Notes on a Two Country Analysis of the Adjustment Process Under Fixed Exchange Rates

Countries are denoted by subscripts 1 and 2. Both countries produce the same basket of goods and income in each is y_1 and y_2 with prices in terms of each country's currency P_1 and P_2. Perfect commodity arbitrage gives:

$$(1) \quad P_1 = e P_2$$

Country 2 is the reserve currency country; thus the total liabilities of its central bank, $\bar{M}_2$, are withheld by its own residents, M_2, or as reserves of country 1, R:

$$(2) \quad \bar{M}_2 = M_2 + R$$

In country 1, its money supply, M_1, plus the net worth of the Central Bank, W, equals the sum of Domestic Credit, C, plus the value of foreign exchange reserves:

$$(3) \quad M_1 = (C-W) + e R$$

and, calling $D = C-W$

$$(4) \quad M_1 = D + e R$$

Money demand in each country is given by

$$(5) \quad M_i^d = P_i k_i y_i \qquad i : 1, 2$$

There is slow adjustment to stock "disequilibrium" in the money market; at any instant the public's desired hoarding rate, H, is proportional to the difference between desired and actual money stocks:

$$(6) \quad H_i = \alpha_i (P_i k_i y_i - M_i)$$

Total expenditure, E_i, in each country equals nominal income minus hoarding, or:

$$(7) \quad E_i = P_i y_i - H_i$$

For world equilibrium in the goods market, the nominal value of world expenditure must equal the world's nominal income:

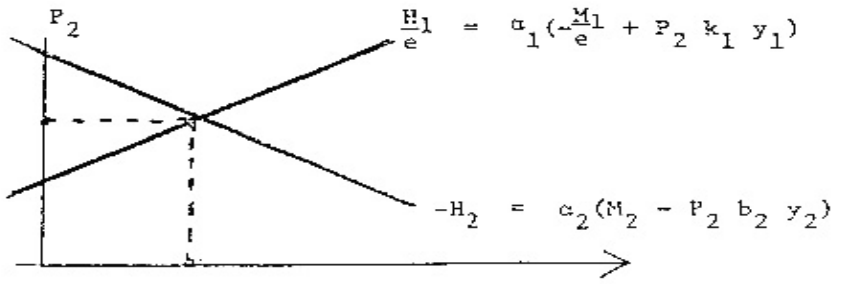
$$(8) \quad E_1 + e E_2 = P_1 y_1 + e P_2 y_2$$

This, in turn implies that the world's total hoarding (in the absence of credit creation) must be zero:

(9) $H_1 + e\,H_2 = 0$, or, using $P_1 = e\,P_2$:

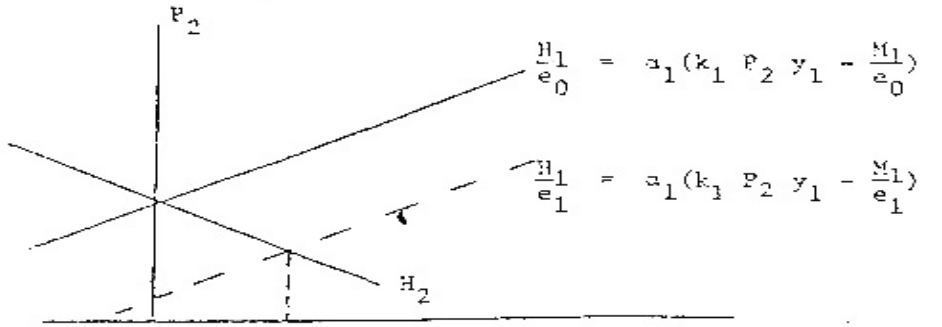
(10) $\alpha_1(\frac{M_1}{e} - P_2\,k_1\,y_1) = -\alpha_2(M_2 - P_2\,k_2\,y_2)$

Figure 1 shows the determination of the equilibrium P_2 given initial M_1 and M_2



As shown in the figure H_1 is positive, so $H_2 < 0$ and country 1 runs a surplus. As reserves accumulate M_1 rises, and M_2 falls shifting the H_1 schedule up and the $-H_2$ schedule down. The effect on P_2 of the redistribution of money is ambiguous but hoarding rates are unambiguously reduced. Eventually both schedules intersect such that no hoarding or dishoarding takes place and the adjustment is completed.

