CHAPTER 11

REAL OPTIONS IN VALUATION

In discounted cash flow valuation, the value of a firm is the present value of the expected cash flows from the assets of the firm. In recent years, this framework has come under some fire for failing to consider the options that are embedded in many firms. For instance, the discounted cash flow value of a young start-up firm in a very large market may not reflect the possibility, small though it might be, that this firm may break out of the pack and become the next Microsoft or Cisco. Similarly, a firm with a patent or a license on a product may be under valued using a discounted cash flow model, because these expected cash flows do not consider the possibility that the patent could become commercially viable and extremely valuable in the future.

In both the examples cited above, discounted cash flow valuation understates the value of the firm, not because the expected cash flows are too low – they reflect the probability of success – but because they ignore the options that these firms have to invest more in the future and take advantage of unexpected success in their businesses. These options are often called real options because the underlying assets are real investments and they might explain, at least in some cases, why discounted cash flow valuations sometimes understate the value of technology firms.

This chapter begins with an introduction to options, the determinants of option value and the basics of option pricing. You will not spend much time on the technicalities of option pricing, though some of the special issues that come up when valuing real options are presented. You then consider the two types of real options you are most likely to confront in the process of valuing technology firms: the option to delay investing in a proprietary technology that might not be viable today and the option to expand the firm to take advantage of unexpected opportunities that emerge in the market served by the firm. In the process, you examine when real options have significant value and have to be considered when valuing a firm, and when the discounted cash flow valuation is sufficient.

Basics of Option Pricing

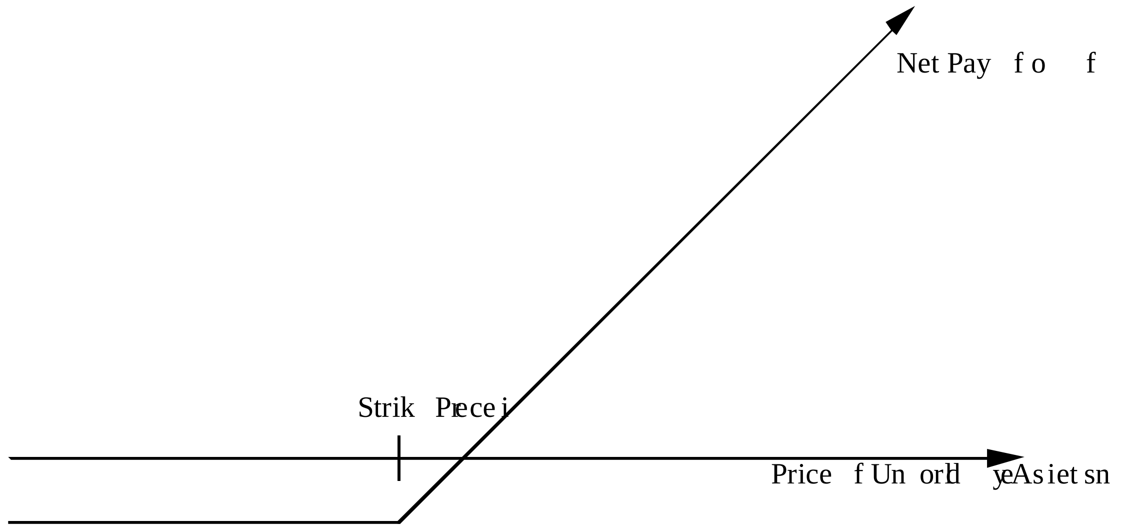
An option provides the holder with the right to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at, or before, the expiration date of the option. Since it is a right and not an obligation, the holder can choose not to exercise the right and allow the option to expire. There are two types of options - call options and put options.

Call and Put Options: Description and Payoff Diagrams

A call option gives the buyer of the option the right to buy the underlying asset at a fixed price, called the strike or the exercise price, any time prior to the expiration date of the option: the buyer pays a price for this right. If at expiration, the value of the asset is less than the strike price, the option is not exercised and expires worthless. If, on the other hand, the value of the asset is greater than the strike price, the option is exercised - the buyer of the option buys the stock at the exercise price and the difference between the asset value and the exercise price comprises the gross profit on the investment. The net profit on the investment is the difference between the gross profit and the price paid for the call initially.

A payoff diagram illustrates the cash payoff on an option at expiration. For a call, the net payoff is negative (and equal to the price paid for the call) if the value of the underlying asset is less than the strike price. If the price of the underlying asset exceeds the strike price, the gross payoff is the difference between the value of the underlying asset and the strike price, and the net payoff is the difference between the gross payoff and the price of the call. This is illustrated in figure 11.1:

Figure 11.1: Payoff on Call Option



A put option gives the buyer of the option the right to sell the underlying asset at a fixed price, again called the strike or exercise price, at any time prior to the expiration date of the option. The buyer pays a price for this right. If the price of the underlying asset is greater than the strike price, the option will not be exercised and will expire worthless. If on the other hand, the price of the underlying asset is less than the strike price, the owner of the put option will exercise the option and sell the stock at the strike price, claiming the difference between the strike price and the market value of the asset as the gross profit -- again, netting out the initial cost paid for the put yields the net profit from the transaction.

A put has a negative net payoff if the value of the underlying asset exceeds the strike price, and has a gross payoff equal to the difference between the strike price and the value of the underlying asset if the asset value is less than the strike price. This is summarized in figure 11.2.

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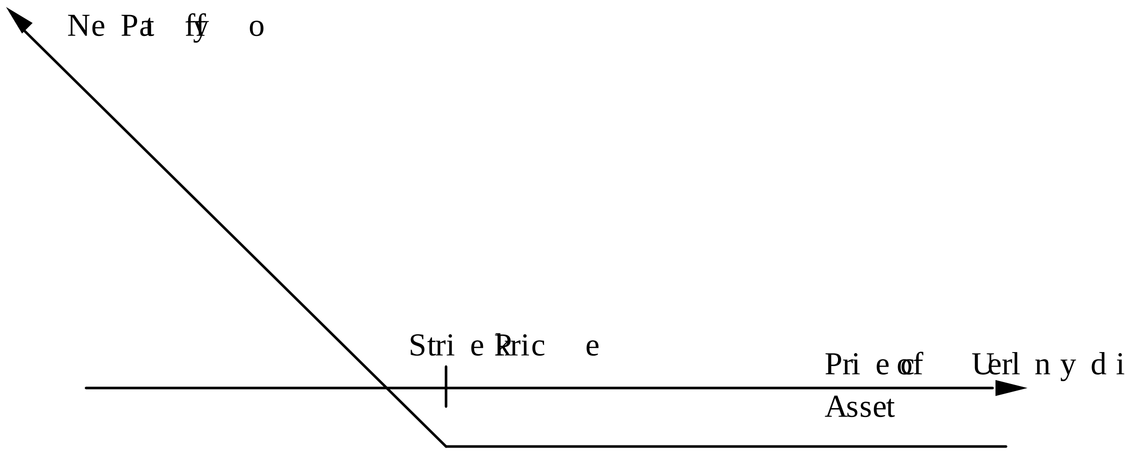


Determinants of Option Value

The value of an option is determined by a number of variables relating to the underlying asset and financial markets.

1. Current Value of the Underlying Asset: Options are assets that derive value from an underlying asset. Consequently, changes in the value of the underlying asset affect the value of the options on that asset. Since calls provide the right to buy the underlying asset at a fixed price, an increase in the value of the asset will increase the value of the calls. Puts, on the other hand, become less valuable as the value of the asset increase.

2. Variance in Value of the Underlying Asset: The buyer of an option acquires the right to buy or sell the underlying asset at a fixed price. The higher the variance in the value of the underlying asset, the greater the value of the option. This is true for both calls and puts. While it may seem counter-intuitive that an increase in a risk measure (variance) should increase value, options are different from other securities since buyers of options can never lose more than the price they pay for them; in fact, they have the potential to earn significant returns from large price movements.

3. Dividends Paid on the Underlying Asset: The value of the underlying asset can be expected to decrease if dividend payments are made on the asset during the life of the option. Consequently, the value of a call on the asset is a decreasing function of the size of expected dividend payments, and the value of a put is an increasing function of expected dividend payments. A more intuitive way of thinking about dividend payments, for call options, is as a cost of delaying exercise on in-the-money options. To see why, consider an option on a traded stock. Once a call option is in the money, exercising the call option will provide the holder with the stock, and entitle him or her to the dividends on the stock in subsequent periods. Failing to exercise the option will mean that these dividends are foregone.

4. Strike Price of Option: A key characteristic used to describe an option is the strike price. In the case of calls, where the holder acquires the right to buy at a fixed price, the value of the call will decline as the strike price increases. In the case of puts, where the holder has the right to sell at a fixed price, the value will increase as the strike price increases.

5. Time To Expiration On Option: Both calls and puts become more valuable as the time to expiration increases. This is because the longer time to expiration provides more time for the value of the underlying asset to move, increasing the value of both types of options. Additionally, in the case of a call, where the buyer has to pay a fixed price at expiration, the present value of this fixed price decreases as the life of the option increases, further increasing the value of the call.

6. Riskless Interest Rate Corresponding To Life Of Option: Since the buyer of an option pays the price of the option up front, an opportunity cost is involved. This cost depends upon the level of interest rates and the time to expiration on the option. The riskless interest rate also enters into the valuation of options when the present value of the exercise price is calculated, since the exercise price does not have to be paid (received) until expiration on calls (puts). Increases in the interest rate will increase the value of calls and reduce the value of puts. Table 11.1 summarizes the variables and their predicted effects on call and put prices.

