CHAPTER 4

THE BASICS OF RISK

When valuing assets and firms, we need to use discount rates that reflect the riskiness of the cash flows. In particular, the cost of debt has to incorporate a default spread for the default risk in the debt and the cost of equity has to include a risk premium for equity risk. But how do we measure default and equity risk, and more importantly, how do we come up with the default and equity risk premiums?

In this chapter, we will lay the foundations for analyzing risk in valuation. We present alternative models for measuring risk and converting these risk measures into "acceptable" hurdle rates. We begin with a discussion of equity risk and present our analysis in three steps. In the first step, we define risk in statistical terms to be the variance in actual returns around an expected return. The greater this variance, the more risky an investment is perceived to be. The next step, which we believe is the central one, is to decompose this risk into risk that can be diversified away by investors and risk that cannot. In the third step, we look at how different risk and return models in finance attempt to measure this non-diversifiable risk. We compare and contrast the most widely used model, the capital asset pricing model, with other models, and explain how and why they diverge in their measures of risk and the implications for the equity risk premium.

In the second part of this chapter, we consider default risk and how it is measured by ratings agencies. In addition, we discuss the determinants of the default spread and why it might change over time. By the end of the chapter, we should have a methodology of estimating the costs of equity and debt for any firm.

What is risk?

Risk, for most of us, refers to the likelihood that in life's games of chance, we will receive an outcome that we will not like. For instance, the risk of driving a car too fast is getting a speeding ticket, or worse still, getting into an accident. Webster's dictionary, in fact, defines risk as "exposing to danger or hazard". Thus, risk is perceived almost entirely in negative terms.

In finance, our definition of risk is both different and broader. Risk, as we see it, refers to the likelihood that we will receive a return on an investment that is different from the return we expected to make. Thus, risk includes not only the bad outcomes, i.e, returns that are lower than expected, but also good outcomes, i.e., returns that are higher than expected. In fact, we can refer to the former as downside risk and the latter is upside risk; but we consider both when measuring risk. In fact, the spirit of our definition of risk in finance is captured best by the Chinese symbols for risk, which are reproduced below:

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The first symbol is the symbol for "danger", while the second is the symbol for "opportunity", making risk a mix of danger and opportunity. It illustrates very clearly the tradeoff that every investor and business has to make – between the higher rewards that come with the opportunity and the higher risk that has to be borne as a consequence of the danger.

Much of this chapter can be viewed as an attempt to come up with a model that best measures the "danger" in any investment and then attempts to convert this into the "opportunity" that we would need to compensate for the danger. In financial terms, we term the danger to be "risk" and the opportunity to be "expected return".

What makes the measurement of risk and expected return so challenging is that it can vary depending upon whose perspective we adopt. When analyzing Boeing's risk, for instance, we can measure it from the viewpoint of Boeing's managers. Alternatively, we can argue that Boeing's equity is owned by its stockholders and that it is their perspective on risk that should matter. Boeing's stockholders, many of whom hold the stock as one investment in a larger portfolio, might perceive the risk in Boeing very differently from Boeing's managers, who might have the bulk of their capital, human and financial, invested in the firm.

In this chapter, we will argue that risk in an investment has to be perceived through the eyes of investors in the firm. Since firms like Boeing often have thousands of investors, often with very different perspectives, we will go further. We will assert that risk has to be measured from the perspective of not just any investor in the stock, but of the **marginal investor**, defined to be the investor most likely to be trading on the stock at any given point in time. The objective in corporate finance is the maximization of firm value and stock price. If we want to stay true to this objective, we have to consider the viewpoint of those who set the stock prices, and they are the marginal investors.

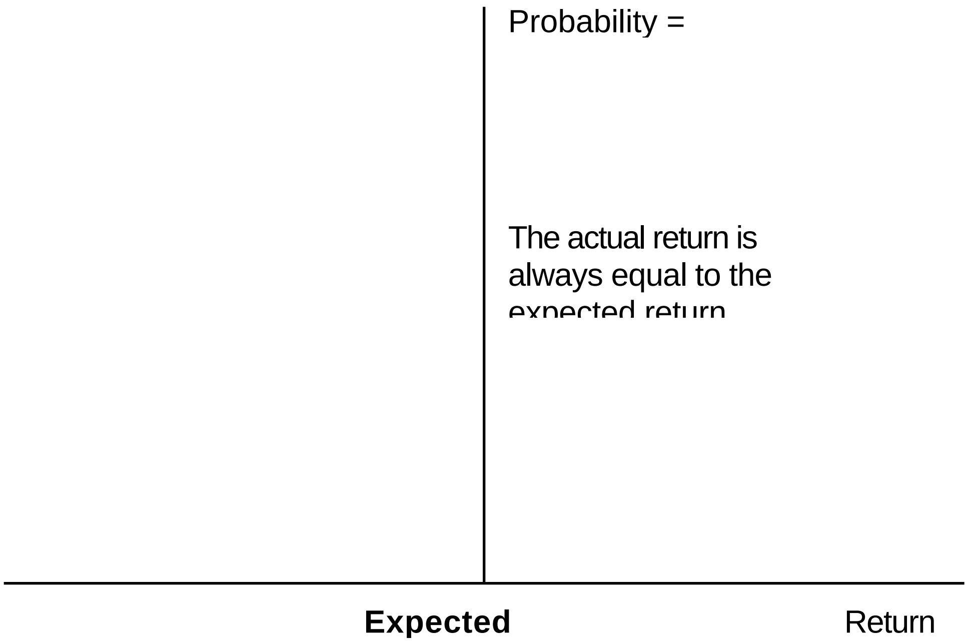
Equity Risk and Expected Return

To demonstrate how risk is viewed in corporate finance, we will present risk analysis in three steps. First, we will define risk in terms of the distribution of actual returns around an expected return. Second, we will differentiate between risk that is specific to one or a few investments and risk that affects a much wider cross section of investments. We will argue that in a market where the marginal investor is well diversified, it is only the latter risk, called **market risk** that will be rewarded. Third, we will look at alternative models for measuring this market risk and the expected returns that go with it.

I. Defining Risk

Investors who buy assets expect to earn returns over the time horizon that they hold the asset. Their actual returns over this holding period may be very different from the expected returns and it is this difference between actual and expected returns that is source of risk. For example, assume that you are an investor with a 1-year time horizon buying a 1-year Treasury bill (or any other default-free one-year bond) with a 5% expected return. At the end of the 1-year holding period, the actual return on this investment will be 5%, which is equal to the expected return. The return distribution for this investment is shown in Figure 4.1.

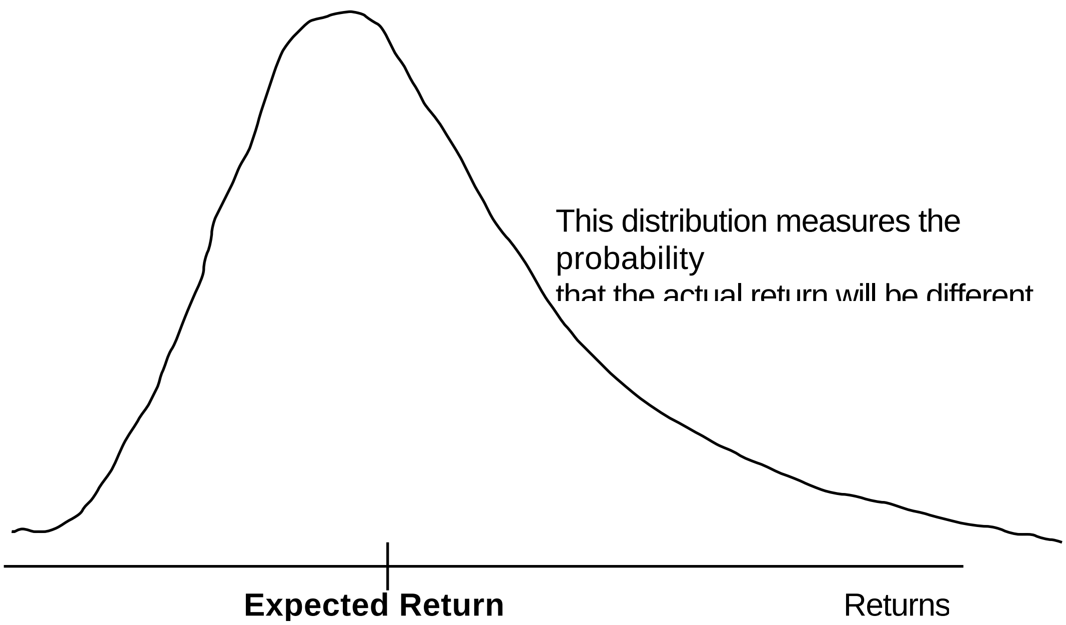
Figure 4.1: Probability Distribution for Riskfree Investment



This is a riskless investment.

To provide a contrast to the riskless investment, consider an investor who buys stock in Boeing. This investor, having done her research, may conclude that she can make an expected return of 30% on Boeing over her 1-year holding period. The actual return over this period will almost certainly not be equal to 30%; it might be much greater or much lower. The distribution of returns on this investment is illustrated in Figure 4.2.

