CHAPTER 2

SHOW ME THE MONEY: THE FUNDAMENTALS OF DISCOUNTED CASH FLOW VALUATION

In the last chapter, you were introduced to the notion that the value of an asset is determined by its expected cash flows in the future. In this chapter, you will begin making this link between value and expected cash flows much more explicit by looking at how to value an asset. You will see that the value of any asset is the present value of the expected cash flow from that asset. This proposition lies at the core of the discounted cash flow approach to valuation. In this chapter, you explore the fundamentals of this approach, starting with an asset with guaranteed cash flows and then moving on to look at assets where there is uncertainty about the future. In the process, you cover the groundwork for how to value a firm, and estimate the inputs that go into the valuation.

Discounted Cash Flow Value

Intuitively, the value of any asset should be a function of three variables - how much it generates in cash flows, when these cash flows are expected to occur, and the uncertainty associated with these cash flows. Discounted cash flow valuation brings all three of these variables together, by computing the value of any asset to be the present value of its expected future cash flows:

$$\text{Value} = \sum_{t=1}^{t=n} \frac{CF_t}{(1+r)^t}$$

where

n = Life of the asset

CF_t = Cash flow in period t

r = Discount rate reflecting the riskiness of the estimated cash flows

The cash flows vary from asset to asset -- dividends for stocks; coupons (interest) and face value for bonds -- and after-tax cash flows for real projects. The discount rate is a function of the riskiness of the estimated cash flows — riskier assets carry higher rates; safer projects carry lower rates.

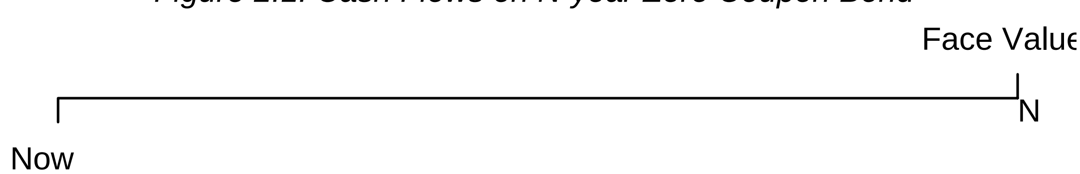
You begin this section by looking at valuing assets that have finite lives (at the end of which they cease to generate cash flows) and you conclude by looking at the more difficult case of assets with infinite lives. You look at firms whose cash flows are known with certainty and conclude by looking at how you can consider uncertainty in valuation.

Valuing an Asset with Guaranteed Cash Flows

The simplest assets to value have cash flows that are guaranteed -- i.e, assets whose promised cash flows are always delivered. Such assets are riskless, and the interest rate earned on them is called a **riskless rate**. The value of such an asset is the present value of the cash flows, discounted back at the riskless rate. Generally speaking, riskless investments are issued by governments that have the power to print money to meet any obligations they otherwise cannot cover. Not all government obligations are not riskless, though, since some governments have defaulted on promised obligations.

The simplest asset to value is a bond that pays no coupon but has a face value that is guaranteed at maturity; this bond is a *default-free zero coupon bond*. Using a time line, you can show the cash flow on this bond as in Figure 2.1.

Figure 2.1: Cash Flows on N-year Zero Coupon Bond



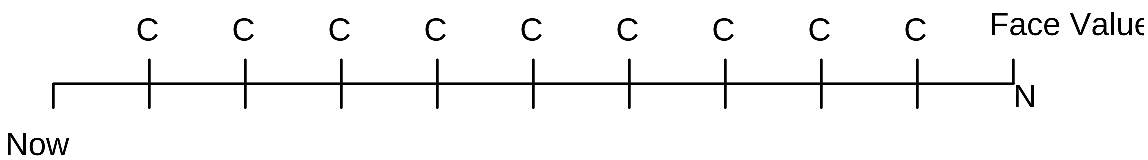
The value of this bond can be written as the present value of a single cash flow discounted back at the riskless rate.

$$\text{Value of Zero Coupon Bond} = \frac{\text{Face Value of Bond}}{(1+r)^N}$$

where r is the riskless rate on the zero-coupon and N is the maturity of the zero-coupon bond. Since the cash flow on this bond is fixed, the value of the bond varies inversely with the riskless rate. As the riskless rate increases, the value of the bond will decrease.

Consider, now, a default-free coupon bond, which has fixed cash flows (coupons) that occur at regular intervals (usually semi annually) and a final cash flow (face value) at maturity. The time line for this bond is shown in Figure 2.2 (with C representing the coupon each period and N being the maturity of the bond).

Figure 2.2: Cash Flows on N-year Coupon Bond



This bond can actually be viewed as a series of zero-coupon bonds, and each can be valued using the riskless rate that corresponds to when the cash flow comes due:

$$\text{Value of Coupon Bond} = \sum_{t=1}^{t=N} \frac{\text{Coupon}}{(1+r_t)^t} + \frac{\text{Face Value of the Bond}}{(1+r_N)^N}$$

where r_t is the interest rate that corresponds to a t-period zero coupon bond and the bond has a life of N periods.

Introducing Uncertainty into Valuation

You have to grapple with two different types of uncertainty in valuation. The first arises in the context of securities like bonds, where there is a promised cash flow to the holder of the bonds in future periods. The risk that these cash flows will not be delivered is called **default risk**; the greater the default risk in a bond, given its cash flows, the less valuable the bond becomes.

The second type of risk is more complicated. When you make equity investments in assets, you are generally not promised a fixed cash flow but are entitled, instead, to whatever cash flows are left over after other claim holders (like debt) are paid; these cash flows are called *residual cash flows*. Here, the uncertainty revolves around what these residual cash flows will be, relative to expectations. In contrast to default risk, where the risk can only result in negative consequences (the cash flows delivered will be less than promised), uncertainty in the context of equity investments can cut both ways. The actual cash flows can be much lower than expected, but they can also be much higher. For the moment, you can label this risk *equity risk* and consider, at least in general terms, how best to deal with it in the context of valuing an equity investment.

Valuing an Asset with Default Risk

You begin this section on how you assess default risk and adjust interest rates for default risk, and then consider how best to value assets with default risk.

Measuring Default Risk and Estimating Default-risk adjusted Rates

When valuing investments where the cash flows are promised, but where there is a risk that they might not be delivered, it is no longer appropriate to use the riskless rate as the discount rate. The appropriate discount rate here includes the riskless rate and an appropriate premium for the default risk called a *default spread*. There are two parts to estimating this spread. The first part is assessing the default risk of an entity. While banks do this routinely when making loans to individuals and businesses, investors buying bonds in firms get some help, at least in the United States, from independent ratings agencies like Standard and Poor's and Moody's. These agencies measure the default risk and give the bonds a rating that measures the default risk. Table 2.1 summarizes the ratings used by Standard and Poor's and Moody's to rate US companies.

Table 2.1: Ratings Description

Standard and Poor's	**Moody's**
AAA The highest debt rating assigned. The borrower's capacity to repay debt is extremely strong.	Aaa Judged to be of the best quality with a small degree of risk.