Table 11.1: Summary of Variables Affecting Call and Put Prices

	Effect on	
Factor	*Call Value*	*Put Value*
Increase in underlying asset's value	Increases	Decreases
Increase in Strike Price	Decreases	Increases
Increase in variance of underlying asset	Increases	Increases
Increase in time to expiration	Increases	Increases
Increase in interest rates	Increases	Decreases
Increase in dividends paid	Decreases	Increases

American Versus European Options: Variables Relating To Early Exercise

A primary distinction between American and European options is that American options can be exercised at any time prior to its expiration, while European options can be exercised only at expiration. The possibility of early exercise makes American options more valuable than otherwise similar European options; it also makes them more difficult to value. There is one compensating factor that enables the former to be valued using models designed for the latter. In most cases, the time premium associated with the remaining life of an option and transactions costs makes early exercise sub-optimal. In other words, the holders of in-the-money options generally gets much more by selling the option to someone else than by exercising the options.

While early exercise is not optimal generally, there are at least two exceptions to this rule. One is a case where the *underlying asset pays large dividends*, thus reducing the value of the asset, and any call options on that asset. In this case, call options may be exercised just before an ex-dividend date, if the time premium on the options is less than the expected decline in asset value as a consequence of the dividend payment. The other exception arises when an investor holds both the underlying asset and *deep in-the-money puts* on that asset at a time when interest rates are high. In this case, the time premium on the put may be less than the potential gain from exercising the put early and earning interest on the exercise price.

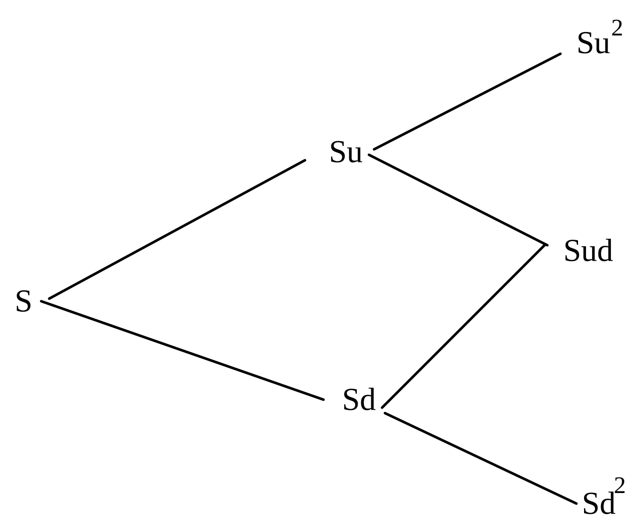
Option Pricing Models

Option pricing theory has made vast strides since 1972, when Black and Scholes published their path-breaking paper providing a model for valuing dividend-protected European options. Black and Scholes used a "replicating portfolio" — a portfolio composed of the underlying asset and the risk-free asset that had the same cash flows as the option being valued— to come up with their final formulation. While their derivation is mathematically complicated, there is a simpler binomial model for valuing options that draws on the same logic.

The Binomial Model

The *binomial option pricing model* is based upon a simple formulation for the asset price process, in which the asset, in any time period, can move to one of two possible prices. The general formulation of a stock price process that follows the binomial is shown in figure 11.3 for a two-period process.

Figure 11.3: General Formulation for Binomial Price Path



In this figure, S is the current stock price; the price moves up to Su with probability p and down to Sd with probability 1-p in any time period.

Creating A Replicating Portfolio

The objective in creating a replicating portfolio is to use a combination of risk-free borrowing/lending and the underlying asset to create the same cash flows as the option being valued. The principles of arbitrage apply here, and the value of the option must be equal to the value of the replicating portfolio. In the case of the general formulation above, where stock prices can either move up to Su or down to Sd in any time period, the replicating portfolio for a call with strike price K will involve borrowing \$B and acquiring of the underlying asset, where:

= Number of units of the underlying asset bought = (C_u - C_d)/(Su - Sd)

where,

C_u = Value of the call if the stock price is Su

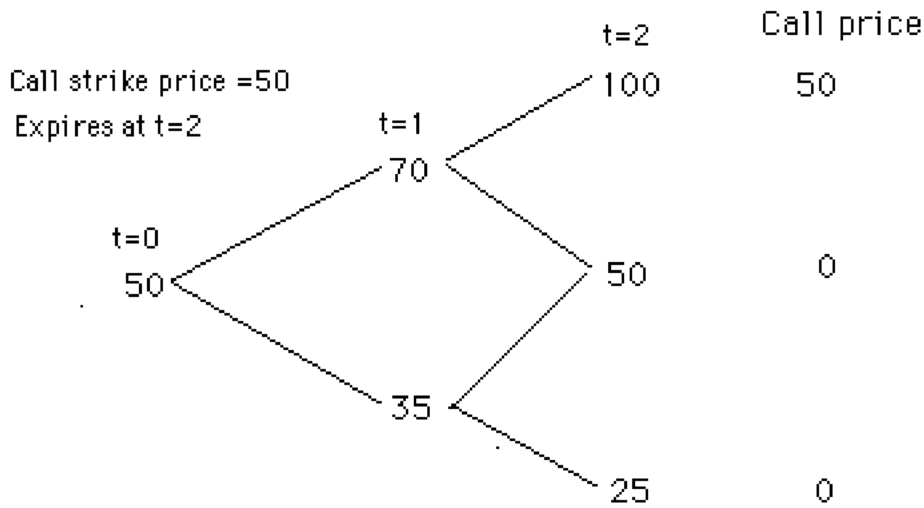
C_d = Value of the call if the stock price is Sd

In a multi-period binomial process, the valuation has to proceed iteratively; i.e., starting with the last time period and moving backwards in time until the current point in time. The portfolios replicating the option are created at each step and valued, providing the values for the option in that time period. The final output from the binomial option pricing model is a statement of the value of the option in terms of the replicating portfolio, composed of Δ shares (option delta) of the underlying asset and risk-free borrowing/lending.

Value of the call = Current value of underlying asset * Option Delta - Borrowing needed to replicate the option

An Example of Binomial valuation

Assume that the objective is to value a call with a strike price of 50, which is expected to expire in two time periods, on an underlying asset whose price currently is 50 and is expected to follow a binomial process:



Now assume that the interest rate is 11%. In addition, define

Δ = Number of shares in the replicating portfolio

B = Dollars of borrowing in replicating portfolio

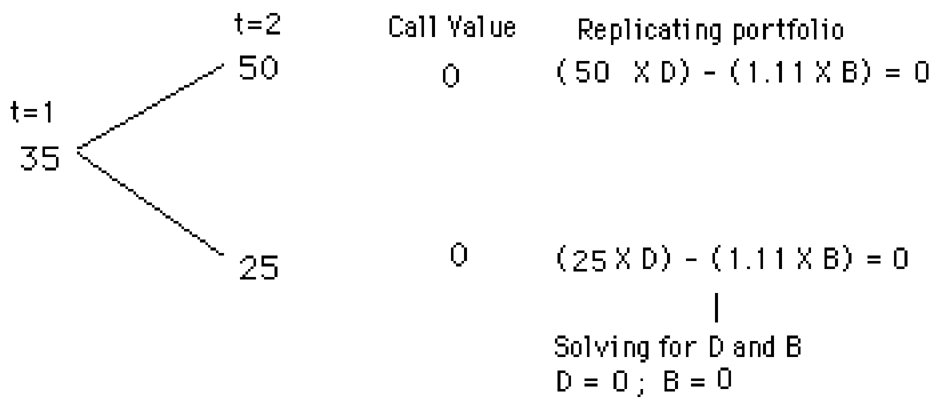
The objective is to combined Δ shares of stock and B dollars of borrowing to replicate the cash flows from the call with a strike price of $ 50. This can be done starting with the last period and working back through the binomial tree.

Step 1: Start with the end nodes and work backward:

Thus, if the stock price is $70 at t=1, borrowing $45 and buying one share of the stock will yield the same cash flows as buying the call. The value of the call at t=1, if the stock price is $70, is therefore:

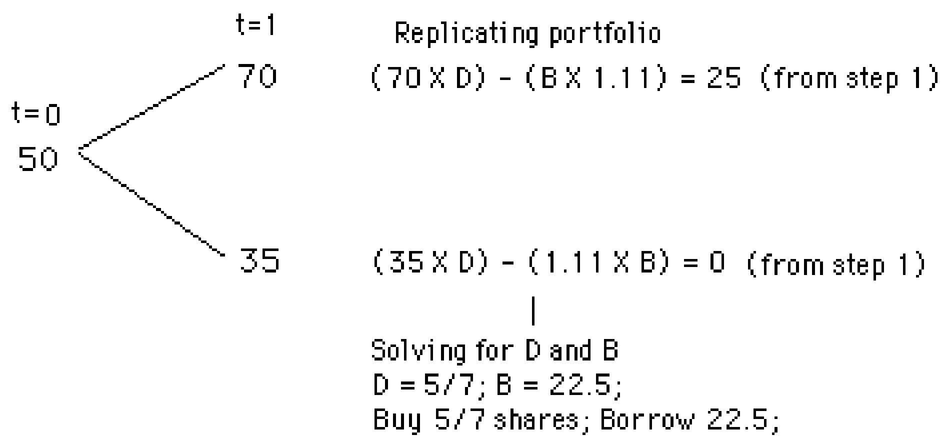
Value of Call = Value of Replicating Position = 70 Δ - B = 70-45 = 25

Considering the other leg of the binomial tree at t=1,



If the stock price is 35 at t=1, then the call is worth nothing.

