.

Figure A1.3. Replicate of the share following the bullish course. In one year’s time, the call will be worth 54.66 million euros. The replicate portfolio will consist of 254.66 million euros in shares and 200 million euros in debt.



The expectations regarding an increase in the share’s price do not affect the value of a replicable call

We have seen that expectations of an increase in the share price do not influence the call’s value. A bullish investor and a bearish investor will agree on the call’s value because if they form today a portfolio with 87.23 million euros in shares and borrowing 69.99 million euros, in one year’s time they will have the same position as with the call, whatever may be the evolution of the share’s future price.

The expectations with respect to an increase in the share price can be included in the formula [1] given in the Appendix in the parameter μ . Figure A1.4 shows the distribution of the return expected for the share’s price by three investors who have identical volatility expectations (30%) but different expectations regarding the return μ : one has $\mu = -5\%$, another has $\mu=0.379\%$ and the other has $\mu=10\%$.

Figure A1.4. Distribution of the share’s return in one year’s time according to 3 different expectations.

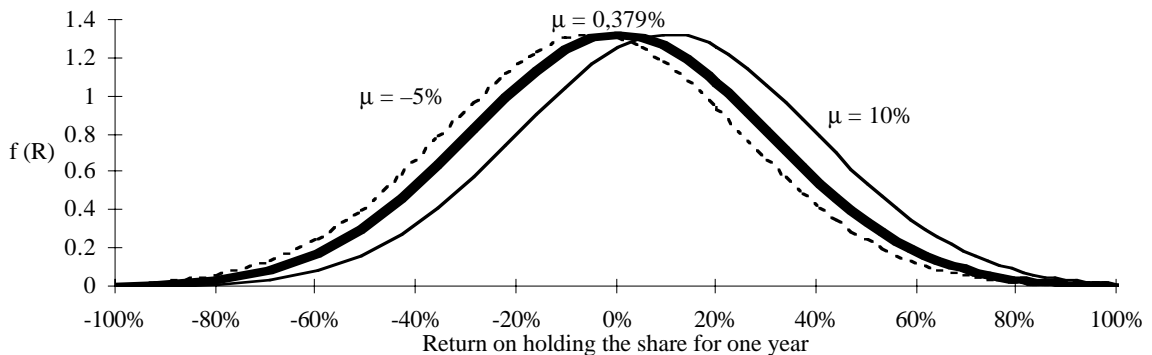


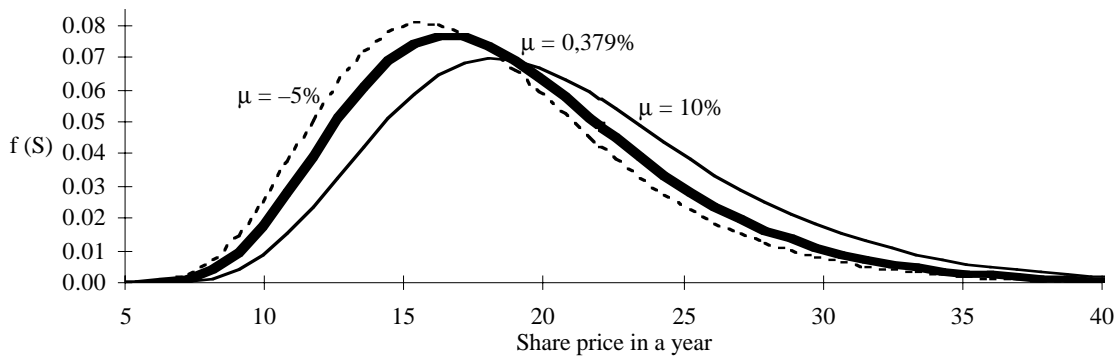
Figure A1.5 shows the three investors' distribution of the share price in one year's time. Using equation [4], the expected value of the share price is 17.91 for the investor with $\mu=-5\%$, 18.90 for the investor with $\mu=0.379\%$, and 20.8087 for the investor with $\mu=10\%$. Note that $18.90 = 18 \times 1.05$. Thus, the investor with $\mu=0.379\%$ expects a return on the share price equal to 5%, which is the risk-free interest rate. This is because $\mu=0.379\%$ meets equation [5]. In spite of their differing expectations about the share's appreciation, all three investors will agree that the option's value is 17.24 million euros.