AA Capacity to repay is strong and differs from the highest quality only by a small amount.	Aa High quality but rated lower than Aaa because margin of protection may not be as large or because there may be other elements of long-term risk.
A Has strong capacity to repay; Borrower is susceptible to adverse effects of changes in circumstances and economic conditions.	A Bonds possess favorable investment attributes but may be susceptible to risk in the future.
BBB Has adequate capacity to repay, but adverse economic conditions or circumstances are more likely to lead to risk.	Baa Neither highly protected nor poorly secured; adequate payment capacity.
BB,B, Regarded as predominantly CCC, speculative, BB being the least CC speculative andd CC the most.	Ba Judged to have some speculative risk. B Generally lacking characteristics of a desirable investment; probability of payment small.
D In default or with payments in arrears.	Caa Poor standing and perhaps in default. Ca Very speculative; often in default. C Highly speculative; in default.

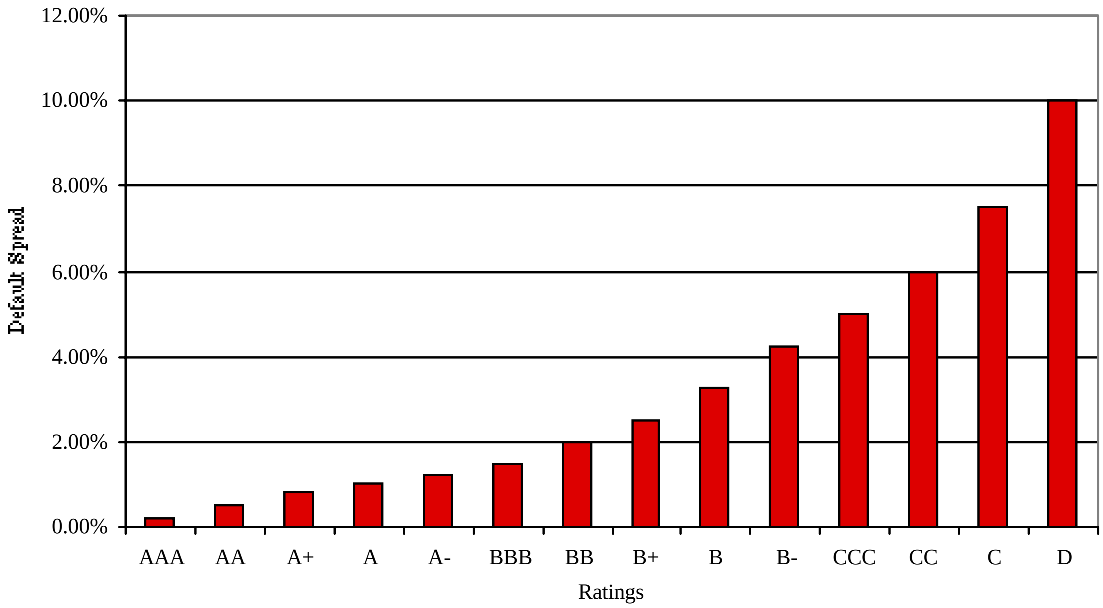
Source: Standard and Poor’s, Moody’s

While ratings agencies do make mistakes, the rating system saves investors a significant amount of cost that would otherwise be expended doing research on the default risk of issuing firms.

The second part of the risk-adjusted discount rate assessment is coming up with the default spread. The demand and supply for bonds within each ratings class determines the appropriate interest rate for that rating. Low rated firms have more default risk and generally have to pay much higher interest rates on their bonds than highly rated firms. The spread itself changes over time, tending to increase for all ratings classes in economic recessions and to narrow for all ratings classes in economic recoveries. Figure 2.3

summarizes default spreads for bonds in S&P's different rating classes as of December 31, 1998:

Figure 2.3: Default Spreads and Ratings
The default spread is the difference between the interest rate on a corporate bond the interest rate on a treasury bond of the same maturity.



Source: www.bondsonline.com

These default spreads, when added to the riskless rate, yield the interest rates for bonds with the specified ratings. For instance, a D rated bond has an interest rate about 10% higher than the riskless rate.

Valuing an Asset with Default Risk

The most common example of an asset with just default risk is a corporate bond, since even the largest, safest companies still have some risk of default. When valuing a corporate bond, you generally make two modifications to the bond valuation approach you developed earlier for a default-free bond. First, you discount the coupons on the corporate bond, even though these no longer represent expected cash flows, but are instead promised cash flows[1]. Second, the discount rate used for a bond with default risk will be higher than that used for default-free bond. Furthermore, as the default risk increases, so will the discount rate used:

$$\text{Value of Corporate Coupon Bond} = \sum_{t=1}^{t=N} \frac{\text{Coupon}}{(1+k_d)^t} + \frac{\text{Face Value of the Bond}}{(1+k_d)^N}$$

where k_d is the market interest rate given the default risk.

Valuing an Asset with Equity Risk

Having valued assets with guaranteed cash flows and those with only default risk, let you now consider the valuation of assets with equity risk. You begin with an introduction to the way to estimate cash flows and to consider equity risk in investments with equity risk, and then you look at how best to value these assets.

Measuring Cash Flows for an Asset with Equity Risk

Unlike the bonds that you valued so far in this chapter, the cash flows on assets with equity risk are not promised cash flows. Instead, the valuation is based upon the *expected cash flows* on these assets over their lives. You need to consider two basic questions: the first relates to how you measure these cash flows, and the second to how to come up with expectations for these cash flows.

To estimate cash flows on an asset with equity risk, first consider the perspective of the the equity investor in the asset. Assume that the equity investor borrowed some of the funds needed to buy the asset. The cash flows to the equity investor will therefore be the cash flows generated by the asset after all expenses and taxes, and also after payments due on the debt. This cash flow, which is after debt payments, operating expenses and taxes, is called the **cash flow to equity investors**. There is also a broader definition of

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[1] When you buy a corporate bond with a coupon rate of 8%, you are promised a payment of 8% of the face value of the bond each period, but the payment may be lower or non-existent, if the company defaults.

cash flow that you can use, where you look at not just the equity investor in the asset, but at the total cash flows generated by the asset for both the equity investor and the lender. This cash flow, which is before debt payments but after operating expenses and taxes, is called the **cash flow to the firm** (where the firm is considered to include both debt and equity investors).

Note that, since this is a risky asset, the cash flows are likely to vary across a broad range of outcomes, some good and some not so positive. To estimate the expected cash flow, you need to consider all possible outcomes in each period, weight them by their relative probabilities[2] and arrive at an expected cash flow for that period.

Measuring Equity Risk and Estimate Risk-Adjusted Discount Rates

When you analyzed bonds with default risk, you noted that the interest rate has to be adjusted to reflect the default risk. This default-risk adjusted interest rate can be considered the **cost of debt** to the investor or business borrowing the money. When analyzing investments with equity risk, you have to make an adjustment to the riskless rate to arrive at a discount rate, but the adjustment must reflect the equity risk rather than the default risk. Furthermore, since there is no longer a promised interest payment, you can think of this rate as a risk-adjusted discount rate rather than an interest rate. This adjusted discount rate is the **cost of equity**.

