## CHAPTER 8

## ESTIMATING RISK PARAMETERS AND COSTS OF FINANCING

In the last chapter, we laid the groundwork for estimating the costs of equity and capital for firms by looking at how best to estimate a riskless rate that operates as a base for all costs, an equity risk premium for estimating the cost of equity and default spreads for estimating the cost of debt. We did not, however, consider how to estimate the risk parameters for individual firms. In this chapter, we will examine the process of estimating risk parameters for individual firms, both for estimating cost of equity and the cost of debt.

For the cost of equity, we will look at the standard process of estimating the beta for a firm and consider alternative approaches. For the cost of debt, we will examine bond ratings as measures of default risk and the determinants of these ratings.

We will close the chapter by bringing together the risk parameter estimates for individual firms and the economy-wide estimates of the riskfree rate and risk premia to estimate a cost of capital for the firm. To do this, we will argue that the sources of capital have to be weighted by their relative market values.

## The Cost of Equity and Capital

Firms raise money from both equity investors and lenders to fund investments. Both groups of investors make their investments expecting to make a return. In chapter 4, we argued that the expected return for equity investors would include a premium for the equity risk in the investment. We label this expected return the cost of equity. Similarly, the expected return that lenders hope to make on their investments includes a premium for default risk and we call that expected return the cost of debt. If we consider all of the financing that the firm takes on, the composite cost of financing will be a weighted average of the costs of equity and debt and this weighted cost is the cost of capital.

We will begin by estimating the equity risk in a firm and using the equity risk to estimate the cost of equity and we follow up by measuring the default risk to estimate a
cost of debt. We will conclude the chapter by determining the weights we should attach to each of these costs to arrive at a cost of capital.

## Cost of Equity

The cost of equity is the rate of return investors require on an equity investment in a firm. The risk and return models described in chapter 4 need a riskless rate and a risk premium (in the CAPM) or premiums (in the APM and multi-factor models), which we estimated in the last chapter. They also need measures of a firm's exposure to market risk in the form of betas. These inputs are used to arrive at an expected return on an equity investment using the CAPM.

$$
\text { Expected Return }=\text { Riskless rate }+ \text { Beta }(\text { Risk Premium })
$$

This expected return to equity investors includes compensation for the market risk in the investment and is the cost of equity. In this section, we will concentrate on the estimation of the beta of a firm. While much of our discussion is directed at the CAPM, it can be extended to apply to the arbitrage pricing and multi factor models, as well.

## Betas

In the CAPM, the beta of an investment is the risk that the investment adds to a market portfolio. In the APM and Multi-factor model, the betas of the investment relative to each factor have to be measured. There are three approaches available for estimating these parameters. The first is to use historical data on market prices for individual investments. The second is to estimate the betas from the fundamental characteristics of the investment. The third is to use accounting data. We describe all three approaches in this section.

## A. Historical Market Betas

The conventional approach for estimating the beta of an investment is a regression of the historical returns on the investment against the historical returns on a market index. For firms that have been publicly traded for a length of time, it is relatively straightforward to estimate returns that an investor would have made on investing in stock in intervals (such as a week or a month) over that period. In theory, these stock returns on the assets should be related to returns on a market portfolio, i.e. a portfolio
that includes all traded assets, to estimate the betas of the assets. In practice, we tend to use a stock index, such as the S\&P 500, as a proxy for the market portfolio, and we estimate betas for stocks against the index.

## Regression Estimates of Betas

The standard procedure for estimating betas is to regress 1 stock returns $\left(\mathrm{R}_{\mathrm{j}}\right)$ against market returns $\left(\mathrm{R}_{\mathrm{m}}\right)$ -

$$
R_{j}=a+b R_{m}
$$

where
$\mathrm{a}=$ Intercept from the regression
$\mathrm{b}=$ Slope of the regression $=\frac{\operatorname{Cov}\left(R_{j}, R_{m}\right)}{\sigma_{m}^{2}}$
The slope of the regression corresponds to the beta of the stock and measures the riskiness of the stock.

The intercept of the regression provides a simple measure of performance of the investment during the period of the regression, when returns are measured against the expected returns from the capital asset pricing model. To see why, consider the following rearrangement of the capital asset pricing model:

$$
R_{j}=R_{f}+\beta\left(R_{m}-R_{f}\right)=R_{f}(1-\beta)+\beta R_{m}
$$

Compare this formulation of the return on an investment to the return equation from the regression:

$$
R_{j}=a+b R_{m}
$$

Thus, a comparison of the intercept (a) to $\mathrm{R}_{\mathrm{f}}(1-\beta)$ should provide a measure of the stock's performance, at least relative to the capital asset pricing model. ${ }^{2}$ In summary, then:
If $\quad a>R_{f}(1-\beta) \quad \ldots . \quad$ Stock did better than expected during regression period.

[^0]$\mathrm{a}=\mathrm{R}_{\mathrm{f}}(1-\beta) \quad \ldots . \quad$ Stock did as well as expected during regression period.
$\mathrm{a}<\mathrm{R}_{\mathrm{f}}(1-\beta) \quad \ldots . \quad$ Stock did worse than expected during regression period.
The difference between a and $\mathrm{R}_{\mathrm{f}}(1-\beta)$ is called Jensen's alpha ${ }^{3}$ and provides a measure of whether the investment in question earned a return greater than or less than its required return, given both market performance and risk. For instance, a firm that earned 15\% during a period, when firms with similar betas earned $12 \%$, will have earned an excess return of $3 \%$; its intercept will also exceed $\mathrm{R}_{\mathrm{f}}(1-\beta)$ by $3 \%$.

The third statistic that emerges from the regression is the $R$ squared $\left(R^{2}\right)$ of the regression. While the statistical explanation of the R squared is that it provides a measure of the goodness of fit of the regression, the economic rationale is that it provides an estimate of the proportion of the risk of a firm that can be attributed to market risk; the balance ( $1-\mathrm{R}^{2}$ ) can then be attributed to firm-specific risk.

The final statistic worth noting is the standard error of the beta estimate. The slope of the regression, like any statistical estimate, may be different from the true value; and the standard error reveals just how much error there could be in the estimate. The standard error can also be used to arrive at confidence intervals for the "true" beta value from the slope estimate.

## Illustration 8.1: Estimating a Regression Beta for Boeing

Boeing is a dominant firm in both the aerospace and defense businesses and has been traded on the NYSE for decades. In assessing risk parameters for Boeing, we compute the returns on the stock and the market index as follows.
(1) The returns to a stockholder in Boeing are computed month by month from January 1996 to December 2000. These returns include both dividends and price appreciation are defined as follows.

[^1]Stock Return $_{\text {Boeing }, \mathrm{j}=}=\frac{\text { Price }_{\text {Boeing }, \mathrm{j}}-\text { Price }_{\text {Boeing }, \mathrm{j}-1}+\text { Dividends }_{\mathrm{j}}}{\text { Price }_{\text {Boeing. } \mathrm{j}-1}}$ where Stock
Return $_{\text {Boeing. } \mathrm{j}}=$ Returns to a stockholder in Boeing in month j
Price $_{\text {Boeing }, \mathrm{j}}=$ Price of Boeing stock at the end of month j
Price $_{\text {Boeing.j }-1}=$ Price of Boeing stock at the end of month $\mathrm{j}-1$ (the previous month)
Dividends $_{\mathrm{j}}=$ Dividends on Boeing stock in month j
Dividends are added to the returns of the month in which stockholders are entitled to the dividend. ${ }^{4}$
(2) The returns on the S\&P 500 market index are computed for each month of the period, using the level of the index at the end of each month and the monthly dividend on the stocks in the index.

where Market $^{\text {Return }}=$ returns of the index in month $j$
Index ${ }_{j}=$ the level of the index at the end of month $j$
Index $_{\mathrm{j}-1}=$ the level of the index at the end of month $\mathrm{j}-1$ (the previous month)
Dividend $_{\mathrm{j}}=$ the dividends paid on the index in month j
While the S\&P 500 and the NYSE Composite are the most widely used indices for U.S. stocks, they are, at best, imperfect proxies for the market portfolio in the CAPM, which is supposed to include all assets.

Figure 8.1 graphs the monthly returns on Boeing against the monthly returns on the S\&P 500 index from January 1996 to December 2000.

[^2]

The regression statistics for Boeing are as follows:
(a) Slope of the regression $=0.56$. This is Boeing's beta, based on monthly returns from 1996 to 2000. Using a different time period for the regression or different return intervals (weekly or daily) for the same period can result in a different beta.
(b) Intercept of the regression $=0.54 \%$. This is a measure of Boeing's performance, when it is compared with $R_{f}(1-\beta)$. The monthly riskless rate (since the returns used in the regression are monthly returns) between 1996 and 2000 averaged $0.4 \%$, resulting in the following estimate for the performance:

$$
\begin{aligned}
& R_{f}(1-\beta)=0.4 \%(1-0.56)=0.18 \% \\
& \text { Intercept }-R_{f}(1-\beta)=0.54 \%-0.18 \%=0.36 \%
\end{aligned}
$$

This analysis suggests that Boeing performed $0.36 \%$ better than expected, when expectations are based on the CAPM and on a monthly basis between January 1996 and December 2000. This results in an annualized excess return of approximately $4.41 \%$.

$$
\begin{aligned}
\text { Annualized Excess Return } & =(1+\text { Monthly Excess Return })^{12}-1 \\
& =(1+0.0036)^{12}-1=4.41 \%
\end{aligned}
$$

Note, however, that this does not imply that Boeing would be a good investment in the future. The performance measure also does not provide a breakdown of how much of this excess return can be attributed to the performance of the entire sector (aerospace and defense) and how much is specific to the firm. To make that breakdown, we would need to compute the excess over the same period for other firms in the aerospace and defense industry and compare them with Boeing's excess return. The difference would be then attributable to firm-specific actions. In this case, for instance, the average annualized excess return on other aerospace/defense firms between 1996 and 2000 was $-0.85 \%$, suggesting that the firm-specific component of performance for Boeing is actually $5.26 \%$. (Firm-specific Jensen's alpha $=4.41 \%-(-0.85 \%))$
(c) $R$ squared of the regression $=9.43 \%$. This statistic suggests that $9.43 \%$ of the risk (variance) in Boeing comes from market sources and that the balance of $90.57 \%$ of the risk comes from firm-specific components. The latter risk should be diversifiable and therefore will not be rewarded with a higher expected return. Boeing's R squared is higher than the median R squared of companies listed on the New York Stock Exchange, which was approximately $19 \%$ in 2000.
(d) Standard Error of Beta Estimate $=0.23$. This statistic implies that the true beta for Boeing could range from 0.33 to 0.79 (subtracting and adding one standard error to beta estimate of 0.56 ) with $67 \%$ confidence and from 0.10 to 1.02 (subtracting and adding two standard error to beta estimate of 0.56 ) with $95 \%$ confidence. While these ranges may seem large, they are not unusual for most U.S. companies. This suggests that we should consider estimates of betas from regressions with caution.

## Using a Service Beta

Most of us who use betas obtain them from an estimation service; Merrill Lynch, Barra, Value Line, Standard and Poor's, Morningstar and Bloomberg are some of the well known services. All these services begin with the regression beta described above and adjust them to reflect what they feel are better estimates of future risk. Although many of these services do not reveal their estimation procedures, Bloomberg is an exception.

Figure 8.2 is the beta calculation page from Bloomberg for Boeing, using the same period as our regression (January 1996 to December 2000):

Figure 8.2: Bloomberg Beta Estimate for Boeing

P059 Equity BETA

## HISTORICAL BETA

Number of points may be insufficient for an accurate beta.

| BA US Equity |  |
| :---: | :---: |
| Relative Index | SPX |
| Period If Monthly |  |
| Range 1/31/96 To | 12/29/00 |
| Market IT Trade |  |
| ADJ BETA | 0.71 |
| RAW BETA | 0.5 ? |
| Alpha(Intercept) | 0.50 |
| R2 (Correlation) | 0.10 |
| Std Dev of Error | 8.17 |
| Std Error of Beta | 0.23 |
| Number of Points | 59 |

ADJ BETA $=(0.67) *$ RAL BETA
$+(0.33) * 1.0$


While the time period used is identical to the one used in our earlier regression, there are subtle differences between this regression and the earlier one in Figure 8.1. First, Bloomberg uses price appreciation in the stock and the market index in estimating betas and ignores dividends ${ }^{5}$. The fact that dividends are ignored does not make much of a difference for a company like Boeing, but it could make a difference for a company that either pays no dividends or pays significantly higher dividends than the market. This explains the mild differences in the intercept ( $0.50 \%$ versus $0.54 \%$ ) and the beta ( 0.57 versus 0.56 ).

Second, Bloomberg also computes what it calls an adjusted beta, which is estimated as follows.

$$
\text { Adjusted Beta }=\text { Raw Beta }(0.67)+1.00(0.33)
$$

[^3]These weights ( 0.67 and 0.33 ) do not vary across stocks and this process pushes all estimated betas toward one. Most services employ similar procedures to adjust betas towards one. In doing so, they are drawing on empirical evidence that suggests that the betas for most companies, over time, tend to move towards the average beta, which is one. This may be explained by the fact that firms get more diversified in their product mix and client base as they get larger.

## Estimation Choices for Beta Estimation

There are three decisions we must make in setting up the regression described above. The first concerns the length of the estimation period. Most estimates of betas, including those by Value Line and Standard and Poor's, use five years of data, while Bloomberg uses two years of data. The trade-off is simple: A longer estimation period provides more data, but the firm itself might have changed in its risk characteristics over the time period. Boeing, during the period of our analysis, acquired both Rockwell and McDonnell Douglas changing its business mix and its basic risk characteristics.