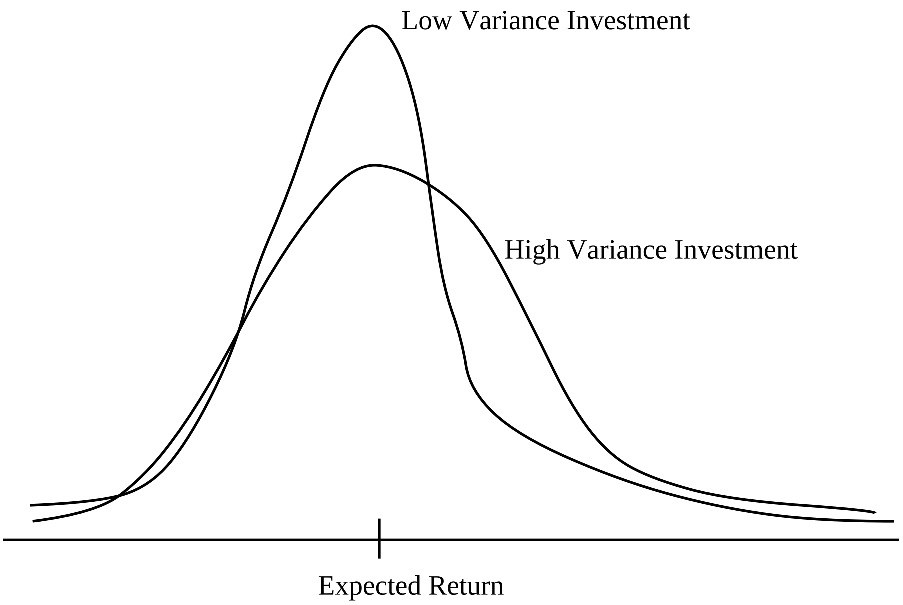
Figure 4.2: Probability Distribution for Risky Investment



In addition to the expected return, an investor now has to consider the following. First, note that the actual returns, in this case, are different from the expected return. The spread of the actual returns around the expected return is measured by the **variance** or **standard deviation** of the distribution; the greater the deviation of the actual returns from expected returns, the greater the variance. Second, the bias towards positive or negative returns is represented by the **skewness** of the distribution. The distribution in Figure 4.2 is positively skewed, since there is a higher probability of large positive returns than large negative returns. Third, the shape of the tails of the distribution is measured by the **kurtosis** of the distribution; fatter tails lead to higher kurtosis. In investment terms, this represents the tendency of the price of this investment to jump (up or down from current levels) in either direction.

In the special case, where the distribution of returns is normal, investors do not have to worry about skewness and kurtosis. Normal distributions are symmetric (no skewness) and defined to have a kurtosis of zero. Figure 4.3 illustrates the return distributions on two investments with symmetric returns.

Figure 4.3: Return Distribution Comparisons



When return distributions take this form, the characteristics of any investment can be measured with two variables – the expected return, which represents the opportunity in the investment, and the standard deviation or variance, which represents the danger. In this scenario, a rational investor, faced with a choice between two investments with the same standard deviation but different expected returns, will always pick the one with the higher expected return.

In the more general case, where distributions are neither symmetric nor normal, it is still conceivable that investors will choose between investments on the basis of only the expected return and the variance, if they possess utility functions[1] that allow them to do so. It is far more likely, however, that they prefer positive skewed distributions to negatively skewed ones, and distributions with a lower likelihood of jumps (lower kurtosis) to those with a higher likelihood of jumps (higher kurtosis). In this world, investors will trade off the good (higher expected returns and more positive skewness) against the bad (higher variance and higher kurtosis) in making investments.

[1] A utility function is a way of summarizing investor preferences into a generic term called 'utility' on the basis of some choice variables. In this case, for instance, we state the investor's utility or satisfaction as a function of wealth. By doing so, we effectively can answer questions such as – Will an investor be twice as happy if he has twice as much wealth? Does each marginal increase in wealth lead to less additional utility than the prior marginal increase? In one specific form of this function, the quadratic utility function, the entire utility of an investor can be compressed into the expected wealth measure and the standard deviation in that wealth.

In closing, we should note that the expected returns and variances that we run into in practice are almost always estimated using past returns rather than future returns. The assumption we are making when we use historical variances is that past return distributions are good indicators of future return distributions. When this assumption is violated, as is the case when the asset's characteristics have changed significantly over time, the historical estimates may not be good measures of risk.

In Practice 4.1: Calculation of standard deviation using historical returns: Boeing and the Home Depot

We will use Boeing and the Home Depot as our investments to illustrate how standard deviations and variances are computed. To make our computations simpler, we will look at returns on an annual basis from 1991 to 1998. To begin the analysis, we first estimate returns for each company for each of these years, in percentage terms, incorporating both price appreciation and dividends into these returns:

$$\text{Return in year n} = \frac{\text{Price at the end of year n - Price at beginning of year n + Dividend in year n}}{\text{Price at the beginning of year n}}$$

Table 4.1 summarizes returns on the two companies.

Table 4.1: Returns on Boeing and the Home Depot: 1991-1998

	Return on Boeing	*Return on The Home Depot*
1991	5.00%	161%
1992	-16%	50.30%
1993	7.80%	-22%
1994	8.70%	16.50%
1995	66.80%	3.80%
1996	35.90%	5.00%
1997	-8.10%	76.20%
1998	-33.10%	107.90%
Sum	67.00%	398.70%

We compute the average and standard deviation in these returns for the two firms, using the information in the table (there are 8 years of data):

Average Return on Boeing$_{91\text{-}98}$ = 67.00%/8 = 8.38%

Average Return on The Home Depot$_{91\text{-}98}$ = 398.70%/8 = 49.84%

The variance is measured by looking at the deviations of the actual returns in each year, for each stock, from the average return. Since we consider both better-than-expected and worse-than-expected deviations in measuring variance, we square the deviations[2].

Table 4.2: Squared Deviations from the Mean

	Return on Boeing	*Return on The Home Depot*	*$(R_B$-Average$(R_B))^2$*	*$(R_{HD}$-Average$(R_{HD}))^2$*
1991	5.00%	161%	0.00113906	1.23571014
1992	-16%	50.30%	0.05941406	2.1391E-05
1993	7.80%	-22%	3.3063E-05	0.51606264
1994	8.70%	16.50%	1.0562E-05	0.11113889
1995	66.80%	3.80%	0.34134806	0.21194514
1996	35.90%	5.00%	0.07576256	0.20104014
1997	-8.10%	76.20%	0.02714256	0.06949814
1998	-33.10%	107.90%	0.17201756	0.33712539
Sum			0.6768675	2.68254188

Following the standard practice for estimating the variances of samples, the variances in returns at the two firms can be estimated by dividing the sum of the squared deviation columns by (n-1), where n is the number of observations in the sample. The standard deviations can be computed to be the squared-root of the variances.

	Boeing	The Home Depot
Variance	$\frac{0.6768675}{8-1} = 0.0967$	$\frac{2.68254188}{8-1} = 0.3832$
Standard Deviation	$0.0967^{0.5} = 0.311$ or 31.1%	$0.3832^{0.5} = 0.619$ or 61.9%

[2] If we do not square the deviations, the sum of the deviations will be zero.

Based upon this data, the Home Depot looks like it was two times more risky than Boeing between 1991 and 1998. What does this tell us? By itself, it provides a measure of how much each these companies' returns in the past have deviated from the average. If we assume that the past is a good indicator of the future, the Home Depot is a more risky investment than Boeing.

optvar.xls: There is a dataset on the web that summarizes standard deviations and variances of stocks in various sectors in the United States.

II. Diversifiable and Non-diversifiable Risk

Although there are many reasons that actual returns may differ from expected returns, we can group the reasons into two categories: firm-specific and market-wide. The risks that arise from firm-specific actions affect one or a few investments, while the risk arising from market-wide reasons affect many or all investments. This distinction is critical to the way we assess risk in finance.