You saw earlier that a firm can be viewed as a collection of assets, financed partly with debt and partly with equity. The composite cost of financing, which comes from both debt and equity, is a weighted average of the costs of debt and equity, with the weights depending upon how much of each financing is used. This cost is labeled the **cost of capital**.

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[2] Note that in many cases, though we might not explicitly state probabilities and outcomes, you are implicitly doing so, when you use expected cash flows.

If the cash flows that you are discounting are cash flows to equity investors, as defined in the previous section, the appropriate discount rate is the cost of equity. If the cash flows are prior to debt payments and therefore to the firm, the appropriate discount rate is the cost of capital.

Valuing an Asset with Equity Risk and Finite Life

Most assets firms acquire have finite lives. At the end of that life, the assets are assumed to lose their operating capacity, though they might still preserve some value. To illustrate, assume that you buy an apartment building and plan to rent the apartments out to earn income. The building will have a finite life, say 30 to 40 years, at the end of which it will have to be torn down and a new building constructed, but the land will continue to have value even if this occurs.

This building can be valued using the cash flows that it will generate, prior to any debt payments, and discounting them at the composite cost of the financing used to buy the building, i.e., the cost of capital. At the end of the expected life of the building, you estimate what the building (and the land it sits on) will be worth and discount this value back to the present, as well. In summary, the value of a finite life asset can be written as:

$$\text{Value of Finite - Life Asset} = \sum_{t=1}^{t=N} \frac{\text{E(Cash flow on Asset}_t)}{(1+k_c)^t} + \frac{\text{Value of Asset at End of Life}}{(1+k_c)^N}$$

where k_c is the cost of capital.

This entire analysis can also be done from your perspective as the sole equity investor in this building. In this case, the cash flow is defined more narrowly as cash flows after debt payments, and the appropriate discount rate becomes the cost of equity. At the end of the building’s life, you look at how much it will be worth but consider only the cash that will be left over after any remaining debt is paid off. Thus, the value of the equity investment in an asset with a fixed life of N years, say an office building, can be written as follows:

$$\text{Value of Equity in Finite - Life Asset} = \sum_{t=1}^{t=N} \frac{\text{E(Cash Flow to Equity}_t)}{(1+\text{k}_e)^t} + \frac{\text{Value of Equity in Asset at End of Life}}{(1+\text{k}_e)^N}$$

where k_e is the rate of return that the equity investor in this asset would demand given the riskiness of the cash flows and the value of equity at the end of the asset's life is the value of the asset net of the debt outstanding on it.

Can you extend the life of the building by reinvesting more in maintaining it? Possibly. If you choose this course of action, however, the life of the building will be longer, but the cash flows to equity and to the firm each period have to be reduced[3] by the amount of the reinvestment needed for maintenance.

Valuing an Asset with an Infinite Life

When you value businesses and firms, as opposed to individual assets, you are often looking at entities that have no finite lives. If they reinvest sufficient amounts in new assets each period, firms could keep generating cash flows forever. In this section, you value assets that have infinite lives and uncertain cash flows.

Equity and Firm Valuation

A firm, as defined here, includes both investments already made -- call these **assets-in-place** -- and investments yet to be made -- these **growth assets**. In addition, a firm can either borrow the funds it needs to make these investments, in which case it is using debt, or raise it from its owners, in the form of equity. Figure 2.4 summarizes this description of a firm in the form of a financial balance sheet:

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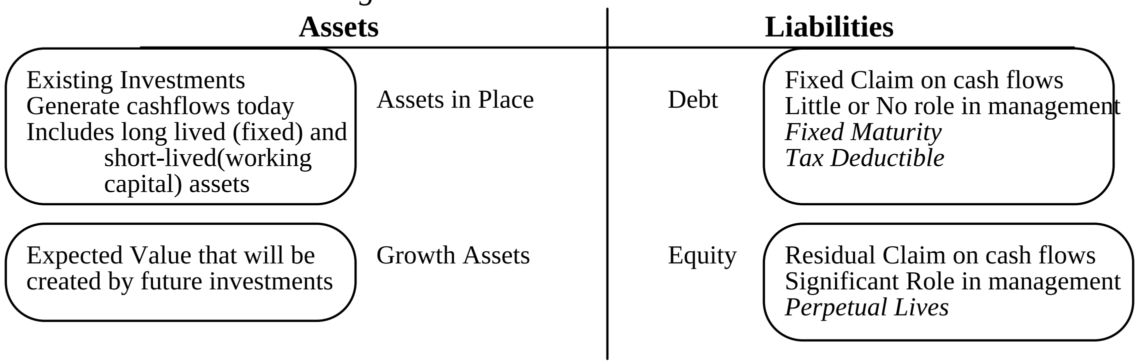
[3] By maintaining the building better, you might also be able to charge higher rents, which may provide an offsetting increase in the cash flows.

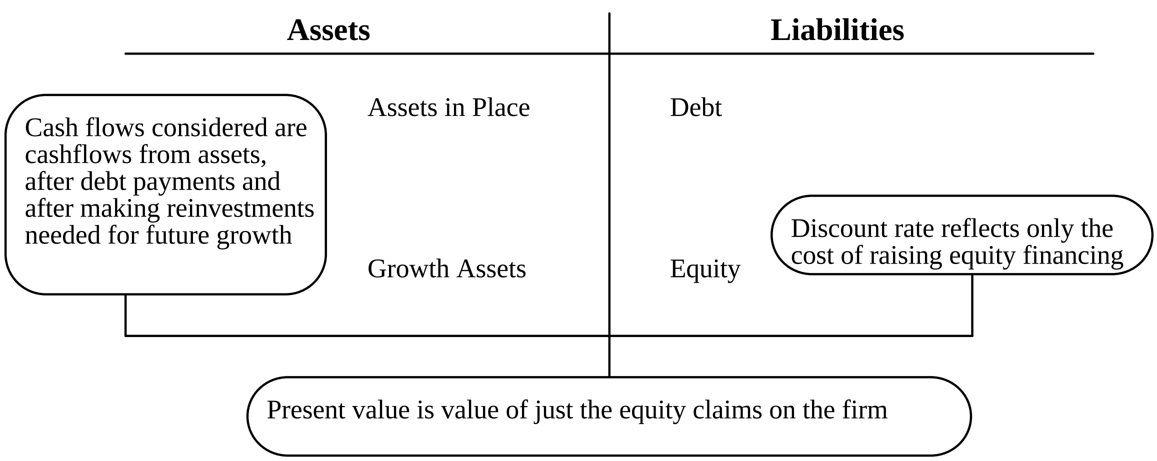
Figure 2.4: A Financial Balance Sheet



Note that while this summary does have some similarities with the accounting balance sheet, there are key differences. The most important one is that here you explicitly consider growth assets when you look at what a firm owns.