It is important to realize that Black and Scholes' formula interpreted as net present value considers $\mu = 0.379\% = \ln(r) - \sigma^2/2$ and discounts the option's expected value $E[\text{Max}(S-K,0)]$ with the risk-free rate r . This is because the option is replicable: the financial outcome of holding the option is identical to buying today 87.23 million euros in shares and borrowing 69.99 million euros.

It is important to stress again that this formula assumes that the option can be replicated, and therefore:

- Considers that $\mu = \ln(r) - \sigma^2 / 2$
- Calculates the present value using the risk-free rate.

Figure A1.5. Distribution of the share price in one year's time according to 3 different expectations.



Differences between a financial option and a real option

The factors that determine the value of a financial option are different from those that affect a real option. These differences in the parameters are shown in Table A1.2.

Table A1.2. Parameters that influence the value of a financial option and of a real option.

FINANCIAL CALL OPTION
Share price
Exercise price
Risk-free interest rate
Share's volatility
Time to exercise
Dividends

REAL CALL OPTION
Expected value of cash flows
Cost of investment
Discount rate with risk
Volatility of expected cash flows
Time to exercise
Cost of holding the option

Its value does not depend on the expected appreciation of the underlying asset

Its value does depend on the expected appreciation of the underlying asset

Appendix 2. Value of a call if it cannot be replicated

If the option cannot be replicated, the call's value is not based on riskless arbitrage but on the valuer's expectations: expectations regarding the appreciation of the underlying asset and expectations regarding the investment's risk. In this situation:

$$P [S_t > K] = N [y - \sigma \sqrt{t}].$$

$$NPV [K / S_t > K] P[S_t > K] = K r_K^{-t} N (y - \sigma \sqrt{t})$$

$$NPV [S_t / S_t > K] P[S_t > K] = S e^{(\mu + \sigma^2/2)t} r_K^{-t} N (y)$$

$$y = [\ln (S / K) + t \mu + t \sigma^2] / [\sigma \sqrt{t}]$$

Non-replicable call = $S e^{(\mu + \sigma^2/2)t} r_K^{-t} N (y) - K r_K^{-t} N (y - \sigma \sqrt{t})$

The first term can be interpreted as the present value of the cash flows that are expected if the option is exercised. The second term is the present value of the investment required to exercise the option.

Table A2.1 shows the value of the option to extract oil in one year's time for different values of μ and r_K . Note that for $\mu = 0.379$ and $r_K = 1.05$, the same value as with Black and Scholes is obtained. This value only makes sense if the option is replicable. If it is not, the option's value also depends on the expected return μ and the discount rate r_K that is appropriate for the project.

Table A2.1. Value of the option to extract with differing expectations for μ and r_K (million euros).
Volatility = 30%.

	μ									
r_K	-5,0%	-2,0%	0,0%	0,379%	1,0%	2,0%	3,0%	4,0%	5,0%	10,0%
1,05	13,00	15,25	16,91	17,24	17,79	18,69	19,64	20,61	21,63	27,22
1,06	12,88	15,11	16,75	17,07	17,62	18,52	19,45	20,42	21,42	26,97
1,07	12,76	14,97	16,59	16,92	17,45	18,35	19,27	20,23	21,22	26,71
1,08	12,64	14,83	16,44	16,76	17,29	18,18	19,09	20,04	21,03	26,47
1,09	12,52	14,69	16,29	16,60	17,13	18,01	18,92	19,86	20,83	26,22
1,10	12,41	14,56	16,14	16,45	16,98	17,84	18,74	19,68	20,64	25,98
1,11	12,30	14,43	16,00	16,31	16,82	17,68	18,58	19,50	20,46	25,75

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