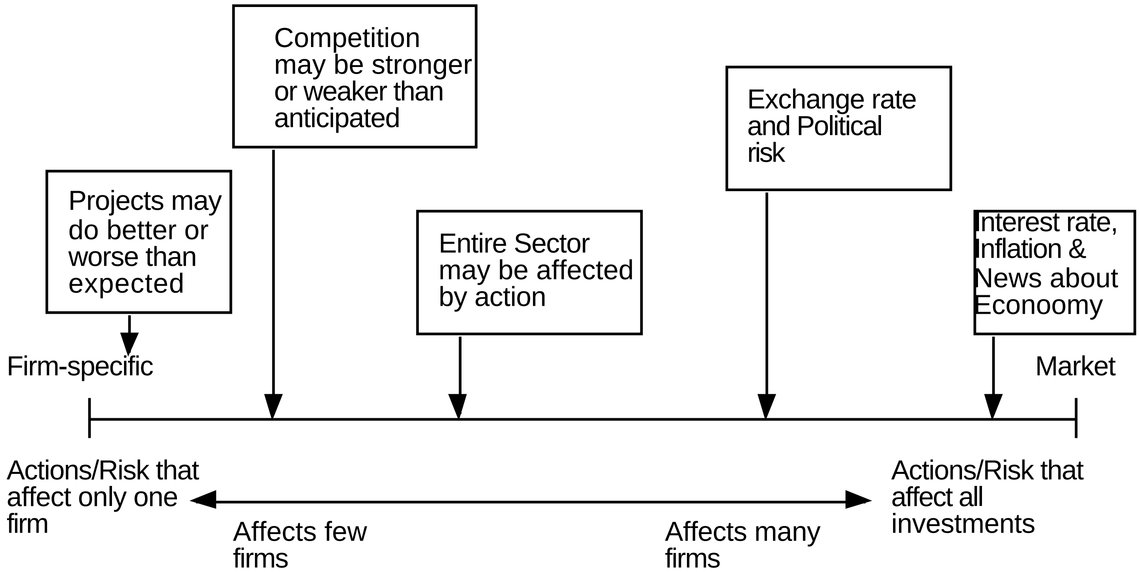
The Components of Risk

When an investor buys stock or takes an equity position in a firm, he or she is exposed to many risks. Some risk may affect only one or a few firms and it is this risk that we categorize as **firm-specific risk**. Within this category, we would consider a wide range of risks, starting with the risk that a firm may have misjudged the demand for a product from its customers; we call this **project risk**. For instance, in the coming chapters, we will be analyzing Boeing's investment in a Super Jumbo jet. This investment is based on the assumption that airlines want a larger airplane and are will be willing to pay a higher price for it. If Boeing has misjudged this demand, it will clearly have an impact on Boeing's earnings and value, but it should not have a significant effect on other firms in the market. The risk could also arise from competitors proving to be stronger or weaker than anticipated; we call this **competitive risk**. For instance, assume that Boeing and Airbus are competing for an order from Quantas, the Australian airline. The possibility that Airbus may win the bid is a potential source of risk to Boeing and perhaps a few of its suppliers. But again, only a handful of firms in the market will be affected by it. Similarly, the Home Depot recently launched an online store to sell its home improvement products. Whether it succeeds or not is clearly important to the Home Depot and its competitors, but it is unlikely to have an impact on the rest of the market. In fact, we would extend our risk measures to include risks that may affect an entire sector but are restricted to that sector; we call this **sector risk**. For instance, a cut in the defense budget in the United States will adversely affect all firms in the defense business, including Boeing, but there should be no significant impact on other sectors, such as food and apparel. What is common across the three risks described above – project, competitive and sector risk – is that they affect only a small sub-set of firms.

There is other risk that is much more pervasive and affects many if not all investments. For instance, when interest rates increase, all investments are negatively affected, albeit to different degrees. Similarly, when the economy weakens, all firms feel the effects, though cyclical firms (such as automobiles, steel and housing) may feel it more. We term this risk **market risk**.

Finally, there are risks that fall in a gray area, depending upon how many assets they affect. For instance, when the dollar strengthens against other currencies, it has a significant impact on the earnings and values of firms with international operations. If most firms in the market have significant international operations, it could well be categorized as market risk. If only a few do, it would be closer to firm-specific risk. Figure 4.4 summarizes the break down or the spectrum of firm-specific and market risks.

Figure 4.4: A Break Down of Risk



Why Diversification reduces or eliminates Firm-specific Risk: An Intuitive Explanation

As an investor, you could invest your entire portfolio in one asset, say Boeing. If you do so, you are exposed to both firm-specific and market risk. If, however, you expand your portfolio to include other assets or stocks, you are diversifying, and by doing so, you can reduce your exposure to firm-specific risk. There are two reasons why diversification reduces or, at the limit, eliminates firm specific risk. The first is that each investment in a diversified portfolio is a much smaller percentage of that portfolio than would be the case if you were not diversified. Thus, any action that increases or decreases the value of only that investment or a small group of investments will have only a small impact on your overall portfolio, whereas undiversified investors are much more exposed to changes in the values of the investments in their portfolios. The second reason is that the effects of firm-specific actions on the prices of individual assets in a portfolio can be either positive or negative for each asset for any period. Thus, in very large portfolios, this risk will average out to zero and will not affect the overall value of the portfolio.

In contrast, the effects of market-wide movements are likely to be in the same direction for most or all investments in a portfolio, though some assets may be affected more than others. For instance, other things being equal, an increase in interest rates will lower the values of most assets in a portfolio. Being more diversified does not eliminate this risk.

A Statistical Analysis Of Diversification Reducing Risk

We can illustrate the effects of diversification on risk fairly dramatically by examining the effects of increasing the number of assets in a portfolio on portfolio variance. The variance in a portfolio is partially determined by the variances of the individual assets in the portfolio and partially by how they move together; the latter is measured statistically with a correlation coefficient or the covariance across investments in the portfolio. It is the covariance term that provides an insight into why and by how much diversification will reduce risk.

Consider a portfolio of two assets. Asset A has an expected return of μ_A and a variance in returns of σ_A^2, while asset B has an expected return of μ_B and a variance in returns of σ_B^2. The correlation in returns between the two assets, which measures how the assets move together, is ρ_{AB}. The expected returns and variance of a two-asset portfolio can be written as a function of these inputs and the proportion of the portfolio going to each asset.

$$\mu_P = w_A \mu_A + (1 - w_A)\mu_B$$

$$\sigma_P^2 = w_A^2 \sigma_A^2 + (1 - w_A)^2 \sigma_B^2 + 2 w_A (1 - w_A) \sigma_A \sigma_B \rho_{AB}$$

where

w_A = Proportion of the portfolio in asset A

The last term in the variance equation is sometimes written in terms of the covariance in returns between the two assets, which is

$$Cov_{AB} = \sigma_A \sigma_B \rho_{AB}$$

The savings that accrue from diversification are a function of the correlation coefficient. Other things remaining equal, the higher the correlation in returns between the two assets, the smaller are the potential benefits from diversification.

Why is the marginal investor assumed to be diversified?

The argument that diversification reduces an investor's exposure to risk is clear both intuitively and statistically, but risk and return models in finance go further. The models look at risk through the eyes of the investor most likely to be trading on the investment at any point in time, i.e. the marginal investor. They argue that this investor, who sets prices for investments, is well diversified; thus, the only risk that he or she cares about is the risk added on to a diversified portfolio or market risk. This argument can be justified simply. The risk in an investment will always be perceived to be higher for an undiversified investor than for a diversified one, since the latter does not shoulder any firm-specific risk and the former does. If both investors have the same expectations about future earnings and cash flows on an asset, the diversified investor will be willing to pay a higher price for that asset because of his or her perception of lower risk. Consequently, the asset, over time, will end up being held by diversified investors.

This argument is powerful, especially in markets where assets can be traded easily and at low cost. Thus, it works well for a stock traded in the United States, since investors can become diversified at fairly low cost. In addition, a significant proportion of the trading in US stocks is done by institutional investors, who tend to be well diversified. It becomes a more difficult argument to sustain when assets cannot be easily traded, or the costs of trading are high. In these markets, the marginal investor may well be undiversified and firm-specific risk may therefore continue to matter when looking at individual investments. For instance, real estate in most countries is still held by investors who are undiversified and have the bulk of their wealth tied up in these investments.

III. Models Measuring Market Risk

While most risk and return models in use in corporate finance agree on the first two steps of the risk analysis process, i.e., that risk comes from the distribution of actual returns around the expected return and that risk should be measured from the perspective of a marginal investor who is well diversified, they part ways when it comes to measuring non-diversifiable or market risk. In this section, we will discuss the different models that exist in finance for measuring market risk and why they differ. We will begin with what still is the standard model for measuring market risk in finance – the capital asset pricing model (CAPM) – and then discuss the alternatives to this model that have developed over the last two decades. While we will emphasize the differences, we will also look at what they have in common.

A. The Capital Asset Pricing Model (CAPM)

The risk and return model that has been in use the longest and is still the standard in most real world analyses is the capital asset pricing model (CAPM). In this section, we will examine the assumptions made by the model and the measures of market risk that emerge from these assumptions.

Assumptions

While diversification reduces the exposure of investors to firm specific risk, most investors limit their diversification to holding only a few assets. Even large mutual funds rarely hold more than a few hundred stocks and many of them hold as few as ten to twenty. There are two reasons why investors stop diversifying. One is that an investor or mutual fund manager can obtain most of the benefits of diversification from a relatively small portfolio, because the marginal benefits of diversification become smaller as the portfolio gets more diversified. Consequently, these benefits may not cover the marginal costs of diversification, which include transactions and monitoring costs. Another reason for limiting diversification is that many investors (and funds) believe they can find under valued assets and thus choose not to hold those assets that they believe to be fairly or over valued.

The capital asset pricing model assumes that there are no transactions costs, all assets are traded and investments are infinitely divisible (i.e., you can buy any fraction of a unit of the asset). It also assumes that everyone has access to the same information and that investors therefore cannot find under or over valued assets in the market place. Making these assumptions allows investors to keep diversifying without additional cost. At the limit, their portfolios will not only include every traded asset in the market but will have identical weights on risky assets The fact that this diversified portfolio includes all traded assets in the market is the reason it is called the **market portfolio**, which should not be a surprising result, given the benefits of diversification and the absence of transactions costs in the capital asset pricing model. If diversification reduces exposure to firm-specific risk and there are no costs associated with adding more assets to the portfolio, the logical limit to diversification is to hold a small proportion of every traded asset in the market. If this seems abstract, consider the market portfolio to be an extremely well diversified mutual fund that holds stocks and real assets, and treasury bills as the riskless asset. In the CAPM, all investors will hold combinations of treasury bills and the same mutual fund[3].