The second estimation issue relates to the return interval. Returns on stocks are available on an annual, monthly, weekly, daily and even on an intra-day basis. Using daily or intra-day returns will increase the number of observations in the regression, but it exposes the estimation process to a significant bias in beta estimates related to nontrading. ${ }^{6}$ For instance, the betas estimated for small firms, which are more likely to suffer from non-trading, are biased downwards when daily returns are used. Using weekly or monthly returns can reduce the non-trading bias significantly. 7 In this case, using weekly returns for 2 years yields a beta estimate for Boeing of only 0.88 , while the monthly beta estimate is 0.96 . The latter is a much more reliable estimate of the firm's beta.

The third estimation issue relates to the choice of a market index to be used in the regression. The standard practice used by most beta estimation services is to estimate the betas of a company relative to the index of the market in which its stock trades. Thus, the

[^4]betas of German stocks are estimated relative to the Frankfurt DAX, British stocks relative to the FTSE, Japanese stocks relative to the Nikkei and U.S. stocks relative to the NYSE composite or the $S \& P 500$. While this practice may yield an estimate that is a reasonable measure of risk for the domestic investor, it may not be the best approach for an international or cross-border investor, who would be better served with a beta estimated relative to an international index. For instance, Boeing's beta between 1993 and 1998 estimated relative to the Morgan Stanley Capital Index, an index that is composed of stocks from different global markets, yields a beta of 0.82 .

To the extent that different services use different estimation periods, different market indices and different beta adjustments, they will often provide different beta estimates for the same firm at the same point in time. While these beta differences are troubling, note that the beta estimates delivered by each of these services comes with a standard error and it is very likely that all the betas reported for a firm fall within the range of standard errors from the regressions.

## Historical Beta Estimate for Companies in Smaller (or Emerging) Markets

The process for estimating betas in markets with fewer stocks listed on them is no different from the process described above, but the estimation choices on return intervals, the market index and the return period can make a much bigger difference in the estimate.

- When liquidity is limited, as it often is in many stocks in emerging markets, the betas estimated using short return intervals tend to be much more biased. In fact, using daily or even weekly returns in these markets will tend to yield betas that are not good measures of the true market risk of the company.
- In many emerging markets, both the companies being analyzed and the market itself change significantly over short periods of time. Using five years of returns, as we did for Boeing, for a regression may yield a beta for a company (and market) that bears little resemblance to the company (and market) as it exists today.
- Finally, the indices that measure market returns in many smaller markets tend to be dominated by a few large companies. For instance, the Bovespa ( the Brazilian index) was dominated for several years by Telebras, which represented almost half the index. Nor is this just a problem with emerging markets. The DAX, the equity
index for Germany, is dominated by Allianz, Deutsche Bank. Siemens and Daimler. When an index is dominated by one or a few companies, the betas estimated against that index are unlikely to be true measures of market risk. In fact, the betas are likely to be close to one for the large companies that dominate the index and wildly variable for all other companies.


## Index Domination and Beta Estimates

There are a number of indices that are dominated by one or a few stocks. One of the most striking cases was the Helisinki Stock Exchange (HEX) in the late 1990s. Nokia, the telecommunications giant represented $75 \%$ of the Helisinki Index, in terms of market value. Not surprisingly, a regression of Nokia against the HEX yielded the results shown in Figure 8.3.

Figure 8.3: Beta Estimate for Nokia



#### Abstract

The regression looks impeccable. In fact, the noise problem that we noted with Boeing, arising from the high standard errors, disappears. The beta estimate has a standard error of 0.03 , but the results are deceptive. The low standard error is the result of a regression of Nokia on itself, since it dominates the index. The beta is meaningless to a typical investor in Nokia, who is likely to be diversified, if not globally, at least across European stocks. Worse still, the betas of all other Finnish stocks against the HEX become betas estimated against Nokia. In fact, the beta of every other Finnish stock at the time of this regression was less than 1 . How is this possible, you might ask, if the average beta is one? It is the weighted average beta that is one, and if Nokia which comprises three quarters of the index has a beta greater than one (which it does), every other stock in the index could well end up with a beta less than one.


## Illustration 8.2: Estimating a Beta for Titan Cements

Titan Cements is a cement and construction company in Greece. Reproduced below in Figure 8.4 is the beta estimate for Titan obtained from a beta service (Bloomberg) from January 1996 to December 2000.

Figure 8.4: Beta Estimate for Titan Cement: Athens Stock Exchange Index

## HISTORICAL BETA



| ADJ BETA | 0.95 |
| :--- | ---: |
| RAW BETA | 0.93 |
| Alphaintercept) | 0.31 |
| R2 (Correlation) | 0.57 |
| Std Dev of Error | 3.85 |
| Std Error of Beta | 0.08 |
| Rumber of Points | 103 |

ADJ BETA $=(0.67) *$ RAN BETA $+(0.33) * 1.0$


Note that the index used is the Athens Stock Index. This is a fairly conventional choice since most services estimate betas against a local index. Based upon this regression, we arrive at the following equation.

$$
\begin{equation*}
\text { Returns }_{\text {Titan Cement }}=0.31 \%+0.93 \text { Returns }_{\text {ASE }} \quad \text { R squared }=57 \% \tag{0.08}
\end{equation*}
$$

The beta for Titan Cements, based upon this regression, is 0.93 . The standard error of the estimate, shown in brackets below, is only 0.08 , but the caveats about narrow indices apply to the Athens Stock Exchange Index.

Drawing on the arguments in the previous section, if the marginal investor in Titan Cements is, in fact, an investor diversified across European companies, the appropriate index would have been a European stock index. The Bloomberg beta calculation with the MS European Index is reported below in Figure 8.5.

Figure 8.5: Beta Estimate for Titan: MSCI Euro Index

## HISTORICAL BETA

## TITK 6月 [quity <br> TITAN CEMENT CO. S.f.

Relative Index HSER
Perlod Weekly

| Range $4 / 23 / 9 y$ |
| :--- |
| Harket Ti Trade | 4/13/01


| ADJ BETA | 0.55 |
| :--- | :--- |
| RAW BETA | 0.33 |
| Alpha(Intercept) | 0.24 |
| R2 (Correlatton) | 0.03 |
| Std Dev of Error | 5.78 |
| Std Error of Beta | 0.20 |
| Number of Polnts | 103 |

```
ADJ BETA = (0.67) }\times\mathrm{ RAN BETA
    +(0.33)*1.0
```



Note the decline in beta to 0.33 and the increase in the standard error of the beta estimate.
In fact, if the marginal investor is globally diversified, Titan Cement's beta (as well as Boeing's beta in the previous illustration) should have been estimated against a global index. Using the Morgan Stanley Capital Index (MSCI), we get the regression beta of 0.33 in Figure 8.6.

Figure 8.6: Beta Estimate For Titan Cement: MSCI Global Index

## HISTORICAL BETA



In fact, the beta estimate and the standard error look very similar to the ones estimated against the European index.

## Estimating the Historical Beta for Private Firms

The historical approach to estimating betas works only for assets that have been traded and have market prices. Private companies do not have a market price history. Consequently, we cannot estimate a regression beta for these companies. Nevertheless, we still need estimates of cost of equity and capital for these companies.

You might argue that this is not an issue because you do not value private companies but you will still be confronted with this issue even when valuing publicly traded firms. Consider, for instance, the following scenarios.

- If you have to value a private firm for an initial public offering, you will need to estimate discount rates for the valuation.
- Even after a firm has gone public, there will be a period of time lasting as long as two years when there will be insufficient data for a regression.
- If you are called upon to value the division of a publicly traded firm that is up for sale, you will not have past prices to draw upon to run a regression.
- Finally, if your firm has gone through significant restructuring - divestitures or recapitalization - in the recent past, regression betas become meaningless because the company itself has changed its risk characteristics.
Thus, regression betas are either unavailable or meaningless in a significant number of valuations.

Some analysts assume that discounted cash flow valuation is not feasible in these scenarios and use multiples. Others make assumptions about discount rates based upon rules of thumb. Neither approach is appealing. In the next section, we will develop an approach for estimating betas that is general enough to apply to all of these companies.
risk.xls. This spreadsheet allows you to run a regression of stock returns against market returns and estimate risk parameters.

## The Limitations of Regression Betas

Much of what we have presented in this section represents an indictment of regression betas. In the case of Boeing, the biggest problem was that the beta had high standard error. In fact, this is not a problem unique to Boeing. Figure 8.7 presents the distribution of standard errors on beta estimates for US companies.

Figure 8.7: Distribution of Standard Errors on Beta Estimates


With the Nokia regression, we seem to cure the standard error problem but at a very large cost. The low standard errors reflect the domination of the index by a stock and result in betas that may be precise but bear no resemblance to true risk.

Changing the market index, the return period and return interval offer no respite. If the index becomes a more representative index, the standard errors on betas will increase, reflecting the fact that more of the risk in the stock is firm-specific. If the beta changes as the return period or interval changes, it creates more uncertainty about the true beta of the company.

In short, regression betas will almost always be either too noisy or skewed by estimation choices to be useful measures of the equity risk in a company. The cost of equity is far too important an input into a discounted cash flow valuation to be left to statistical chance.

## B. Fundamental Betas

A second way to estimate betas is to look at the fundamentals of the business. The beta for a firm may be estimated from a regression but it is determined by decisions the firm has made on what business to be in, how much operating leverage to use in the business and by the degree to which the firm uses financial leverage. In this section, we
will examine an alternative way of estimating betas for firms, where we are less reliant on historical betas and more cognizant of their fundamental determinants.

## Determinants of Betas

The beta of a firm is determined by three variables -(1) the type of business or businesses the firm is in, (2) the degree of operating leverage of the firm and (3) the firm's financial leverage. Although we will use these determinants to find betas in the capital asset pricing model, the same analysis can be used to calculate the betas for the arbitrage pricing and the multi-factor models as well.

Type of Business
Since betas measure the risk of a firm relative to a market index, the more sensitive a business is to market conditions, the higher its beta. Thus, other things remaining equal, cyclical firms can be expected to have higher betas than non-cyclical firms. Companies involved in housing and automobiles, two sectors of the economy which are very sensitive to economic conditions, should have higher betas than companies in food processing and tobacco, which are relatively insensitive to business cycles.

We can extend this view to a company's products. The degree to which a product's purchase is discretionary will affect the beta of the firm manufacturing the product. Firms whose products are much more discretionary to their customers should have higher betas than firms whose products are viewed as necessary or less discretionary. Thus, the beta of Procter and Gamble, which sells diapers and daily household products, should be lower than the beta of Gucci, which manufactures luxury products.

Degree of Operating Leverage
The degree of operating leverage is a function of the cost structure of a firm and is usually defined in terms of the relationship between fixed costs and total costs. A firm that has high fixed costs relative to total costs is said to have high operating leverage. A firm with high operating leverage will also have higher variability in operating income than would a firm producing a similar product with low operating leverage. Other things remaining equal, the higher variance in operating income will lead to a higher beta for the firm with high operating leverage.

Can firms change their operating leverage? While some of a firm's cost structure is determined by the business it is in (an energy utility has to build expensive power plants and airlines have to lease expensive planes), firms in the United States have become increasingly inventive in lowering the fixed cost component in their total costs. For instance, firms have made cost structures more flexible by

- negotiating labor contracts that emphasize flexibility and allow the firm to make its labor costs more sensitive to its financial success;
- entering into joint venture agreements, where the fixed costs are borne or shared by someone else; and
- sub-contracting manufacturing and outsourcing, which reduce the need for expensive plant and equipment.

While the arguments for such actions may be couched in terms of offering competitive advantage and flexibility, they do also reduce the operating leverage of the firm and its exposure to market risk.

While operating leverage affects betas, it is difficult to measure the operating leverage of a firm, at least from the outside, since fixed and variable costs are often aggregated in income statements. It is possible to get an approximate measure of the operating leverage of a firm by looking at changes in operating income as a function of changes in sales.

Degree of Operating leverage $=\%$ Change in Operating Profit $/ \%$ Change in Sales For firms with high operating leverage, operating income should change more than proportionately when sales change.

## Size, Growth and Betas

Generally, smaller firms with higher growth potential are viewed as riskier than larger, more stable firms. While the rationale for this argument is clear when talking about total risk, it becomes more difficult to see when looking at market risk or betas. Should a smaller software firm have a higher beta than a larger software firm? One reason to believe that it should is operating leverage. If there is a set-up cost associated with investing in infrastructure or economies of scale, smaller firms will have higher fixed costs than larger firms, leading in turn to higher betas for these firms.

With growth firms, the argument for higher betas rests on the notion of discretionary versus non-discretionary purchases. For a high growth firm to deliver on its growth, new customers have to adopt the product or existing customers have to buy more of the product. Whether they do so or not will depend, in large part, on how well-off they feel. This, in turn, will make the profits of high growth firms much more dependent on how well the economy is doing, thus increasing their betas.

## Degree of Financial Leverage

Other things remaining equal, an increase in financial leverage will increase the beta of the equity in a firm. Intuitively, we would expect that the fixed interest payments on debt result in high net income in good times and low or negative net income in bad times. Higher leverage increases the variance in net income and makes equity investment in the firm riskier. If all the firm's risk is borne by the stockholders (i.e., the beta of debt is zero) ${ }^{8}$ and debt has a tax benefit to the firm, then,

$$
\beta_{L}=\beta_{u}\left(1+(1-t)\left(\frac{D}{E}\right)\right)
$$

where
$\beta_{\mathrm{L}}=$ Levered Beta for equity in the firm
$\beta_{u}=$ Unlevered beta of the firm (i.e., the beta of the firm without any debt)
$t=$ Corporate tax rate
D/E = Debt/Equity Ratio
Intuitively, we expect that as leverage increases (as measured by the debt to equity ratio), equity investors bear increasing amounts of market risk in the firm, leading to higher betas. The tax factor in the equation measures the tax deductibility of interest payments.
${ }^{8}$ This formula was originally developed by Hamada in 1972. There are two common modifications. One is to ignore the tax effects and compute the levered beta as:

$$
\beta_{L}=\beta_{u}\left(1+\frac{D}{E}\right)
$$

If debt has market risk (i.e., its beta is greater than zero), the original formula can be modified to take it into account. If the beta of debt is $\beta_{D}$, the beta of equity can be written as:

$$
\beta_{L}=\beta_{u}\left(1+(1-t)\left(\frac{D}{E}\right)\right)-\beta_{D}(1-t)\left(\frac{D}{E}\right)
$$

The unlevered beta of a firm is determined by the types of the businesses in which it operates and its operating leverage. It is often also referred to as the asset beta since it is determined by the assets owned by the firm. Thus, the levered beta, which is also the beta for an equity investment in a firm or the equity beta, is determined both by the riskiness of the business it operates in and by the amount of financial leverage risk it has taken on.