A devaluation by country 1 lowers the initial value of real cash balances (for a given P_2) and thus generates a positive hoarding rate. In figure 2 the $\frac{H_1}{e}$ schedule is shifted to the right from its initial equilibrium level as e rises from e_0 to e_1



The effect of the devaluation is thus to generate a positive surplus for country 1. As reserves accumulate, however, M_1 rises and M_2 falls until a new long-run equilibrium is attained with zero balance of payments surplus although country 1 must now have a

larger reserve stock. The new equilibrium P_2 must be lower; this is so because M_2 has fallen due to the reserve gain by country 1. Since M_1 is higher, however, P_1 must have risen and thus the fall in P_2 is less than the increase in e.

The long-run effects can be computed by solving for $H_1 = H_2 = 0$ and making use of (2) and (4) describing the money supply processes; letting an (*) denote the long-run value of a variable:

$$(11) \quad P_1^* = \frac{D + e\,\bar{M}_2}{k_1\,y_1 + k_2\,y_2} \quad ; \quad \text{thus}$$

P_1^* is directly proportional to the world's supply of money (in terms of 1's currency) and inversely proportional to the world's real money demand. Long-run money supply in country 1 is:

$$(12) \quad M_1^* = \frac{k_1\,y_1}{k_1\,y_1 + k_2\,y_2} \cdot (D + e\,\bar{M}_2), \text{ a}$$

fraction of total world money supply, this fraction being the share of 1's real money demand in the world's real money demand: a measure of the country's size which we will denote ψ_1. The long-run effect of a devaluation on M_1^* can be obtained by differentiating (12) keeping in mind that

$$(13) \quad dD = -R\,de$$

since we assume that the capital gains from the devaluation on the existing stock of reserves are not monetized. The net effect is then

$$\frac{dP_1^*}{P_1^*} = \frac{dM_1^*}{M_1^*} = (1 - \psi_1)\,\frac{de}{e}$$

Thus, the domestic inflationary effect of the devaluation is inversely proportional to the country's size. For a very small country $\psi_1 = 0$ so $\frac{dP_1^*}{P_1^*} = \frac{de}{e}$ and the proportional reserve gain is the largest since $\frac{dM_1^*}{M_1^*} = \frac{de}{e}$. As the country grows larger both the inflationary impact and proportional reserve gain decrease and, in the limit, as $\psi_1 = 1$, $\frac{dM_1^*}{M_1^*} = \frac{dP_1^*}{P_1^*} = 0$.

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The basic reference for this Section is:

Dornbusch, R.: "Currency Depreciation, Hoarding and Relative Prices", Journal of Political Economy, July/Aug. 1973.

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Notes on Growth Equilibrium and World Inflation

The basic reference for this Section is H.G. Johnson: "The Monetary Approach to Balance of Payments Theory" in Frenkel and Johnson.

We assume that the money supply of each country is backed by monetary gold (R) and Domestic Credit (D). All exchange rates between monies and gold are fixed and equal to unity. All countries produce the same basket of goods whose price is P in terms of gold (and also in terms of each domestic currency since exchange rates are assumed equal to unity).

