1 Staggered Pricing model

Staggered pricing

- Method of Calvo
- In each period, a fraction $1 - \rho$ of firms can choose their price
- In other periods, the price either
  - stays fixed
  - is adjusted by some rule of thumb
- Firms have some market power they can exploit

Structure of economy

- Households
  - buy final good for consumption
  - buy final good for investment
- Final good firms
  - Buy intermediate goods from intermediate goods firms
  - bundles goods together to make final good
  - are perfectly competitive
Intermediate goods firms
- Produce a differentiated intermediate good
- Have some market power to set price
- Use capital and labor from factor market
- Can produce excess profits or losses

Final good firm
- Combines the $k \in [0,1]$ intermediate goods to make final good

Production technology (bundler) is
$$Y_t = \left[ \int_0^1 Y_t(k) \frac{\psi-1}{\psi} \, dk \right]^\frac{1}{\psi}$$

- $\psi$ is the elasticity of substitution in production

Firm maximizes
$$\text{profits}_t = P_t Y_t - \int_0^1 P_t(k) Y_t(k) \, dk$$

Final good firm
- Final good firm maximizes
$$\max_{\{Y_t(k)\}} P_t \left[ \int_0^1 Y_t(k) \frac{\psi-1}{\psi} \, dk \right]^\frac{1}{\psi} - \int_0^1 P_t(k) Y_t(k) \, dk$$

- first order condition
$$P_t \left[ \int_0^1 Y_t(k) \frac{\psi-1}{\psi} \, dk \right]^\frac{1}{\psi} Y_t(k)^\frac{1}{\psi} = P_t(k)$$

- this simplifies to a demand function for good $k$
$$Y_t(k) = Y_t \left( \frac{P_t}{P_t(k)} \right)^\psi$$

Putting demand into the bundler function
$$Y_t = \left[ \int_0^1 \left[ Y_t \left( \frac{P_t}{P_t(k)} \right)^\psi \right] \frac{\psi-1}{\psi} \, dk \right]^\frac{\psi}{\psi-1} = Y_t \left[ \int_0^1 \left( \frac{P_t}{P_t(k)} \right)^{\psi-1} \, dk \right]^\frac{\psi}{\psi-1}$$
• which can be written as

\[ \frac{1}{P_t} = [\int_0^1 \left( \frac{1}{P_t(k)} \right)^{\psi-1} dk]^{\frac{1}{\psi-1}} \]

• final goods pricing rule

\[ P_t = \left[ \int_0^1 P_t(k)^{1-\psi} dk \right]^{\frac{1}{1-\psi}} \]

Intermediate goods firms

• Some common rules of thumb

  – 1) keeping prices the same as the last updating

  \[ P_t(k) = P_{t-1}(k) \]

  – 2) updating prices by a stationary state gross inflation rate \( \pi \),

  \[ P_t(k) = \pi P_{t-1}(k) \]

  – 3) updating prices by the one period lagged realized gross inflation rate, \( \pi_{t-1} = P_{t-1}/P_{t-2} \),

  \[ P_t(k) = \pi_{t-1} P_{t-1}(k) \]

Intermediate goods firms

• Choose \( P_t^*(k) \), to maximize

\[ E_t \sum_{i=0}^{\infty} \beta^i \left[ P_t^*(k) Y_{t+i} \left( \frac{P_{t+i}}{P_t^*(k)} \right)^{\psi} - P_{t+i} \rho_{t+i} K_{t+i}(k) - P_{t+i} \omega_{t+i} H_{t+i}(k) \right] \]

• subject to the production technology,

\[ Y_{t+i} \left( \frac{P_{t+i}}{P_t^*(k)} \right)^{\psi} = \lambda_{t+i} K_{t+i}(k) H_{t+i}^{1-\theta}(k) \]

Intermediate goods firms

• A firm that is maximizing above is also minimizing costs
Minimization problem
\[
\min_{K_{t+i}(k), H_{t+i}(k)} r_{t+i}K_{t+i}(k) + w_{t+i}H_{t+i}(k)
\]
subject to the production technology
\[
Y_{t+i}(k) = \lambda_{t+i}K_{t+i}^\theta H_{t+i}^{1-\theta}(k)
\]

First order conditions
\[
\frac{(1-\theta)r_t}{\theta w_t} = \frac{H_t(k)}{K_t(k)}
\]

Demand equations for factors
\[
H_{t+i}(k) = \frac{Y_{t+i}(k)}{\lambda_{t+i}} \left[ \frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^\theta
\]
and
\[
K_{t+i}(k) = \frac{Y_{t+i}(k)}{\lambda_{t+i}} \left[ \frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^{\theta-1}
\]

Intermediate goods firms: Costs

Put these demand equations into cost equation
\[
\frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[ \frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^\theta Y_{t+i}(k)
\]

Marginal costs are
\[
\frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[ \frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^\theta
\]

Intermediate goods firms: maximization problem

Rewriting costs get
\[
\max_{P_t^*(k)} E_t \sum_{i=0}^{\infty} (\beta \rho)^i \left[ P_t^*(k)Y_{t+i} \left( \frac{P_{t+i}}{P_t^*(k)} \right) \right]^\psi
\]