Since financial leverage multiplies the underlying business risk, it stands to reason that firms that have high business risk should be reluctant to take on financial leverage. It also stands to reason that firms that operate in stable businesses should be much more willing to take on financial leverage. Utilities, for instance, have historically had high debt ratios but have not had high betas, mostly because their underlying businesses have been stable and fairly predictable.

## Illustration 8.3: Effects of Leverage on betas: Boeing

From the regression for the period from 1996 to 2000, Boeing had a historical beta of 0.56 . Since this regression uses stock prices of Boeing over this period, we begin by estimating the average debt/equity ratio between 1996 and 2000, using market values for debt and equity.

Average Debt/Equity Ratio between 1996 and $2000=15.56 \%$
The beta over the 1996-2000 period reflects this average leverage. To estimate the unlevered beta over the period, we used a corporate tax rate of $35 \%$.

Unlevered Beta $=\frac{\text { Current Beta }}{1+(1-\text { tax rate })(\text { Average Debt/Equity })}=\frac{0.56}{1+(1-0.35)(0.1556)}=0.51$

The unlevered beta for Boeing over the $1996-2000$ period is 0.51 . The levered beta at different levels of debt can then be estimated.

Levered Beta $=$ Unlevered Beta1 $+((1-$ tax rate $)($ Debt/ Equity $))$
For instance, if Boeing were to decrease its debt equity ratio to $10 \%$, its equity beta will be:

Levered Beta $(@ 10 \% \mathrm{D} / \mathrm{E})=0.51(1+(1-0.35)(0.10))=0.543$
If the debt equity ratio were raised to $25 \%$, the equity beta would be
Levered Beta $(@ 25 \% \mathrm{D} / \mathrm{E})=0.51(1+(1-0.35)(0.25))=0.59$
Table 8.1 summarizes the beta estimates for different levels of financial leverage ranging from 0 to $90 \%$ debt.

Table 8.1: Financial Leverage and Betas

| Debt to Capital | Debt/Equity Ratio | Beta | Effect of Leverage |
| :---: | :---: | :---: | :---: |
| $0.00 \%$ | $0.00 \%$ | 0.51 | 0.00 |
| $10.00 \%$ | $11.11 \%$ | 0.55 | 0.04 |
| $20.00 \%$ | $25.00 \%$ | 0.59 | 0.08 |
| $30.00 \%$ | $42.86 \%$ | 0.65 | 0.14 |
| $40.00 \%$ | $66.67 \%$ | 0.73 | 0.22 |
| $50.00 \%$ | $100.00 \%$ | 0.84 | 0.33 |
| $60.00 \%$ | $150.00 \%$ | 1.00 | 0.50 |
| $70.00 \%$ | $233.33 \%$ | 1.28 | 0.77 |
| $80.00 \%$ | $400.00 \%$ | 1.83 | 1.32 |
| $90.00 \%$ | $900.00 \%$ | 3.48 | 2.98 |

As Boeing's financial leverage increases, the beta increases concurrently.
levbeta.xls.This spreadsheet allows you to estimate the unlevered beta for a firm and compute the betas as a function of the leverage of the firm.

## Bottom Up Betas

Breaking down betas into their business risk and financial leverage components provides us with an alternative way of estimating betas in which we do not need past prices on an individual firm or asset.

To develop this alternative approach, we need to introduce an additional property of betas that proves invaluable. The beta of two assets put together is a weighted average of the individual asset betas, with the weights based upon market value. Consequently,
the beta for a firm is a weighted average of the betas of all the different businesses it is in. We can estimate the beta for a firm in five steps.

- Step 1: We identify the business or businesses the firm operates in.
- Step 2: We find other publicly traded firms in these businesses and obtain their regression betas, which we use to compute an average beta for the firms, and their financial leverage.
- Step 3: We estimate the average unlevered beta for the business, by unlevering the average beta for the firm by their average debt to equity ratio. Alternatively, we could estimate the unlevered beta for each firm and then compute the average of the unlevered betas. The first approach is preferable because unlevering an erroneous regression beta is likely to compound the error.

$$
\text { Unlevered Beta }_{\text {Business }}=\frac{\text { Beta }_{\text {comparable firms }}}{1+(1-t)(\text { D/E ratiocomparable firms })}
$$

- Step 4: To estimate an unlevered beta for the firm that we are analyzing, we take a weighted average of the unlevered betas for the businesses it operates in, using the proportion of firm value derived from each business as the weights. If values are not available, we use operating income or revenues as weights. This weighted average is called the bottom-up unlevered beta.

$$
\text { Unlevered Beta }_{\text {firm }}=\sum_{j=1}^{i=k} \text { Unlevered Beta }_{\mathrm{j}} * \text { Value Weight }_{\mathrm{j}}
$$

where the firm is assumed to operating in k different businesses.

- Step 5: Finally, we estimate the current market values of debt and equity of the firm and use this debt to equity ratio to estimate a levered beta.
The betas estimated using this process are called bottom-up betas.


## The Case for Bottom Up Betas

At first sight, the use of bottom up betas may seem to leave us exposed to all of the problems we noted with regression betas. After all, the betas for other publicly traded firms in the business are obtained from regressions. Notwithstanding these bottom up betas represent a significant improvement on regression betas for the following reasons.

- While each regression beta is estimated with standard error, the average across a number of regression betas will have much lower standard error. The intuition is simple. A high standard error on a beta estimate indicates that it can be significantly higher or lower than the true beta. Averaging across these errors results in an average beta that is far more precise than the individual betas that went into it. In fact, if the estimation errors on individual firm betas are uncorrelated across firms, the savings in standard error can be stated as a function of the average standard error and the number of firms in the sample.

$$
\text { Standard } \text { Error }_{\text {Botom-up beta }}=\frac{\text { Average Standard Error }}{\text { Comparable firms }}
$$

where n is the number of firms in the sample. Thus, if the average standard error in beta estimates for software firms is 0.50 and the number of software firms is 100 , the standard error of the average beta is only $0.05(0.50 / \sqrt{100})$.

- A bottom-up beta can be adapted to reflect actual changes in a firm's business mix and expected changes in the future. Thus, if a firm divested a major portion of its operations last week, the weights on the businesses can be modified to reflect the divestiture. The same can be done with acquisitions. In fact, a firm's strategic plans to enter new businesses in the future can be brought into the beta estimates for future periods.
- Firms do change their debt ratios over time. While regression betas reflect the average debt to equity ratio maintained by the firm during the regression period, bottom-up betas use the current debt to equity ratio. If a firm plans to change its debt to equity ratio in the future, the beta can be adjusted to show these changes.
- Finally, bottom-up betas wean us from our dependence on historical stock prices. While we do need these prices to get betas for comparable firms, all we need for the firm being analyzed is a breakdown of the businesses it is in. Thus, bottom-up betas can be estimated for private firms, divisions of business and stocks that have just started trading in financial markets.

Computational Details
While the idea behind bottom-up betas is fairly simple, there are several computational details that are deserving of attention.
a. Defining Comparable firms: First, we have to decide how narrowly we want to define a business. Consider, for instance, a firm that manufactures entertainment software. We could define the business as entertainment software and consider only companies that primarily manufacture entertainment software to be comparable firms. We could go even further and define comparable firms as firms manufacturing entertainment software with revenues similar to that of the company being analyzed. While there are benefits to narrowing the comparable firm definition, there is a large cost. Each additional criterion added on to the definition of comparable will mean that fewer firms make the list and the savings in standard error that comprise the biggest benefit to bottom-up betas become smaller. A common sense principle should therefore come into play. If there are hundreds of firms in a business, as there are in the software business, you can afford to be more selective. If there are relatively few firms, not only do you have to become less selective, you might have to broaden the definition of comparable to bring in other firms into the mix.
b. Estimating Betas: Once the comparable firms in a business have been defined, you have to estimate the betas for these firms. While it would be best to estimate the regressions for all of these firms against a common and well diversified equity index, it is usually easier to use service betas that are available for each of these firms. These service betas may be estimated against different indices. For instance, if you define your business to be global telecommunications and obtain betas for global telecomm firms from Bloomberg, these betas will be estimated against their local indices. This is usually not a fatal problem, especially with large samples, since errors in the estimates tend to average out.
c. Averaging Method: The average beta for the firms in the sector can be computed in one of two ways. We could use market-weighted averages, but the savings in standard error that we touted in the earlier section will be muted, especially if there are one or two very large firms in the sample. We could estimate the simple
average of the betas of the companies, thus weighting all betas equally. The process weighs in the smallest firms in the sample disproportionately but the savings in standard error are likely to be maximized. There is also the issue of whether the firm being analyzed should be excluded from the group when computing the average. While the answer is yes, there will make little or no difference in the final estimate if there are more than 15 or 20 comparable firms.
d. Controlling for differences: In essence, when we use betas from comparable firms, we are assuming that all firms in the business are equally exposed to business risk and have similar operating leverage. Note that the process of levering and unlevering of betas allows us to control for differences in financial leverage. If there are significant differences in operating leverage - cost structure - across companies, the differences in operating leverage can be controlled for as well. This would require that we estimate a business beta, where we take out the effects of operating leverage from the unlevered beta.

$$
\text { Business Beta }=\frac{\text { Unlevered Beta }}{1+(1-\text { tax rate })(\text { Fixed Costs/ Variable Costs })}
$$

Note the similarity to the adjustment for financial leverage; the only difference is that both fixed and variable costs are eligible for the tax deduction and the tax rate is therefore no longer a factor. The business beta can then be relevered to reflect the differences in operating leverage across firms.

betas.xls: This dataset on the web has updated betas and unlevered betas by business sector in the United States.

Illustration 8.3: Estimating a Bottom-up Beta for Vans Shoes - January 2001
Vans Shoes is a shoe manufacturing firm with a market capitalization of \$191 million. To estimate the bottom up beta for Vans Shoes, we consider the betas of all publicly traded shoe manufacturers in Table 8.2.

Table 8.2: Betas and Relevant Statistics: Publicly Traded Shoe Companies

| Company Name | Beta | Market D/E | Tax Rate | Fixed/Variable |
| :--- | ---: | ---: | ---: | ---: |
| Barry (R.G.) | 1.00 | $40.51 \%$ | $36.89 \%$ | $75.66 \%$ |
| Brown Shoe | 0.80 | $106.64 \%$ | $37.06 \%$ | $61.41 \%$ |
| Candie's Inc. | 1.20 | $75.86 \%$ | $0.00 \%$ | $29.78 \%$ |
| Converse Inc | 0.60 | $653.46 \%$ | $0.00 \%$ | $39.64 \%$ |
| Deckers Outdoor Corp | 0.80 | $82.43 \%$ | $0.00 \%$ | $62.52 \%$ |
| Florsheim Group Inc | 0.65 | $96.79 \%$ | $32.47 \%$ | $79.03 \%$ |
| K-Swiss Inc. | 0.65 | $0.69 \%$ | $40.94 \%$ | $56.92 \%$ |
| Kenneth Cole 'A' | 1.05 | $0.29 \%$ | $39.50 \%$ | $56.97 \%$ |
| LaCrosse Footwear Inc | 0.55 | $81.15 \%$ | $39.25 \%$ | $30.36 \%$ |
| Maxwell Shoe Inc | 0.75 | $2.24 \%$ | $33.28 \%$ | $20.97 \%$ |
| NIKE Inc. 'B' | 0.90 | $9.47 \%$ | $39.50 \%$ | $46.07 \%$ |
| Reebok Int'l | 1.05 | $171.90 \%$ | $32.28 \%$ | $35.03 \%$ |
| Rocky Shoes \& Boots Inc. | 0.80 | $93.51 \%$ | $0.00 \%$ | $26.89 \%$ |
| Saucony Inc | 0.15 | $34.93 \%$ | $31.11 \%$ | $49.33 \%$ |
| Shoe Carnival | 0.85 | $2.18 \%$ | $39.97 \%$ | $35.03 \%$ |
| Stride Rite Corp. | 0.80 | $0.00 \%$ | $36.80 \%$ | $48.23 \%$ |
| Timberland Co. 'A' | 1.10 | $15.23 \%$ | $32.00 \%$ | $49.50 \%$ |
| Vulcan Int'L | 0.65 | $3.38 \%$ | $5.61 \%$ | $11.92 \%$ |
| Wellco Enterprises Inc. | 0.60 | $48.89 \%$ | $0.00 \%$ | $11.52 \%$ |
| Weyco Group | 0.30 | $11.91 \%$ | $35.74 \%$ | $24.69 \%$ |
| Wolverine World Wide | 1.35 | $44.37 \%$ | $32.62 \%$ | $32.31 \%$ |
| Average (Simple) | 0.79 | $75.04 \%$ | $25.95 \%$ | $42.08 \%$ |

[NOTE: I have $42.08 \%$ for the Fixed/Variable average. Suggestion: Instead of changing the calculations, just decrease Wolverine World Wide to $22.00 \%$ to make the average 41.59\%.]

In addition to the betas for each firm, we report the market debt to equity ratio, the effective tax rate and a measure of operating leverage obtained by dividing S,G\&A expenses (which we consider fixed) by other operating expenses (which we consider variable). We can estimate the unlevered beta for the business using the averages for these values.