Investor Portfolios in the CAPM

If every investor in the market holds the identical market portfolio, how exactly do investors reflect their risk aversion in their investments? In the capital asset pricing model, investors adjust for their risk preferences in their allocation decision, where they decide how much to invest in a riskless asset and how much in the market portfolio. Investors who are risk averse might choose to put much or even all of their wealth in the riskless asset. Investors who want to take more risk will invest the bulk or even all of their wealth in the market portfolio. Investors, who invest all their wealth in the market portfolio and are still desirous of taking on more risk, would do so by borrowing at the riskless rate and investing more in the same market portfolio as everyone else.

These results are predicated on two additional assumptions. First, there exists a riskless asset, where the expected returns are known with certainty. Second, investors can lend and borrow at the same riskless rate to arrive at their optimal allocations. While lending at the riskless rate can be accomplished fairly simply by buying treasury bills or bonds, borrowing at the riskless rate might be more difficult to do for individuals. There are variations of the CAPM that allow these assumptions to be relaxed and still arrive at the conclusions that are consistent with the model.

Measuring the Market Risk of an Individual Asset

The risk of any asset to an investor is the risk added by that asset to the investor's overall portfolio. In the CAPM world, where all investors hold the market portfolio, the risk to an investor of an individual asset will be the risk that this asset adds on to the market portfolio. Intuitively, if an asset moves independently of the market

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[3] The significance of introducing the riskless asset into the choice mix, and the implications for portfolio choice were first noted in Sharpe (1964) and Lintner (1965). Hence, the model is sometimes called the

portfolio, it will not add much risk to the market portfolio. In other words, most of the risk in this asset is firm-specific and can be diversified away. In contrast, if an asset tends to move up when the market portfolio moves up and down when it moves down, it will add risk to the market portfolio. This asset has more market risk and less firm-specific risk. Statistically, this added risk is measured by the covariance of the asset with the market portfolio.

Measuring the Non-Diversifiable Risk

In a world in which investors hold a combination of only two assets – the riskless asset and the market portfolio – the risk of any individual asset will be measured relative to the market portfolio. In particular, the risk of any asset will be the risk that it adds on to the market portfolio. To arrive at the appropriate measure of this added risk, assume that σ_m^2 is the variance of the market portfolio prior to the addition of the new asset and that the variance of the individual asset being added to this portfolio is σ_i^2. The market value portfolio weight on this asset is w_i, and the covariance correlation in returns between the individual asset and the market portfolio is Cov_{im}. The variance of the market portfolio prior to and after the addition of the individual asset can then be written as

Variance prior to asset i being added = σ_m^2

Variance after asset i is added = $\sigma_{m'}^2 = w_i^2\sigma_i^2 + (1 - w_i)^2\sigma_m^2 + 2w_i(1 - w_i)Cov_{im}$

The market value weight on any individual asset in the market portfolio should be small (w_i is very close to 0) since the market portfolio includes all traded assets in the economy. Consequently, the first term in the equation should approach zero, and the second term should approach σ_m^2, leaving the third term (Cov_{im}, the covariance) as the measure of the risk added by individual asset i.

Standardizing Covariances

The covariance is a percentage value and it is difficult to pass judgment on the relative risk of an investment by looking at this value. In other words, knowing that the

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Sharpe-Lintner model.

covariance of Boeing with the Market Portfolio is 55% does not provide us a clue as to whether Boeing is riskier or safer than the average asset. We therefore standardize the risk measure by dividing the covariance of each asset with the market portfolio by the variance of the market portfolio. This yields a risk measure called the **beta** of the asset:

$$\text{Beta of an asset i} = \frac{\text{Covariance of asset i with Market Portfolio}}{\text{Variance of the Market Portfolio}} = \frac{Cov_{im}}{\sigma_m^2}$$

Since the covariance of the market portfolio with itself is its variance, the beta of the market portfolio, and by extension, the average asset in it, is one. Assets that are riskier than average (using this measure of risk) will have betas that are greater than 1 and assets that are less riskier than average will have betas that are less than 1. The riskless asset will have a beta of 0.

Getting Expected Returns

The fact that every investor holds some combination of the riskless asset and the market portfolio leads to the next conclusion: the expected return of an asset is linearly related to the beta of the asset. In particular, the expected return of an asset can be written as a function of the risk-free rate and the beta of that asset.

$$E(R_i) = R_f + \beta_i \left(E(R_m) - R_f\right)$$

where,

$E(R_i)$ = Expected Return on asset i

R_f = Risk-free Rate

$E(R_m)$ = Expected Return on market portfolio

β_i= Beta of investment i

To use the capital asset pricing model, we need three inputs. While we will look at the estimation process in far more detail in the next chapter, each of these inputs is estimated as follows:

- The riskless asset is defined to be an asset for which the investor knows the expected return with certainty for the time horizon of the analysis.

- The risk premium is the premium demanded by investors for investing in the market portfolio, which includes all risky assets in the market, instead of investing in a riskless asset.
- The beta, which we defined as the covariance of the asset divided by the variance of the market portfolio, measures the risk added on by an investment to the market portfolio.

In summary, in the capital asset pricing model, all the market risk is captured in the beta, measured relative to a market portfolio, which at least in theory should include all traded assets in the market place held in proportion to their market value.

B. The Arbitrage Pricing Model

The restrictive assumptions on transactions costs and private information in the capital asset pricing model and the model's dependence on the market portfolio have long been viewed with skepticism by both academics and practitioners. Ross (1976) suggested an alternative model for measuring risk called the arbitrage pricing model (APM).

Assumptions

If investors can invest risklessly and earn more than the riskless rate, they have found an arbitrage opportunity. The premise of the arbitrage pricing model is that investors take advantage of such arbitrage opportunities, and in the process, eliminate them. If two portfolios have the same exposure to risk but offer different expected returns, investors will buy the portfolio that has the higher expected returns, sell the portfolio with the lower expected returns and earn the difference as a riskless profit. To prevent this arbitrage from occurring, the two portfolios have to earn the same expected return.

Like the capital asset pricing model, the arbitrage pricing model begins by breaking risk down into firm-specific and market risk components. As in the capital asset pricing model, firm specific risk covers information that affects primarily the firm. Market risk affects many or all firms and would include unanticipated changes in a number of economic variables, including gross national product, inflation, and interest rates. Incorporating both types of risk into a return model, we get:

$$R = E(R) + m + \varepsilon$$

where R is the actual return, E(R) is the expected return, *m* is the market-wide component of unanticipated risk and ε is the firm-specific component. Thus, the actual return can be different from the expected return, either because of market risk or firm-specific actions.

The Sources of Market-Wide Risk

While both the capital asset pricing model and the arbitrage pricing model make a distinction between firm-specific and market-wide risk, they measure market risk differently. The CAPM assumes that market risk is captured in the market portfolio, whereas the arbitrage pricing model allows for multiple sources of market-wide risk and measures the sensitivity of investments to changes in each source. In general, the market component of unanticipated returns can be decomposed into economic factors:

$$\begin{aligned} R &= E(R) + m + \alpha \\ &= R + (\beta_j F_j + \beta_2 F_2 + \ldots + \beta_n F_n) + \varepsilon \end{aligned}$$

where

β_j = Sensitivity of investment to unanticipated changes in factor j

F_j = Unanticipated changes in factor j

Note that the measure of an investment's sensitivity to any macro-economic factor takes the form of a beta, called a **factor beta**. In fact, this beta has many of the same properties as the market beta in the CAPM.

The Effects of Diversification

The benefits of diversification were discussed earlier, in the context of our break down of risk into market and firm-specific risk. The primary point of that discussion was that diversification eliminates firm-specific risk. The arbitrage pricing model uses the same argument and concludes that the return on a portfolio will not have a firm-specific component of unanticipated returns. The return on a portfolio can be written as the sum of two weighted averages: the anticipated returns in the portfolio and the market factors.

$$\begin{aligned} R_p = {} & (w_1 R_1 + w_2 R_2 + \ldots + w_n R_n) + (w_1 R_{1,1} + w_2 R_{1,2} + \ldots + w_n R_{1,n}) F_1 + \\ & + (w_2 R_{2,1} + w_2 R_{2,2} + \ldots + w_n R_{2,n}) F_2 + \ldots \end{aligned}$$

where,

w_j = Portfolio weight on asset j

R_j = Expected return on asset j

$\beta_{i,j}$ = Beta on factor i for asset j

Expected Returns and Betas

The final step in this process is estimating an expected return as a function of the betas specified above. To do this, we should first note that the beta of a portfolio is the weighted average of the betas of the assets in the portfolio. This property, in conjunction with the absence of arbitrage, leads to the conclusion that expected returns should be linearly related to betas. To see why, assume that there is only one factor and three portfolios. Portfolio A has a beta of 2.0 and an expected return on 20%; portfolio B has a beta of 1.0 and an expected return of 12%; and portfolio C has a beta of 1.5 and an expected return on 14%. Note that the investor can put half of his wealth in portfolio A and half in portfolio B and end up with a portfolio with a beta of 1.5 and an expected return of 16%. Consequently no investor will choose to hold portfolio C until the prices of assets in that portfolio drop and the expected return increases to 16%. By the same rationale, the expected returns on every portfolio should be a linear function of the beta. If they were not, we could combine two other portfolios, one with a higher beta and one with a lower beta, to earn a higher return than the portfolio in question, creating an opportunity for arbitrage. This argument can be extended to multiple factors with the same results. Therefore, the expected return on an asset can be written as

$$E(R) = R_f + \beta_1 \left[E(R_1) - R_f\right] + \beta_2 \left[E(R_2) - R_f\right] + ... + \beta_n \left[E(R_n) - R_f\right]$$

where

R_f = Expected return on a zero-beta portfolio

$E(R_j)$ = Expected return on a portfolio with a factor beta of 1 for factor j and zero for all other factors.