- \[
-\frac{w_{t+i}}{(1-\theta)\lambda_{t+i}} \left[ \frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right]^\theta Y_{t+i} \left( \frac{P_{t+i}}{P_t^*(k)} \right)^\psi
\]
Intermediate goods firms: maximization problem

- FOC is

\[
0 = E_t \sum_{i=0}^{\infty} (\beta \rho)^i Y_{t+i}(k) [1 - \psi + \frac{\psi P_{t+i} w_{t+i}}{P_t^* (k) (1-\theta) \lambda_{t+i}} \left( \frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right)^\theta]
\]

- Price setting rule is

\[
P_t^*(k) = \frac{\psi}{\psi - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta \rho)^i P_{t+i} Y_{t+i}(k) \frac{w_{t+i}}{(1-\theta) \lambda_{t+i}} \left( \frac{r_{t+i}(1-\theta)}{w_{t+i}\theta} \right)^\theta}{E_t \sum_{i=0}^{\infty} (\beta \rho)^i Y_{t+i}(k)}
\]

- \( \frac{\psi}{\psi - 1} \) is the markup of discounted stream of expected nominal costs divided by discounted stream of expected output.

Price process

- All firms fixing prices at date \( t \) set the same price.
- All set to \( P_t^* (1 - \rho) \) firms.
- Recall that firms that don’t fix price keep old one (\( \rho \) firms).
- Prices follow updating process of

\[
P_t^{1-\psi} = \rho P_{t-1}^{1-\psi} + (1 - \rho) (P_t^*)^{1-\psi}
\]

The households

- Unit mass of households.
- Max

\[
E_0 \sum_{i=0}^{\infty} \beta^i \left[ \ln c_i + Bh_i \right]
\]

- subject to a cash in advance constraint

\[
P_t c_t^i = m_{t-1}^i + (g_t - 1) M_{t-1}
\]
and a real budget constraint,

\[ k_{i+1}^t + \frac{m_i^t}{P_t} = w_t h_t^i + r_t k_t^i + \xi_t^i (1 - \delta) k_t^i \]

The households

- First order conditions for family \( i \) are

\[
\frac{B}{w_t} = E_t \left[ \frac{B \beta}{w_{t+1}} (r_{t+1} + (1 - \delta)) \right]
\]

and

\[
-E_t \left[ \frac{\beta}{c_{t+1}^i P_{t+1}} \right] = \frac{B}{w_t P_t}
\]

Equilibrium conditions

- Aggregation across families gives

\[
C_t = c_t^i, \\
K_t = k_t^i, \\
H_t = h_t^i, \\
and \\
M_t = m_t^i.
\]

- Aggregating labor across intermediate firms

\[
H_t = \int_0^1 H_t(k) dk.
\]

Equilibrium conditions

- Use the demand for labor

\[
H_t(k) = \frac{Y_t(k)}{\lambda_t} \left[ \frac{r_t(1 - \theta)}{w_t \theta} \right]^{\theta},
\]

- equilibrium condition for the labor market in period \( t \) is

\[
H_t = \int_0^1 H_t(k) dk = \frac{1}{\lambda_t} \left[ \frac{r_t(1 - \theta)}{w_t \theta} \right]^{\theta} \int_0^1 Y_t(k) dk.
\]
• For the market for capital,

\[ K_t = \int_0^1 K_t(k)dk \]

• Using the demand for capital

\[ K_t(k) = \frac{Y_t(k)}{\lambda_t} \left[ \frac{r_t(1 - \theta)}{w_t \theta} \right]^{\theta-1} \]

• the equilibrium condition for the capital market in period \( t \) is

\[ K_t = \int_0^1 K_t(k)dk = \frac{1}{\lambda_t} \left[ \frac{r_t(1 - \theta)}{w_t \theta} \right]^{\theta-1} \int_0^1 Y_t(k)dk \]

• Note integral of output (not bundler)

Equilibrium conditions

• Aggregate excess profits paid to the families in period \( t \) are

\[ P_t \xi_t = P_t \int_0^1 \xi_t di = P_t \int_0^1 profits(k)dk \]

\[ = \int_0^1 P_t(k)Y_t(k)dk - P_t \frac{w_t}{(1 - \theta)\lambda_t} \left[ \frac{r_t(1 - \theta)}{w_t \theta} \right]^{\theta} \int_0^1 Y_t(k)dk \]

• Since the final good firms are perfectly competitive and make no profits,

\[ P_t Y_t = \int_0^1 P_t(k)Y_t(k)dk \]

• Substituting this into the total profits equation for period \( t \), and removing the price level \( P_t \), gives excess profits as

\[ \xi_t = \left( Y_t - \frac{w_t}{(1 - \theta)\lambda_t} \left[ \frac{r_t(1 - \theta)}{w_t \theta} \right]^{\theta} \int_0^1 Y_t(k)dk \right) \]