Average Beta $=0.79$
Average Debt to Equity Ratio $=75.04 \%$
Using the effective tax rate of $25.95 \%$, we can estimate the unlevered beta.
Unlevered Beta $_{\text {Vans }}=\frac{0.79}{1+(1-0.2595)(0.7504)}=0.5081$

The beta for Vans Shoes can then be obtained, using the firm's marginal tax rate of $34.06 \%$ and market debt to equity ratio of $9.41 \%$.
Unlevered Beta ${ }_{\text {Vans }}=0.5081(1+(1-0.3406)(0.0941))=0.5397$
This levered beta is based on the implicit assumption that all shoe manufacturers have similar operating leverage. In fact, we could adjust the unlevered beta for the average fixed cost/variable cost ratio for the business and then relever back at the operating leverage for Vans Shoes.
Average Fixed Cost $/$ Variable Cost Ratio $=42.08 \%$
Business Beta $=\frac{\text { Unlevered Beta }}{1+\frac{\text { Fixed Costs }}{\text { Variable Costs }}}=\frac{0.5081}{1+0.4208}=0.3576$
We can then use Van's fixed cost to variable cost ratio of $31.16 \%$ to estimate an adjusted unlevered and levered beta.

Unlevered Beta $_{\text {Vans }}=0.3576(1+0.3116)=0.4691$
Levered Beta $_{\text {Vans }}=0.4691(1+(1-0.3406)(0.0941))=0.4981$
By having a debt to equity ratio and an operating leverage that is lower than the average for the industry, Vans Shoes ends up with a beta much lower than that of the industry.

## Illustration 8.4: Estimating a Bottom-up Beta for Boeing: September 2000

Boeing has undergone a significant change in both its business mix and its financial leverage over the last 5 years. Not only did it acquire Rockwell and McDonnell Douglas, giving it a major foothold in the defense business, but it borrowed substantial amounts to make these acquisitions. Since these events have occurred in the last few years, the historical regression beta does not fully reflect the effects of these changes. To estimate Boeing's beta today, we broke its business into two areas.

- Commercial Aircraft, which is Boeing's core business of manufacturing commercial jet aircraft and providing related support services.
- Information, Space and Defense Systems (ISDS), which includes research, development, production and support of military aircraft, helicopters and missile systems.

Each of these areas of business has very different risk characteristics and we estimated the unlevered beta for each business by looking at comparable firms in each business. Table 8.3 summarizes these estimates.

Table 8.3: Estimating Unlevered Betas for Boeing's Business Areas

| Segment | Revenues | Value/Sales <br> Ratiofor <br> Segment | Estimated <br> Value | Unlevered <br> Beta | Segment <br> Weight | Weighted <br> Beta |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Commercial <br> Aircraft | $\$ 26,929$ | 1.12 | 30,160 | 0.91 | $70.39 \%$ | 0.6405 |
| ISDS | $\$ 18,125$ | 0.70 | 12,688 |  | 0.8 | $29.61 \%$ |

For ISDS, we used 17 firms that derived the bulk of their revenues from defense contracting and computed the average beta and debt/equity ratio for these firms. The unlevered beta was computed using these averages. For commercial aircraft, there are no truly comparable firms. We looked at Boeing's own beta prior to its expansion in the defense business and computed the unlevered beta using this estimate. The values for each of the divisions was estimated using the revenues ${ }^{9}$ from each segment and a typical revenue multiple ${ }^{10}$ for that type of business. The unlevered beta for Boeing as a company in 2000 can be estimated by taking a value-weighted average of the betas of each of the different business areas. This is reported in the last column to be 0.8774 .

The equity beta can then be estimated using the current financial leverage for Boeing as a firm. Combining the market value of equity of $\$ 55.2$ billion and the market value of debt of $\$ 7.85$ billion, using a $35 \%$ tax rate for the firm, we arrive at the current beta for Boeing.

[^5]$$
\text { Equity Beta for Boeing }=0.8774\left(1+(1-0.35)\left(\frac{7.85}{55.2+7.85}\right)\right)=0.9585
$$

This is very different from the historical beta of 0.56 that we obtained from the regression, but it is, in our view, a much truer reflection of the risk in Boeing.

## Illustration 8.5: Estimating a Bottom-up Beta for Titan Cements - January 2000

To estimate a beta for Titan Cements, we began by defining comparable firms as other cement companies in Greece but found only one comparable firm. When we expanded the list to include cement companies across Europe, we increased our sample to nine firms. Since we did not see any reason to restrict our comparison to just European firms, we decided to look at the average beta for cement companies globally. There were 108 firms in this sample with an average beta of 0.99 , an average tax rate of $34.2 \%$ and an average debt to equity ratio of $27.06 \%$. We used these numbers to arrive at an unlevered beta of 0.84 .
Unlevered Beta for cement companies $=\frac{0.99}{1+(1-0.342)(0.2706)}=0.84$
We then used Titan's market values of equity ( 566.95 million Gdr) and debt (13.38 million GDr) to estimate a levered beta for its equity:
Levered Beta $=0.84\left(1+(1-0.2414)\left(\frac{13.38}{566.95}\right)\right)=0.86$
We used a tax rate of $24.14 \%$ in this calculation.

## How well do betas travel?

Often, when analyzing firms in small or emerging markets, we have to estimate betas by looking at firms in the same business but traded on other markets. This is what we did when estimating the beta for Titan Cements. Is this appropriate? Should the beta for a steel company in the United States be comparable to that of a steel company in Indonesia? We see no reason why it should not. But the company in Indonesia has much more risk, you might argue. We do not disagree, but the fact that we use similar betas does not mean that we believe that the costs of equity are identical across all steel companies. In fact, using the approach described in the last chapter, the risk premium used to estimate the cost of equity for the Indonesian company will incorporate a country risk
premium, whereas the cost of equity for the U.S. company will not. Thus, even if the betas used for the two companies are identical, the cost of equity for the Indonesian company will be much higher.

There are a few exceptions to this proposition. Recall that one of the key determinants of betas is the degree to which a product or service is discretionary. It is entirely possible that products or services that are discretionary in one market (and command high betas) may be non-discretionary in another market (and have low betas). For instance, phone service is viewed as a non-discretionary product in most developed markets, but is a discretionary product in emerging markets. Consequently, the average beta estimated by looking at telecom firms in developed markets will understate the true beta of a telecomm firm in an emerging market. Here, the comparable firms should be restricted to include only telecomm firms in emerging markets.

## Calculating Betas after A Major Restructuring

The bottom-up process of estimating betas provides a solution when firms go through a major restructuring that change both their business mix and leverage. In these cases, the regression betas are misleading because they do not reflect fully the effects of these changes. Boeing's beta, estimated using the bottom up approach, is likely to provide a more precise estimate than the historical beta from a regression of Boeing's stock prices, given Boeing's acquisitions of Rockwell and McDonnell Douglas and its increase in leverage. In fact, a firm's beta can be estimated even before the restructuring becomes effective using the bottom-up approach. In the illustration that follows, for instance, we estimate Boeing's beta just before and after its acquisition of McDonnell Douglas, allowing for the changes in both the business mix and the leverage.

## Illustration 8.6: Beta of a Firm After an Acquisition: Boeing and McDonnell Douglas

In 1997, Boeing announced that it was acquiring McDonnell Douglas, another company involved in the aerospace and the defense business. At the time of the acquisition, the two firms had the following market values and betas:

| Company | Beta | Debt | Equity | Firm Value |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Boeing | 0.95 | $\$ 3,980$ | $\$ 32,438$ | $\$ 36,418$ |  |


| McDonnell Douglas | 0.90 | $\$$ | 2,143 | $\$$ | 12,555 | $\$$ | 14,698 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Note that the market values of equity used for the two firms reflect the market values after the acquisition announcement and reflect the acquisition price agreed upon for McDonnell Douglas shares.

In order to evaluate the effects of the acquisition on Boeing's beta, we first examine the effects of the merger on the business risk of the combined firm, by estimating the unlevered betas of the two companies and calculating the combined firm's unlevered beta.

$$
\text { Boeings unlevered beta }=\frac{0.95}{1+(0.65)\left(\frac{3980}{32438}\right)}=0.88
$$

$$
\text { McDonnell Douglas unlevered beta }=\frac{0.90}{1+(0.65)\left(\frac{2143}{12555}\right)}=0.81
$$

The unlevered beta for the combined firm can be calculated as the weighted average of the two unlevered betas, with the weights based upon the market values of the two firms.

$$
\text { Unlevered Beta for combined firm }=0.88\left(\frac{36,418}{51,116}\right)+0.81\left(\frac{14,698}{51,116}\right)=0.86
$$

Boeing's acquisition of McDonnell Douglas was accomplished by issuing new stock in Boeing to cover the value of McDonnell Douglas's equity of $\$ 12,555$ million. Since no new debt was used to finance the deal, the debt outstanding in the firm after the acquisition is just the sum of the debt outstanding at the two companies before the acquisition.

$$
\begin{aligned}
\text { Debt } & =\text { McDonnell Douglas Old Debt }+ \text { Boeing's Old Debt } \\
& =\$ 3,980+\$ 2,143=\$ 6,123 \text { million } \\
\text { Equity } & =\text { Boeing's Old Equity }+ \text { New Equity used for Acquisition } \\
& =\$ 32,438+\$ 12,555=\$ 44,993 \text { million }
\end{aligned}
$$

The debt/equity ratio can then be computed as follows -
D/E Ratio $=\$ 6,123 / 44,993=13.61 \%$
This debt/equity ratio in conjunction with the new unlevered beta for the combined firm yields a new beta of

New Beta $=0.86(1+(0.65)(0.1361))=0.94$

## C. Accounting Betas

A third approach is to estimate the market risk parameters from accounting earnings rather than from traded prices. Thus, changes in earnings at a division or a firm, on a quarterly or annual basis, can be related to changes in earnings for the market, in the same periods, to arrive at an estimate of an accounting beta to use in the CAPM. While the approach has some intuitive appeal, it suffers from three potential pitfalls. First, accounting earnings tend to be smoothed out relative to the underlying value of the company, as accountants spread expenses and income over multiple periods. This results in betas that are "biased down", especially for risky firms, or "biased up", for safer firms. In other words, betas are likely to be closer to 1.00 for all firms using accounting data.

Second, accounting earnings can be influenced by non-operating factors, such as changes in depreciation or inventory methods and by allocations of corporate expenses at the divisional level. Finally, accounting earnings are measured, at most, once every quarter and often only once every year, resulting in regressions with few observations and not much explanatory power (low R-squared, high standard errors).

## Illustration 8.7: Estimating Accounting Betas —Defense Division of Boeing - 1995

Having operated in the defense business for decades, Boeing has a record of its profitability. These profits are reported in Table 8.4, together with earnings changes for companies in the S\&P 500 going back to 1980.

Table 8.4: Earnings on Defense Business - Boeing

| Year | S\& P 500 | Boeing Defense <br> Business |
| :---: | :---: | :---: |
| 1980 | $-2.10 \%$ | $-12.70 \%$ |
| 1981 | $-6.70 \%$ | $-35.56 \%$ |
| 1982 | $-45.50 \%$ | $27.59 \%$ |
| 1983 | $37.00 \%$ | $159.36 \%$ |
| 1984 | $41.80 \%$ | $13.11 \%$ |
| 1985 | $-11.80 \%$ | $-26.81 \%$ |


| 1986 | $7.00 \%$ | $-16.83 \%$ |
| :---: | :---: | :---: |
| 1987 | $41.50 \%$ | $20.24 \%$ |
| 1988 | $41.80 \%$ | $18.81 \%$ |
| 1989 | $2.60 \%$ | $-29.70 \%$ |
| 1990 | $-18.00 \%$ | $-40.00 \%$ |
| 1991 | $-47.40 \%$ | $-35.00 \%$ |
| 1992 | $64.50 \%$ | $10.00 \%$ |
| 1993 | $20.00 \%$ | $-7.00 \%$ |
| 1994 | $25.30 \%$ | $11.00 \%$ |

## Source: Bloomberg

Regressing the changes in profits in the defense division ( $\Delta$ Earnings $_{\text {Defense }}$ ) against changes in profits for the S\&P $500\left(\Delta\right.$ Earnings $\left._{\text {S\&P }}\right)$ yields the following:
$\Delta$ Earnings $_{\text {Defense }}=-0.03+0.65 \Delta$ Earnings $_{\text {s\&P }}$
Based upon this regression, the beta for the defense division is 0.65 . We can now estimate the cost of equity for the defense division, with a riskless rate of $5 \%$ and a risk premium of $5.5 \%$, as follows.

Cost of Equity for the Defense Division $=5 \%+0.65(5.5 \%)=8.58 \%$
accbeta.xls: This spreadsheet allows you to estimate the accounting beta on a division or firm.

spearn.xls: This dataset on the web has earnings changes, by year, for the S\&P 500 going back to 1960 .

Market, Bottom-up and Accounting Betas: Which one do we use?
For most publicly traded firms, betas can be estimated using accounting data or market data or from the bottom-up approach. Since the betas will almost never be the same using these different approaches, the question is which one do we use? We would almost never use accounting betas, for all the reasons specified above. We are almost as
reluctant to use historical market betas for individual firms because of the standard errors in beta estimates, the failures of the local indices (as is the case with most emerging market companies) and the inability of these regressions to reflect the effects of major changes in the business mix and financial risk at the firm. Bottom-up betas, in our view, provide us with the best beta estimates because:

1. They allow us to consider changes in business and financial mix, even before they occur.
2. They use average betas across large numbers of firms, which tend to be less noisy than individual firm betas.
3. They allow us to calculate betas by area of business for a firm, which is useful both in the context of investment analysis and valuation.

## From Betas to Cost of Equity

Having estimated the riskless rate, the risk premium(s) and the beta(s), we can now estimate the expected return from investing in equity at any firm. In the CAPM, this expected return can be written as:

Expected Return $=$ Riskless Rate + Beta * Expected Risk Premium
where the riskless rate would be the rate on a long-term government bond, the beta would be either the historical, fundamental or accounting betas described above and the risk premium would be either the historical premium or an implied premium.