The terms in the brackets can be considered to be risk premiums for each of the factors in the model.

The capital asset pricing model can be considered to be a special case of the arbitrage pricing model, where there is only one economic factor driving market-wide returns and the market portfolio is the factor.

$$E(R) = R_f + \beta_m (E(R_m) - R_f)$$

The APM in Practice

The arbitrage pricing model requires estimates of each of the factor betas and factor risk premiums in addition to the riskless rate. In practice, these are usually estimated using historical data on asset returns and a *factor analysis*. Intuitively, in a factor analysis, we examine the historical data looking for common patterns that affect broad groups of assets (rather than just one sector or a few assets). A factor analysis provides two output measures:

1. It specifies the number of common factors that affected the historical return data
2. It measures the beta of each investment relative to each of the common factors and provides an estimate of the actual risk premium earned by each factor.

The factor analysis does not, however, identify the factors in economic terms. In summary, in the arbitrage pricing model, the market risk is measured relative to multiple unspecified macroeconomic variables, with the sensitivity of the investment relative to each factor being measured by a beta. The number of factors, the factor betas and factor risk premiums can all be estimated using the factor analysis.

C. Multi-factor Models for risk and return

The arbitrage pricing model's failure to identify the factors specifically in the model may be a statistical strength, but it is an intuitive weakness. The solution seems simple: Replace the unidentified statistical factors with specific economic factors and the resultant model should have an economic basis while still retaining much of the strength of the arbitrage pricing model. That is precisely what multi-factor models try to do.

Deriving a Multi-Factor Model

Multi-factor models generally are determined by historical data, rather than economic modeling. Once the number of factors has been identified in the arbitrage pricing model, their behavior over time can be extracted from the data. The behavior of the unnamed factors over time can then be compared to the behavior of macroeconomic variables over that same period to see whether any of the variables is correlated, over time, with the identified factors.

For instance, Chen, Roll, and Ross (1986) suggest that the following macroeconomic variables are highly correlated with the factors that come out of factor analysis: industrial production, changes in default premium, shifts in the term structure, unanticipated inflation, and changes in the real rate of return. These variables can then be correlated with returns to come up with a model of expected returns, with firm-specific betas calculated relative to each variable.

$$E(R) = R_f + \beta_{GNP}\left[E(R_{GNP}) - R_f\right] + \beta_I\left[E(R_I) - R_f\right] + \ldots + \beta_\partial\left[E(R_\partial) - R_f\right]$$

where

β_{GNP} = Beta relative to changes in industrial production

$E(R_{GNP})$ = Expected return on a portfolio with a beta of one on the industrial production factor and zero on all other factors

β_I = Beta relative to changes in inflation

$E(R_I)$ = Expected return on a portfolio with a beta of one on the inflation factor and zero on all other factors

The costs of going from the arbitrage pricing model to a macroeconomic multi-factor model can be traced directly to the errors that can be made in identifying the factors. The economic factors in the model can change over time, as will the risk premia associated with each one. For instance, oil price changes were a significant economic factor driving expected returns in the 1970s but are not as significant in other time periods. Using the wrong factor or missing a significant factor in a multi-factor model can lead to inferior estimates of expected return.

In summary, multi-factor models, like the arbitrage pricing model, assume that market risk can be captured best using multiple macro economic factors and betas relative to each. Unlike the arbitrage pricing model, multi factor models do attempt to identify the macro economic factors that drive market risk.

D. Regression or Proxy Models

All the models described so far begin by defining market risk in broad terms and then developing models that might best measure this market risk. All of them, however, extract their measures of market risk (betas) by looking at historical data. There is a final class of risk and return models that start with the returns and try to explain differences in returns across stocks over long time periods using characteristics such as a firm's market value or price multiples[4]. Proponents of these models argue that if some investments earn consistently higher returns than other investments, they must be riskier. Consequently, we could look at the characteristics that these high-return investments have in common and consider these characteristics to be indirect measures or proxies for market risk.

Fama and French, in a highly influential study of the capital asset pricing model in the early 1990s, noted that actual returns between 1963 and 1990 have been highly correlated with book to price ratios[5] and size. High return investments, over this period, tended to be investments in companies with low market capitalization and high book to price ratios. Fama and French suggested that these measures be used as proxies for risk and report the following regression for monthly returns on stocks on the NYSE:

$$R_t = 1.77\% - 0.11 \ln(MV) + 0.35 \ln\left(\frac{BV}{MV}\right)$$

where

MV = Market Value of Equity

BV/MV = Book Value of Equity / Market Value of Equity

The values for market value of equity and book-price ratios for individual firms, when plugged into this regression, should yield expected monthly returns.

A Comparative Analysis of Risk and Return Models

Figure 4.5 summarizes all the risk and return models in finance, noting their similarities in the first two steps and the differences in the way they define market risk.

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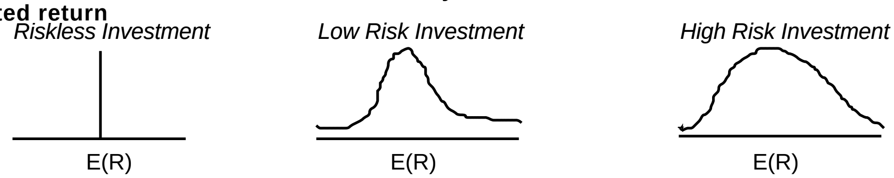
[4] A price multiple is obtained by dividing the market price by its earnings or its book value. Studies indicate that stocks that have low price to earnings multiples or low price to book value multiples earn higher returns than other stocks.

[5] The book to price ratio is the ratio of the book value of equity to the market value of equity.

Figure 4.5: Risk and Return Models in Finance

Step 1: Defining Risk

The risk in an investment can be measured by the variance in actual returns around an expected return



Step 2: Differentiating between Rewarded and Unrewarded Risk

Risk that is specific to investment (Firm Specific)
Can be diversified away in a diversified portfolio
1. each investment is a small proportion of portfolio
2. risk averages out across investments in portfolio

Risk that affects all investments (Market Risk)
Cannot be diversified away since most assets are affected by it.

The marginal investor is assumed to hold a "diversified" portfolio. Thus, only market risk will be rewarded and priced.

Step 3: Measuring Market Risk

The CAPM	**The APM**	**Multi-Factor Models**	**Proxy Models**
If there is 1. no private information 2. no transactions cost the optimal diversified portfolio includes every traded asset. Everyone will hold this<u>market portfolio</u> **Market Risk = Risk added by any investment to the market portfolio:**	If there are no arbitrage opportunities then the market risk of any asset must be captured by betas relative to factors that affect all investments. **Market Risk = Risk exposures of any asset to market factors**	Since market risk affects most or all investments, it must come from macro economic factors. **Market Risk = Risk exposures of any asset to macro economic factors.**	In an efficient market, differences in returns across long periods mus be due to market risk differences. Looking for variables correlated with returns should then give us proxies for this risk. **Market Risk = Captured by the Proxy Variable(s)**
Beta of asset relative to Market portfolio (from a regression)	Betas of asset relative to unspecified market factors (from a factor analysis)	Betas of assets relative to specified macro economic factors (from a regression)	Equation relating returns to proxy variables (from a regression)

As noted in Figure 4.9, all the risk and return models developed in this chapter make some assumptions in common. They all assume that only market risk is rewarded and they derive the expected return as a function of measures of this risk. The capital asset pricing model makes the most restrictive assumptions about how markets work but arrives at the simplest model, with only one factor driving risk and requiring estimation. The arbitrage pricing model makes fewer assumptions but arrives at a more complicated model, at least in terms of the parameters that require estimation. The capital asset pricing model can be considered a specialized case of the arbitrage pricing model, where there is only one underlying factor and it is completely measured by the market index. In general, the CAPM has the advantage of being a simpler model to estimate and to use, but it will underperform the richer APM when an investment is sensitive to economic factors not well represented in the market index. For instance, oil company stocks, which derive most of their risk from oil price movements, tend to have low CAPM betas and low expected returns. Using an arbitrage pricing model, where one of the factors may measure oil and other commodity price movements, will yield a better estimate of risk and higher expected return for these firms[6].