In the section on valuing assets with equity risk, you encountered the notions of cash flows to equity and cash flows to the firm. You saw that cash flows to equity are cash flows after debt payments, all expenses and reinvestment needs have been met. In the context of a business, you can use the same definition to measure the cash flows to its equity investors. These cash flows, when discounted back at the cost of equity for the business, yields the value of the equity in the business. This is illustrated in Figure 2.5:

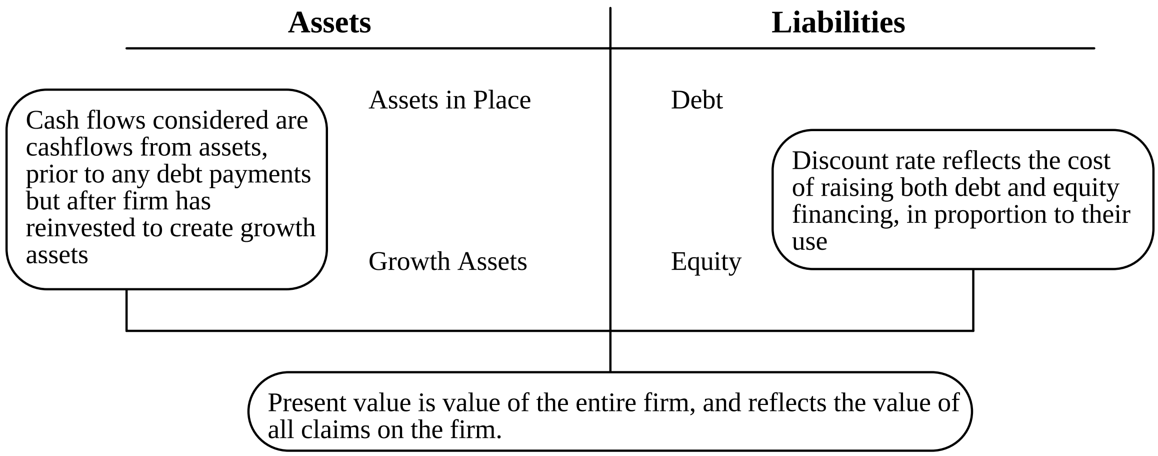
Figure 2.5: Equity Valuation



Note that the definition of both cash flows and discount rates is consistent – they are both defined in terms of the equity investor in the business.

There is an alternative approach in which, instead of valuing the equity stake in the asset or business, you can look at the value of the entire business. To do this, you look at the collective cash flows not just to equity investors but also to lenders (or bondholders in the firm). The appropriate discount rate is the cost of capital, since it reflects both the cost of equity and the cost of debt. The process is illustrated in Figure 2.6.

Figure 2.6: Firm Valuation



Note again that you are defining both cash flows and discount rates consistently, to reflect the fact that you are valuing not just the equity portion of the investment but the investment itself.

Dividends and Equity Valuation

When valuing equity investments in publicly traded companies, you could argue that the only cash flows investors in these investments get from the firm are dividends. Therefore, the value of the equity in these investments can be computed as the present value of expected dividend payments on the equity.

$$\text{Value of Equity (Only Dividends)} = \sum_{t=1}^{t=\infty} \frac{E(\text{Dividend}_t)}{(1+k_e)^t}$$

The mechanics are similar to those involved in pricing a bond, with dividend payments replacing coupon payments, and the cost of equity replacing the interest rate on the bond. The fact that equity in a publicly traded firm has an infinite life, however, indicates that you cannot arrive at closure on the valuation without making additional assumptions.

a. Stable (and Constant) Growth Scenario

One way in which you might be able to estimate the value of the equity in a firm is by assuming that the dividends, starting today, will grow at a constant rate forever. If you do that, you can estimate the value of the equity using the present value formula for a perpetually growing cash flow. In fact, the value of equity will be

$$\text{Value of Equity (Dividends growing at a constant rate forever)} = \frac{\text{E(Dividend next period)}}{(k_e - g_n)}$$

This model, which is called the **Gordon growth model**, is simple but limited, since it can value only companies that pay dividends, and only if these dividends are expected to grow at a constant rate forever. The reason this is a restrictive assumption is that no asset or firm's cash flows can grow forever at a rate higher than the growth rate of the economy. If it did, the firm would become the economy. Therefore, the constant growth rate is constrained to be less than or equal to the economy's growth rate. For valuations of firms in US dollars, this puts an upper limit on the growth rate of approximately 5-6%[4]. This constraint will also ensure that the growth rate used in the model will be less than the discount rate.

b. High Growth Scenario

What happens if you have to value a stock whose dividends are growing at 15% a year? The solution is simple. You value the stock in two parts. In the first part, you estimate the expected dividends each period for as long as the growth rate of this firm's dividends remains higher than the growth rate of the economy, and sum up the present value of the dividends. In the second part, you assume that the growth rate in dividends will drop to a stable or constant rate forever sometime in the future. Once you make this

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[4] The nominal growth rate of the US economy through the nineties has been about 5%. The growth rate of the global economy, in nominal US dollar terms, has been about 6% over that period.

assumption, you can apply the Gordon growth model to estimate the present value of all dividends in stable growth. This present value is called the **terminal price** and represents the expected value of the stock in the future, when the firm becomes a stable growth firm. The present value of this terminal price is added to the present value of the dividends to obtain the value of the stock today.

$$\text{Value of Equity with high-growth dividends} = \sum_{t=1}^{t=N} \frac{\text{E(Dividends}_t)}{(1+k_e)^t} + \frac{\text{Terminal Price}_N}{(1+k_e)^N}$$

where N is the number of years of high growth and the terminal price is based upon the assumption of stable growth beyond year N.

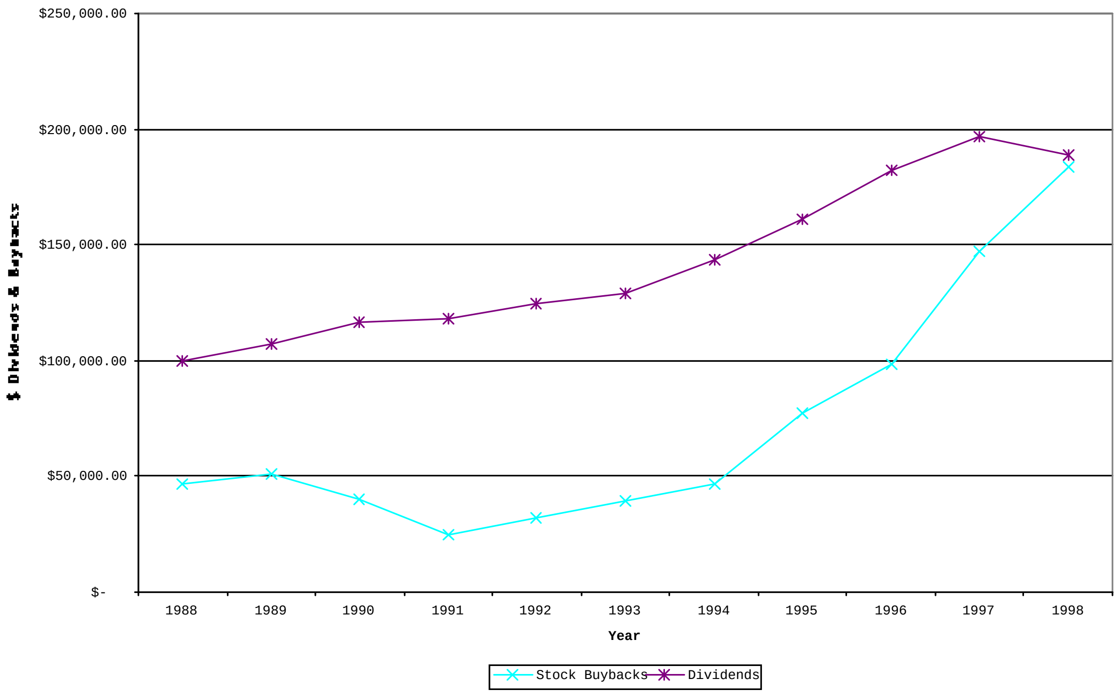
$$\text{Terminal Price} = \frac{\text{E(Dividend}_{N+1})}{(k_e - g_n)}$$