In the arbitrage pricing and multi-factor model, the expected return would be written as follows:

Expected Return $=$ Riskless Rate $+\sum_{\mathrm{j}-1}^{\mathrm{j}=\mathrm{n}} \beta_{\mathrm{j}} *$ Risk Premium ${ }_{\mathrm{j}}$
where the riskless rate is the long term government bond rate, $\beta_{j}$ is the beta relative to factor j estimated using historical data or fundamentals and Risk Premium $\mathrm{j}_{\mathrm{j}}$ is the risk premium relative to factor j , estimated using historical data.

The expected return on an equity investment in a firm, given its risk, has strong implications for both equity investors in the firm and the managers of the firm. For equity investors, it is the rate they need to earn to be compensated for the risk they have taken in investing in the equity of the firm. If, after analyzing an investment, they conclude they
cannot make this return, they would not buy this investment; alternatively, if they decide they can make a higher return, they would make the investment. For managers in the firm, the return that investors need to make to break even on their equity investments becomes the return they have to try to deliver to keep these investors from becoming restive and rebellious. Thus, it becomes the rate they have to beat in terms of returns on their equity investments in projects. In other words, this is the cost of equity to the firm.

Illustration 8.8: Estimating the Cost of Equity for Boeing - June 2000
Now that we have an estimate of beta of 0.9585 for Boeing, based upon the bottom-up estimates, we can estimate its cost of equity. To make the estimate, we used the prevailing treasury bond rate of $5 \%$ and a historical risk premium of $5.51 \%$.

Cost of Equity $=5.00 \%+0.9585(5.51 \%)=10.28 \%$
There are two points making about this estimate. The first is that the cost of equity would have been significantly lower, if we had chosen to use the implied equity premium at the time of this analysis which was about $2.87 \%$ (See chapter 7).

Cost of Equity $=5.00 \%+0.9585(2.87 \%)=7.75 \%$
The second point is that we are not considering the exposure that Boeing has to emerging market risk from its business. If the exposure is significant, we should be adding a country risk premium to the cost of equity estimate.

## Illustration 8.9: Estimating the Cost of Equity for Embraer - March 2001

Embraer is a Brazilian aerospace firm. To estimate its cost of equity, we first estimated the unlevered beta by looking at aerospace firms globally.

Unlevered Beta for aerospace firms $=0.87$
Embraer's debt to equity ratio ${ }^{11}$ at the time of this analysis was $2.45 \%$, resulting in a levered beta for Embraer:

Levered beta for Embraer $=0.87(1+(1-0.33) 0.0245)=0.88$
To estimate the cost of equity for Embraer in U.S. dollar terms, we began with the treasury bond rate of $5 \%$ at the time of the analysis, but incorporated the country risk

[^6]associated with Brazil into the risk premium. Using the approach described in chapter 7, we estimated a country risk premium of $10.24 \%$. In conjunction with a mature market risk premium of $5.59 \%$, estimated for the United States, this yields a cost of equity of 18.93\%.

Cost of Equity for Embraer $=5 \%+0.88(5.51 \%+10.24 \%)=18.86 \%$
Again, there are several points that are worth making on this estimate. The first is that this cost of equity can be expected to change over time as Brazil matures as a market and country risk declines. The second is that we have assumed that betas measure exposure to country risk. A company like Embraer that derives the bulk of its revenues outside Brazil could argue that it is less exposed to country risk. We could have derived $\lambda$ as a measure of exposure to country risk for Embraer by looking at the proportion of its revenues that it derives in Brazil and comparing it to the proportion of revenues derived by a typical company in Brazil. In 2000, for instance, this would have yielded the following:
$\lambda_{\text {Embraer }}=\frac{\text { Proportion of Revenues from Brazil }{ }_{\text {Embraer }}}{\text { Proportion of Revenues from Brazil }{ }_{\text {Typical Brazilian firm }}}=\frac{9 \%}{60 \%}=0.15$
Using this measure of exposure to country risk, Embraer would have had a much lower cost of equity.
Cost of Equity in U.S. dollars
$=$ Riskfree rate + Beta (Mature Market Risk Premium ) $+\lambda$ (Country Risk Premium)
$=5 \%+0.88(5.51 \%)+0.15(10.24 \%)=11.39 \%$
The final point is that the cost of equity in dollar terms can be converted into a nominal real cost of equity fairly simply by considering the differences in expected inflation rates in Brazil and the United States. For instance, if the expected inflation rate in Brazil is $10 \%$ and the differential inflation rate in the United States is $2 \%$, the cost of equity in nominal real is as follows:

$$
\begin{aligned}
\text { Cost of Equity }_{\text {Nominal Real }} & =\left(1+\text { Cost of Equity }_{S}\right)\left(\frac{1+\text { Inflation Rate }_{\text {Brazil }}}{1+\text { Inflation Rate }_{\text {US }}}\right)-1 \\
& =(1+0.1139)\left(\frac{1.10}{1.02}\right)-1=20.12 \%
\end{aligned}
$$

Implicitly, we assume that real riskfree rates around that world are the same with this approach and that the risk premium scales up with inflation as well. The alternative is to
estimate a cost of equity from scratch, beginning with a nominal real risk free rate (which was $14 \%$ at the time of this analysis) and adding the premiums from above:

Cost of Equity ${ }_{\text {Nominal Real }}$
$=$ Riskfree rate + Beta (Mature Market Risk Premium ) $+\lambda$ (Country Risk Premium)
$=14 \%+0.88(5.51 \%)+0.15(10.24 \%)=20.39 \%$
Substituting in a real riskfree rate in the equation above would yield a real cost of equity.

## Cost of Equity and a Small Firm Premium

In chapter 6, we presented evidence of a small firm premium - small market-cap stocks earn higher returns than large market-cap stocks with equivalent betas. The magnitude and persistence of the small firm premium can be viewed as evidence that the capital asset pricing model understates the risk of smaller companies, and that a cost of equity based purely upon a CAPM beta will therefore yield too low a number for these firms. There are some analysts who argue that you should therefore add a premium on to the estimated cost of equity for smaller firms. Since small cap stocks have earned about $2 \%$ more than large cap stocks over the last few decades, you could consider this a reasonable estimate of the small firm premium. To estimate the cost of equity for a small cap stock with a beta of 1.2 (assuming a riskfree rate of $5.1 \%$ and a market risk premium of $4 \%$ ), for instance, you would do the following:
Cost of Equity for small-cap stock $=$ Riskfree Rate + Beta $*$ Market Risk Premium + Small Cap Premium $=5.1 \%+1.2 * 4 \%+2 \%=11.9 \%$

We would introduce two notes of caution with this approach. First, it opens the door to a series of adjustments that you could make to the cost of equity, reflecting the numerous inefficiencies that we cited in chapter 6. For instance, you could estimate a low PE premium, a low price to book premium and a high dividend yield premium and add them all to the cost of equity. If our objective in valuation is to uncover market mistakes, it would be a mistake to start off with the presumption that markets are right in their assessments in the first place. Second, a better way of considering the small firm premium would be to identify the reasons for the premium and then develop more direct measures of risk. For instance, assume that the higher risk of small cap stocks comes from the higher operating leverage that these firms have, relative to their larger competitors. You could adjust the betas for operating leverage (as we did a few pages ago for Van Shoes) and use the higher betas for small firms.

## From Cost of Equity to Cost of Capital

While equity is undoubtedly an important and indispensable ingredient of the financing mix for every business, it is but one ingredient. Most businesses finance some or much of their operations using debt or some security that is a combination of equity and debt. The costs of these sources of financing are generally very different from the cost of equity and the cost of financing for a firm should reflect their costs as well, in proportion to their use in the financing mix. Intuitively, the cost of capital is the weighted average of the costs of the different components of financing -- including debt, equity and hybrid securities -- used by a firm to fund its financial requirements. In this section, we examine the process of estimating the cost of financing other than equity and the weights for computing the cost of capital.

## Calculating the Cost of Debt

The cost of debt measures the current cost to the firm of borrowing funds to finance projects. In general terms, it is determined by the following variables:
(1) The riskless rate: As the riskless increases, the cost of debt for firms will also increase.
(2) The default risk (and associated default spread) of the company: As the default risk of a firm increases, the cost of borrowing money will also increase. In chapter 7, we looked at how the default spread has varied across time and can vary across maturity.
(3) The tax advantage associated with debt: Since interest is tax deductible, the after-tax cost of debt is a function of the tax rate. The tax benefit that accrues from paying interest makes the after-tax cost of debt lower than the pre-tax cost. Furthermore, this benefit increases as the tax rate increases.

After-tax cost of debt $=$ Pre-tax cost of debt ( $1-$ tax rate $)$
In this section, we will focus on how best to estimate the default risk in a firm and to convert that default risk into a default spread that can be used to come up with a cost of debt.

## Estimating the Default Risk and Default Spread of a firm

The simplest scenario for estimating the cost of debt occurs when a firm has long term bonds outstanding that are widely traded. The market price of the bond, in conjunction with its coupon and maturity can serve to compute a yield that we use as the
cost of debt. For instance, this approach works for a firm like AT\&T that has dozens of outstanding bonds that are liquid and trade frequently.

Many firms have bonds outstanding that do not trade on a regular basis. Since these firms are usually rated, we can estimate their costs of debt by using their ratings and associated default spreads. Thus, Boeing with a AA rating can be expected to have a cost of debt approximately $0.50 \%$ higher than the treasury bond rate, since this is the spread typically paid by AA rated firms.

Some companies choose not to get rated. Many smaller firms and most private businesses fall into this category. While ratings agencies have sprung up in many emerging markets, there are still a number of markets where companies are not rated on the basis of default risk. When there is no rating available to estimate the cost of debt, there are two alternatives:

1. Recent Borrowing History: Many firms that are not rated still borrow money from banks and other financial institutions. By looking at the most recent borrowings made by a firm, we can get a sense of the types of default spreads being charged the firm and use these spreads to come up with a cost of debt.
2. Estimate a synthetic rating: An alternative is to play the role of a ratings agency and assign a rating to a firm based upon its financial ratios; this rating is called a synthetic rating. To make this assessment, we begin with rated firms and examine the financial characteristics shared by firms within each ratings class. To illustrate, table 8.5 lists the range of interest coverage ratios for small manufacturing firms in each $\mathrm{S} \& \mathrm{P}$ ratings class ${ }^{12}$.

Table 8.5: Interest Coverage Ratios and Ratings: Low Market Cap Firms

| Interest Coverage Ratio | Rating | Spread |
| :---: | :---: | :---: |
| $>12.5$ | AAA | $0.75 \%$ |
| $9.5-12.5$ | AA | $1.00 \%$ |
| $7.5-9.5$ | $\mathrm{~A}+$ | $1.50 \%$ |
| $6-7.5$ | A | $1.80 \%$ |
| $4.5-6$ | $\mathrm{~A}-$ | $2.00 \%$ |

[^7]| $3.5-4.5$ | BBB | $2.25 \%$ |
| :---: | :---: | :---: |
| $3-3.5$ | BB | $3.50 \%$ |
| $2.5-3$ | B+ | $4.75 \%$ |
| $2-2.5$ | B | $6.50 \%$ |
| $1.5-2$ | B- | $8.00 \%$ |
| $1.25-1.5$ | CCC | $10.00 \%$ |
| $0.8-1.25$ | CC | $11.50 \%$ |
| $0.5-0.8$ | C | $12.70 \%$ |
| $<0.5$ | D | $14.00 \%$ |

Now consider a small firm that is not rated but has an interest coverage ratio of 6.15. Based on this ratio, we would assess a "synthetic rating" of A for the firm.

The interest coverage ratios tend to be lower for larger firms, for any given rating. Table 8.6 summarizes these ratios:

Table 8.6: Interest Coverage Ratios and Ratings: High Market Cap Firms

| Interest Coverage Ratio | Rating | Spread |
| :---: | :---: | :---: |
| $>8.5$ | AAA | $0.75 \%$ |
| $6.5-8.5$ | AA | $1.00 \%$ |
| $5.5-6.5$ | $\mathrm{~A}+$ | $1.50 \%$ |
| $4.25-5.5$ | A | $1.80 \%$ |
| $3-4.25$ | $\mathrm{~A}-$ | $2.00 \%$ |
| $2.5-3$ | BBB | $2.25 \%$ |
| $2-2.5$ | BB | $3.50 \%$ |
| $1.75-2$ | $\mathrm{~B}+$ | $4.75 \%$ |
| $1.5-1.75$ | B | $6.50 \%$ |
| $1.25-1.5$ | $\mathrm{~B}-$ | $8.00 \%$ |
| $0.8-1.25$ | CCC | $10.00 \%$ |
| $0.65-0.8$ | CC | $11.50 \%$ |
| $0.2-0.65$ | C | $12.70 \%$ |
| $<0.2$ | D | $14.00 \%$ |

This approach can be expanded to allow for multiple ratios and qualitative variables, as well. Once a synthetic rating is assessed, it can be used to estimate a default spread which when added to the riskfree rate yields a pre-tax cost of debt for the firm.

## Extending the Synthetic Ratings Approach

By basing the rating on the interest coverage ratio alone, we run the risk of missing the information that is available in the other financial ratios used by ratings agencies. The
approach described above can be extended to incorporate other ratios. The first step would be to develop a score based upon multiple ratios. For instance, the Altman Z score, which is used as a proxy for default risk, is a function of five financial ratios, which are weighted to generate a Z score. The ratios used and their relative weights are usually based upon empirical evidence on firm defaults. The second step is to relate the level of the score to a bond rating, much as we have done in the table above with interest coverage ratios.

In making this extension, though, note that complexity comes at a cost. While credit or Z scores may, in fact, yield better estimates of synthetic ratings than those based upon interest coverage ratios, changes in ratings arising from these scores are much more difficult to explain than those based upon interest coverage ratios. That is the reason we prefer the flawed but simpler ratings that we get from interest coverage ratios.