Which of these models works the best? Is beta a good proxy for risk and is it correlated with expected returns? The answers to these questions have been debated widely in the last two decades. The first tests of the CAPM suggested that betas and returns were positively related, though other measures of risk (such as variance) continued to explain differences in actual returns. This discrepancy was attributed to limitations in the testing techniques. In 1977, Roll, in a seminal critique of the model's tests, suggested that since the market portfolio could never be observed, the CAPM could never be tested, and all tests of the CAPM were therefore joint tests of both the model and the market portfolio used in the tests. In other words, all that any test of the CAPM could show was that the model worked (or did not) given the proxy used for the market portfolio. It could therefore be argued that in any empirical test that claimed to reject the CAPM, the rejection could be of the proxy used for the market portfolio rather than of the model itself. Roll noted that there was no way to ever prove that the CAPM worked and thus no empirical basis for using the model.

Fama and French (1992) examined the relationship between betas and returns between 1963 and 1990 and concluded that there is no relationship. These results have been contested on three fronts. First, Amihud, Christensen, and Mendelson (1992), used the same data, performed different statistical tests and showed that differences in betas did, in fact, explain differences in returns during the time period. Second, Kothari and Shanken (1995) estimated betas using annual data, instead of the shorter intervals used in many tests, and concluded that betas do explain a significant proportion of the differences in returns across investments. Third, Chan and Lakonishok (1993) looked at a much longer time series of returns from 1926 to 1991 and found that the positive relationship between betas and returns broke down only in the period after 1982. They also find that betas are a useful guide to risk in extreme market conditions, with the riskiest firms (the

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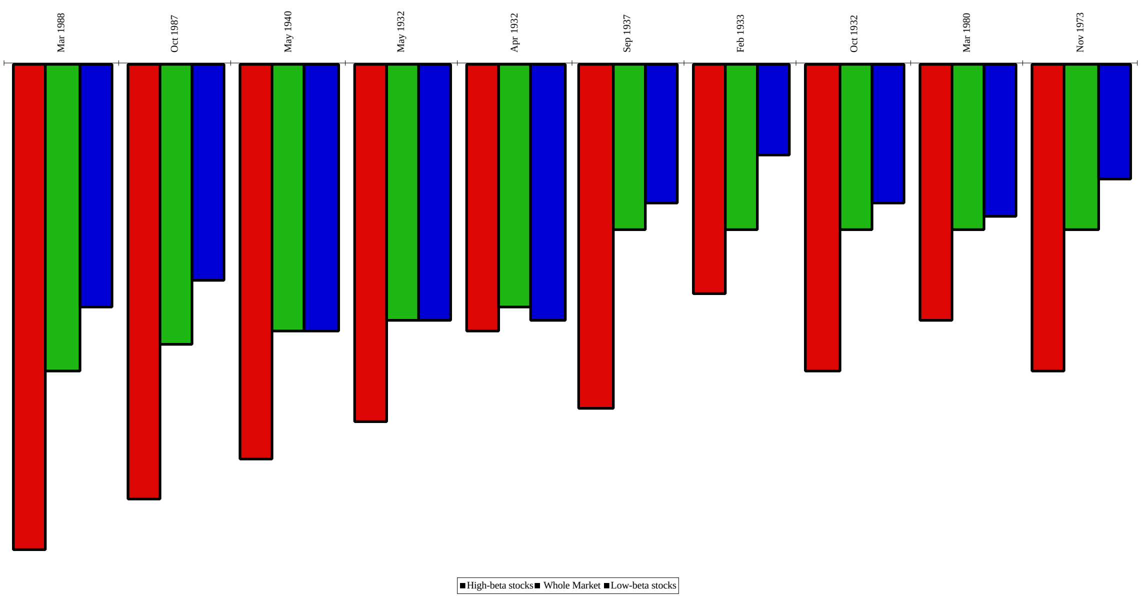
[6] Weston and Copeland used both approaches to estimate the cost of equity for oil companies in 1989 and came up with 14.4% with the CAPM and 19.1% using the arbitrage pricing model.

10% with highest betas) performing far worse than the market as a whole, in the ten worst months for the market between 1926 and 1991 (See Figure 4.6).

Figure 4.6: Returns and Betas: Ten Worst Months between 1926 and 1991



Source: Chan and Lakonishok

While the initial tests of the APM suggested that they might provide more promise in terms of explaining differences in returns, a distinction has to be drawn between the use of these models to explain differences in past returns and their use to predict expected returns in the future. The competitors to the CAPM clearly do a much better job at explaining past returns since they do not constrain themselves to one factor, as the CAPM does. This extension to multiple factors does become more of a problem when we try to project expected returns into the future, since the betas and premiums of each of these factors now have to be estimated. Because the factor premiums and betas are themselves volatile, the estimation error may eliminate the benefits that could be gained by moving from the CAPM to more complex models. The regression models that were offered as an alternative also have an estimation problem, since the variables that work best as proxies for market risk in one period (such as market capitalization) may not be the ones that work in the next period.

Ultimately, the survival of the capital asset pricing model as the default model for risk in real world applications is a testament to both its intuitive appeal and the failure of more complex models to deliver significant improvement in terms of estimating expected returns. We would argue that a judicious use of the capital asset pricing model, without an over reliance on historical data, is still the most effective way of dealing with risk in modern corporate finance.

Models of Default Risk

The risk that we have discussed hitherto in this chapter relates to cash flows on investments being different from expected cash flows. There are some investments, however, in which the cash flows are promised when the investment is made. This is the case, for instance, when you lend to a business or buy a corporate bond; the borrower may default on interest and principal payments on the borrowing. Generally speaking, borrowers with higher default risk should pay higher interest rates on their borrowing than those with lower default risk. This section examines the measurement of default risk and the relationship of default risk to interest rates on borrowing.

In contrast to the general risk and return models for equity, which evaluate the effects of market risk on expected returns, models of default risk measure the consequences of firm-specific default risk on promised returns. While diversification can be used to explain why firm-specific risk will not be priced into expected returns for equities, the same rationale cannot be applied to securities that have limited upside potential and much greater downside potential from firm-specific events. To see what we mean by limited upside potential, consider investing in the bond issued by a company. The coupons are fixed at the time of the issue and these coupons represent the promised cash flow on the bond. The best case scenario for you as an investor is that you receive the promised cash flows; you are not entitled to more than these cash flows even if the company is wildly successful. All other scenarios contain only bad news, though in varying degrees, with the delivered cash flows being less than the promised cash flows. Consequently, the expected return on a corporate bond is likely to reflect the firm-specific default risk of the firm issuing the bond.

The Determinants of Default Risk

The default risk of a firm is a function of two variables. The first is the firm's capacity to generate cash flows from operations and the second is its financial obligations – including interest and principal payments[7]. Firms that generate high cash flows relative to their financial obligations should have lower default risk than firms that generate low cash flows relative to their financial obligations. Thus, firms with significant existing investments, which generate relatively high cash flows, will have lower default risk than firms that do not.

In addition to the magnitude of a firm's cash flows, the default risk is also affected by the volatility in these cash flows. The more stability there is in cash flows the lower the default risk in the firm. Firms that operate in predictable and stable businesses will have lower default risk than will other similar firms that operate in cyclical or volatile businesses.

Most models of default risk use financial ratios to measure the cash flow coverage (i.e., the magnitude of cash flows relative to obligations) and control for industry effects to evaluate the variability in cash flows.