Monetary equilibrium for a typical country requires:

$$(1) \quad M = R + D = P\,k\,y$$

Differentiating (1) with respect to time we obtain the following expression for the percentage change in the country's reserves: (where $\hat{x} = \frac{1}{x}\frac{dx}{dt}$)

$$(2) \quad \hat{R} = \frac{\hat{P} + \hat{y}}{\alpha_R} - \frac{\alpha_D}{\alpha_R}\hat{D}, \qquad \alpha_D = D/M, \quad \alpha_R = R/M$$

The world rate of inflation, $\hat{P}$, is in turn determined by the difference between the growth rate of the world money supply and world's real income. The world money supply is

$$M_W = D_W + R_W$$

where $D_W = \Sigma D_i$ $\qquad i: 1....N$

$$R_W = \Sigma R_i$$

and its growth rate is then:

$$(3) \quad \hat{M}_W = \alpha_{DW}\,\hat{D}_W + \alpha_{RW}\,\hat{R}_W$$

the world rate of inflation is then:

$$(4) \quad \hat{P} = \alpha_{DW}\,\hat{D}_W + \alpha_{RW}\,\hat{P}_W - \hat{y}_W$$

Substituting (4) into (2) we then get:

$$(5) \quad \hat{R} = \left(\frac{\alpha_{RW}}{\alpha_W}\right)\hat{R}_W + \frac{1}{\alpha_R}\left(\alpha_{DW}\,\hat{D}_W - \alpha_D\,\hat{D}\right) + \frac{1}{\alpha_R}\left(\hat{y} - \hat{y}_W\right)$$

which shows the country's balance of payments as a function of rates of change in the country and world income and domestic credit plus the net growth of the world reserves. With identical reserve ratios everywhere ($\alpha_R = \alpha_{RW}$ so $\alpha_D = \alpha_{DW}$), (5) becomes:

$$(6) \quad \hat{R} = \hat{R}_W + \frac{\alpha_D}{\alpha_R}(\hat{D}_W - \hat{D}) + \frac{1}{\alpha_R}(\hat{y} - \hat{y}_W)$$

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$\hat{R}$ & $\hat{R}_W$ in balanced growth:

$$\hat{D}_W = \hat{R}_W$$

$$\hat{D} = \hat{R}$$

So: $$\hat{D}\left(1 + \frac{\alpha_D}{\alpha_R}\right) = \hat{R}_W\left(\frac{1}{\alpha_R}\right) + \frac{1}{\alpha_R}(\hat{y} - \hat{y}_W)$$

$$\hat{D} = \hat{R}_W + (\hat{y} - \hat{y}_W)$$

and $$\pi = \hat{R}_W - \hat{y}_W.$$

The basic reference for this section is Frankel, J . and Rodriguez, C., "Portfolio Equilibrium and the Balance of Payments: A Monetary Approach," American Economic Review, September 1975, A SIMPLE PORTFOLIO-BALANCE MODEL OF A SMALL OPEN ECONOMY.

All goods produced in the country are also produced abroad and their price in terms of foreign exchange is fixed since the country is small and cannot affect world prices by its own actions. We can thus group all goods into a single composite good called y whose price in terms of foreign exchange is unity by assumption. Given full employment at home, the output of y is fixed and equal to the domestic income of the country, $\bar{y}$. For simplicity we assume investment is zero at all times. The domestic price of the composite good is P. Because of perfect arbitrage in goods markets, the domestic currency price of the good must equal the foreign currency price times the exchange rate, E, so P = E (assuming the foreign price is unity).

Domestic consumption is assumed to depend on domestic income (which is a constant) and the real value of assets. Domestic residents hold domestic cash balances (M) and foreign securities (savings accounts at foreign banks which are denominated in terms of the foreign currency) which yield a fixed interest, r, per unit of time. The total value in terms of foreign exchange of foreign securities held is denoted D. Since the foreign price of goods is unity, D also measures the real value of the stock of foreign securities in terms of goods. The real value of domestic money holdings is m = M/P = M/E since P = E. The total real value of assets held is then: a = M/P + D and consumption is an increasing function of a : c = c(a). If consumption falls below the domestic output of goods the remaining is net exports abroad, the trade balance surplus denoted b_T. Notice that b_T can be negative if consumption exceeds domestic output. We do not need a foreign demand for domestic goods since they are the same as abroad and the country is so small that the rest of