## Estimating a Tax Rate

To estimate the after-tax cost of debt, we consider the fact that interest expenses are tax deductible to the firm. While the computation is fairly simple and requires that we multiply the pre-tax cost by (1- tax rate), the question of what tax rate to use can be a difficult one to answer because we have so many choices. For instance, firms often report an effective tax rate, estimated by dividing the taxes due by the taxable income. The effective tax rate, though, is usually very different from the marginal tax rate, which is the rate at which the last or the next dollar of income is taxed. Since interest expenses save taxes at the margin (they are deducted from the last or the next dollar of income), the right tax rate to use is the marginal tax rate.

The other caveat to keep in mind is that interest creates a tax benefit only if a firm has enough income to cover the interest expenses. Firms that have operating losses will not get a tax benefit, at least in the year of the loss, from interest expenses. The after-tax cost of debt will be equal to the pre-tax cost of debt in that year. If you expect the firm to make money in future years, you would need to adjust the after-tax cost of debt for taxes.

We will return to this issue and examine it in more detail in chapter 10 , where we will look at the same issue in the context of estimating after-tax cash flows.

Illustration 8.10: Estimating the Cost of Debt: Boeing in June 2000
Boeing is rated AA by S\&P. Using the typical default spreads for AA rated firms, we could estimate the pre-tax cost for Boeing by adding the default spread ${ }^{13}$ of $1.00 \%$ to the riskless rate of $5 \%$.

Pre-tax cost of debt Actual Rating $=5.00 \%+1.00 \%=6.00 \% \%$
Boeing has an effective tax rate of $27 \%$ but we use a marginal tax rate of $35 \%$, which is the federal marginal corporate tax rate to estimate the after-tax cost of debt for Boeing.

After-tax cost of debt $=6.00 \%(1-.35)=3.90 \%$
We could also compute a synthetic rating for Boeing, based upon its interest coverage ratio from 1999. Based upon its operating income of $\$ 1,720$ million in 1999 and interest expense of $\$ 453$ million in of that year, we would have estimated an interest coverage ratio:

Interest coverage ratio Boeing $=1720 / 453=3.80$
Using Table 8.6, we would have assigned a synthetic rating of A- to Boeing. Based upon default spreads prevailing in June 2000, this would have resulted in a default spread of $2.00 \%$ and a pre-tax cost of debt of $7.00 \%$ for the firm.

## Estimating the Cost of Debt for an Emerging Market firm

In general, there are three problems that we run into when assessing the cost of debt for emerging market firms. The first is that most of these firms are not rated, leaving us with no option but to estimate the synthetic rating (and associated costs). The second is that the synthetic ratings may be skewed by differences in interest rates between the emerging market and the United States. Interest coverage ratios will usually decline as interest rates increase and it may be far more difficult for a company in an emerging market to achieve the interest coverage ratios of companies in developed markets. Finally, the existence of country default risk level hangs over the cost of debt of firms in that market.

The second problem can be fixed fairly simply by either modifying the tables developed using U.S. firms or restating the interest expenses (and interest coverage ratios)
in dollar terms. The question of country risk is a thornier one. Conservative analysts often assume that companies in a country cannot borrow at a rate lower than the country can borrow at. With this reasoning, the cost of debt for an emerging market company will include the country default spread for the country.
Cost of debt ${ }_{\text {Emerging Market company }}=$ Riskless Rate + Country Default Spread + Company Default Spread $_{\text {Synthetic Rating }}$
The counter to this argument is that companies may be safer than the countries that they operate in and that they bear only a portion or perhaps even none of the country default spread.

## Illustration 8.11: Estimating the Cost of Debt: Embraer in December 2000

To estimate Embraer's cost of debt, we first estimate a synthetic rating for the firm. Based upon its operating income of $\$ 810$ million and interest expenses of $\$ 28$ million in 2000, we arrived at an interest coverage ratio of 28.93 and an AAA rating. While the default spread for AAA rated bonds was only $0.75 \%$, there is the added consideration that Embraer is a Brazilian firm. Since the Brazilian government bond traded at a spread of $5.37 \%$ at the time of the analysis, you could argue that every Brazilian company should pay this premium, in addition to its own default spread. With this reasoning, the pre-tax cost of debt for Embraer in U.S. dollars (assuming a treasury bond rate is $5 \%$ ) can be calculated:
Cost of Debt $=$ Riskfree rate + Default spread for country + Default Spread for firm $=5 \%+5.37 \%+0.75 \%=11.12 \%$
Using a marginal tax rate of $33 \%$, we can estimate an after-tax cost of debt for Embraer:
After-tax cost of debt $=11.12 \%(1-.33)=7.45 \%$
With this approach, the cost of debt for a firm can never be lower than the cost of debt for the country in which it operates. Note, though, that Embraer gets a significant portion of its revenues in dollars from contracts with non-Brazilian airlines. Consequently, it could reasonably argue that it is less exposed to risk than the Brazilian government and should therefore command a lower cost of debt.

[^8]ratings.xls: This spreadsheet allows you to estimate the synthetic rating and cost of debt for any firm.

## Calculating the Cost of Hybrid Securities

While debt and equity represent the fundamental financing choices available for firms, there are some types of financing that share characteristics with both debt and equity. These are called hybrid securities. In this section, we consider how best to estimate the costs of such securities.

## Cost of Preferred Stock

Preferred stock shares some of the characteristics of debt - the preferred dividend is pre-specified at the time of the issue and is paid out before common dividend - and some of the characteristics of equity - the payments of preferred dividend are not tax deductible. If preferred stock is viewed as perpetual, the cost of preferred stock can be written as follows:

$$
\mathrm{k}_{\mathrm{ps}}=\frac{\text { Preferred Dividend per share }}{\text { Market Price per preferred share }}
$$

This approach assumes the dividend is constant in dollar terms forever and that the preferred stock has no special features (convertibility, callability, etc.). If such special features exist, they will have to be valued separately to estimate the cost of preferred stock. In terms of risk, preferred stock is safer than common equity, because preferred dividends are paid before dividends on common equity. It is, however, riskier than debt since interest payments on debt are made prior to preferred dividend payments. Consequently, on a pre-tax basis, it should command a higher cost than debt and a lower cost than equity.

Illustration 8.12: Calculating the Cost Of Preferred Stock: General Motors Co.
In March 1995, General Motors had preferred stock that paid a dividend of $\$ 2.28$ annually and traded at $\$ 26.38$ per share. The cost of preferred stock can be estimated as follows:

$$
\text { Cost of preferred stock }=\frac{\text { Preferred Dividend per share }}{\text { Preferred Stock Price }}=\frac{2.28}{26.38}=8.64 \%
$$

At the same time, GM's cost of equity, using the CAPM, was $13 \%$, its pre-tax cost of debt was $8.25 \%$ and its after-tax cost of debt was $5.28 \%$. Not surprisingly, its preferred stock was less expensive than equity but much more expensive than debt.

## Calculating the Cost of Other Hybrid Securities

A convertible bond is a bond that can be converted into equity, at the option of the bondholder. A convertible bond can be viewed as a combination of a straight bond (debt) and a conversion option (equity). Instead of trying to calculate the cost of these hybrid securities individually, we can break down hybrid securities into their debt and equity components and treat the components separately.

Illustration 8.13: Breaking down a convertible bond into debt and equity components: Amazon Inc

In 1999, Amazon Inc, the online retailer, issued convertible bonds with a coupon rate of $4.75 \%$ and a ten-year maturity. Since the firm was losing money, it was rated CCC+ by S\&P and would have had to pay $11 \%$ if it had issued straight bonds at the same time. The bonds were issued at a price that was $98 \%$ of par and the total par value of the convertible bond issue was $\$ 1.25$ billion. The convertible bond can be broken down into straight bond and conversion option components.

Straight Bond component $=$ Value of a straight $4.75 \%$ coupon bond due in 10 years with an $11 \%$ interest rate $=\$ 636$ (assuming semi-annual coupons)

Conversion Option $=\$ 980-\$ 636=\$ 344$
The straight bond component of $\$ 636$ is treated as debt and has the same cost as the rest of debt. The conversion option of $\$ 344$ is treated as equity and has the same cost of equity as other equity issued by the firm. For the entire bond issue of $\$ 1.25$ billion, the value of debt is $\$ 811$ million, and the value of equity is $\$ 439$ million.

## Calculating the Weights of Debt and Equity Components

Now that we have the costs of debt, equity and hybrid securities, we have to estimate the weights that should be attached to each. Before we discuss how best to estimate weights, we define what we include in debt. We then make the argument that
weights used should be based upon market value and not book value. This is so because the cost of capital measures the cost of issuing securities - stocks as well as bonds - to finance projects and these securities are issued at market value, not at book value.

## What is debt?

The answer to this question may seem obvious since the balance sheet for a firm shows the outstanding liabilities of a firm. There are, however, limitations with using these liabilities as debt in the cost of capital computation. The first is that some of the liabilities on a firm's balance sheet, such as accounts payable and supplier credit, are not interest bearing. Consequently, applying an after-tax cost of debt to these items can provide a misleading view of the true cost of capital for a firm. The second is that there are items off the balance sheet that create fixed commitments for the firm and provide the same tax deductions that interest payments on debt do. The most prominent of these offbalance sheet items are operating leases. In chapter 3, we contrasted operating and capital leases and noted that operating leases are treated as operating expenses rather than financing expenses. Consider, though, what an operating lease involves. A retail firms leases a store space for 12 years and enters into a lease agreement with the owner of the space agreeing to pay a fixed amount each year for that period. We do not see much difference between this commitment and borrowing money from a bank and agreeing to pay off the bank loan over 12 years in equal annual installments.

There are therefore two adjustments we will make when we estimate how much debt a firm has outstanding.

- We will consider only interest bearing debt rather than all liabilities. We will include both short term and long term borrowings in debt.
- We will also capitalize operating leases and treat these expenditures as financing expenses.


## Capitalizing Operating Leases

Converting operating lease expenses into a debt equivalent is straightforward. The operating lease commitments in future years, which are revealed in the footnotes to the financial statements for US firms, should be discounted back at a rate that reflects their status as unsecured and fairly risky debt. As an approximation, using the firm's current
pre-tax cost of borrowing as the discount rate yields a good estimate of the value of operating leases.

Outside the United States, firms do not have to reveal their operating lease commitments in future periods. When this is the case, you can get a reasonably close estimate of the estimated debt value of operating leases by assuming that an annuity equal to the current year's operating lease payment for a period that reflects a typical lease period (8 to 10 years).

There is one final issue relating to capitalization. Earlier in this chapter, we argued that the interest coverage ratio could be used to estimate a synthetic rating for a firm that is not rated. For firms with little in terms of conventional debt and substantial operating leases, the interest coverage ratio used to estimate a synthetic rating has to be adapted to include operating lease expenses.

Modified Interest Coverage Ratio $=$ (EBIT + Current year's Operating Lease Expense)/(Interest Expenses + Current year's Operating Lease Expense)

This ratio can then be used in conjunction with Tables 8.5 and 8.6 to estimate a synthetic rating.

## Illustration 8.14: The Debt Value of Operating Leases: Boeing in June 2000

Boeing has both conventional debt and operating lease commitments. In this illustration, we will estimate the "debt value" of Boeing's operating leases by taking the present value of operating lease expenses over time. To compute the present value of operating leases in Table 8.7, we use the pre-tax cost of borrowing for the firm which we estimated to be $6.0 \%$ in Illustration 8.10.

Table 8.7: Debt Value of Operating Leases

| Year | Operating Lease <br> Expense |  | Present Value <br> at $6.0 \%$ |  |
| :---: | :--- | ---: | :--- | ---: |
| 1 | $\$$ | 205 | $\$$ | 193.40 |
| 2 | $\$$ | 167 | $\$$ | 146.83 |
| 3 | $\$$ | 120 | $\$$ | 100.75 |
| 4 | $\$$ | 86 | $\$$ | 68.12 |


| 5 | $\$$ | 61 | $\$$ |
| :---: | :--- | :--- | :--- |
| Yr $6-15$ | $\$$ | -28 |  |
| PV of Operating Lease Expenses | $\$$ | 556.48 |  |

Thus, Boeing has $\$ 556$ million more in debt than is reported in the balance sheet.
Oplease.xls: This spreadsheet allows you to convert operating lease expenses into debt.

## Book Value versus Market Value Debt ratios

There are three standard arguments against using market value and none of them are convincing. First, there are some financial managers who argue that book value is more reliable than market value because it is not as volatile. While it is true that book value does not change as much as market value, this is more a reflection of book value's weakness rather than its strength since the true value of the firm changes over time as both firmspecific and market information is revealed. We would argue that market value, with its volatility, is a much better reflection of true value than book value. ${ }^{14}$

Second, the defenders of book value also suggest that using book value rather than market value is a more conservative approach to estimating debt ratios. This assumes that market value debt ratios are always lower than book value debt ratios, an assumption not based on fact. Furthermore, even if the market value debt ratios are lower than the book value ratios, the cost of capital calculated using book value ratios will be lower than those calculated using market value ratios, making them less conservative estimates, not more. To illustrate this point, assume that the market value debt ratio is $10 \%$, while the book value debt ratio is $30 \%$, for a firm with a cost of equity of $15 \%$ and an after-tax cost of debt of $5 \%$. The cost of capital can be calculated as follows:

With market value debt ratios: $\quad 15 \%(0.9)+5 \%(0.1)=14 \%$
With book value debt ratios: $15 \%(.7)+5 \%(.3)=12 \%$

[^9]Third, it is claimed that lenders will not lend on the basis of market value, but this claim again seems to be based more upon perception than fact. Any homeowner who has taken a second mortgage on a house that has appreciated in value knows that lenders do lend on the basis of market value. It is true, however, that the greater the perceived volatility in the market value of an asset, the lower is the borrowing potential on that asset.

## Estimating the Market Values of Equity and Debt

The market value of equity is generally the number of shares outstanding times the current stock price. If there other equity claims in the firm such as warrants and management option, these should also be valued and added on to the value of the equity in the firm.