Limitations of Dividend Discount Models

The dividend discount model was the first of the discounted cash flow models used in practice. While it does bring home key fundamental concepts about valuation, it does have serious limitations, especially in the context of technology firms. The biggest problem, contrary to popular opinion, is not that these firms do not pay dividends. Given the high growth and reinvestment needs exhibited by these firms, this may be, in fact, what you would expect them to do. It is that they do not pay dividends or do not pay as much as they can in dividends, even when they have the cash flows to do so.

Dividends are discretionary, and are determined by managers. If managers have excess cash, they can choose to pay a dividend but they can also choose to hold the cash or buy back stock. In the United States, the option of buying back stock has become an increasingly attractive one to many firms. Figure 2.7 summarizes dividends paid and equity repurchases at U.S. corporations between 1989 and 1998.

Figure 2.7: Stock Buybacks and Dividends: Aggregate for US Firms -



Source: Compustat database (1998)

It is worth noting that while aggregate dividends at all US firms have grown at a rate of about 7.29% a year over this 10-year period, stock buybacks have grown 16.53% a year. In another interesting shift, the proportion of cash returned to stockholders in the form of stock buybacks has climbed from 32% in 1989 to almost 50% in 1998.

The shift has been even more dramatic at technology firms, as is evidenced by two facts about them:

1. Of the 1340 firms classified as technology firms by Morningstar in 1999, only 74 paid dividends. Of these, only 15 had dividend yields that exceeded 1%. Collectively, these firms paid out less than $2 billion in dividends in 1999.
2. In 1999, technology firms collectively bought back $ 21.2 billion, more than ten times what they paid in dividends.

The net effect of using dividend discount models to value technology firms is a significant understatement in their value.

Illustration 2.1: Valuing a technology stock with the dividend discount model: Hewlett Packard

Hewlett Packard (HP) reported earnings per share of $ 3.00 in 1999 and paid out dividends of $0.60. Assume that HP's earnings will grow 16% a year for the next 10 years, and that the dividend payout ratio (dividends as a percent of earnings) will remain at 20% for that period. Also assume that HP's cost of equity is 10.40% for that period. The following table summarizes the expected dividends per share for the next 10 years, and the present value of these dividends:

Table 2.: Expected Dividends per share*

Year	*EPS*	*DPS*	*PV of DPS at 10.40%*
1	$3.48	$0.70	$0.63
2	$4.04	$0.81	$0.66
3	$4.68	$0.94	$0.70
4	$5.43	$1.09	$0.73
5	$6.30	$1.26	$0.77
6	$7.31	$1.46	$0.81
7	$8.48	$1.70	$0.85
8	$9.84	$1.97	$0.89
9	$11.41	$2.28	$0.94
10	$13.23	$2.65	$0.98
PV of Dividends =			$7.96

After year 10, you expect Hewlett Packard's earnings to grow 6% a year, and its dividend payout ratio to increase to 60%. Assuming that the cost of equity remains unchanged at 10.4%, you can estimate the price at the end of year 10 (terminal price):

Expected Earnings per share in year 11 = EPS_{10} (1 + growth rate in year 11)

= $13.23 (1.06) = $14.03

Expected Dividends per share in year 11 = EPS_{11} (Payout $Ratio_{11}$)

= $14.03 (0.60) = $ 8.42

Terminal Price = DPS_{11} / (Cost of $equity_{11}$ – Growth $rate_{11}$)

= $8.42/(.104- .06) = $191.30

The present value of this terminal price should be added on to the present value of the dividends during the first 10 years to yield a dividend discount model value for HP:

Value per share of HP = $7.96 + $191.30/$1.104^{10}$ = $79.08

Since HP was trading at $131 per share at the time of this valuation, the dividend discount model at least would suggest that HP is over valued.

ddmst.xls: This spreadsheet allows you to value a stable growth dividend paying stock, using a dividend discount model.

ddm2st.xls: This spreadsheet allows you to value a dividend paying stock, using a 2-stage dividend discount model.

A Broader Measure of Cash Flows to Equity

To counter the problem of firms not paying out what they can afford to in dividends, you might consider a broader definition of cash flow which you can call **free cash flow to equity**, defined as the cash left over after operating expenses, interest expenses, net debt payments and reinvestment needs. **Net debt payments** refer to the difference between new debt issued and repayments of old debt. If the new debt issued exceeds debt repayments, the free cash flow to equity will be higher. In **reinvestment needs**, you include any investments that the firm has to make in long-term assets (such as land, buildings, equipment and research, for a technology firm) and short term assets (such as inventory and accounts receivable) to generate future growth.

Free Cash Flow to Equity (FCFE) = Net Income – Reinvestment Needs – (Debt Repaid – New Debt Issued)

Think of this as potential dividends, or what the company could have paid out in dividend. To illustrate, in 1998, the Motorola's free cash flow to equity using this definition was:

$FCFE_{Boeing}$ = Net Income – Reinvestment Needs – (Debt Repaid – New Debt Issued)

= $ 1,614 million - $1,876 million – (8 – 246 million) = - $ 24 million

Clearly, Motorola did not generate positive cash flows after reinvesment needs and net debt payments. Surprisingly, the firm did pay a dividend, albeit a small one. Any dividends paid by the Motorola during 1998 had to be financed with existing cash balances, since the free cash flow to equity is negative.

Valuation of Free Cash Flows to Equity

Once the free cash flows to equity have been estimated, the process of estimating value parallels the dividend discount model. To value equity in a firm where the free cash flows to equity are growing at a constant rate forever, you use the present value equation to estimate the value of cash flows in perpetual growth:

$$\text{Value of Equity in Infinite - Life Asset} = \frac{E(FCFE_t)}{(k_e - g_n)}$$

All the constraints relating to the magnitude of the constant growth rate used that you discussed in the context of the dividend discount model, continue to apply here.