the world will act as a residual buyer or seller provided only that the purchasing power relation, $P = E$, holds. The disposition of domestic output is then:

$$(1) \quad \bar{y} = c(a) + b_T$$

The total income available for spending to households equals domestic income (equal to the value of domestic output) plus net interest payments from abroad, r.D. Consumers can use this income for three purposes: consumption expenditures, acquisition of cash balances or acquisition of foreign securities. Thus, the following relationship must hold:

$$(2) \quad \bar{y} + r.D = c(a) + \dot{m} + \dot{D}$$

where $\dot{m}$ is the rate of acquisition of real cash balances and $\dot{D}$ is the rate of acquisition of foreign securities. This relation can be transformed into a balance of payments equilibrium condition by noticing that under fixed exchange rates $\dot{m}$ equals the rate of Central Bank purchases of foreign exchange (the balance of payments surplus, denoted by B), r.D is the service account surplus (denoted by b_S) and $-\dot{D}$ is the country's capital account surplus (denoted by b_C). Substituting (1) into (2) and using the three definitions given above, (2) implies:

$$B = b_T + b_S + b_C \text{ , a well-known relation.}$$

We postulate that at any moment the public has a desired portfolio composition ratio between real money holdings and foreign securities holdings. This ratio is assumed to depend negatively on the rate of interest paid by foreign securities:

$$(3) \quad m/D = z(r)$$

Since r is constant so is z(r) and thus the public will always have a constant ratio between money and securities holdings. Portfolio equilibrium is always

assured to the extent the Central Bank has enough foreign exchange reserves and exchanges domestic for foreign money at the fixed exchange rate. For example, if thepublic wants less money and more foreign securities they take the money to the Central Bank andobtain foreign exchange with which they buy the securities from foreigners. A foreign exchange crisis may takeplace if the public wants to obtain more foreign exchange than what the Central Bank has. In that case restrictions on foreign exchange purchases could be imposed (capital controls) or the currency may be devalued thus reducing the purchasing power of domestic money in terms of foreign exchange and at the same time reducing the real value of cash balances because of the higher price level.

To the extent that the public is on portfolio balance so (3) holds, the rates of acquisition of both assets must also be porportional to z(r); thus

$$(4) \quad \dot{m} = z(r)\,\dot{D}$$

Using (4) into (2) we obtain:

$$(5) \quad \dot{D} = [\bar{y} - c(m+D) + rD] \,/\, (1+z(r))$$

Since m = z.D, (5) describes the rate of acquisition of foreign securities as a function only of the existing stock of foreign securities (and the other parameters, namely $\bar{y}$ and r which are constant throughout the analysis). We assume that there is some value of D such that there will be no new acquisition ($\dot{D} = 0$) and thus the economy will be at rest with constant stocks of D and m provided no new shocks take place. Such a steady state will be stable provided that if for any reason the public's holdings of D are suddenly increased then the rate of acquisition of D becomes negative (that is, the public gets rid of the extra addition of D and returns to the original holdings, although this process will take some time). We will assume the steady state equilibrium is stable and thus that there is some value of D, D*, such that if D is larger