The market value of debt is usually more difficult to obtain directly, since very few firms have all their debt in the form of bonds outstanding trading in the market. Many firms have non-traded debt, such as bank debt, which is specified in book value terms but not market value terms. A simple way to convert book value debt into market value debt is to treat the entire debt on the books as one coupon bond, with a coupon set equal to the interest expenses on all the debt and the maturity set equal to the face-value weighted average maturity of the debt, and then to value this coupon bond at the current cost of debt for the company. Thus, the market value of $\$ 1$ billion in debt, with interest expenses of $\$ 60$ million and a maturity of 6 years, when the current cost of debt is $7.5 \%$ can be estimated as follows:
Estimated Market Value of Debt $=60\left(\frac{1-\frac{1}{1.075^{6}}}{0.075}\right)+\frac{1,000}{1.075^{6}}=\$ 930$ million

## Gross Debt versus Net Debt

Gross debt refers to all debt outstanding in a firm. Net debt is the difference between gross debt and the cash balance of the firm. For instance, a firm with $\$ 1.25$ billion in interest bearing debt outstanding and a cash balance of $\$ 1$ billion has a net debt balance of $\$ 250$ million. The practice of netting cash against debt is common in both Latin America and Europe and the debt ratios are usually estimated using net debt.

It is generally safer to value a firm based upon gross debt outstanding and to add the cash balance outstanding to the value of operating assets to arrive at the firm value. The interest payment on total debt is then entitled to the tax benefits of debt and we can assess the effect of whether the company invests its cash balances efficiently on value.

In some cases, especially when firms maintain large cash balances as a matter of routine, analysts prefer to work with net debt ratios. If we choose to use net debt ratios, we have to be consistent all the way through the valuation. To begin, the beta for the firm should be estimated using a net debt ratio rather than a gross debt ratio. The cost of equity that emerges from the beta estimate can be used to estimate a cost of capital, but the market value weight on debt should be based upon net debt. Once we discount the cash flows of the firm at the cost of capital, we should not add back cash. Instead, we should subtract the net debt outstanding to arrive at the estimated value of equity.

Implicitly, when we net cash against debt to arrive at net debt ratios, we are assuming that cash and debt have roughly similar risk. While this assumption may not be outlandish when analyzing highly rated firms, it becomes much shakier when debt becomes riskier. For instance, the debt in a BB rated firm is much riskier than the cash balance in the firm and netting out one against the other can provide a misleading view of the firm's default risk. In general, using net debt ratios will overstate the value of riskier firms.

Illustration 8.15: Difference between market value and book value debt ratios - Boeing in June 2000

In this illustration we contrast the book values of debt and equity with the market values. For debt, we estimate the market value of debt using the book value of debt, the interest expense on the debt, the average maturity of the debt and the pre-tax cost of debt for each firm. For Boeing, the book value of debt is $\$ 6,972$ million, the interest expense on the debt is $\$ 453$ million, the average maturity of the debt is 13.76 years and the pre-tax cost of debt is $6.00 \%$. The estimated market value is as follows:

Estimated MV of Boeing Debt $=453\left(\frac{1-\frac{1}{1.06^{13.76}}}{0.06}\right)+\frac{6,972}{1.06^{13.76}}=\$ 7,291$
To this, we need to add the present value of operating leases of $\$ 556$ million to arrive at a total market value for debt of $\$ 7,847$ million.

The book value of equity for Boeing was $\$ 12,316$ million while the market value of equity was $\$ 55,197$ million. The debt ratios in market value and book value terms are computed.

|  | Market | Book |
| :--- | :--- | :--- |
| Debt to Equity | $\frac{7847}{55197}=14.22 \%$ | $\frac{6972}{12316}=56.61 \%$ |
| Debt/(Debt + Equity) | $\frac{7847}{7847+55197}=12.45 \%$ | $\frac{6972}{6972+12316}=36.15 \%$ |

The market debt ratio is significantly lower than the book debt ratio.
wacccalc.xls: This spreadsheet allows you to convert book values of debt into market values.

## Estimating the Cost of Capital

Since a firm can raise its money from three sources -- equity, debt and preferred stock -- the cost of capital is defined as the weighted average of each of these costs. The cost of equity $\left(\mathrm{k}_{\mathrm{e}}\right)$ reflects the riskiness of the equity investment in the firm, the after-tax cost of debt $\left(\mathrm{k}_{\mathrm{d}}\right)$ is a function of the default risk of the firm and the cost of preferred stock $\left(k_{p s}\right)$ is a function of its intermediate standing in terms of risk between debt and equity. The weights on each of these components should reflect their market value proportions since these proportions best measure how the existing firm is being financed. Thus if E, D and PS are the market values of equity, debt and preferred stock, respectively, the cost of capital can be written as follows:

$$
\text { Cost of Capital }=\mathrm{k}_{\mathrm{e}}\left(\frac{\mathrm{E}}{\mathrm{D}+\mathrm{E}+\mathrm{PS}}\right)+\mathrm{k}_{\mathrm{d}}\left(\frac{\mathrm{D}}{\mathrm{D}+\mathrm{E}+\mathrm{PS}}\right)+\mathrm{k}_{\mathrm{ps}}\left(\frac{\mathrm{PS}}{\mathrm{D}+\mathrm{E}+\mathrm{PS}}\right)
$$

Illustration 8.16: Estimating Cost of Capital - Boeing

Having estimated the costs of debt and equity in earlier illustrations and the market value debt ratio in Illustration 8.15, we can put them together to arrive at a cost of capital for Boeing.
Cost of Equity $=10.28 \%$ (from Illustration 8.8)
Cost of Debt $=3.90 \%$ (from Illustration 8.10)
Market Value Debt ratio $=12.45 \%$ (from Illustration 8.15)
Cost of capital $=10.28 \%(0.8755)+3.90 \%(0.1245)=9.49 \%$

## Illustration 8.17: Estimating Cost of Capital - Embraer in January 2001

To estimate a cost of capital for Embraer, we again draw on the estimates of cost of equity and cost of debt we obtained in prior illustrations. The cost of capital will be estimated using net debt all the way through (for the levered betas, interest coverage ratios and debt ratios) and in U.S. dollars.

Cost of equity $=18.86 \%$ (from illustration 8.9)
After-tax cost of debt $=7.45 \%($ from illustration 8.11 $)$
Market Value of Debt $=1,328$ million BR
Cash and Marketable Securities $=1,105$ million BR
Market Value of Equity $=9,084$ million BR
The cost of capital for Embraer is estimated below:
Net Debt $=1,328$ million $-1,105$ million $=223$ million
Cost of Capital $=18.86 \%\left(\frac{9084}{9084+223}\right)+7.45 \%\left(\frac{223}{9084+223}\right)=18.59 \%$
To convert this into a nominal real cost of capital, we would apply the differential inflation rates ( $10 \%$ in Brazil and $2 \%$ in the US)

$$
\begin{aligned}
& \text { Cost of Capital }_{\text {NominalBR }}=\left(1+\text { Cost of Capital }_{\mathrm{S}}\right)\left(\frac{\text { Inflation Rate }_{\text {Brazil }}}{\text { Inflation Rate }_{\mathrm{US}}}\right)-1 \\
&=(1.1859)\left(\frac{1.10}{1.02}\right)-1=27.89 \%
\end{aligned}
$$

## Best Practices at Firms

We have spent this chapter discussing what firms should do when it comes to estimating the cost of capital. What do they actually do? Bruner, Eades, Harris and Higgins surveyed 27 well regarded corporations and their findings are summarized in Table 8.8.

Table 8.8: Current Practices for Estimating Cost of Capital

| Cost of capital item | Current Practices |
| :---: | :---: |
| Cost of Equity | - $81 \%$ of firms used the capital asset pricing model to estimate the cost of equity, $4 \%$ used a modified capital asset pricing model and $15 \%$ were uncertain about how they estimated the cost of equity. <br> - $70 \%$ of firms used 10-year treasuries or longer as the riskless rate, $7 \%$ used 3 to 5 -year treasuries and $4 \%$ used the treasury bill rate. <br> - $52 \%$ used a published source for a beta estimate, while $30 \%$ estimated it themselves. <br> - There was wide variation in the market risk premium used, with $37 \%$ using a premium between 5 and $6 \%$. |
| Cost of Debt | - $52 \%$ of firms used a marginal borrowing rate and a marginal tax rate, while $37 \%$ used the current average borrowing rate and the effective tax rate. |
| Weights for Debt and Equity | - $59 \%$ used market value weights for debt and equity in the cost of capital, $15 \%$ used book value weights and $19 \%$ were uncertain about what weights they used. |

Source: Bruner, Eades, Harris and Higgins (1998)

## Summary

When we analyze the investments of a firm or assess its value, we need to know the cost that the firm faces in raising equity, debt and capital. The risk and return models described in the previous chapter can be used to estimate the costs of equity and capital for a firm.

Building on the premise that the cost of equity should reflect the riskiness of equity to investors in the firm, there are three basic inputs we need to estimate the cost of equity for any firm. The riskless rate is the expected return on an investment with no default risk and no reinvestment risk. Since much of the analysis in corporate finance is long term, the riskless rate should be the interest rate on a long term government bond. The risk premium measures what investors demand as a premium for investing in risky investments instead of riskless investments. This risk premium, which can vary across investors, can be estimated either by looking at past returns on stocks and government securities or by looking at how the market prices stocks currently. The beta for a firm is conventionally measured using a regression of returns on the firm's stock against returns on a market index. This approach yields imprecise beta estimates and we are better off estimating betas by examining the betas of the businesses that the firm operates in.

The cost of capital is a weighted average of the costs of the different components of financing, with the weights based on the market values of each component. The cost of debt is the market rate at which the firm can borrow, adjusted for any tax advantages of borrowing. The cost of preferred stock, on the other hand, is the preferred dividend.

The cost of capital is useful at two levels. On a composite basis, it is what these firms have to make collectively on their investments to break even. It is also the appropriate discount rate to use to discount expected future cash flows to arrive at an estimate of firm value.

## Problems

1. In December 1995, Boise Cascade's stock had a beta of 0.95 . The treasury bill rate at the time was $5.8 \%$ and the treasury bond rate was $6.4 \%$.
a. Estimate the expected return on the stock for a short term investor in the company.
b. Estimate the expected return on the stock for a long term investor in the company.
c. Estimate the cost of equity for the company.
2. (Continuing problem 1) Boise Cascade also had debt outstanding of $\$ 1.7$ billion and a market value of equity of $\$ 1.5$ billion; the corporate marginal tax rate was $36 \%$.
a. Assuming that the current beta of 0.95 for the stock is a reasonable one, estimate the unlevered beta for the company.
b. How much of the risk in the company can be attributed to business risk and how much to financial leverage risk?
3. Biogen Inc., as biotechnology firm, had a beta of 1.70 in 1995. It had no debt outstanding at the end of that year.
a. Estimate the cost of equity for Biogen, if the treasury bond rate is $6.4 \%$.
b. What effect will an increase in long term bond rates to $7.5 \%$ have on Biogen's cost of equity?
c. How much of Biogen's risk can be attributed to business risk?
4. Genting Berhad is a Malaysian conglomerate, with holding in plantations and tourist resorts. The beta estimated for the firm, relative to the Malaysian stock exchange, is 1.15 , and the long term government borrowing rate in Malaysia is $11.5 \%$.
a. Estimate the expected return on the stock.
b. If you were an international investor, what concerns, if any, would you have about using the beta estimated relative to the Malaysian Index? If you do, how would you modify the beta?
5. You have just done a regression of monthly stock returns of HeavyTech Inc., a manufacturer of heavy machinery, on monthly market returns over the last five years and come up with the following regression:

$$
\mathrm{R}_{\text {HeavyTech }}=0.5 \%+1.2 \mathrm{R}_{\mathrm{M}}
$$

The variance of the stock is $50 \%$ and the variance of the market is $20 \%$. The current T.Bill rate is $3 \%$ (It was $5 \%$ one year ago). The stock is currently selling for $\$ 50$, down $\$ 4$ over the last year, and has paid a dividend of $\$ 2$ during the last year and expects to pay a dividend of $\$ 2.50$ over the next year. The NYSE composite has gone down $8 \%$ over the last year with a dividend yield of $3 \%$. HeavyTech Inc. has a tax rate of $40 \%$.
a. What is the expected return on HeavyTech over the next year?
b. What would you expect HeavyTech's price to be one year from today?
c. What would you have expected HeavyTech's stock returns to be over the last year?
d. What were the actual returns on HeavyTech over the last year?
e. HeavyTech has $\$ 100$ million in equity and $\$ 5$ million in debt. It plans to issue $\$ 50$ million in new equity and retire $\$ 50$ million in debt. Estimate the new beta.
6. Safecorp, which owns and operates grocery stores across the United States, currently has \$50 million in debt and $\$ 100$ million in equity outstanding. Its stock has a beta of 1.2. It is planning a leveraged buyout, where it will increase its debt/equity ratio to 8 . If the tax rate is $40 \%$, what will the beta of the equity in the firm be after the LBO?
7. Novell, which had a market value of equity of $\$ 2$ billion and a beta of 1.50 , announced that it was acquiring WordPerfect, which had a market value of equity of $\$ 1$ billion and a beta of 1.30. Neither firm had any debt in its financial structure at the time of the acquisition and the corporate tax rate was $40 \%$.
a. Estimate the beta for Novell after the acquisition, assuming that the entire acquisition was financed with equity.
b. Assume that Novell had to borrow the $\$ 1$ billion to acquire WordPerfect. Estimate the beta after the acquisition.
8. You are analyzing the beta for Hewlett Packard and have broken down the company into four broad business groups with market values and betas for each group.

| Business Group | Market Value of Equity | Beta |
| :--- | :--- | :--- |
| Mainframes | $\$ 2.0$ billion | 1.10 |
| Personal Computers | $\$ 2.0$ billion | 1.50 |
| Software | $\$ 1.0$ billion | 2.00 |

Printers
$\$ 3.0$ billion
1.00
a. Estimate the beta for Hewlett Packard as a company. Is this beta going to be equal to the beta estimated by regressing past returns on HP stock against a market index. Why or Why not?
b. If the treasury bond rate is $7.5 \%$, estimate the cost of equity for Hewlett Packard.