Step 2: Move backward to the earlier time period and create a replicating portfolio that provides the cash flows the option provides.



In other words, borrowing \$22.5 and buying 5/7 of a share provides the same cash flows as a call with a strike price of \$50. The value of the call, therefore, has to be the same as the value of this position.

Value of Call = Value of replicating position = 5/7 X Current stock price - \$ 22.5 = \$ 13.20

The Determinants of Value

The binomial model provides insight into the determinants of option value. The value of an option is not determined by the expected price of the asset but by its current price, which, of course, reflects expectations about the future. This is a direct consequence of arbitrage. If the option value deviates from the value of the replicating portfolio, investors can create an arbitrage position, i.e., one that requires no investment, involves no risk, and delivers positive returns. To illustrate, if the portfolio that replicates the call costs more than the call does in the market, an investor could buy the call, sell the replicating portfolio and be guaranteed the difference as a profit. The cash flows on the two positions offset each other, leading to no cash flows in subsequent periods. The option value also increases as the time to expiration is extended, as the price movements (u and d) increase, and with increases in the interest rate.

The Black-Scholes Model

While the binomial model provides an intuitive feel for the determinants of option value, it requires a large number of inputs, in terms of expected future prices at each node. The Black-Scholes model is not an alternative to the binomial model; rather, it is one limiting case of the binomial.

The binomial model is a discrete-time model for asset price movements, including a time interval (t) between price movements. As the time interval is shortened, the limiting distribution, as t approaches 0, can take one of two forms. If as t approaches 0, price changes become smaller, the limiting distribution is the normal distribution and the price process is a continuous one. If as t approaches 0, price changes remain large, the limiting distribution is the Poisson distribution, i.e., a distribution that allows for price jumps. The Black-Scholes model applies when the limiting distribution is the normal distribution,[1] and it explicitly assumes that the price process is continuous and that there are no jumps in asset prices.

The Model

The version of the model presented by Black and Scholes was designed to value European options, which were dividend-protected. Thus, neither the possibility of early exercise nor the payment of dividends affects the value of options in this model.

The value of a call option in the Black-Scholes model can be written as a function of the following variables:

S = Current value of the underlying asset

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[1] Stock prices cannot drop below zero, because of the limited liability of stockholders in publicly listed firms. Hence, stock prices, by themselves, cannot be normally distributed, since a normal distribution requires some probability of infinitely negative values. The distribution of the natural logs of stock prices is assumed to be log-normal in the Black-Scholes model. This is why the variance used in this model is the variance in the log of stock prices.

K = Strike price of the option

t = Life to expiration of the option

r = Riskless interest rate corresponding to the life of the option

σ^2 = Variance in the ln(value) of the underlying asset

The model itself can be written as:

Value of call = $S\,N(d_1) - K\,e^{-rt}\,N(d_2)$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{\sigma^2}{2})\,t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma \quad t$$

The process of valuation of options using the Black-Scholes model involves the following steps:

Step 1: The inputs to the Black-Scholes are used to estimate d_1 and d_2.

Step 2: The cumulative normal distribution functions, $N(d_1)$ and $N(d_2)$, corresponding to these standardized normal variables are estimated.

Step 3: The present value of the exercise price is estimated, using the continuous time version of the present value formulation:

Present value of exercise price = $K\,e^{-rt}$

Step 4: The value of the call is estimated from the Black-Scholes model.

The Replicating Portfolio in the Black-Scholes

The determinants of value in the Black-Scholes are the same as those in the binomial - the current value of the stock price, the variability in stock prices, the time to expiration on the option, the strike price, and the riskless interest rate. The principle of replicating portfolios that is used in binomial valuation also underlies the Black-Scholes model. In fact, embedded in the Black-Scholes model is the replicating portfolio.

Value of call = $S\,N(d_1)$ $- K\,e^{-rt}\,N(d_2)$

Buy $N(d_1)$ shares Borrow $K\ e^{-rt}\ N(d_2)$

N(d1), which is the number of shares that are needed to create the replicating portfolio is called the *option delta.* This replicating portfolio is self-financing and has the same value as the call at every stage of the option's life.

Model Limitations and Fixes

The version of the Black-Scholes model presented above does not take into account the possibility of early exercise or the payment of dividends, both of which impact the value of options. Adjustments exist, which, while not perfect, provide partial corrections to value.

1. Dividends

The payment of dividends reduces the stock price. Consequently, call options become less valuable and put options more valuable as dividend payments increase. One approach to dealing with dividends to estimate the present value of expected dividends paid by the underlying asset during the option life and subtract it from the current value of the asset to use as “S” in the model. Since this becomes impractical as the option life becomes longer, you can use an alternate approach. If the dividend yield (y = dividends/ current value of the asset) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

$$C = S\ e^{-yt}\ N(d_1) - K\ e^{-rt}\ N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \quad t$$

From an intuitive standpoint, the adjustments have two effects. First, the value of the asset is discounted back to the present at the dividend yield to take into account the expected drop in value from dividend payments. Second, the interest rate is offset by the dividend yield to reflect the lower carrying cost from holding the stock (in the replicating portfolio). The net effect will be a reduction in the value of calls, with the adjustment, and an increase in the value of puts.

2. Early Exercise

The Black-Scholes model is designed to value European options, i.e. options that cannot be exercised until the expiration day. Most of the options that you analyze are American options, which can be exercised anytime before expiration. Without working through the mechanics of valuation models, an American option should always be worth at least as much and generally more than a European option because of the early exercise option. There are three basic approaches for dealing with the possibility of early exercise. The first is to continue to use the unadjusted Black-Scholes, and regard the resulting value as a floor or conservative estimate of the true value. The second approach is to value the option to each potential exercise date. With options on stocks, this basically requires that you value options to each ex-dividend day and choose the maximum of the estimated call values. The third approach is to use a modified version of the binomial model to consider the possibility of early exercise.

While it is difficult to estimate the prices for each node of a binomial, there is a way in which variances estimated from historical data can be used to compute the expected up and down movements in the binomial. To illustrate, if σ^2 is the variance in ln(stock prices), the up and down movements in the binomial can be estimated as follows:

$$u = \text{Exp}\ [(r - \sigma^2/2)(T/m) + \quad (\sigma^2 T/m)]$$

$$d = \text{Exp}\ [(r - \sigma^2/2)(T/m) - \quad (\sigma^2 T/m)]$$

where u and d are the up and down movements per unit time for the binomial, T is the life of the option and m is the number of periods within that lifetime. Multiplying the stock price at each stage by u and d will yield the up and the down prices. These can then be used to value the asset.

3. The Impact Of Exercise On The Value Of The Underlying Asset

The derivation of the Black-Scholes model is based upon the assumption that exercising an option does not affect the value of the underlying asset. This may be true for listed options on stocks, but it is not true for other types of options. For instance, the exercise of warrants increases the number of shares outstanding and brings fresh cash into the firm, both of which will affect the stock price.[2] The expected negative impact (dilution) of exercise decreases the value of warrants compared to otherwise similar call options. The adjustment for dilution in the Black-Scholes to the stock price is fairly simple. The stock price is adjusted for the expected dilution from the exercise of the options. In the case of warrants, for instance:

Dilution-adjusted $S = (S\ n_S + W\ n_W) / (n_S + n_w)$

where

S = Current value of the stock n_W = Number of warrants outstanding

W = Market value of warrants outstanding n_S = Number of shares outstanding

When the warrants are exercised, the number of shares outstanding increases, reducing the stock price. The numerator reflects the market value of equity, including both stocks and warrants outstanding. The reduction in S will reduce the value of the call option.

There is an element of circularity in this analysis, since the value of the warrant is needed to estimate the dilution-adjusted S and the dilution-adjusted S is needed to estimate the value of the warrant. This problem can be resolved by starting the process off with an estimated value of the warrant (say, the exercise value), and then iterating with the new estimated value for the warrant until there is convergence.

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[2] Warrants are call options issued by firms, either as part of management compensation contracts or to raise equity.

Valuing Puts

The value of a put can be derived from the value of a call with the same strike price and the same expiration date through an arbitrage relationship that specifies that:

$$C - P = S - K e^{-rt}$$

where C is the value of the call and P is the value of the put (with the same life and exercise price).

This arbitrage relationship can be derived fairly easily and is called *put-call parity*. To see why put-call parity holds, consider creating the following portfolio:

(a) Sell a call and buy a put with exercise price K and the same expiration date "t"

(b) Buy the stock at current stock price S

The payoff from this position is riskless and always yields K at expiration (t). To see this, assume that the stock price at expiration is S*:

Position	*Payoffs at t if S*>K*	*Payoffs at t if S*<K*
Sell call	-(S*-K)	0
Buy put	0	K-S*
Buy stock	S*	S*
Total	*K*	*K*

Since this position yields K with certainty, its value must be equal to the present value of K at the riskless rate ($K e^{-rt}$).

$$S+P-C = K e^{-rt}$$

$$C - P = S - K e^{-rt}$$

This relationship can be used to value puts. Substituting the Black-Scholes formulation, with the dividend adjustment, for the value of an equivalent call,

$$\text{Value of put} = S e^{-yt} (N(d_1) - 1) - K e^{-rt} (N(d_2) - 1)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

optlt.xls: This spreadsheet allows you to estimate the value of an option, using the dividend-adjusted Black-Scholes model.