Bond Ratings and Interest rates

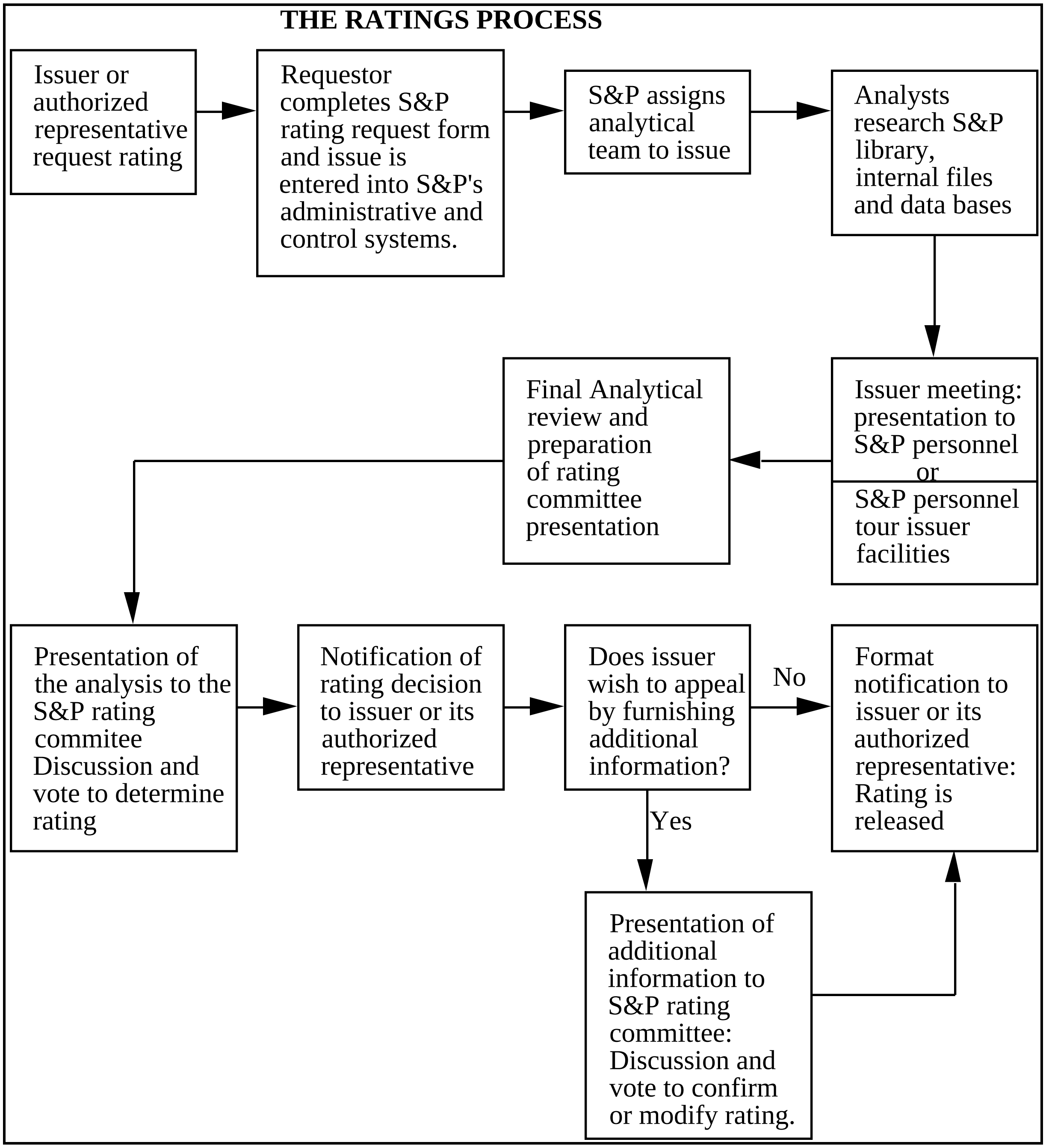
The most widely used measure of a firm's default risk is its bond rating, which is generally assigned by an independent ratings agency. The two best known are Standard and Poor's and Moody's. Thousands of companies are rated by these two agencies and their views carry significant weight with financial markets.

The Ratings Process

The process of rating a bond usually starts when the issuing company requests a rating from a bond ratings agency. The ratings agency then collects information from both publicly available sources, such as financial statements, and the company itself and makes a decision on the rating. If the company disagrees with the rating, it is given the

[7] Financial obligation refers to any payment that the firm has legally obligated itself to make, such as interest and principal payments. It does not include discretionary cash flows, such as dividend payments or new capital expenditures, which can be deferred or delayed, without legal consequences, though there may be economic consequences.

opportunity to present additional information. This process is presented schematically for one ratings agency, Standard and Poors (S&P), in Figure 4.7.



The ratings assigned by these agencies are letter ratings. A rating of AAA from Standard and Poor's and Aaa from Moody's represents the highest rating granted to firms that are viewed as having the lowest default risk. As the default risk increases, the ratings decrease toward D for firms in default (Standard and Poor’s). A rating at or above BBB by Standard and Poor’s is categorized as investment grade, reflecting the view of the ratings agency that there is relatively little default risk in investing in bonds issued by these firms.

Determinants of Bond Ratings

The bond ratings assigned by ratings agencies are primarily based upon publicly available information, though private information conveyed by the firm to the rating agency does play a role. The rating assigned to a company's bonds will depend in large part on financial ratios that measure the capacity of the company to meet debt payments and generate stable and predictable cash flows. While a multitude of financial ratios exist, table 4.6 summarizes some of the key ratios used to measure default risk.

Table 4.6: Financial Ratios used to measure Default Risk

Ratio	**Description**
Pretax Interest Coverage	$\frac{\text{Pretax Income from Continuing Operations} + \text{Interest Expense}}{\text{Gross Interest}}$
EBITDA Interest Coverage	$\frac{\text{EBITDA}}{\text{Gross Interest}}$
Funds from Operations / Total Debt	$\frac{\text{Net Income from Continuing Operations} + \text{Depreciation}}{\text{Total Debt}}$
Free Operating Cashflow/ Total Debt	$\frac{\left(\text{Funds from Operations} - \text{Capital Expenditures} - \text{Change in Working Capital}\right)}{\text{Total Debt}}$
Pretax Return on Permanent Capital	$\frac{\text{Pretax Income from Continuing Operations} + \text{Interest Expense}}{\left(\text{Average of Beginning of the year and End of the year of long and short term debt, minority interest and Shareholders Equity}\right)}$)
Operating Income/Sales	$\frac{\left(\text{Sales} - \text{COGS(before depreciation)} - \text{Selling Expenses} - \text{Administrative Expenses} - \text{R\&D Expenses}\right)}{\text{Sales}}$

Long Term Debt/ Capital	$\frac{\text{Long Term Debt}}{\text{Long Term Debt} + \text{Equity}}$
Total Debt/Capitalization	$\frac{\text{Total Debt}}{\text{Total Debt} + \text{Equity}}$

Source: Standard and Poors

There is a strong relationship between the bond rating a company receives and its performance on these financial ratios. Table 4.7 provides a summary of the median ratios[8] from 1998 to 2000 for different S&P ratings classes for manufacturing firms.

Table 4.7: Financial Ratios by Bond Rating: 1998-2000

	AAA	*AA*	*A*	*BBB*	*BB*	*B*	*CCC*
EBIT interest cov. (x)	17.5	10.8	6.8	3.9	2.3	1.0	0.2
EBITDA interest cov.	21.8	14.6	9.6	6.1	3.8	2.0	1.4
Funds flow/total debt	105.8	55.8	46.1	30.5	19.2	9.4	5.8
Free oper. cash flow/total debt (%)	55.4	24.6	15.6	6.6	1.9	–4.5	-14.0
Return on capital (%)	28.2	22.9	19.9	14.0	11.7	7.2	0.5
Oper.income/sales (%)	29.2	21.3	18.3	15.3	15.4	11.2	13.6
Long-term debt/capital (%)	15.2	26.4	32.5	41.0	55.8	70.7	80.3
Total Debt/ Capital (%)	26.9	35.6	40.1	47.4	61.3	74.6	89.4
Number of firms	10	34	150	234	276	240	23

Source: Standard and Poors

Note that the pre-tax interest coverage ratio (EBIT) and the EBITDA interest coverage ratio are stated in terms of times interest earned, whereas the rest of the ratios are stated in percentage terms.

Not surprisingly, firms that generate income and cash flows significantly higher than debt payments, that are profitable and that have low debt ratios are more likely to be highly rated than are firms that do not have these characteristics. There will be individual firms whose ratings are not consistent with their financial ratios, however, because the ratings agency does add subjective judgments into the final mix. Thus, a firm which performs poorly on financial ratios but is expected to improve its performance dramatically over the next period may receive a higher rating than is justified by its current financials. For most firms, however, the financial ratios should provide a reasonable basis for guessing at the bond rating.

ratingfins.xls: There is a dataset on the web that summarizes key financial ratios by bond rating class for the United States in the most recent period for which the data is available.

Bond Ratings and Interest Rates

The interest rate on a corporate bond should be a function of its default risk, which is measured by its rating. If the rating is a good measure of the default risk, higher rated bonds should be priced to yield lower interest rates than would lower rated bonds. In fact, in chapter 5, we will define the difference between the interest rate on a bond with default risk and a default-free government bond to be the default spread. This default spread will vary by maturity of the bond and can also change from period to period, depending on economic conditions. In chapter 7, we will consider how best to estimate these default spreads and how they might vary over time.

Summary

Risk, as we define it in finance, is measured based upon deviations of actual returns on an investment from its' expected returns. There are two types of risk. The first, which we call equity risk, arises in investments where there are no promised cash flows, but there are expected cash flows. The second, default risk, arises on investments with promised cash flows.

On investments with equity risk, the risk is best measured by looking at the variance of actual returns around the expected returns, with greater variance indicating

[8] See the Standard and Poor's online site: http://www.standardandpoors.com/ratings/criteria/index.htm

greater risk. This risk can be broken down into risk that affects one or a few investments, which we call firm specific risk, and risk that affects many investments, which we refer to as market risk. When investors diversify, they can reduce their exposure to firm specific risk. By assuming that the investors who trade at the margin are well diversified, we conclude that the risk we should be looking at with equity investments is the market risk. The different models of equity risk introduced in this chapter share this objective of measuring market risk, but they differ in the way they do it. In the capital asset pricing model, exposure to market risk is measured by a market beta, which estimates how much risk an individual investment will add to a portfolio that includes all traded assets. The arbitrage pricing model and the multi-factor model allow for multiple sources of market risk and estimate betas for an investment relative to each source. Regression or proxy models for risk look for firm characteristics, such as size, that have been correlated with high returns in the past and use these to measure market risk. In all these models, the risk measures are used to estimate the expected return on an equity investment. This expected return can be considered the cost of equity for a company.