(smaller) than D* there will be net sales (purchases) of foreign securities. Such D* is depicted in Figure 1 as the vertical line in the (m, D) plane. The straight line through the origin represents the desired composition ratio between money and securities and since this ratio is always attained the economy must travel along this line in the direction indicated by the arrows. At point a, for example, the public will be holding m_a and D_a of money and securities and since D_a is less than D*, D is growing and so is m ; thus both m and D tend to move northeast along the portfolio balance locus. In the transition from point a and the steady state equilibrium (point a*), the economy accumulates cash balances (so there is a surplus in the balance of payments) and accumulates foreign securities (a capital account deficit). Thus, the stock of real assets is increasing and so is consumption. In the steady state both capital flows and accumulation of cash balances cease and thus the balance of payments is in equilibrium (B = 0) and the capital account is balanced (b_C = 0). It follows that in the steady state the trade balance has to equal the service account deficit or, in our case where the country is a net creditor to the rest of the world, the long run trade deficit equals the long run service account surplus. This long run equilibrium trade deficit does not imply any deterioration of the economic position of the country but simply reflects that the economy is receiving interest payments from abroad and thus can afford to spend permanently in excess of its income. While at point a* the trade account must be in deficit (always assuming that D* is positive, otherwise there should be a trade surplus) at the short run equilibrium position at point a the trade account may be in surplus or deficit. There are two ways to see this: at a assets are below their long run value and thus consumption is below its long run level; the long run trade deficit is long run consumption minus $\bar{y}$ and it is positive; however, at a consumption is below long run consumption and thus the trade deficit is smaller than its long run level and maybe even negative. What is sure is that as time

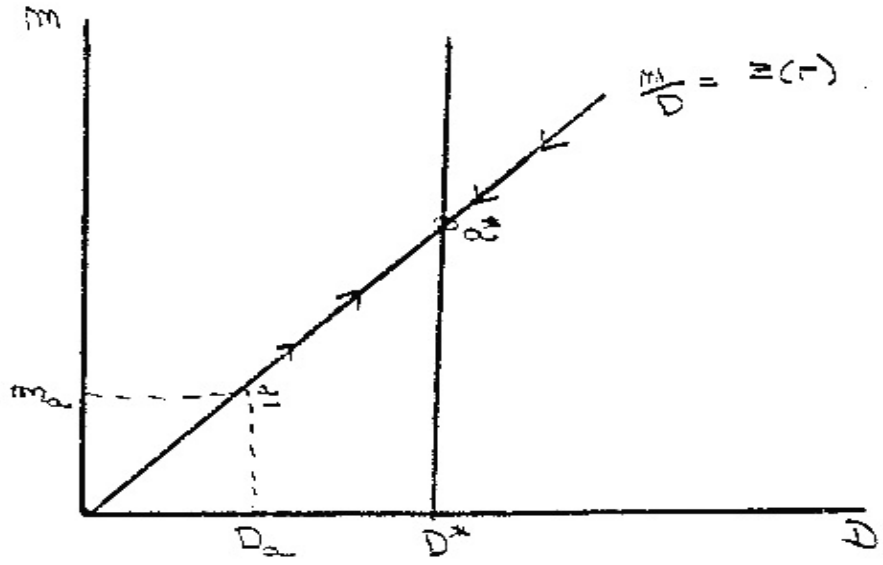

Figure 1.