A Few Caveats On Applying Option Pricing Models

The option pricing models described in the preceding chapter can be used to value any asset that has the characteristics of an option, with some caveats. In subsequent sections, you apply option pricing theory in a variety of contexts. In many of the cases described, the options being valued are not on financially traded assets (such as stocks or commodities) but are real options (such as those on projects). There are a few caveats on the application of option pricing models to these cases and some adjustments might need to be made to these models.

1. *The Underlying Asset Is Not Traded*: Option pricing theory, as presented in both the binomial and the Black-Scholes models, is built on the premise that a replicating portfolio can be created using the underlying asset and riskless lending or borrowing. While this is a perfectly justifiable assumption in the context of listed options on traded stocks, it becomes less defensible when the underlying asset is not traded, and arbitrage is, therefore, not feasible. When the options valued are on assets that are not traded, the values from option pricing models have to be interpreted with caution.

2. *The Price Of The Asset Follows A Continuous Process*: As noted earlier, the Black-Scholes option pricing model is derived under the assumption that the underlying asset's price process is continuous (i.e., there are no price jumps). If this assumption is violated, as it is with many real options, the model will underestimate the value of deep out-of-the-money options. One solution is to use higher variance estimates to value deep out-of-the-money options and lower variance estimates for at-the-money or in-the-money options;

another is to use an option pricing model that explicitly allows for price jumps, though the inputs to these models are often difficult to estimate.[3]

3. The Variance Is Known And Does Not Change Over The Life Of The Option: The assumption option pricing models make that the variance is known and does not change over the option lifetime is not unreasonable when applied to listed short-term options on traded stocks. When option pricing models are used to value long-term real options, however, there are problems with this assumption, since the variance is unlikely to remain constant over extended periods of time and may in fact be difficult to estimate in the first place. Again, modified versions of the option pricing model exist that allow for changing variances, but they require that the process by which variance changes be modeled explicitly.

4. Exercise Is Instantaneous: The option pricing models are based upon the premise that the exercise of an option is instantaneous. This assumption may be difficult to justify with real options, however; exercise may require building a plant or constructing an oil rig, for example, actions that do not happen in an instant. The fact that exercise takes time also implies that the true life of a real option is often less than the stated life. Thus, while a firm may own the rights to an oil reserve for the next ten years, the fact that it takes several years to extract the oil reduces the life of the natural resource option the firm owns.

Barrier, Compound and Rainbow Options

So far in the discussion of option pricing, you have not considered more complicated options that often arise in analysis. In this section, you consider three

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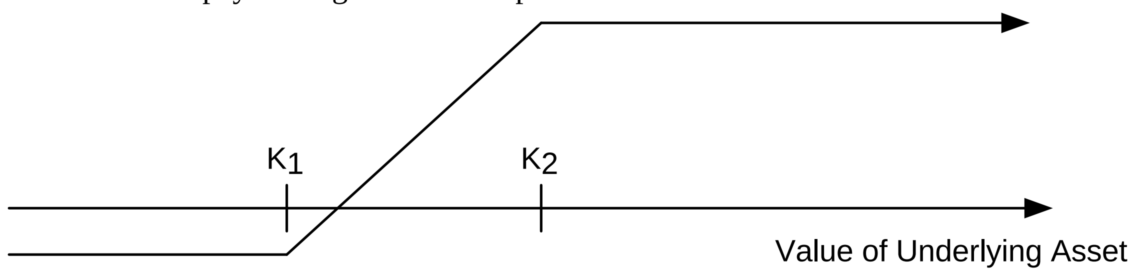
[3] Jump process models that incorporate the Poisson process require inputs on the probability of price jumps, the average magnitude, and the variance, all of which can be estimated, but with a significant amount of noise.

variations on the simple option. The first is a barrier option, where the option value is capped if the price of the underlying asset exceeds a pre-specified level. The second is the compound option, which is an option on an option. The third is a rainbow option, where there is more than one source of uncertainty affecting the value of the option.

Capped and Barrier Options

Assume that you have a call option with a strike price of \$ 25 on an asset. In an unrestricted call option, the payoff on this option will increase as the underlying asset's price increases above \$ 25. Assume, however, that if the price reaches \$ 50, the payoff is capped at \$ 25. The payoff diagram on this option is as follows:



This option is called a *capped call*. Notice, also, that once the price reaches \$ 50, there is no time premium associated with the option anymore and that the option will therefore be exercised. Capped calls are part of a family of options called *barrier options*, where the payoff on and the life of the option is a function of whether the underlying asset reaches a certain level during a specified period.

The value of a capped call will always be lower than the value of the same call without the payoff limit. A simple approximation of this value can be obtained by valuing the call twice, once with the given exercise price, and once with the cap, and taking the difference in the two values. In the above example, then, the value of the call with an exercise price of K_1 and a cap at K_2 can be written as:

Value of Capped Call = Value of call ($K=K_1$) - Value of call ($K=K_2$)

Barrier options can take many forms. In a *knockout option*, an option ceases to exist if the option reaches a certain price. In the case of a call option, this knock-out price is usually set below the strike price, and this option is called a *down-and-out option*. In the case of a put option, the knock-out price will be set above the exercise price and this option is called an *up-and-out option*. Like the capped call, these options will be worth less than their unrestricted counterparts. Many real options have limits on potential upside and/or knock-out provisions, and ignoring these limits can result in the overstatement of the value of these options.

Compound Options

Some options derive their value, not from an underlying asset, but from other options. These options are called *compound options*. Compound options can take any of four forms - a call on a call, a put on a put, a call on a put and a put on a call. Geske (1979) developed the analytical formulation for valuing compound options by replacing the standard normal distribution used in a simple option model with a bivariate normal distribution in the calculation.

In the context of real options, the compound option process can get complicated. Consider, for instance, the option to expand a project that you consider in the next section. While this option is valued using a simple option pricing model, in reality, there could be multiple stages in expansion, with each stage representing an option for the following stage. In this case, you will under value the option by considering it as a simple rather than a compound option.

Notwithstanding this discussion, the valuation of compound options become progressively more difficult as you add more options to the chain. In this case, rather than wreck the valuation on the shoals of estimation error, it may be better to accept the conservative estimate that is provided with a simple valuation model as a floor on the value.

Rainbow Options

In a simple option, the only source of uncertainty is the price of the underlying asset. There are some options that derive their value from two or more sources of uncertainty, and these options are called *rainbow options*. Using the simple option pricing model to value such options can lead to biased estimates of value. As an example, consider an undeveloped oil reserve as an option, where the firm that owns the reserve has the right to develop the reserve. Here, there are two sources of uncertainty. The first is obviously the price of oil, and the second is the quantity of oil that is in the reserve.

To value this undeveloped reserve, you can make the simplifying assumption that you know the quantity of the reserves with certainty. In reality, however, uncertainty about the quantity will affect the value of this option and make the decision to exercise more difficult[4].

The Option to Delay

Projects are typically analyzed based upon their expected cash flows and discount rates at the time of the analysis; the net present value computed on that basis is a measure of its value and acceptability at that time. Expected cash flows and discount rates change over time, however, and so does the net present value. Thus, a project that has a negative net present value now may have a positive net present value in the future. In a competitive environment, in which individual firms have no special advantages over their competitors in taking projects, this may not seem significant. In an environment in which a project can be taken by only one firm (because of legal restrictions or other barriers to

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[4] The analogy to a listed option on a stock would be the case where you do not know what the stock price is with certainty when you exercise the option. The more uncertain you are about the stock price, the more margin for error you have to give yourself when you exercise the option to ensure that you are in fact earning a profit.

entry to competitors), however, the changes in the project's value over time give it the characteristics of a call option.

The Payoff Diagram on the Option to Delay

In the abstract, assume that a project requires an initial up-front investment of X, and that the present value of expected cash inflows computed right now is V. The net present value of this project is the difference between the two:

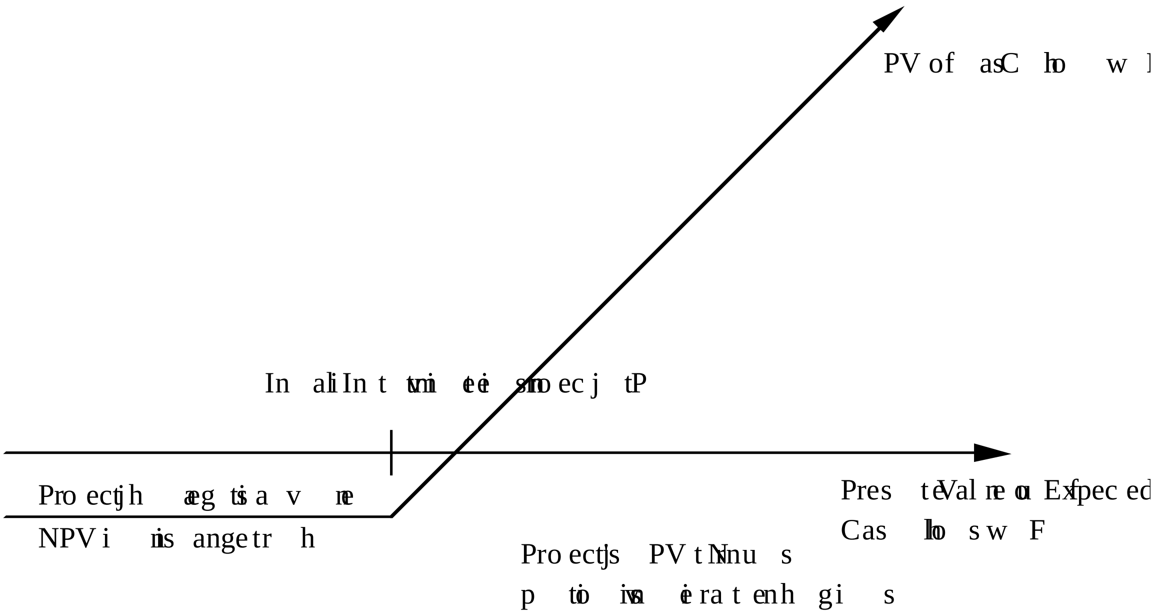
$$NPV = V - X$$

Now assume that the firm has exclusive rights to this project for the next n years, and that the present value of the cash inflows may change over that time, because of changes in either the cash flows or the discount rate. Thus, the project may have a negative net present value right now, but it may still be a good project if the firm waits. Defining V again as the present value of the cash flows, the firm's decision rule on this project can be summarized as follows:

If	$V > X$	Take the project: Project has positive net present value
	$V < X$	Do not take the project: Project has negative net present value

If the firm does not take the project, it incurs no additional cash flows, though it will lose what it originally invested in the project. This relationship can be presented in a payoff diagram of cash flows on this project, as shown in Figure 11.4, assuming that the firm holds out until the end of the period for which it has exclusive rights to the project:

Figure 1.14: The Option to Delay a Project



Note that this payoff diagram is that of a call option — the underlying asset is the project, the strike price of the option is the investment needed to take the project; and the life of the option is the period for which the firm has rights to the project. The present value of the cash flows on this project and the expected variance in this present value represent the value and variance of the underlying asset.

Valuing the Option to Delay

On the surface, the inputs needed to apply the option pricing model to valuing the option to delay are the same as those needed for any option. You need the value of the underlying asset, the variance in that value, the time to expiration on the option, the strike price, the riskless rate and the equivalent of the dividend yield (cost of delay). Actually estimating these inputs for product patent valuation can be difficult, however.

Value Of The Underlying Asset

In the case of product options, the underlying asset is the project itself. The current value of this asset is the present value of expected cash flows from initiating the project now, not including the up-front investment, which can be obtained by doing a standard capital budgeting analysis. There is likely to be a substantial amount of noise in the cash flow estimates and the present value, however. Rather than being viewed as a problem, this uncertainty should be viewed as the reason for why the project delay option has value. If the expected cash flows on the project were known with certainty and were not expected to change, there would be no need to adopt an option pricing framework, since there would be no value to the option.

Variance in the value of the asset

As noted in the prior section, there is likely to be considerable uncertainty associated with the cash flow estimates and the present value that measures the value of the asset now, partly because the potential market size for the product may be unknown, and partly because technological shifts can change the cost structure and profitability of the product. The variance in the present value of cash flows from the project can be estimated in one of three ways.

- If similar projects have been introduced in the past, the variance in the cash flows from those projects can be used as an estimate. This may be the way that a firm like Intel might estimate the variance associated with introducing a new computer chip.
- Probabilities can be assigned to various market scenarios, cash flows estimated under each scenario and the variance estimated across present values. Alternatively, the probability distributions can be estimated for each of the inputs into the project analysis - the size of the market, the market share and the profit margin, for instance - and simulations used to estimate the variance in the present values that emerge. This approach tends to work best when there are only one or two sources[5] of significant uncertainty about future cash flows.

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[5] In practical terms, the probability distributions for inputs like market size and market share can often be obtained from market testing.

- The variance in firm value of firms involved in the same business (as the project being considered) can be used as an estimate of the variance. Thus, the average variance in firm value of firms involved in the software business can be used as the variance in present value of a software project.

The value of the option is largely derived from the variance in cash flows - the higher the variance, the higher the value of the project delay option. Thus, the value of a option to invest in a project in a stable business will be less than the value of one in an environment where technology, competition and markets are all changing rapidly.

Exercise Price On Option

A project delay option is exercised when the firm owning the rights to the project decides to invest in it. The cost of making this investment is the exercise price of the option. The underlying assumption is that this cost remains constant (in present value dollars) and that any uncertainty associated with the product is reflected in the present value of cash flows on the product.

Expiration Of The Option And The Riskless Rate

The project delay option expires when the rights to the project lapse; investments made after the project rights expire are assumed to deliver a net present value of zero as competition drives returns down to the required rate. The riskless rate to use in pricing the option should be the rate that corresponds to the expiration of the option. While the life of the option can be estimated easily when firms have the explicit right to a project (through a license or a patent, for instance), it becomes far more difficult to obtain when firms only have a competitive advantage to take a project. Since competitive advantages fade over time, the number of years for which the firm can be expected to have these advantages is the life of the option.

Cost of Delay (Dividend Yield)

There is a cost to delaying taking a project, once the net present value turns positive. Since the project rights expire after a fixed period, and excess profits (which are the source of positive present value) are assumed to disappear after that time as new competitors emerge, each year of delay translates into one less year of value-creating cash flows.[6] If the cash flows are evenly distributed over time, and the life of the patent is n years, the cost of delay can be written as:

$$\text{Annual cost of delay} = \frac{1}{n}$$

Thus, if the project rights are for 20 years, the annual cost of delay works out to 5% a year. Note, though, that this cost of delay rises each year , to 1/19 in year 2, 1/18 in year 3 and so on, making the cost of delaying exercise larger over time.

optvar.xls: There is a dataset on the web that summarizes standard deviations in equity and firm value, by industry, for firms in the United States.

Illustration 11.1: Valuing the Option to Delay a Project

Assume that you have, or are interested in acquiring, the exclusive rights to market a new product that will make it easier for people to access their e-mail on the road. If you do acquire the rights to the product, you estimate that it will cost you $ 500 million up-front to set up the infrastructure needed to provide the service. Based upon your current projections, you believe that the service will generate only $ 100 million in after-tax cash flows each year. In addition, you expect to operate without serious competition for the next 5 years.

[6] A value-creating cashflow is one that adds to the net present value because it is in excess of the required return for investments of equivalent risk.

The net present value of this project can be computed by taking the present value of the expected cash flows over the next 5 years. Assuming a discount rate of 15% (based on the riskiness of this project), you obtain the following net present value for the project:

NPV of project = - 500 million + $ 100 million (PV of annuity, 15%, 5 years)

= - 500 million + $ 335 million = - $ 165 million

This project has a negative net present value.

The biggest source of uncertainty on this project is the number of people who will be interested in this product. While the current market tests indicate that you will capture a relatively small number of business travelers as your customers, the test also indicates a possibility that the potential market could get much larger over time. In fact, a simulation of the project's cash flows yields a standard deviation of the 42% in the present value of the cash flows, with an expected value of $ 335 million.

To value the exclusive rights to this project, you first define the inputs to the option pricing model:

Value of the Underlying Asset (S) = PV of Cash Flows from Project if introduced now

= $ 335 million

Strike Price (K) = Initial Investment needed to introduce the product = $ 500 million

Variance in Underlying Asset's Value = $0.42^2 = 0.1764$

Time to expiration = Period of exclusive rights to product = 5 years

Dividend Yield = 1/Life of the patent = 1/5 = 0.20

Assume that the 5-year riskless rate is 5%. The value of the option can be estimated as follows:

Call Value= 335 $\exp^{(-0.2)(5)}$ (0.2250) -500 ($\exp^{(-0.05)(5)}$ (0.0451)= $ 10.18 million

The rights to this product, which has a negative net present value if introduced today, is $ 10.18 million. Note though that the likelihood that this project will become viable before expiration is low (4.5%-22.5%) as measured by N(d1) and N(d2).

Practical Considerations

While it is quite clear that the option to delay is embedded in many projects, there are several problems associated with the use of option pricing models to value these options. First, the *underlying asset in this option, which is the project, is not traded*, making it difficult to estimate its value and variance. The value can be estimated from the expected cash flows and the discount rate for the project, albeit with error. The variance is more difficult to estimate, however, since you are attempting to estimate a variance in project value over time.

Second, the *behavior of prices over time may not conform to the price path assumed by the option pricing models.* In particular, the assumption that value follows a diffusion process, and that the variance in value remains unchanged over time, may be difficult to justify in the context of a project. For instance, a sudden technological change may dramatically change the value of a project, either positively or negatively.

Third, *there may be no specific period for which the firm has rights to the project.* Unlike the example above, in which the firm had exclusive rights to the project for 20 years, often the firm's rights may be less clearly defined, both in terms of exclusivity and time. For instance, a firm may have significant advantages over its competitors, which may, in turn, provide it with the virtually exclusive rights to a project for a period of time. The rights are not legal restrictions, however, and could erode faster than expected. In such cases, the expected life of the project itself is uncertain and only an estimate. In the valuation of the rights to the product, in the previous section, you used a life for the option of 5 years, but competitors could in fact enter sooner than anticipated. Alternatively, the barriers to entry may turn out to be greater than expected, and allow the firm to earn excess returns for longer than 5 years. Ironically, uncertainty about the expected life of the option can increase the variance in present value, and through it, the expected value of the rights to the project.

delay.xls: This spreadsheet allows you to estimate the value of the option to delay a project.