The full model

• FOCs of household

\[ \frac{1}{w_t} = E_t \left[ \frac{\beta}{w_{t+1}} (r_{t+1} + (1 - \delta)) \right], \]

\[ -E_t \left[ \frac{\beta}{C_{t+1}P_{t+1}} \right] = \frac{B}{w_t P_t}, \]
• the aggregated cash-in-advance constraint,
\[ P_tC_t = g_t M_{t-1}, \]
• aggregated family’s real budget constraint,
\[ K_{t+1} + \frac{M_t}{P_t} = w_t H_t + r_t K_t + \xi_t + (1 - \delta) K_t. \]

The full model
• all income goes to the families and sums to output
\[ Y_t = w_t H_t + r_t K_t + \xi_t, \]
• family’s real budget constraint will be used as
\[ K_{t+1} + \frac{M_t}{P_t} = Y_t + (1 - \delta) K_t. \]
• Price setting equation
\[ P_t^* (k) = \frac{\psi}{\psi - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta \rho)^i P_{t+i} Y_{t+i} (k) \frac{w_{t+i}}{(1-\theta) \lambda_{t+i}} \left[ \frac{r_{t+i}(1-\theta)}{w_{t+i}} \right]^\theta}{E_t \sum_{i=0}^{\infty} (\beta \rho)^i Y_{t+i} (k)}. \]
• cost minimization,
\[ \frac{(1 - \theta) r_t}{\theta w_t} = \frac{H_t (k)}{K_t (k)}. \]
• aggregate production equation,
\[ \int_0^1 Y_t (k) dk = \lambda_t H_t^\theta K_t^{1-\theta}. \]
• rule determining the final good price in period \( t \),
\[ P_t^{1-\psi} = \rho P_{t-1}^{1-\psi} + (1 - \rho) (P_t^*)^{1-\psi}. \]
• money supply growth rule
\[ M_t = g_t M_{t-1}, \]
• stochastic process for money growth and technology,
\[ \ln \lambda_t = \gamma \ln \lambda_{t-1} + \varepsilon_t^\lambda \]
\[ \ln g_t = \pi \ln g_{t-1} + \varepsilon_t^{\pi} \]
Interesting points of stationary state

- Much is the same as before, except
- The rule for determining the final goods price gives
  \[ P_1 = P_1 + (1 - \rho) \left( \frac{P^*(k)}{P(k)} \right)^{1-\psi} \]
- or that
  \[ P = P^*(k) = P(k) \]
- Putting this into the demand function for intermediate good \( k \) gives
  \[ Y(k) = Y \left( \frac{P}{P(k)} \right)^\psi = Y \]

Interesting points of stationary state

- The price setting rule gives
  \[ P^*(k) = \frac{\psi}{\psi - 1} \frac{1}{1 - \beta \rho} \frac{P}{(1 - \theta) \lambda} \left[ \frac{\tau (1 - \theta)}{\pi \theta} \right]^\theta \]
  \[ = \frac{\psi}{\psi - 1} \frac{P}{(1 - \theta) \lambda} \left[ \frac{\tau (1 - \theta)}{\pi \theta} \right]^\theta \]
- or a relation for wages and rentals
  \[ \frac{\psi}{\psi - 1} = \frac{1}{\frac{\pi}{(1 - \theta)} \left[ \frac{\tau (1 - \theta)}{\pi \theta} \right]^\theta} \]
- wages are
  \[ \bar{w} = \left[ \frac{(\psi - 1)(1 - \theta)^{1 - \theta} \beta \theta^\theta}{\psi \theta^\theta} \right]^{\frac{1}{1 - \theta}} \]

Interesting points of stationary state

- Dividends are
  \[ \bar{\xi} = Y \left( 1 - \frac{\bar{w}}{(1 - \theta)} \left[ \frac{\tau (1 - \theta)}{\pi \theta} \right]^\theta \right) = \frac{Y}{\psi} \]
- Output is not yet determined. Using the real budget constraint, the first order condition for consumption, and the results above
  \[ -\frac{\beta \bar{w}}{B} = \bar{w} \left[ \frac{\tau (1 - \theta)}{\pi \theta} \right]^\theta \frac{\bar{Y}}{\psi} + \left( \frac{\tau (1 - \theta)}{\pi \theta} \right)^{\theta - 1} \bar{Y}, \]
• This can be rewritten as

\[ \bar{Y} = \frac{-\beta \bar{W}}{B \left( \frac{\tau (1-\theta)}{\bar{W} \psi} \right) + \frac{1}{\psi} + (\tau - \delta) \left( \frac{\tau (1-\theta)}{\bar{W} \psi} \right)^{\theta-1}}. \]

Stationary states

• \( \beta = .99, B = -2.5805, \delta = .025, \theta = .36. \) Gali argues that \( \rho = .75 \) and \( \psi = 11 \), a value that gives a 10% markup in the stationary state.

• Stationary states for this model are

<table>
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<tr>
<th>Variable</th>
<th>( \tau )</th>
<th>( \bar{w} )</th>
<th>( \bar{Y} )</th>
<th>( H )</th>
<th>( K )</th>
<th>( \xi )</th>
<th>( \bar{C} )</th>
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<td>.0929</td>
<td>.7836</td>
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