On investments with default risk, risk is measured by the likelihood that the promised cash flows might not be delivered. Investments with higher default risk should have higher interest rates and the premium that we demand over a riskless rate is the default premium. For most US companies, default risk is measured by rating agencies in the form of a company rating; these ratings determine, in large part, the interest rates at which these firms can borrow. Even in the absence of ratings, interest rates will include a default premium that reflects the lenders' assessments of default risk. These default-risk adjusted interest rates represent the cost of borrowing or debt for a business

Problems

1. The following table lists the stock prices for Microsoft from 1989 to 1998. The company did not pay any dividends during the period

Year	Price
1989	$ 1.20
1990	$ 2.09
1991	$ 4.64
1992	$ 5.34
1993	$ 5.05
1994	$ 7.64
1995	$ 10.97
1996	$ 20.66
1997	$ 32.31
1998	$ 69.34

a. Estimate the average annual return you would have made on your investment.

b. Estimate the standard deviation and variance in the annual returns.

c. If you were investing in Microsoft today, would you expect the historical standard deviations and variances to continue to hold? Why or why not?

2. Unicom is a regulated utility serving Northern Illinois. The following table lists the stock prices and dividends on Unicom from 1989 to 1998.

Year	Price	Dividends
1989	$ 36.10	$ 3.00
1990	$ 33.60	$ 3.00
1991	$ 37.80	$ 3.00
1992	$ 30.90	$ 2.30
1993	$ 26.80	$ 1.60
1994	$ 24.80	$ 1.60
1995	$ 31.60	$ 1.60

1996	$	28.50	$	1.60
1997	$	24.25	$	1.60
1998	$	35.60	$	1.60

a. Estimate the average annual return you would have made on your investment.

b. Estimate the standard deviation and variance in the annual returns.

c. If you were investing in Unicom today, would you expect the historical standard deviations and variances to continue to hold? Why or why not?

3. The following table summarizes the annual returns you would have made on two companies – Scientific Atlanta, a satellite and data equipment manufacturer, and AT&T, the telecomm giant, from 1988 to 1998.

Year	*Scientific Atlanta*	*AT&T*
1989	80.95%	58.26%
1990	-47.37%	-33.79%
1991	31%	29.88%
1992	132.44%	30.35%
1993	32.02%	2.94%
1994	25.37%	-4.29%
1995	-28.57%	28.86%
1996	0.00%	-6.36%
1997	11.67%	48.64%
1998	36.19%	23.55%

a. Estimate the average and standard deviation in annual returns in each company.

b. Estimate the covariance and correlation in returns between the two companies.

c. Estimate the variance of a portfolio composed, in equal parts, of the two investments.

4. You are in a world where there are only two assets, gold and stocks. You are interested in investing your money in one, the other or both assets. Consequently you collect the following data on the returns on the two assets over the last six years.

	Gold	Stock Market
Average return	8%	20%
Standard deviation	25%	22%
Correlation	-0.4	

a. If you were constrained to pick just one, which one would you choose?

b. A friend argues that this is wrong. He says that you are ignoring the big payoffs that you can get on gold. How would you go about alleviating his concern?

c. How would a portfolio composed of equal proportions in gold and stocks do in terms of mean and variance?

d. You now learn that GPEC (a cartel of gold-producing countries) is going to vary the amount of gold it produces with stock prices in the US. (GPEC will produce less gold when stock markets are up and more when it is down.) What effect will this have on your portfolios? Explain.

5. You are interested in creating a portfolio of two stocks – Coca Cola and Texas Utilities. Over the last decade, an investment in Coca Cola stock would have earned an average annual return of 25% with a standard deviation in returns of 36%. An investment in Texas Utilities stock would have earned an average annual return of 12%, with a standard deviation of 22%. The correlation in returns across the two stocks is 0.28.

a. Assuming that the average and standard deviation, estimated using past returns, will continue to hold in the future, estimate the average returns and standard deviation of a portfolio composed 60% of Coca Cola and 40% of Texas Utilities stock.

b. Estimate the minimum variance portfolio.

c. Now assume that Coca Cola's international diversification will reduce the correlation to 0.20, while increasing Coca Cola's standard deviation in returns to 45%. Assuming all of the other numbers remain unchanged, answer (a) and (b).

6. Assume that you have half your money invested in Times Mirror, the media company, and the other half invested in Unilever, the consumer product giant. The expected returns and standard deviations on the two investments are summarized below:

	Times Mirror	Unilever
Expected Return	14%	18%

Standard Deviation	25%	40%

Estimate the variance of the portfolio as a function of the correlation coefficient (Start with –1 and increase the correlation to +1 in 0.2 increments).

7. You have been asked to analyze the standard deviation of a portfolio composed of the following three assets:

Investment	Expected Return	Standard Deviation
Sony Corporation	11%	23%
Tesoro Petroleum	9%	27%
Storage Technology	16%	50%

You have also been provided with the correlations across these three investments:

	Sony	Tesoro	Storage Tech
Sony	1.00	-0.15	0.20
Tesoro	-0.15	1.00	-0.25
Storage Tech	0.20	-0.25	1.00

Estimate the variance of a portfolio equally weighted across all three assets.

9. Assume that the average variance of return for an individual security is 50 and that the average covariance is 10. What is the expected variance of a portfolio of 5, 10, 20, 50 and 100 securities. How many securities need to be held before the risk of a portfolio is only 10% more than the minimum?

10. Assume you have all your wealth (a million dollars) invested in the Vanguard 500 index fund and that you expect to earn an annual return of 12% with a standard deviation in returns of 25%. Since you have become more risk averse, you decide to shift $200,000 from the Vanguard 500 index fund to treasury bills. The T.bill rate is 5%. Estimate the expected return and standard deviation of your new portfolio.

11. Every investor in the capital asset pricing model owns a combination of the market portfolio and a riskless asset. Assume that the standard deviation of the market portfolio is 30% and that the expected return on the portfolio is 15%. What proportion of the following investor's wealth

would you suggest investing in the market portfolio and what proportion in the riskless asset? (The riskless asset has an expected return of 5%)

a. an investor who desires a portfolio with no standard deviation

b. an investor who desires a portfolio with a standard deviation of 15%

c. an investor who desires a portfolio with a standard deviation of 30%

d. an investor who desires a portfolio with a standard deviation of 45%

e. an investor who desires a portfolio with an expected return of 12%

12. The following table lists returns on the market portfolio and on Scientific Atlanta, each year from 1989 to 1998.

Year	*Scientific Atlanta*	*Market Portfolio*
1989	80.95%	31.49%
1990	-47.37%	-3.17%
1991	31%	30.57%
1992	132.44%	7.58%
1993	32.02%	10.36%
1994	25.37%	2.55%
1995	-28.57%	37.57%
1996	0.00%	22.68%
1997	11.67%	33.10%
1998	36.19%	28.32%

a. Estimate the covariance in returns between Scientific Atlanta and the market portfolio.

b. Estimate the variances in returns on both investments.

c. Estimate the beta for Scientific Atlanta.

13. United Airlines has a beta of 1.50. The standard deviation in the market portfolio is 22% and United Airlines has a standard deviation of 66%

a. Estimate the correlation between United Airlines and the market portfolio.

b. What proportion of United Airlines’ risk is market risk?

14. You are using the arbitrage pricing model to estimate the expected return on Bethlehem Steel, and have derived the following estimates for the factor betas and risk premia:

Factor	Beta	Risk Premia
1	1.2	2.5%
2	0.6	1.5%
3	1.5	1.0%
4	2.2	0.8%
5	0.5	1.2%

a. Which risk factor is Bethlehem Steel most exposed to? Is there any way, within the arbitrage pricing model, to identify the risk factor?

b. If the riskfree rate is 5%, estimate the expected return on Bethlehem Steel.

c. Now assume that the beta in the capital asset pricing model for Bethlehem Steel is 1.1 and that the risk premium for the market portfolio is 5%. Estimate the expected return using the CAPM.

d. Why are the expected returns different between the two models?

15. You are using the multi-factor model to estimate the expected return on Emerson Electric, and have derived the following estimates for the factor betas and risk premia:

Macro-economic Factor	*Measure*	*Beta*	*Risk Premia* (R_{factor}-R_f)
Level of Interest rates	T.bond rate	0.5	1.8%
Term Structure	T.bond rate – T.bill rate	1.4	0.6%
Inflation rate	CPI	1.2	1.5%
Economic Growth	GNP Growth rate	1.8	4.2%

With a riskless rate of 6%, estimate the expected return on Emerson Electric.

16. The following equation is reproduced from the study by Fama and French of returns between 1963 and 1990.

$$R_t = 0.0177 - 0.11 \ln(MV) + 0.35 \ln\left(\frac{BV}{MV}\right)$$

where MV is the market value of equity in hundreds of millions of dollar and BV is the book value of equity in hundreds of millions of dollars. The return is a monthly return.

a. Estimate the expected annual return on Lucent Technologies. The market value of equity is $240 billion and the book value of equity is $13.5 billion.

b. Lucent Technologies has a beta of 1.55. If the riskless rate is 6%, and the risk premium for the market portfolio is 5.5%, estimate the expected return.

c. Why are the expected returns different under the two approaches?