Implications for Project Analysis and Valuation

Several interesting implications emerge from the analysis of the option to delay a project as an option, especially in the context of technology firms. First, a project may have a negative net present value based upon expected cash flows currently, but the rights to that project may still be valuable because of the option characteristics. Second, a project may have a positive net present value but still not be accepted right away because the firm may gain by waiting and accepting the project in a future period, for the same reasons that investors do not always exercise an option just because it is in the money. This is more likely to happen if the firm has the rights to the project for a long time, and the variance in project inflows is high. To illustrate, assume that a firm has the patent rights to produce a new type of disk drive for computer systems and that building a new plant will yield a positive net present value right now. If the technology for manufacturing the disk drive is in flux, however, the firm may delay taking the project in the hopes that the improved technology will increase the expected cash flows and consequently the value of the project. It has to weigh this off against the cost of delaying taking the project, which will be the cash flows that will be forsaken by not taking the project. Third, factors that can make a project less attractive in a static analysis can actually make the rights to the project more valuable. As an example, consider the effect of uncertainty about how long the firm will be able to operate without competition and earn excess returns. In a static analysis, increasing this uncertainty increases the riskiness of the project and may make it less attractive. When the project is viewed as an option, an increase in the uncertainty may actually make the option more valuable, not less.

What are the implications for discounted cash flow valuation? Consider a firm that owns the rights to a specific technological product (hardware or software) which is not viable today and, in fact, is expected to become viable in the future. On a discounted cash flow basis, these rights are worthless and add no value to the firm. Considered as options, however, these rights have value and should be considered when valuing the firm. In the next section, you will look at patents in this light and attempt to assign a value to them.

Valuing a Patent

A product patent provides a firm with the right to develop and market a product. The firm will do so only if the present value of the expected cash flows from the product sales exceed the cost of development, however, as shown in Figure 11.5. If this does not occur, the firm can shelve the patent and not incur any further costs. If I is the present value of the costs of developing the product and V is the present value of the expected cash flows from development, the payoffs from owning a product patent can be written as:

Payoff from owning a product patent = V - I if V> I

= 0 if V I

Thus, a product patent can be viewed as a call option, where the product is the underlying asset.

Figure 11.5: Payoff to Introducing Product

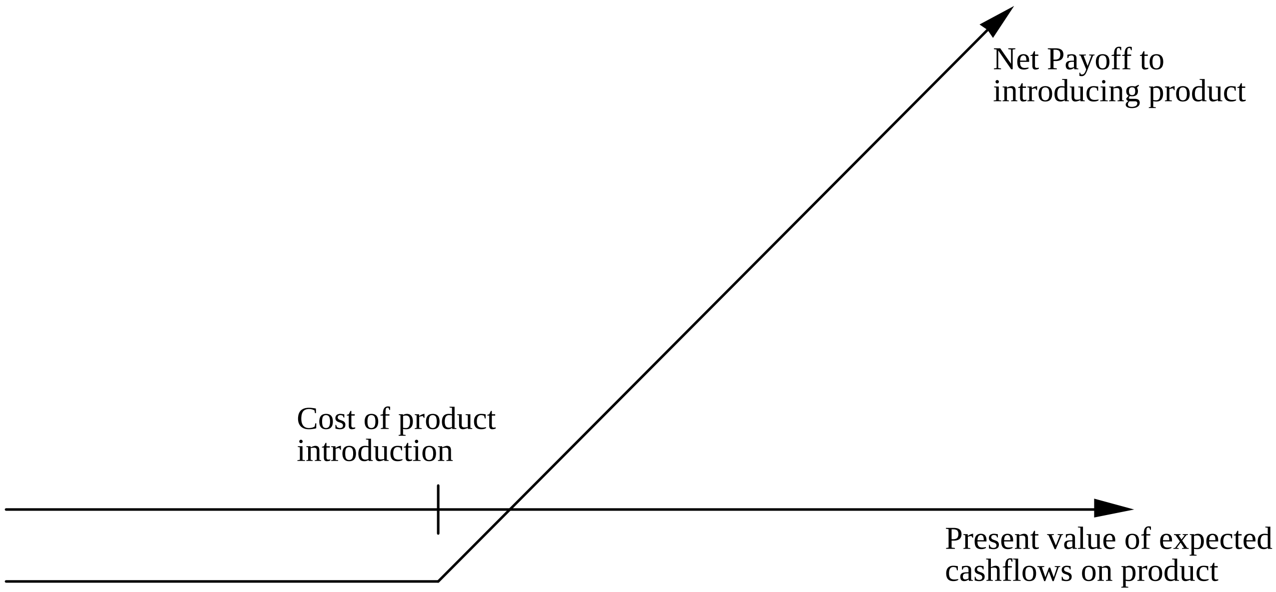

Illustration 11.2: Valuing a Patent: Avonex in 1997

Biogen is a bio-technology firm with a patent on a drug called Avonex, which has passed FDA approval to treat multiple sclerosis. Assume that you are trying to value the patent to Biogen and that you arrive at the following estimates for use in the option pricing model:

- An internal analysis of the drug today, based upon the potential market and the price that the firm can expect to charge, yields a present value of cash flows of $ 3.422 billion, prior to considering the initial development cost.
- The initial cost of developing the drug for commercial use is estimated to be $2.875 billion, if the drug is introduced today.
- The firm has the patent on the drug for the next 17 years, and the current long-term treasury bond rate is 6.7%.
- While it is difficult to do reasonable simulations of the cash flows and present values, the average variance in firm value for publicly traded bio-technology firms is 0.224.
- It is assumed that the potential for excess returns exists only during the patent life, and that competition will wipe out excess returns beyond that period. Thus, any delay in introducing the drug, once it becomes viable, will cost the firm one year of patent-

protected excess returns. (For the initial analysis, the cost of delay will be 1/17, next year it will be 1/16, the year after 1/15 and so on.)

Based on these assumptions, you obtain the following inputs to the option pricing model.

Present Value of Cash Flows from Introducing the Drug Now = S = $ 3.422 billion

Initial Cost of Developing Drug for Commercial Use (today) = K = $ 2.875 billion

Patent Life = t = 17 years

Riskless Rate = r = 6.7% (17-year T.Bond rate)

Variance in Expected Present Values =σ^2 = 0.224

Expected Cost of Delay = y = 1/17 = 5.89%

These yield the following estimates for d and N(d):

d1 = 1.1362   N(d1) = 0.8720

d2 = -0.8512   N(d2) = 0.2076

Plugging back into the option pricing model, you get:

Value of the patent = 3,422 exp(-0.0589)(17) (0.8720) - 2,875 (exp(-0.067)(17) (0.2076)= $ 907 million

To provide a contrast, the net present value of this project is only $ 547 million:

NPV = $3,422 million - $ 2,875 million = $ 547 million

The time premium on this option suggests that the firm will be better off waiting rather than developing the drug immediately, the cost of delay notwithstanding[7]. However, the cost of delay will increase over time, and make exercise (development) more likely.

product.xls: This spreadsheet allows you to estimate the value of a patent, using an option pricing model.

From Patent Value to Firm Value

If you can value the patents owned by a firm as options, how can you incorporate this estimate into firm value? The value of a firm that derives its value primarily from commercial products that emerge from its patents can be written as a function of three variables:

- the cash flows it derives from patents that it has already converted into commercial products,
- the value of the patents that it already possesses that have not been commercially developed and
- the expected value of any patents that the firm can be expected to generate in future periods from new patents that it might obtain as a result of its research.

Value of Firm = Value of commercial products + Value of existing patents + (Value of New patents that will be obtained in the future – Cost of obtaining these patents)

The value of the first component can be estimated using traditional cash flow models. The expected cash flows from existing products can be estimated for their commercial lives and discounted back to the present at the appropriate cost of capital to arrive at the value of these products. The value of the second component can be obtained using the option pricing model described earlier to value each patent. The value of the third component will be based upon your perceptions of a firm's research capabilities. In the special case, where the expected cost of research and development in future periods is equal to the value of the patents that will be generated by this research, the third component will become zero. In the more general case, firms such as Cisco and Pfizer

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[7] This decision could change if the firm believes that one of its competitors is close to obtaining approval for a rival drug to treat MS. In that case, the cost of delay will rise making early exercise (commercially developing the product) more likely.

that have a history of generating value from research will derive positive value from this component as well.

How would the estimate of value obtained using this approach contrast with what you obtain in a traditional discounted cash flow model? In traditional discounted cash flow valuation, the second and the third components of value are captured in the expected growth rate in cash flows. Firms such as Cisco are allowed to grow at much higher rates for longer periods because of the technological edge they possess and their research prowess. In contrast, the approach described in this section looks at each patent separately and allows for the option component of value explicitly.

The biggest limitation of the option-based approach is the information that is needed to put it in practice. To value each patent separately, you need access to proprietary information that is usually available only to managers of the firm. In fact, some of the information, such as the expected variance to use in option pricing may not even be available to insiders and will have to be estimated for each patent separately.

Given these limitations, the real option approach should be used to value small firms with one or two patents and little in terms of established assets. A good example would be Biogen in 1997, which was valued in the last section. For firms such as Cisco and Lucent that have significant assets in place and dozens of patents, discounted cash flow valuation is a more pragmatic choice.