Estimate the cost of equity for each division. Which cost of equity would you use to value the printer division?
c. Assume that HP divests itself of the mainframe business and pays the cash out as a dividend. Estimate the beta for HP after the divestiture. (HP had $\$ 1$ billion in debt outstanding.)
9. The following table summarizes the percentage changes in operating income, percentage changes in revenue and betas for four pharmaceutical firms.

| Firm | \% Change in Revenue | \% Change in Operating Income | Beta |
| :--- | :--- | :--- | :--- |
| PharmaCorp | $27 \%$ | $25 \%$ | 1.00 |
| SynerCorp | $25 \%$ | $32 \%$ | 1.15 |
| BioMed | $23 \%$ | $36 \%$ | 1.30 |
| Safemed | $21 \%$ | $40 \%$ | 1.40 |

a. Calculate the degree of operating leverage for each of these firms.
b. Use the operating leverage to explain why these firms have different betas.
10. A prominent beta estimation service reports the beta of Comcast Corporation, a major cable TV operator, to be 1.45. The service claims to use weekly returns on the stock over the prior five years and the NYSE composite as the market index to estimate betas. You replicate the regression using weekly returns over the same period and arrive at a beta estimate of 1.60 . How would you reconcile the two estimates?
11. Battle Mountain is a mining company, which mines gold, silver and copper in mines in South America, Africa and Australia. The beta for the stock is estimated to be 0.30 . Given the volatility in commodity prices, how would you explain the low beta?
12. You have collected returns on AnaDone Corporation (AD Corp.), a large diversified manufacturing firm, and the NYSE index for five years:

| Year | AD Corp | NYSE |
| :--- | :--- | :--- |
| 1981 | $10 \%$ | $5 \%$ |
| 1982 | $5 \%$ | $15 \%$ |
| 1983 | $-5 \%$ | $8 \%$ |
| 1984 | $20 \%$ | $12 \%$ |
| 1985 | $-5 \%$ | $-5 \%$ |

a. Estimate the intercept (alpha) and slope (beta) of the regression.
b. If you bought stock in AD Corp. today, how much would you expect to make as a return over the next year? [The six-month T.Bill rate is $6 \%$ ]
c. Looking back over the last five years, how would you evaluate AD's performance relative to the market?
d. Assume now that you are an undiversified investor and that you have all of your money invested in AD Corporation. What would be a good measure of the risk that you are taking on? How much of this risk would you be able to eliminate if you diversify?
e. AD is planning to sell off one of its divisions. The division under consideration has assets which comprise half of the book value of AD Corporation and $20 \%$ of the market value. Its beta is twice the average beta for AD Corp (before divestment). What will the beta of AD Corporation be after divesting this division?
13. You run a regression of monthly returns of Mapco Inc, an oil and gas producing firm, on the S\&P 500 index and come up with the following output for the period 1991 to 1995.

Intercept of the regression $=0.06 \%$
Slope of the regression $=0.46$
Standard error of X-coefficient $=0.20$
R squared $=5 \%$
There are 20 million shares outstanding and the current market price is $\$ 2$ per share. The firm has $\$ 20$ million in debt outstanding. (The firm has a tax rate of $36 \%$.)
a. What would an investor in Mapco's stock require as a return if the T.Bond rate is $6 \%$ ?
b. What proportion of this firm's risk is diversifiable?
c. Assume now that Mapco has three divisions, of equal size (in market value terms). It plans to divest itself of one of the divisions for $\$ 20$ million in cash and acquire another for $\$ 50$ million (It will borrow $\$ 30$ million to complete this acquisition). The division it is divesting is in a business line where the average unlevered beta is 0.20 and the division it is acquiring is in a business line where the average unlevered beta is 0.80 . What will the beta of Mapco be after this acquisition?
14. You have just run a regression of monthly returns of American Airlines (AMR) against the S\&P 500 over the last five years. You have misplaced some of the output and are trying to derive it from what you have.
a. You know the R squared of the regression is 0.36 and that your stock has a variance of $67 \%$. The market variance is $12 \%$. What is the beta of AMR?
b. You also remember that AMR was not a very good investment during the period of the regression and that it did worse than expected (after adjusting for risk) by $0.39 \%$ a month for the five years of the regression. During this period, the average riskfree rate was $4.84 \%$. What was the intercept on the regression?
c. You are comparing AMR Inc. to another firm which also has an R squared of 0.48. Will the two firms have the same beta? If not, why not?
15. You have run a regression of monthly returns on Amgen, a large biotechnology firm, against monthly returns on the S\&P 500 index, and come up with the following output.

$$
\mathrm{R}_{\text {stock }}=3.28 \%+1.65 \mathrm{R}_{\text {Market }} \quad \mathrm{R}^{2}=0.20
$$

The current one-year treasury bill rate is $4.8 \%$ and the current thirty-year bond rate is $6.4 \%$. The firm has 265 million shares outstanding selling for $\$ 30$ per share.
a. What is the expected return on this stock over the next year?
b. Would your expected return estimate change if the purpose was to get a discount rate to analyze a thirty-year capital budgeting project?
c. An analyst has estimated, correctly, that the stock did $51.10 \%$ better than expected, annually, during the period of the regression. Can you estimate the annualized riskfree rate that she used for her estimate?
d. The firm has a debt/equity ratio of $3 \%$, and faces a tax rate of $40 \%$. It is planning to issue $\$ 2$ billion in new debt and acquire a new business for that amount with the same risk level as the firm's existing business. What will the beta be after the acquisition?
16. You have just run a regression of monthly returns on MAD Inc., a newspaper and magazine publisher, against returns on the $\mathrm{S} \& \mathrm{P} 500$ and arrived at the following result.

$$
\mathrm{R}_{\mathrm{MAD}}=-0.05 \%+1.20 \mathrm{R}_{\mathrm{S} \& \mathrm{P}}
$$

The regression has an R-squared of $22 \%$. The current T.Bill rate is $5.5 \%$ and the current T.Bond rate is $6.5 \%$. The riskfree rate during the period of the regression was $6 \%$. Answer the following questions relating to the regression.
a. Based upon the intercept, how well or badly did MAD do, relative to expectations, during the period of the regression?
b. You now realize that MAD Inc. went through a major restructuring at the end of last month (which was the last month of your regression) and made the following changes:

- The firm sold off its magazine division, which had an unlevered beta of 0.6 , for \$20 million.
- It borrowed an additional $\$ 20$ million and bought back stock worth $\$ 40$ million.
- After the sale of the division and the share repurchase, MAD Inc. had \$40 million in debt and $\$ 120$ million in equity outstanding.
If the firm's tax rate is $40 \%$, re-estimate the beta, after these changes.

17. Time Warner Inc., the entertainment conglomerate, has a beta of 1.61. Part of the reason for the high beta is the debt left over from the leveraged buyout of Time by Warner in 1989, which amounted to $\$ 10$ billion in 1995. The market value of equity at Time Warner in 1995 was also $\$ 10$ billion. The marginal tax rate was $40 \%$.
a. Estimate the unlevered beta for Time Warner.
b. Estimate the effect of reducing the debt ratio by $10 \%$ each year for the next two years on the beta of the stock.
18. Chrysler, the automotive manufacturer, had a beta of 1.05 in 1995. It had $\$ 13$ billion in debt outstanding in that year and 355 million shares trading at $\$ 50$ per share. The firm had a cash balance of $\$ 8$ billion at the end of 1995 . The marginal tax rate was $36 \%$.
a. Estimate the unlevered beta of the firm.
b. Estimate the effect of paying out a special dividend of $\$ 5$ billion on this unlevered beta.
c. Estimate the beta for Chrysler after the special dividend.
19. You are trying to estimate the beta of a private firm that manufactures home appliances. You have managed to obtain betas for publicly traded firms that also manufacture home appliances.

| Firm | Beta | Debt | MV of Equity |
| :--- | :--- | :--- | :--- |
| Black \& Decker | 1.40 | $\$ 2,500$ | $\$ 3,000$ |
| Fedders Corp. | 1.20 | $\$ 5$ | $\$ 200$ |
| Maytag Corp. | 1.20 | $\$ 540$ | $\$ 2250$ |
| National Presto | 0.70 | $\$ 8$ | $\$ 300$ |
| Whirlpool | 1.50 | $\$ 2900$ | $\$ 4000$ |

The private firm has a debt equity ratio of $25 \%$, and faces a tax rate of $40 \%$. The publicly traded firms all have marginal tax rates of $40 \%$ as well.
a. Estimate the beta for the private firm.
b. What concerns, if any, would you have about using betas of comparable firms?
20. As the result of stockholder pressure, RJR Nabisco is considering spinning off its food division. You have been asked to estimate the beta for the division and decide to do so by obtaining the beta of comparable publicly traded firms. The average beta of comparable publicly traded firms is 0.95 and the average debt/equity ratio of these firms is $35 \%$. The division is expected to have a debt ratio of $25 \%$. The marginal corporate tax rate is $36 \%$.
a. What is the beta for the division?
b. Would it make any difference if you knew that RJR Nabisco had a much higher fixed cost structure than the comparable firms used here?
21. Southwestern Bell, a phone company, is considering expanding its operations into the media business. The beta for the company at the end of 1995 was 0.90 and the debt/equity ratio was 1 . The media business is expected to be $30 \%$ of the overall firm value in 1999 and the average beta of comparable firms is 1.20 ; the average debt/equity ratio for these firms is $50 \%$. The marginal corporate tax rate is $36 \%$.
a. Estimate the beta for Southwestern Bell in 1999, assuming that it maintains its current debt/equity ratio.
b. Estimate the beta for Southwestern Bell in 1999, assuming that it decides to finance its media operations with a debt/equity ratio of $50 \%$.
22. The chief financial officer of Adobe Systems, a growing software manufacturing firm, has approached you for some advice regarding the beta of his company. He subscribes to a service which estimates Adobe System's beta each year and he has noticed that the beta estimates have gone down every year since 1991: 2.35 in 1991 to 1.40 in 1995. He would like the answers to the following questions:
a. Is this decline in beta unusual for a growing firm?
b. Why would the beta decline over time?
c. Is the beta likely to keep decreasing over time?
23. You are analyzing Tiffany, an upscale retailer, and find that the regression estimate of the firm's beta is 0.75 ; the standard error for the beta estimate is 0.50 . You also note that the average unlevered beta of comparable specialty retailing firms is 1.15 .
a. If Tiffany has a debt/equity ratio of $20 \%$, estimate the beta for the company based upon comparable firms. (The tax rate is $40 \%$.)
b. Estimate a range for the beta from the regression.
c. Assume that Tiffany's is rated BBB and that the default spread for BBB rated firms is $1 \%$ over the treasury bond rate. If the treasury bond rate is $6.5 \%$, estimate the cost of capital for the firm.
24. You have been asked to estimate the cost of capital for NewTel, a telecomm firm. The firm has the following characteristics:

- There are 100 million shares outstanding trading at $\$ 250$ per share.
- The firm has a book value of ten-year debt of $\$ 10$ billion and interest expenses of $\$ 600$ million on the debt. The firm is not rated, but it had operating income of $\$ 2.5$ billion last year. Firms with an interest coverage ratio of 3.5 to 4.5 were rated BBB.
- The unlevered beta of other telecomm firms is 0.80

The treasury bond rate is $6 \%$, and the tax rate for the firm is $35 \%$.
a. Estimate the market value of debt for this firm
b. Based upon the synthetic rating, estimate the cost of debt for this firm.
c. Estimate the cost of capital for this firm.


[^0]:    ${ }^{1}$ The appendix to this chapter provides a brief overview of ordinary least squares regressions.
    ${ }^{2}$ The regression is calculated using returns in excess of the riskless rate for both the stock and the market. In this case, the intercept of the regression should be zero if the actual returns equal the expected returns from the CAPM, greater than zero if the stock does better than expected and less than zero if it does worse than expected.

[^1]:    ${ }^{3}$ The terminology is confusing, since the intercept of the regression is sometimes also called the alpha and is sometimes compared to zero as a measure of risk-adjusted performance. The intercept can be compared to zero only if the regression is run with excess returns for both the stock and the index; the riskless rate has to be subtracted from the raw return in each month for both.

[^2]:    ${ }^{4}$ The stock has to be bought by the day called the ex-dividend day for investors to be entitled to dividends. The returns in a month include dividends if the ex-dividend day is in that month.

[^3]:    ${ }^{5}$ This is done purely for computational convenience.

[^4]:    ${ }^{6}$ The non-trading bias arises because the returns in non-trading periods are zero (even though the market may have moved up or down significantly in those periods). Using these non-trading period returns in the regression will reduce the correlation between stock returns and market returns and, ultimately, the beta of the stock.
    7 The bias can also be reduced using statistical techniques suggested by Dimson and Scholes-Williams.

[^5]:    ${ }^{9}$ Note that Boeing breaks its business down in its financial statements into these two segments. We could have used operating income or EBITDA and a typical multiple to arrive at value.
    ${ }^{10}$ To estimate these multiples, we looked at the market value of publicly traded firms relative to their revenue. The multiple we used was 1.0 for the commercial aircraft business and 0.6 for the defense business.

[^6]:    ${ }^{11}$ We used net debt (the difference between gross debt and cash) in making this estimate. See the note later in the chapter about when this practice is appropriate and when it is not.

[^7]:    ${ }^{12}$ This table was developed in 1999 and 2000, by listing out all rated firms, with market capitalization lower than $\$ 2$ billion, and their interest coverage ratios, and then sorting firms based upon their bond ratings. The ranges were adjusted to eliminate outliers and to prevent overlapping ranges.

[^8]:    ${ }^{13}$ We used the default spread of $1.00 \%$ from table 7.6.

[^9]:    14 There are some who argue that stock prices are much more volatile than the underlying true value. Even if this argument is justified (and it has not conclusively been shown to be so), the difference between market value and true value is likely to be much smaller than the difference between book value and true value.