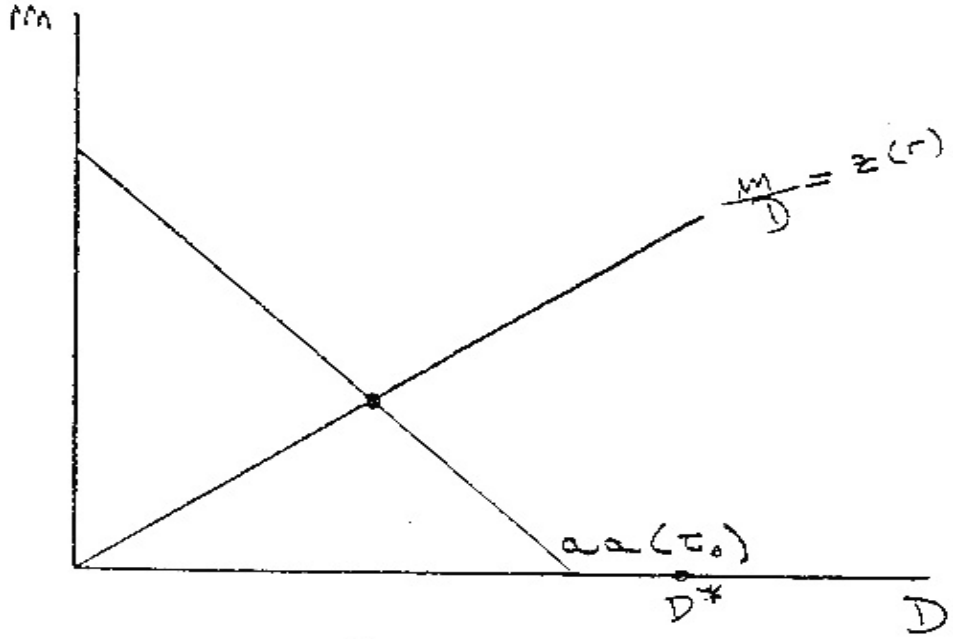

Figure 2

passes the trade position deteriorates as assets and thus consumption increase; thus if there is a trade surplus at $\underline{a}$, as assets grow the surplus will be reduced, become zero eventually and then turn into deficit until the long run deficit is attained at a*.

What determines where the economy will be at any instant along the portfolio equilibrium line? The crucial variable is the total holdings of real assets households have at any instant. Notice that while households can adjust the composition of their portfolio instantaneously, they can alter the size of it only through time by spending less than their income. Thus, at any instant the stock of assets is given as a result of past savings decisions and the choices open to households are two: the composition of their portfolio and how much of their income to consume, the remaining becoming the net addition to their stock of assets (which can be negative if they decide to consume in excess of their income).

In figure 2, the portfolio composition decision implies that m and D must be on the $m = z(r)D$ line while the asset constraint implies that the sum of m plus D must equal to the prevailing value of assets at that moment. Thus the line $aa(t_o)$ shows all the feasible combinations of money and securities open to households given the value of assets $a(t_o)$. Portfolio equilibrium is then obtained with the combination of a m and D at the intersection of both schedules. Since at that equilibrium the stock of foreign exchange is shown to be less than D* we know that households will be spending on goods by less than their income and thus that the stock of assets will be growing. As assets grow the aa line shifts to the right and the equilibrium point moves northeast along the $m = zD$ schedule. Since during the transition D raises, the service account surplus increases continuously. A typical path for all balance of payments accounts and total assets is shown in figure 3.