In the more general case, where free cash flows to equity are growing at a rate higher than the growth rate of the economy, the value of the equity can be estimated again in two parts. The first part is the present value of the free cash flows to equity during the high growth phase, and the second part is the present value of the terminal value of equity, estimated based on the assumption that the firm will reach stable growth sometime in the future.

$$\text{Value of Equity with high growth FCFE} = \sum_{t=1}^{t=N} \frac{E(FCFE_t)}{(1+k_e)^t} + \frac{\text{Terminal Value of Equity}_N}{(1+k_e)^N}$$

With the FCFE approach, you have the flexibility you need to value equity in any type of business or publicly traded company.

Illustration 2.2: Valuing Equity using FCFE – Hewlett Packard

Consider the case of Hewlett Packard. The last illustration valued HP using a dividend discount model, but added the caveat that HP might not be paying out what it can afford to in dividends. HP had net income in 1999 was $3491 million, and reinvested about 50% of this net income. Assume that HP's reinvestment needs will continue to be 50% of earnings for the next 10 years (while it generates 16% growth in earnings each year) and that net debt issued will be 10% of the reinvestment. Table 2.2 summarizes the free cash flows to equity at the firm for this period and computes the present value of these cash flows at the Home Depot's cost of equity of 9.78%.

Table 2.2: Value of FCFE

Year	*Net Income*	*Reinvestment*	*Net Debt Paid (Issued)*	*FCFE*	*PV of FCFE*
1	$4,050	$2,025	($202)	$2,227	$2,017
2	$4,697	$2,349	($235)	$2,584	$2,120
3	$5,449	$2,725	($272)	$2,997	$2,227
4	$6,321	$3,160	($316)	$3,477	$2,340
5	$7,332	$3,666	($367)	$4,033	$2,459
6	$8,505	$4,253	($425)	$4,678	$2,584
7	$9,866	$4,933	($493)	$5,426	$2,715
8	$11,445	$5,722	($572)	$6,295	$2,852
9	$13,276	$6,638	($664)	$7,302	$2,997
10	$15,400	$7,700	($770)	$8,470	$3,149
	PV of FCFE during high growth phase				$25,461

Note that since more debt is issued than paid, net debt issued increases the free cash flows to equity each year. To estimate the terminal price, assume that net income will grow 6% a year forever after year 10. Since lower growth require less reinvestment, assume that the reinvestment rate after year 10 will be 40% of net income; net debt issued will remain 10% of reinvestment.

$FCFE_{11}$ = Net $Income_{11}$ – $Reinvestment_{11}$ – Net Debt Paid $(Issued)_{11}$

= $ 15,400 (1.06) – $ 15,400 (1.06) (0.40) – (-653) = $ 9,142 million

Terminal $Price_{10}$ = $FCFE_{11}/(k_e – g)$ = $ 9,142 / (.104 - .06) = $ 207,764 million

The value of equity today can be computed as the sum of the present values of the free cash flows to equity during the next 10 years and the present value of the terminal value at the end of the 10th year.

Value of Equity today = \$ 25,461 million + \$ 207,764/$(1.104)^{10}$ = \$ 102,708 million

On a free cash flow to equity basis, you would value the equity at the Hewlett Packard at \$ 102.708 billion. Dividing by the number of shares outstanding (997.231 million) yields a value per share:

Value per share of HP = \$ 102,708/997.231 = \$ 102.99

The value per share is higher than the dividend discount model value of \$79.08 but it is still lower than the market price of \$131 per share.

From Valuing Equity to Valuing the Firm

A firm is more than just its equity investors. It has other claim holders, including bondholders and banks. When you value the firm, therefore, you consider cash flows to all of these claim holders. You can define the **free cash flow to the firm** as being the cash flow left over after operating expenses, taxes and reinvestment needs, but before any debt payments (interest or principal payments).

Free Cash Flow to Firm (FCFF) = After-tax Operating Income – Reinvestment Needs

The two differences between FCFE and FCFF become clearer when you compare their definitions. The free cash flow to equity begins with net income, which is after interest expenses and taxes, whereas the free cash flow to the firm begins with after-tax operating income, which is before interest expenses. Another difference is that the FCFE is after net debt payments, whereas the FCFF is before net debt.

What exactly does the free cash flow to the firm measure? On the one hand, it measures the cash flows generated by the assets before any financing costs are considered and thus is a measure of operating cash flow. On the other, the free cash flow to the firm is the cash flow used to service all claim holders' needs for cash – interest and principal to debt holders and dividends and stock buybacks to equity investors.

The General Valuation Model

Once the free cash flows to the firm have been estimated, the process of computing value follows a familiar path. If valuing a firm or business with free cash flows growing at a constant rate forever, you can use the perpetual growth equation:

$$\text{Value of Firm with FCFF growing at constant rate} = \frac{E(FCFF_1)}{(k_c - g_n)}$$

There are two key distinctions between this model and the constant-growth FCFE model used earlier. The first is that you consider cash flows before debt payments in this model, whereas you used cash flows after debt payments when valuing equity. The second is that you then discount these cash flows back at a composite cost of financing, i.e., the cost of capital to arrive at the value of the firm, while you used the cost of equity as the discount rate when valuing equiy.

To value firms where free cash flows to the firm are growing at a rate higher than that of the economy, you can modify this equation to consider the present value of the cash flows until the firm is in stable growth. To this present value, add the present value of the terminal value, which captures all cash flows in stable growth.

$$\text{Value of high - growth business} = \sum_{t=1}^{t=N} \frac{E(FCFF_t)}{(1+k_c)^t} + \frac{\text{Terminal Value of Business}_N}{(1+k_c)^N}$$

Illustration 2.3: Valuing an Asset with Stable Growth

Assume now that Hewlett Packard is interested in selling its printer division. Assume that the division reported cash flows before debt payments but after reinvestment needs of $ 400 million in 1999, and the cash flows are expected to grow 5% a year in the long term. The cost of capital for the division is 9%. The division can be valued as follows:

Value of Division = $ 400 (1.05) / (.09 - .05) = $ 10,500 million

Illustration 2.4: Valuing a Firm in High Growth:

Diebold is a technology firm that provides systems, software and services to the financial services, education and health care businesses. In 1999, the firm reported a free cash flow to the firm of $ 100 million. Assume that these free cash flows will grow at 15% a year for the next 5 years and at 5% thereafter. Diebold has a cost of capital of 11%. The value of Deibold as a firm can then be estimated in Table 2.3:

Table 2.3: Value of Diebold

Year	*Expected FCFF*	*Terminal Value*	*PV of Cash flow*
1	$ 115.00		$ 103.60
2	$ 132.25		$ 107.34
3	$ 152.09		$ 111.21
4	$ 174.90		$ 115.21
5	$ 201.14	$ 3,519.88	$ 2,208.24
		PV of Cashflows =	$ 2,645.60

The terminal value is estimated using the free cash flow to the firm in year 6, the cost of capital of 11% and the expected constant growth rate of 5% as follows:

Terminal Value = $ 201.14 (1.05)/(.11-.05) = $ 3,519.88 million

It is then discounted back to the present to get the value of the firm today shown above as $ 2,645.60 million.