Illustration 11.3: Valuing Biogen as a firm

In illustration 11.2, you valued the patent that Biogen owns on Avonex as a call option and estimated a value of $ 907 million. To value Biogen as a firm, you would need to consider the two other components of value:

- Biogen had two commercial products (a drug to treat Hepatitis B and Intron) at the time of this valuation that it had licensed to other pharmaceutical firms. The license fees on these products were expected to generate $ 50 million in after-tax cash flows each year for the next 12 years. To value these cash flows, which were guaranteed contractually, the riskless rate of 6.7% was used:

Present Value of License Fees = $ 50 million (1 – $(1.067)^{-12}$)/.067 = $ 403.56 million

- Biogen continued to fund research into new products, spending about $ 100 million on R&D in the most recent year. These R&D expenses were expected to grow 20% a year for the next 10 years, and 5% thereafter. While it was difficult to forecast the specific patents that would emerge from this research, it was assumed that every dollar invested in research would create $ 1.25 in value in patents[8] (valued using the option pricing model described above) for the next 10 years, and break even after that (i.e., generate $ 1 in patent value for every $ 1 invested in R&d). There was a significant amount of risk associated with this component and the cost of capital was estimated to be 15%.[9] The value of this component was then estimated as follows:

$$\text{Value of Future Research} = \sum_{t=1}^{t=\infty} \frac{(\text{Value of Patents}_t \text{ - } \text{R\&D}_t)}{(1+r)^t}$$

The table below summarizes the value of patents generated each period and the R&D costs in that period. Note that there is no surplus value created after the tenth year:

Table 11.: Value of New Research*

Year	*Value of Patents generated*	*R&D Cost*	*Excess Value*	*Present Value at 15%*
1	$ 150.00	$ 120.00	$ 30.00	$ 26.09
2	$ 180.00	$ 144.00	$ 36.00	$ 27.22
3	$ 216.00	$ 172.80	$ 43.20	$ 28.40
4	$ 259.20	$ 207.36	$ 51.84	$ 29.64
5	$ 311.04	$ 248.83	$ 62.21	$ 30.93
6	$ 373.25	$ 298.60	$ 74.65	$ 32.27
7	$ 447.90	$ 358.32	$ 89.58	$ 33.68

[8] To be honest, this is not an estimate based upon any significant facts other than Biogen's history of success in coming up with new products.

8	$ 537.48	$ 429.98	$ 107.50	$ 35.14
9	$ 644.97	$ 515.98	$ 128.99	$ 36.67
10	$ 773.97	$ 619.17	$ 154.79	$ 38.26
				$ 318.30

The total value created by new research is $ 318.3 million.

The value of Biogen as a firm is the sum of all three components – the present value of cash flows from existing products, the value of Avonex (as an option) and the value created by new research:

Value = CF: Commerical products + Value: Undeveloped patents + Value: Future R&D

= $ 403.56 million + $ 907 million + $ 318.30 million = $1628.86 million

Since Biogen had no debt outstanding, this value was divided by the number of shares outstanding (35.50 million) to arrive at a value per share:

Value per share = $ 1,628.86 million / 35.5 = $ 45.88

The Option to Expand

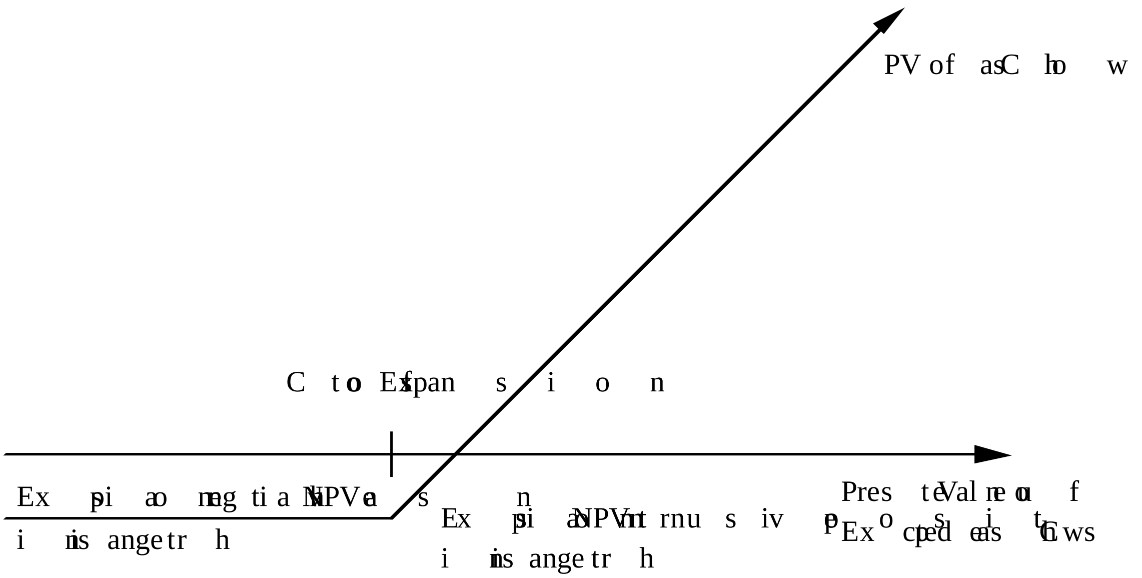
In some cases, firms invest in projects because doing so allows them either to invest in other projects or to enter other markets in the future. In such cases, it can be argued that the initial projects are options allowing the firm to invest in other projects, and the firm should therefore be willing to pay a price for such options. A firm may accept a negative net present value on the initial project because of the possibility of high positive net present values on future projects.

To examine this option using the framework developed earlier, assume that the present value of the expected cash flows, as estimated today, from entering the new market or taking the new project is V, and the total investment needed to enter this market or take this project is X. Further, assume that the firm has a fixed time horizon, at

[9] This discount rate was estimated by looking at the costs of equity of young publicly traded bio-technology firms with little or no revenue from commercial products.

the end of which it has to make the final decision on whether or not to take advantage of this expansion opportunity. Finally, assume that the firm cannot move forward on this opportunity if it does not take the initial project. This scenario implies the option payoffs shown in Figure 7.

Figure 7: The Option to Expand a Project



As you can see, at the expiration of the fixed time horizon, the firm will enter the new market or take the new project if the present value of the expected cash flows at that point in time exceeds the cost of entering the market. If the expansion opportunity never has positive net present value, the firm loses the cost of acquiring the option, which is the negative net present value on the initial investment.

Illustration 11.4: Valuing an Option to Expand: Amazon

Assume that Amazon is considering creating a Spanish version of its web site and expanding into the Mexican market. It is estimated that the cost of creating this site will be $ 500 million, and that the present value of the expected cash flows from the investment will be only $ 300 million. In other words, this venture considered on a stand-alone basis has a negative net present value of $ 200 million.

Assume, however, that by investing in this site and expanding into Mexico today, Amazon acquires the option to expand into the much larger Latin American market anytime over the next 10 years. The cost of expansion will be \$ 1 billion, and it will be undertaken only if the present value of the expected cash flows exceeds this value. At the moment, the present value of the expected cash flows from the expansion is believed to be only \$ 850 million; thus, the expansion would not make economic sense today. Amazon still does not know much about the Latin American market, and there is considerable uncertainty about this estimate of present value. The variance in this estimate, estimated based upon the variance of publicly traded internet ventures in Latin America, is 0.20.

The value of the option to expand can now be estimated, by defining the inputs to the option pricing model as follows:

Value of the Underlying Asset (S) = PV of Cash Flows from Expansion into Latin America, if done now =\$ 850 million

Strike Price (K) =Cost of Expansion into Latin America = \$1,000 million

Variance in Underlying Asset's Value = 0.20

Time to expiration = Period for which expansion option applies = 10 years

Assume that the ten-year riskless rate is 6%. The value of the option can be estimated as follows:

Call Value= 850 (0.8453) -1000 (exp$^{(-0.06)(10)}$ (0.3454)= \$ 528.94 million

This value can be added on to the net present value of the original project under consideration.

NPV of Mexican venture = -\$ 500 million + \$ 300 million = -\$200 million

Value of Option to Expand = \$528.94 million

NPV of investment with expansion option = -\$200 mil+ \$ 528.94 mil = \$ 328.94 mil

Amazon should invest in the Mexican venture even though it has a negative net present value, because the option to expand into Latin America that emerges from it has such high value.

Practical Considerations

The practical considerations associated with estimating the value of the option to expand are similar to those associated with valuing the option to delay. In most cases, firms with options to expand have no specific time horizon by which they have to make an expansion decision, making these open-ended options, or, at best, options with uncertain lives. Even in those cases where a life can be estimated for the option, neither the size nor the potential market for the product may be known, and estimating either can be problematic. To illustrate, consider the Amazon example discussed above. At the end of 10 years, it is assumed that Amazon has to decide whether or not to expand into Latin America. It is entirely possible that this time frame is not specified at the time the initial investment is made. Furthermore, it is assumed that both the cost and the present value of expansion are known initially. In reality, the firm may not have good estimates for either before opening the first store, since it does not have much information on the underlying market.

Implications for Valuation

Is there an option to expand embedded in some firms that can lead to these firms to trade at a premium over their discounted cash flow values? You can, at least in theory, see a rationale for making this argument for a small, high-growth firm in a large and evolving market. The discounted cash flow valuation is based upon expected cash flows and expected growth and these expectations should reflect the probability that the firm could be hugely successful (or a huge failure). What the expectations might fail to consider is that, in the event of success, the firm could invest more, add new products or expand into new markets and augment this success. This is the real option that is creating the additional value.