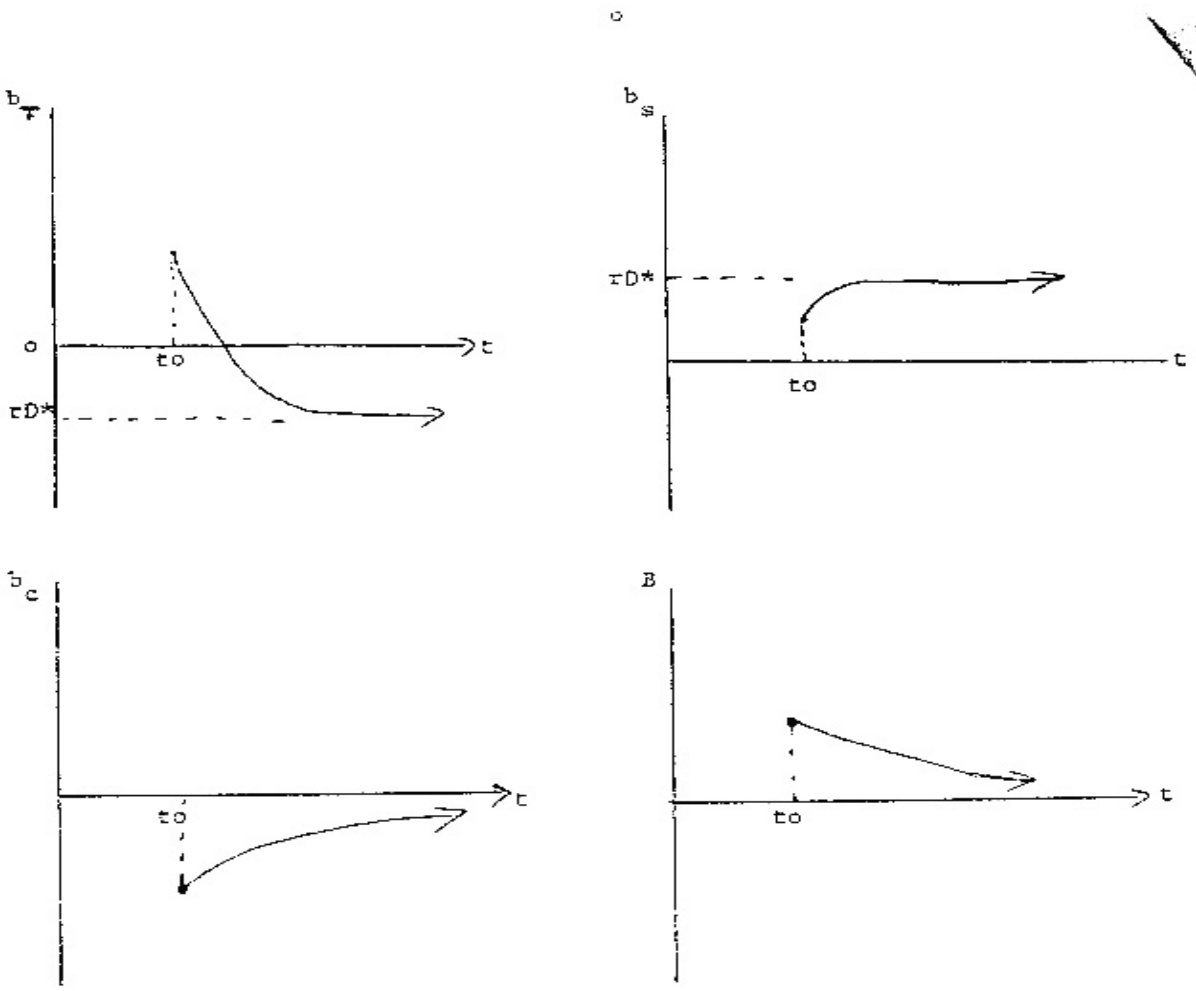

Figure 3

Monetary Policy

So far the Central Bank has played the passive role of exchange domestic money and foreign exchange at the fixed exchange rate.The Central Bank is free, however, to to print domestic currency and put it in circulation by means other than foreign exchange purchases. Assume, for example, that the government decides to print, say, 10 per cent more nominal cash balances at one instant of time and distribute it among the public as a once and for all transfer. The value of real assets of

individuals is then at that same instant increased by the amount of the real value of the newly issued cash balances. Notice that this is an instantaneous jump in the value of real assets as opposed to the continuous rate of change derived previously which was due to differences between the rates of income and absorption of goods (in fact the instantaneous gift of money to the public may be treated as if their income rate were infinity for an instant and all of it is accumulated in assets). Note that all of the increase in the value of real assets came in the form of domestic money and thus, if individuals were before in portfolio balance, now they will be holding a larger money/foreign securities ratio than desired. As a consequence, individuals will try to restore portfolio balance by selling some of their excess domestic money back to the Central Bank in order to get foreign exchange with which to buy foreign securities. Thus, after the increase in the quantity of domestic money, some of it is sold back to the Central Bank and there is a reserve loss and an instantaneous acquisition of foreign securities. After these initial transactions designed to restore portfolio balance for the new higher value of real assets, other transactions will continue to take place through time: assuming initially the economy was at the steady state equilibrium, now assets exceed their long run value. The economy will slowly decumulate those excess assets through time and during this transition since assets exceed their long run value consumption will also; thus, the trade account deficit is larger during the transition. As consumption (absorption) exceeds national income during this period, both money and foreign securities fall as the current account is in deficit; thus, the public sells back the foreign securities purchased originally and gives back to the Central Bank the remaining domestic money which they had kept after the initial increase. With the proceeds of the sales of foreign securities abroad and money to the Central Bank, the public gets the foreign exchange with which to finance