Note that this is not the value of the equity of the firm. To get to the value of the equity, you need to subtract out from $2,646 million the value of all non-equity claims in the firm. Diebold had debt outstanding of $138.25 million at the end of 1999. Subtracting this from the value of the firm would yield the value of equity at the firm:

Value of equity at Diebold = $2,646 - $ 138 = $2,508 million

Dividing by the number of shares outstanding gives you the value per share:

Value per share at Diebold = $2,508 million/ 71.172 million = $37.17

The stock was trading at $29.625 at the time of this analysis (July 2000).

What is different about technology stocks?

The value of any asset is a function of the cash flows generated by that asset, the life of the asset, the expected growth in the cash flows and the riskiness associated with the cash flows. If the value of a technology firm is also determined by these same variables, what is different about them? From a conceptual standpoint, you can argue that there is very little that is different. From an estimation standpoint, however, there are a number of problems that are, if not specific to technology firms, more serious when valuing these firms.

These estimation issues can be understood in the context of the four inputs that go into any firm valuation - cash flows, growth, discount rates and asset life - in this section. You build on each of these inputs separately in the next four chapters.

I. Estimate Cash Flow to the Firm

The cash flow to the firm that you would like to estimate should be both after taxes and after all reinvestment needs have been met. Since a firm includes both debt and equity investors, the cash flow to the firm should be before interest and principal payments on debt.

The cash flow to the firm can be measured in two ways. One is to add up the cash flows to all of the different claim holders in the firm. Thus, the cash flows to equity investors (which take the form of dividends or stock buybacks) are added to the cash flows to debt holders (interest payments, net of the tax benefit,and net debt payments) to arrive at the cash flow. The other approach to estimating cash flow to the firm, which should yield equivalent results, is to estimate the cash flows to the firm prior to debt payments but after reinvestment needs have been met:

EBIT (1 - tax rate)

– (Capital Expenditures - Depreciation)

– Change in Non-cash Working Capital

= Free Cash Flow to the Firm

The difference between capital expenditures and depreciation (net capital expenditures) and the increase in non-cash working capital represent the reinvestments made by the firm to generate future or contemporaneous growth.

Another way of presenting the same equation is to cumulate the net capital expenditures and working capital change into one number, and state it as a percentage of the after-tax operating income. This ratio of reinvestment to after-tax operating income is called the reinvestment rate, and the free cash flow to the firm can be written as:

Free Cash Flow to the Firm = EBIT (1-t) (1 – Reinvestment Rate)

Note that the reinvestment rate can exceed 100%[5], if the firm has substantial reinvestment needs. If that occurs, the free cash flow to a firm will be negative even though after-tax operating income is positive.

What is unique about technology firms? First, some older technology firms and many newer technology firms have negative operating income, leading to negative free cash flows. Even among technology firms that have positive operating income, you sometimes see negative free cash flows, largely because of the prevalence of large reinvestment needs. While the presence of negative free cash flows, by itself, is not a problem for firm valuation, more of the value of these firms has to come from future cash flows and especially the terminal value. Second, there are significant problems associated with how operating income and reinvestment is measured by accountants at technology firms. The biggest capital expenditure for most technology firms is in research and development and this expense is treated as an operating expense for accounting purposes. This leads to a mis-measurement of both the operating income of the firm and its capital expenditures.

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[5] In practical terms, this firm will have to raise external financing, either from debt or equity or both, to cover the excess reinvestment.

II. Expected Growth

In valuation, it is the expected future cash flows that determine value. While the definition of the cash flow, described in the last section, still holds, it is the forecasts of earnings, net capital expenditures and working capital that will yield these cash flows. One of the most significant inputs into any valuation is the expected growth rate in operating income. While you could use past growth or consider analyst forecasts to make this estimate, the fundamentals that drive growth are simple. The expected growth in operating income is a product of a firm's reinvestment rate, i.e., the proportion of the after-tax operating income that is invested in net capital expenditures and changes in non-cash working capital, and the quality of these reinvestments, measured as the return on the capital invested. For a firm that has a steady and sustainable return on capital on its investments, the expected growth rate in operating income can be written as:

$$\text{Expected Growth}_{\text{EBIT}} = \text{Reinvestment Rate} * \text{Return on Capital}$$

where,

$$\text{Reinvestment Rate} = \frac{\text{Capital Expenditure} - \text{Depreciation} + \Delta \text{ Non-cash WC}}{\text{EBIT}(1 - \text{tax rate})}$$

$$\text{Return on Capital} = \text{EBIT}(1\text{-t}) / \text{Capital Invested}$$

Both measures should be forward looking and the return on capital should represent the expected return on capital on future investments. Having said that, it is often based upon the firm's return on capital on assets in place, where the book value of capital is assumed to measure the capital invested in these assets. Implicitly, you can assume then that the current accounting return on capital is a good measure of the true returns earned on assets in place, and that this return is a good proxy for returns that will be made on future investments.

There are again reasons why this computation may not work for technology firms. The first reason is related to the treatment of research and development expenses as operating rather than capital expenses, leading to both reinvestment rates and returns on capital at technology firms that do not reflect reality. Second, the computation relating growth to reinvestment rates and returns on capital cannot be applied unadjusted to estimate growth at companies that are reporting operating losses (such as Amazon or Ariba) or at companies that have returns on capital that are expected to change over time. Since most technology firms fall into one or another of these exceptions, you have to develop variations that allow you to estimate growth at firms such as these.

III. Discount Rate

The expected cashflows need to be discounted back at a rate that reflects the cost of financing these assets. The cost of capital is a composite cost of financing that reflects the costs of both debt and equity, and their relative weights in the financing structure:

$$\text{Cost of Capital} = k_{equity} \text{ (Equity/(Debt+Equity)} + k_{debt} \text{ (Debt/(Debt + Equity)}$$

where the cost of equity represents the rate of return required by equity investors in the firm, and the cost of debt measures the current cost of borrowing, adjusted for the tax benefits of borrowing. The weights on debt and equity have to be market value weights.

While the definition of cost of capital is no different for technology firms than it is for other firms, there are three areas of difference. One is that many technology firms are disproportionately dependent upon equity for their financing, leading to costs of capital that are very close their costs of equity[6]. When technology firms do borrow money, they tend to issue hybrid securities, such as convertible bonds, that share characteristics with debt and equity.The second is that the parameters of the cost of capital computation (the costs of equity and debt, as well as the debt ratio) can be expected to change over time, as the firm becomes larger and more stable. This will result in costs of capital that will be different from year to year. The third is that the estimation of the costs of equity and

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[6] Start-up technology firms can be the exceptions to this rule, often using substantial amounts of bank debt and hybrid securities to raise capital.

debt, which tend to be dependent upon historical data, can be more difficult with technology firms, which often have short and volatile histories.