If you can estimate the value of this option to expand, the value of a firm can be written as the sum of two components – a discounted cash flow value based upon expected cash flows and a value associated with the option to expand:

Value of firm = Discounted Cash flow Value + Option to Expand

The option pricing approach adds rigor to this argument by estimating the value of the option to expand, and it also provides insight into those occasions when it is most valuable. In general, the option to expand is clearly more valuable for more volatile businesses with higher returns on projects (such as biotechnology or computer software), than in stable businesses with lower returns (such as housing, utilities or automobile production).

Again, though, you have to be careful not to double count the value of the option. If you use a higher growth rate than would be justified based upon expectations because of the option to expand, you have already counted the value of the option in the discounted cash flow valuation. Adding an additional component to reflect the value of the option would be double counting.

Illustration 11.5: Considering the value of the option to expand

Consider the discounted cash flow valuation of Rediff.com presented in chapter 7. Rediff.com was valued at \$ 474 million, based upon its expected cash flows in the internet portal business. Assume that in buying Rediff.com, you are in fact buying an option to expand in the online market in India. This market is a small one now, but could potentially be much larger in five or ten years.

In more specific terms, assume that Rediff.com has the option to enter the internet retailing business in India in the future. The cost of entering this business is expected to be \$ 1 billion, and based on current expectations, the present value of the cash flows that would be generated by entering this business today is only $ 500 million. Based upon current expectations of the growth in the Indian e-commerce business, this investment clearly does not make sense.

There is substantial uncertainty about future growth in online retailing in India and the overall performance of the Indian economy. If the economy booms and the online market grows faster than expected over the next 5 years, Rediff.com might be able to create value from entering this market. If you leave the cost of entering the online retailing business at $ 1 billion, the present value of the cash flows would have to increase above this value for Rediff to enter this business and add value. The standard deviation in the present value of the expected cash flows (which is currently $ 500 million) is assumed to be 50%.

The value of the option to expand into internet retailing can now be estimated using an option pricing model, with the following parameters:

S = Present Value of the expected cash flows from entering market today = $ 500 million

K = Cost of entering the market today = $ 1 billion

σ^2 = Variance in the present value of expected cash flows = $0.5^2 = 0.25$

r = 5.8% (This is a five year treasury bond rate: the analysis is being done in U.S dollar terms)

t = 5 years

The value of the option to expand can be estimated as follows:

Option to Expand = 500 (0.5786) -1000 (exp$^{(-0.058)(5)}$ (0.1789)= $ 155.47 million

Why does the option expire in 5 years? If the online retail market in India expands beyond this point in time, it is assumed that there will be other potential entrants into this market and that Rediff.com will have no competitive advantages and hence no good reason for entering this market. If the online retail market in India expands sooner than expected, it is assumed that Rediff.com, as one of the few recognized names in the market, will be able to parlay its brand name and the visitors to its portal to establish competitive advantages.

The value of Rediff.com as a firm can now be estimated as the sum of the discounted cash flow value of $ 474 million and the value of the option to expand into the retail market ($ 155 million). It is true that the discounted cash flow valuation is based upon a high growth rate in revenues, but all of this growth is assumed to occur in the internet portal business and not in online retailing.

expand.xls: This spreadsheet allows you to estimate the value of the option to expand an investment or project.

When are real options valuable? Some Key Tests

While the argument that some or many investments have valuable strategic or expansion options embedded in them has great allure, there is a danger that this argument will be used to justify poor investments. In fact, acquirers have long justified huge premiums on acquisitions on synergistic and strategic grounds. To prevent real options from falling into the same black hole, you need to be more rigorous in your assessment of the value of real options.

Quantitative Estimation

When real options are used to justify a decision, the justification has to be in more than qualitative terms. In other words, managers who argue for taking a project with poor returns or paying a premium on an acquisition on the basis of real options, should be required to value these real options and show, in fact, that the economic benefits exceed the costs. There will be two arguments made against this requirement. The first is that real options cannot be easily valued, since the inputs are difficult to obtain and often noisy. The second is that the inputs to option pricing models can be easily manipulated to back up whatever the conclusion might be. While both arguments have some basis, an estimate with error is better than no estimate at all, and the process of quantitatively trying to estimate the value of a real option is, in fact, the first step to understanding what drives it value.

Key Tests

Not all investments have options embedded in them, and not all options, even if they do exist, have value. To assess whether an investment creates valuable options that need to be analyzed and valued, three key questions need to be answered affirmatively.

1. *Is the first investment a prerequisite for the later investment/expansion? If not, how necessary is the first investment for the later investment/expansion?* Consider the earlier analysis of the value of a patent. A firm cannot generate patents without investing in research or paying another firm for the patents. Clearly, the initial investment here (spending on R&D or acquiring the patent from someone else) is required for the firm to have the second investment. Now, consider the Amazon investment in its Mexican venture and the option to expand into the Latin American market later. The initial store investment allows Amazon to build a Spanish web site and learn more about this market, but it does not give them any exclusive rights to expand into the larger market. Unlike the patent illustration, the initial investment is not a prerequisite for the second, though management might view it as such. The connection gets even weaker when you look at one firm acquiring another to have the option to be able to enter a large market. Acquiring an internet service provider to have a foothold in the internet retailing market would be an example of such a transaction.

- *Does the firm have an exclusive right to the later investment/expansion? If not, does the initial investment provide the firm with significant competitive advantages on subsequent investments?* The value of the option ultimately derives not from the cash flows generated by then second and subsequent investments, but from the excess returns generated by these cash flows. The greater the potential for excess returns on the second investment, the greater the value of the option in the first investment. The potential for excess returns is closely tied to how much of a competitive advantage the first investment provides the firm when it takes subsequent investments. At one extreme, again, consider investing in research and development to acquire a patent. The patent gives the firm that owns it the exclusive rights to produce that product, and if the market potential is large, the right to the excess returns from the project. At the other extreme, the firm might get no competitive advantages on subsequent investments, in which case, it is questionable as to whether there can be any excess returns on these investments. In reality, most investments will fall in the continuum between these two extremes, with greater competitive advantages being associated with higher excess returns and larger option values.

- *How sustainable are the competitive advantages?* In a competitive market place, excess returns attract competitors, and competition drives out excess returns. The more sustainable the competitive advantages possessed by a firm, the greater will be the value of the options embedded in the initial investment. The sustainability of competitive advantages is a function of two forces. The first is the *nature of the competition*; other things remaining equal, competitive advantages fade much more quickly in sectors where there are aggressive competitors. The second is the nature of the competitive advantage. If the resource controlled by the firm is finite and scarce (as is the case with natural resource reserves and vacant land), the competitive advantage is likely to be sustainable for longer periods. Alternatively, if the competitive advantage comes from being the first mover in a market or technological expertise, it will come under assault far sooner. The most direct way of reflecting this in the value of the option is in its life; the life of the option can be set to the period of competitive advantage and only the excess returns earned over this period counts towards the value of the option.

These are tough tests, and you can see that using a real option argument to justify paying large premiums for new technology firms is questionable. You do not need to buy a dot.com firm to partake in the e-commerce market in the future and there is no clear competitive advantage that dot.com firms that exist today are likely to have in this future market. It is true that firms like Yahoo! and Amazon.com will be much better known than other firms in the e-commerce market five or ten years from now (assuming that they survive that long) than their competitors, and that this may give them a brand name component that may allow them to earn excess returns. The question is whether these excess returns can be sustained, given how easy it is for competition to emerge online from both upstart new ventures and established brick-and-mortar businesses.

There might be a stronger rationale for using a real option argument to justify a premium for a small B2B business like Ariba or a small telecomm company like Qualcomm, where a first mover may be able to get its technology to be adopted as the baseline technology for the business. You could argue that investing in these firms today allows you to share in the expansion opportunities that will emerge if this occurs, and that the winning firm will be have a competitive advantage over others in the market.

Conclusion

The value of a firm, or an investment in a traditional discounted cash flow framework, is the present value of the expected cash flows. In the process, however, you might ignore the options to delay or expand that are often embedded in firms.

There are two types of options that can influence the value of a technology firm. The first is the option to delay investing in a technology or project. When a firm has the exclusive rights to a project, even one with a negative net present value, it can hold back on investing until the project becomes an attractive one, and choose not to invest if this never happens. Consequently, the value of the rights to invest in this type of investment will often exceed the discounted cash flow value of the investment, and can be estimated using an option pricing model. In fact, the value of a patent or patents owned by a firm can be estimated using the same approach and added on to the value of the cash flows generated by the more conventional assets of the firm to arrive at firm value.

The second type of option is the option to expand into a new product, market or business as a consequence of an initial investment. In this case, the value of the option to expand can be estimated based upon the expected volatility in the cash flows from expansion and the cost of the expansion. In some cases, the option to expand can have sufficient value to allow firms to invest in project that have negative net present value. In fact, this argument has been used by some analysts as a justification for paying premiums over discounted cash flow values for technology stocks and large premiums on acquisitions.

While real options can exist and have substantial value, you have to be cautious in using them as justification for large premiums over traditional value measures. In particular, a real option will have substantial value only if the first investment is a prerequisite for the second investment and if it creates significant and sustainable competitive advantages on the second investment. While a new technology firm indeed has the option to expand into the e-commerce market, it has neither exclusive rights to do so, nor does it have any significant and sustainable competitive advantages over others who might decide to come into the market later. Consequently, the real option to expand has little or no value.