the higher rate of purchases of goods from abroad. Eventually assets are restored to their original long run value, the stock held of foreign securities and money is the same as before the increase in money. However, during all the transition the rate of consumption has been higher than before; what was given in exchange, or was it a free lunch? The answer is that the higher consumption rate during the transition was financed with the reserve losses of the Central Bank as the public exchanged the excess money in order to buy foreign goods. Thus, the expansion in money supply allowed this economy to consume in excess of its income but at the expense of the Central Bank's stock of foreign exchange reserves; this, of course, is a situation which cannot be repeated indefinitely since at some moment the stock of foreign exchange will be depleted.

In terms of figure 4, the initial position before the monetary increase was at point a*. After the monetary increase the asset constraint shifts to the right by the value of the increased money holdings (equal to a*b). Since the public is now out of portfolio balance, an instantaneous sale of eb of domestic money for ed of foreign exchange (which is used to buy an equal amount of foreign securities) takes place and portfolio equilibrium is restored at point d. At d, however, holdings of foreign securities exceed their long run level D* and m and D fall through time along the m = zD line towards point a*.

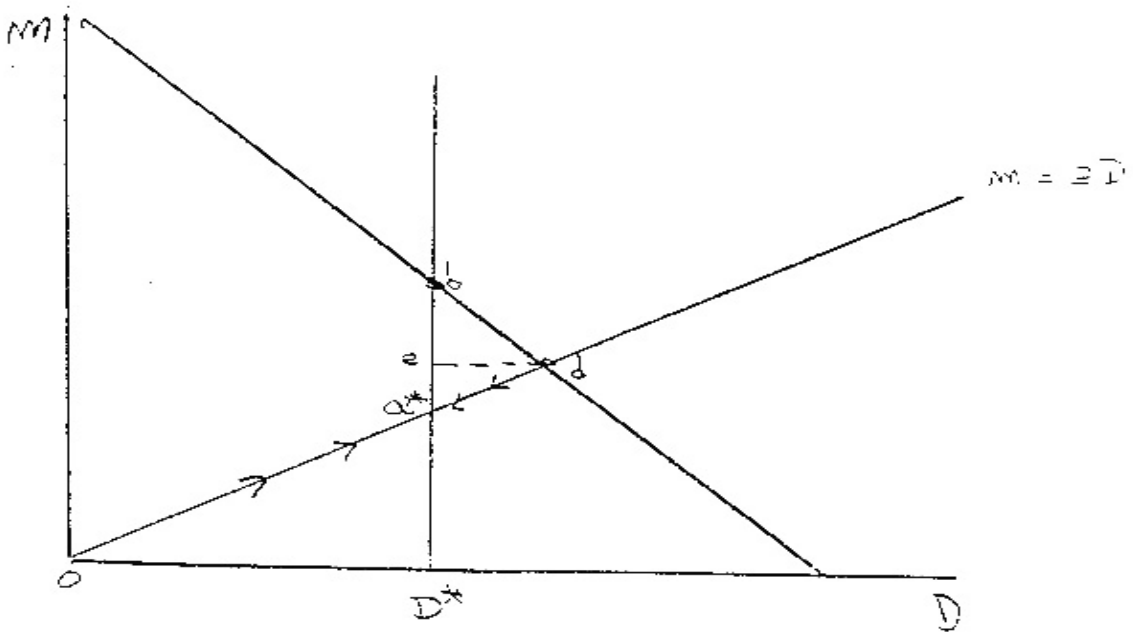

Figure 4