IV. Asset Life

Publicly traded firms do not have finite lives. Given that you cannot estimate cash flows forever, you can generally impose closure in valuation models by stopping your estimation of cash flows sometime in the future and then computing a terminal value that reflects all cash flows beyond that point. A number of different approaches exist for computing the terminal value, including the use of multiples. The approach that is most consistent with a discounted cash flow model is one where you assume that cash flows, beyond the terminal year, will grow at a constant rate forever, in which case the terminal value can be estimated as follows:

Terminal $value_n$ = $FCFF_{n+1}$ / (Cost of $Capital_{n+1}$ - g_n)

where the cost of capital and the growth rate in the model are sustainable forever. It is this fact, i.e., that they are constant forever, that allows you to put some reasonable constraints on them. Since no firm can grow forever at a rate higher than the growth rate of the economy in which it operates, the stable growth rate cannot be greater than the overall growth rate of the economy. In the same vein, stable growth firms should be of average risk. Finally, the relationship between growth and reinvestment rates noted earlier can be used to generate the free cash flow to the firm in the first year of stable growth:

$$\text{Terminal Value} = \frac{EBIT_{n+1}(1-t)\left(1 - \frac{g_n}{ROC_n}\right)}{(WACC_n - g_n)}$$

where the ROC_n is the return on capital that the firm can sustain in stable growth. In the special case where ROC is equal to the cost of capital, this estimate simplifies to become the following:

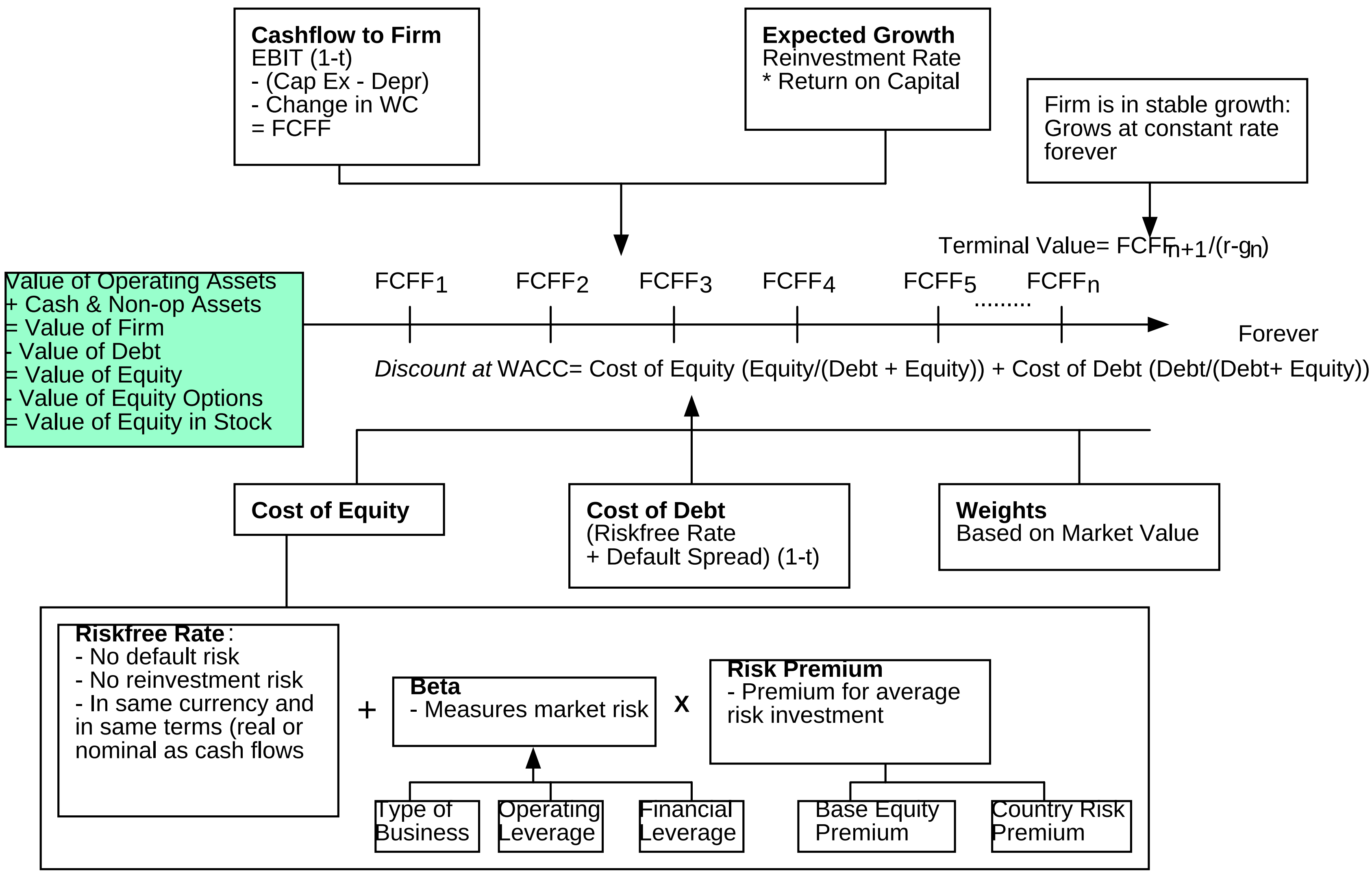
$$\text{Terminal Value}_{ROC=WACC} = \frac{EBIT_{n+1}(1-t)}{WACC_n}$$

Thus, in every discounted cash flow valuation, there are two critical assumptions you need to make on stable growth. The first relates to when the firm that you are valuing will become a stable growth firm, if it is not one already. The second relates to what the characteristics of the firm will be in stable growth, in terms of return on capital and cost of capital. These assumptions are both more difficult to make and more crucial to valuations, when you are looking at technology firms.

V. Bringing it All Together

To value any firm, you begin by estimating how long high growth will last, how high the growth rate will be during that period and the cash flows during the period. You end by estimating a terminal value and discounting all of the cash flows, including the terminal value, back to the present to estimate the value of the firm. Figure 2.8 summarizes the process and the inputs in a discounted cash flow model.

Figure 2.8: Discounted Cash Flow Valuation



Conclusion

The value of an asset is the present value of the expected cash flows generated by it. This simple principle can be used to value any type of asset, ranging from one with guaranteed cash flows (riskless) to one with uncertain cash flows. The cash flow on an asset can be measured prior to debt payments (in which case it is categorized as a cash flow to the firm) or after debt payments (when it is called cash flow to equity). If the cash flows are prior to debt payments, i.e., they are cash flows to the firm, they should be discounted at the cost of capital. If the cash flows are after debt payments, i.e., they are cash flows to equity, they should be discounted at the cost of equity.

Firms are different from individual assets, because their lives are not restricted. Consequently, you need to compute the cash flows on firms forever in order to value them. Since this is an impossible task, you estimate cash flows for a future year, and then develop a measure of value at the end of period. This value is called the terminal value and can account for a large portion of the value of the asset.

In summary, then, the value of a firm is a function of four variables- the cash flows from assets in place (existing investments), the expected growth in these cash flows, the length of the period over which the firm can sustain excess returns and the cost of capital. In the chapters to come, you consider each of these inputs with special emphasis on technology firms.